Basic Electronics
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Lecture – 44
Op-amp filters

Welcome back to Basic Electronics. In this class, we will look at op-amp filters. For some simple circuit, we will work out the transfer function and figure out the type of a filter. For more complex circuits, we will describe the functionality with the help of plots obtained with circuit simulation. Let us get started.

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In practice, it is not possible to realize the transfer functions that we have been discussing, and therefore the ideal filter response is approximated with a suitable H of j omega which can then be obtained with circuit elements. For example, H of s could be 1 over this fifth order polynomial n s and this could be a fifth order low-pass filter provided of course, we choose these coefficients appropriately, not any fifth order function, will do we have to choose the function appropriately.

Some commonly used approximations or polynomials are the Butterworth, Chebyshev, and Bessel and elliptic functions. We will look at some of these coefficients for these filters are listed in filter handbooks. So, filters are fairly well studied several decades earlier in fact, and lot of information is available already in the literature there are handbooks which give all those coefficients that we require. There are also programs for filter design available on the internet.

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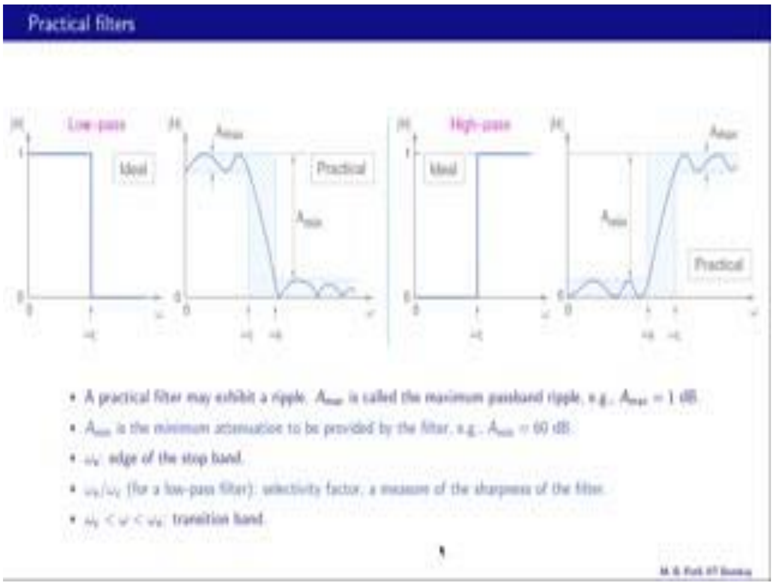

Let us get familiar with some definitions now. Here is the ideal low-pass filter response and here is what we would get in practice. Here is the ideal high-pass filter response and here is what we would get in practice. A practical filter may execute a ripple like this one; here we have a flat response, but here the transfer function magnitude is varying with frequency that is called a ripple. A max is called the maximum pass band ripple - this range here; and of course, it is small something like 1 dB typically, for some filters it is just 0 dB. So, we actually have a flat magnitude of H in this region.

A min is the minimum attenuation to be provided by the filter this quantity here. And ideally of course, we would like that to be infinity. In practice, we might get something like 60 dB that depends of course, on the order of the filter. Omega s is called as a edge of the stop band - this one here and this one here. Omega s by omega c for a low-pass filter, the ratio of these two is called the selectivity factor and that is measure of the sharpness of the filter. If omega s and omega c are very close that means, our response is going to be much better, we going to have very sharp drop here in mod of H and that of course is preferred. This range from omega c to omega s is called the transition band. This one here and of course, whatever we said about the low-pass filter has counter parts also in the high-pass filter s. So, what we are going to do now is to look at some specific filter types and their transfer functions and then we should be able to relate their transfer function to these definitions that we are looked at.

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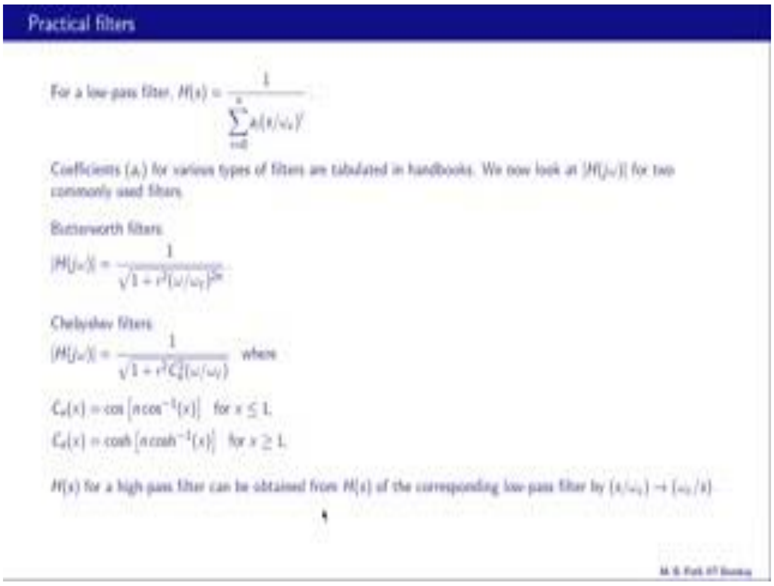

Let us look at some practical filters for a low-pass filter H of s is 1 is over this polynomial inverse which is normally written in terms of s over omega c, omega c being the cut off frequency. And these coefficients a i for various types of filters are tabulated in hand books, we do not need to reinvent a wheel.

Let us now look at mod of H for two commonly used filters namely, but Butterworth filters and Chebyshev filters. For Butterworth filter of order n, this is mod of H as a function of omega; and for Chebyshev filter of order n this is mod of H as a function of omega. Here this C n is actually a function given here. Now, this is the low-pass filter. We can make up the H of s for high-pass filter simply by using this transformation. So, s over omega c should be replaced by omega c by s in this expression, and we get a high-pass filter with the same cut off frequency. Now, these actual polynomials we are not discussing here, because they can be found in hand books.

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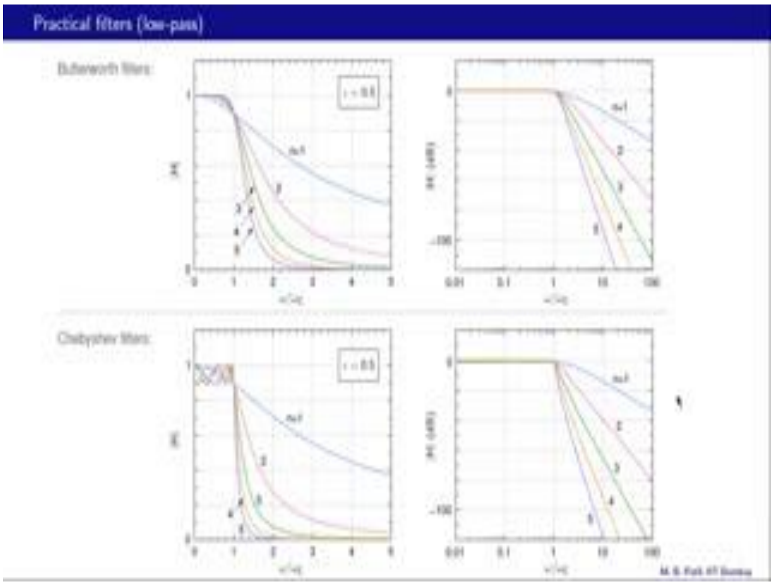

Here are some examples of Butterworth filters and Chebyshev filters. In the plots on the left, these two V plot mod H on a linear scale versus omega also on a linear scale and this omega is normalized with respect to the cut off frequency omega c. What is the major difference between these two plots, we see that there is a ripple here in the Chebyshev filters, whereas in the butter worth filters there is no ripple here in the pass band.

Now, in practice the plots on the right are more useful where we plot mod H in dB versus omega on a logarithmic scale, where they more useful because from these slopes we can immediate tell what the order of the filter would be. If the slope is minus 20 dB per decade then we know that the order is 1; if it is minus 40 dB per decade the order is 2 and so on. And higher the order better it is of course, because that brings us closer to our ideal filter; ideally we would like that to go to infinity at this point go to minus infinity at that point, but of course, if we use a higher order filter, it requires more components for implementation. So, there is some trade off.

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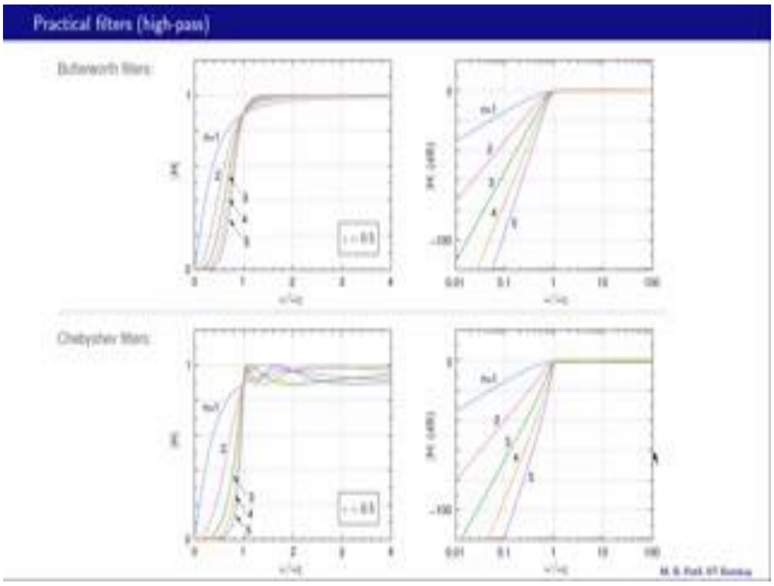

Here are the corresponding plots for a Butterworth filters and Chebyshev filters, for high-pass filters. And they are very similar to the low-pass case except of course that the functionality is now different; the high frequencies are passed and low frequencies are rejected. Once again in this plot where mod H is in dB and omega is on log scale, we can make out the order of the filter from the slope here. So, this slope would be 20 dB per decade that would be 40 dB per decade, 60 dB per decade and so on.

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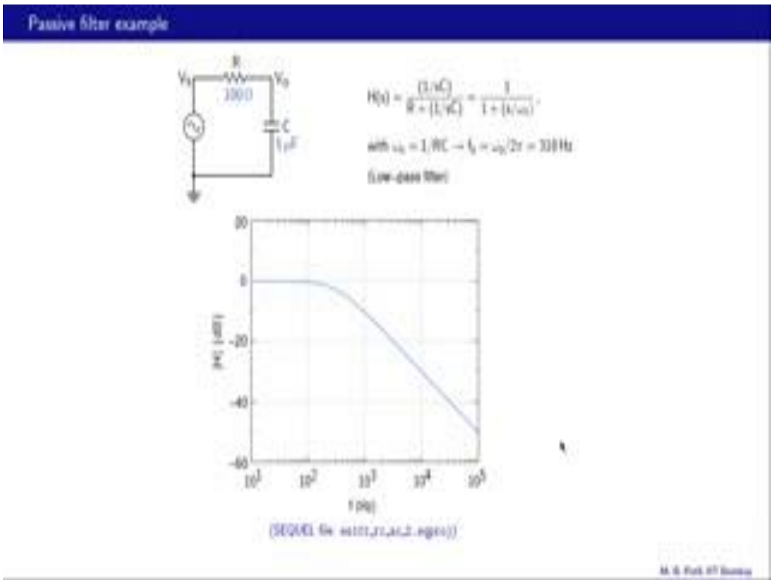

Let us now look at how a filter can be implemented in practice. We will first look at passive filters, which use registers, capacitors, inductors no transistors or op-amps. And then we will look at op-amp filters later and see what the advantages of using op-amps are. Here is an example and the transfer function is 1 over s C, which is the impedance of the capacitor divided by R plus 1 over s C and that can be re written as 1 over 1 plus s by omega 0. Omega 0 is 1 over R C and the corresponding, f 0 - the cut off frequency in hertz is omega 0 by 2 pi. And with these component values it turns to be about three hundred eighteen hertz. So, this is a low-pass filter and in fact, we have looked at it earlier.

Here is the magnitude plot mod H in dB versus f in hertz on a log scale. So, the gain is 0 dB here, the transfer function magnitude is 0 dB here, because for small values of omega, this term is small and H of s tends to 1. This is the higher frequency part, and as we would expect mod of H keeps going down as the frequency increases. And if we relate these two with the bode approximation that we have looked at earlier the inter section of these asymptotes, this asymptote here, the low frequency asymptote and the high frequency asymptote would actually give use the cut-off frequency. So, the cut off frequency would be somewhere here and that is 318 hertz.

The sequel file for this example is available, and you can take a look at it. What is the order of this filter? Let us look at the slope here, if we go from 10 is to 4 to 10 is to 5, we go from there to there and that is 20 dB. So, the slope here is minus 20 dB per decade and therefore the order of this filter is 1 and that is also apparent because we have first order polynomial in s here, so that is something that we would have expected.

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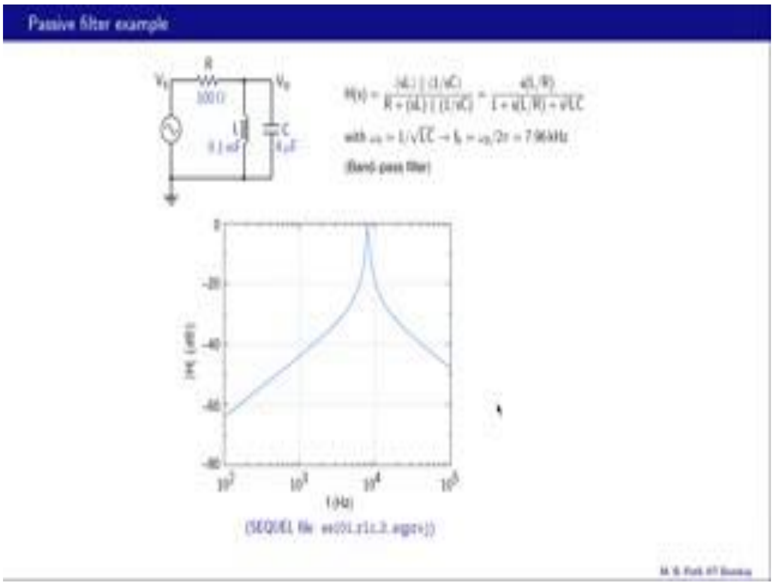

Here is another passive filter example and it uses an inductor now. The transfer function is derived here essentially V o is obtained by voltage division. These two are in parallel, so s l is the impedance of the inductor 1 over s C is the impedance of the capacitor, those two in parallel divided by R plus that parallel combination. And when we simplify things, we get this expression here. This turns out to be a band-pass filter with omega naught equal to 1 over square root L C or f 0 equal to about 8 kilo hertz in this case with these component values. So, it will pass frequencies in this band here and reject frequencies which are too low or too high.

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Let us now discuss op-amp filters also called active filters as opposed to passive filters, which use registers, capacitors, inductors. Op-amp filters have certain advantages they can be designed without using inductors, so that is the significant advantage because inductors are bulky and expensive; inductors also exhibit non-linear behaviour, which arises from the core that they use and that is undesirable you know filter circuit. With op-amps, a filter circuit can be designed with the pass-band gain as opposed to passive filter if you recall we have seen this R C low-pass filter in which the pass-band gain was one. With an op-amp filter, we can have a pass band gain more than one and we will look at some examples.

Op-amp filters can be easily incorporated within an integrated circuit that is another advantage. However, there are situations in which passive filters are still used. Here are two situations; high frequencies at which op-amps do not have sufficient gain that is one situation, in which we would like to use passive filters; second situation - high power which op-amps cannot handle, so that is another case. So, with this background, now let us look at some op-amp filter circuits, how they operate, and how they do the filtering action.

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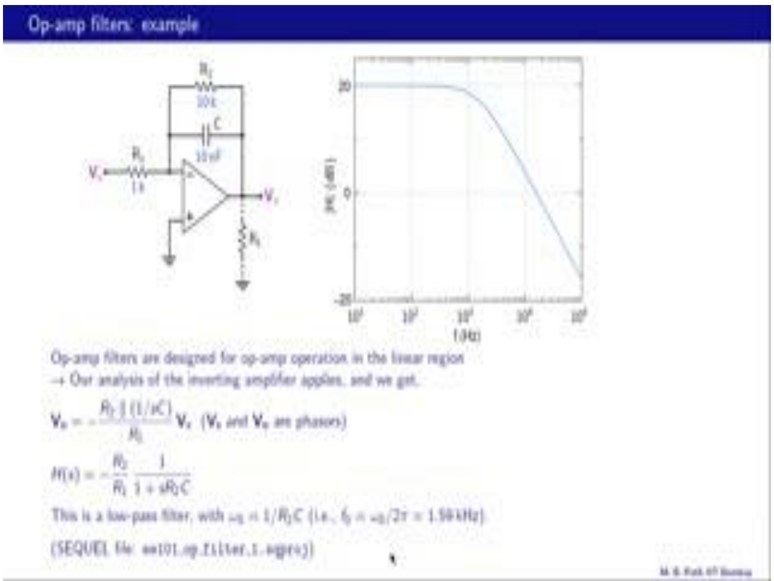

Here is a simple filter circuit using an op-amp. How do we go about analysing it, op-amp filters are designed for op-amp operation in the linear region, and we know lot about the linear region already namely V plus and V minus are approximately equal and these op-amp input currents are 0. And in fact, this circuit is very much like our inverting amplifier except our R 2 is now replaced with these impedances which is R 2 and parallel with C. So, the gain then would be V o by V s remember V o and V s are phasors zone is minus Z 2 by Z 1; Z 2 is R 2 parallel 1 over s C - the impedance of the capacitor divided by Z 1 which is R 1. And if we simplify this we get H of s equal to minus R 2 by R 1 times over 1 plus R 2 C.

Now, this part provides the pass band gain and this is the filtering action and in fact we have looked at the plot of this second expression the magnitude as well as phase. So, this is a low-pass filter with omega naught equal to 1 over R 2 C that is f naught in this particular case is omega 0 by 2 pi which is 1.6 kilo hertz. So, this circuit will function as a low-pass filter with the cut off frequency of 1.6 kilo hertz and that is the plot of mod H in dB versus f on a log scale. It has a gain of 20, 20 dB in the pass-band as opposed to the passive R C low-pass filter that we have seen which had a gain of 0 dB in the pass band. And the corner frequency is given by the intersection of this low frequency asymptote and the high frequency asymptote and that is 1.6 kilo hertz. These two should intersect add a frequency which is 1.6 kilo hertz you can check that out.

This is the first order filter and you can see that from this function as well as from the plot here; this slop is minus 20 dB per decade. There is the circuit file available to you. So, you can change let say the C value or R 1 value and predict what is going to happen and then see it that indeed happens.

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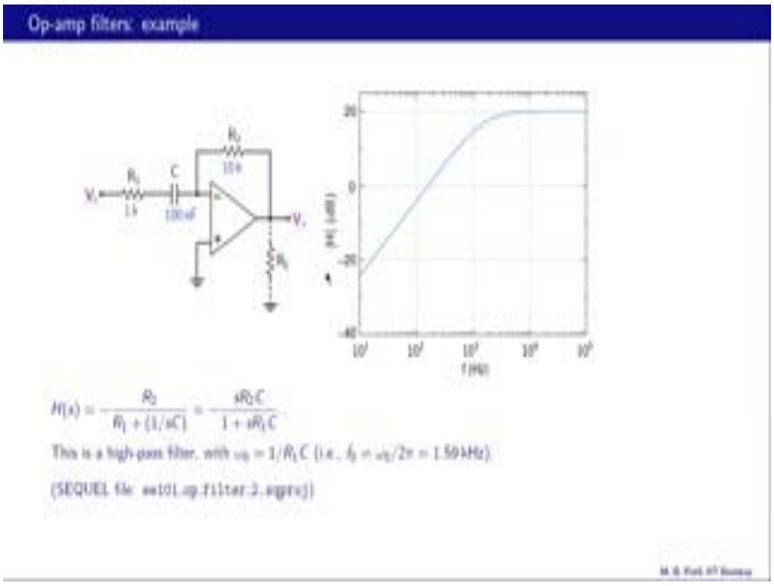

Here is another op-amp filter and this filter is also like an inverting amplifier with Z 2 equal to R 2 and Z 1 equal to the series combination of R 1 and C. So, the transfer function V o by V s would be minus Z 2 by Z 1, Z 2 is R 2 and Z 1 is R 1 plus 1 over s C, and when we simplify that we get this expression here. And this turns out to be a high-pass filter with the corner frequency omega 0 equal to 1 over R 1 C; and f 0 in hertz then is omega 0 by 2 pi and that turns out to be about 1.6 kilo hertz for the component values that we have in this example.

And since we have already looked at bode plots earlier you can take this as an exercise plot the contribution of each of these terms that is R 2 C s and 1 over one plus s R 1 C. You can obtain the bode magnitude plot for each of these terms and then combine the three contributions to get the overall magnitude bode plot for this filter circuit, and that should turn out to be the high-pass filter function.

Let us now look at the plot of H versus omega, the magnitude of H versus omega and that is what we have. This is the exact magnitude versus frequency plot. And if your boding plot is correct then you should get two asymptotes one like that and the other like that. And those two asymptotes should meet at corner frequency that is 1.6 kilo hertz. So, this is 1 kilo hertz, that is 2 kilo hertz and 1.6 kilo hertz is somewhere between the two. So, this is our pass band and notice that there is a pass band gain of 20 dB on the circuit

Now, is this something that we could have estimated without doing this calculation? The answer is yes. What is this 20 dB, it is the gain in the high frequency region and when frequencies are high the impedance of this capacitor is very small. This is 0 and then this circuit is nothing but an inverting amplifier and the gain is minus 10 k divided by 1 k that is 10 in magnitude and that corresponds to 20 dB. Now in the low frequency region, what do we expect this slope to be let us look at this transfer function at low frequencies omega is small. So, therefore, this is R 1 C is small, and we can ignore that and therefore, our H of s is proportional to s; that means, H of j omega is proportional to omega and that gives us a slope of 20 dB per decade.

So, let us check that is happening, let us change the frequency from let us say 10 is to 1 hertz to 10 is to 2 hertz that means, we are going up in frequency by a decade. And as a result we are going from this mod of H to this mod of H and that is 20 dB. So, the slope is indeed 20 dB per decade. Here is the sequel file, you can play with these parameters, the component values, figure out what this plot should look like, and then run the simulation and then check whether your prediction is correct.

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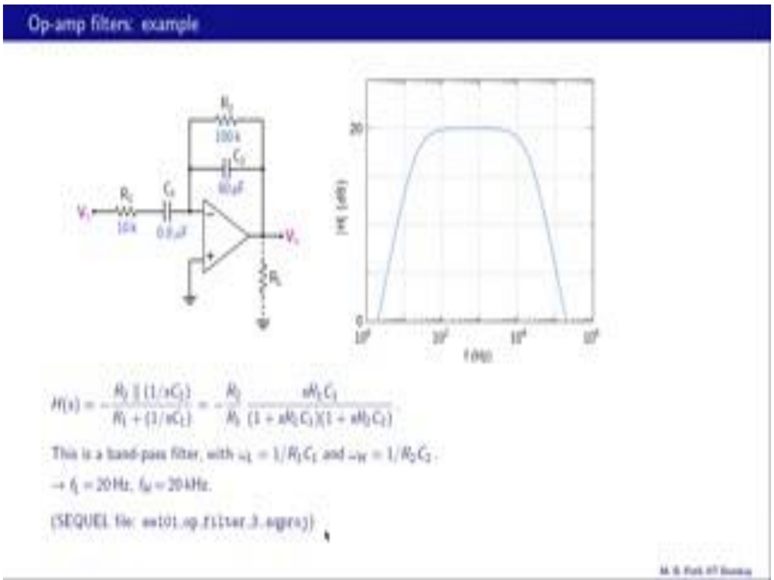

Another circuit of the same type looks like an inverting amplifier. Our Z 2 now is R 2 in parallel with 1 over s C 2; and Z 1 is R 1 in series with 1 over s C 1. So, the transfer function then is minus Z 2 by Z 1; and when we simplify it, we get this expression here. So, now this circuit has got two poles one at 1 over R 1 C 1 and the other at R 2 C 2. And it turns out that for the component values that we have here this will function like a band-pass filter with omega L equal to 1 over R 1 C 1 and omega H equal to 1 over R 2 C 2. If you substitute the numbers f L turns out to be 20 hertz and f H turn turns out to be 20 kilo hertz that is the plot, that is our pass band, and our f l is given by the intersection of this asymptote and that asymptote.

So, that should be something like 20 hertz, this is 1 hertz, 10 hertz, 20 hertz is here, so that is where these two asymptotes should intersect. What about the high cut-off frequency f H we look at the intersection of this asymptote with that one and that should happen at 20 kilo hertz this is 10 kilo hertz, this is 20 kilo hertz. Again the circuit file is available.

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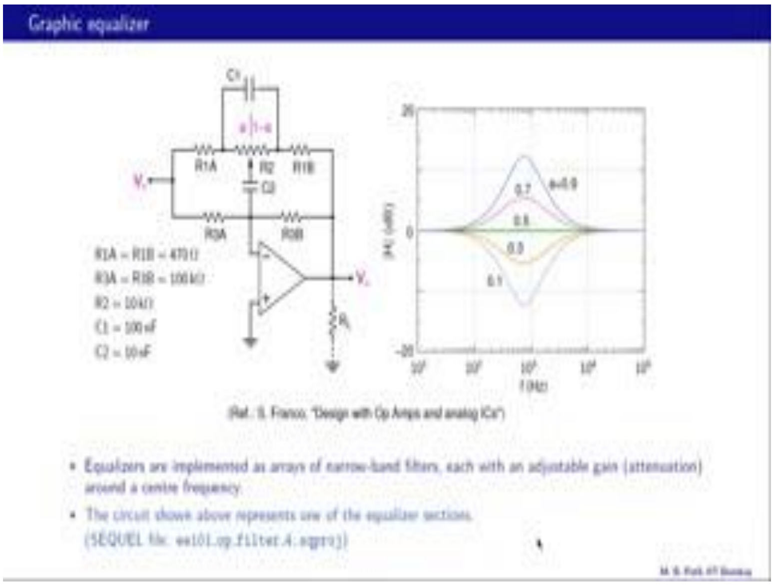

Here is a filter circuit, which has applications in graphic equalizer and here is the reference from which it is taken. This book also has several other interesting circuits as well as many practical tips. So, it is a good book to have in case you want to pursue electronics further. So, how does it work, we have a plot here the fraction of this total resistance on the left is a, and the fraction on the right is 1 minus a. So, by changing this wiper we are essentially changing this a here when a is 0.5, V o by V s is 1 in magnitude and that gives us H equal to 0 dB. So, that means, there is neither any attenuation nor gain from the input to output. When a is smaller than 1, these two values here then the output gets attenuated and when a is larger than one these two values here the output gets amplified. So, by changing this wiper position, we can change the frequency response of this circuit very substantially from this flat response to this band reject filter or towards band-pass filter.

So, this circuit is used in graphic equalizer. Equalizers are implemented as arrays of narrow band filters, and this section is then just one of these filters. It has got adjustable gain or attenuation around a centre frequency. In this case, the centre frequency is somewhere here. So, in the graphic equalizer, we will have several of these circuits with different central frequencies. Here is the sequel circuit file, you can change the value of a, and see that this happens.

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This is a graphic equalizer, and this picture is taken from the internet, but we should clarify that it is not a commercial advertisement. Now, these are those narrow band filters that we mentioned in the last slide; this one is centred around 30 hertz, this one is centred around 60 hertz, 120, 200 etcetera all the way up to 12.8 kilo hertz. So, this covers more or less the frequencies that we can here. And this slider here corresponds to the wiper that we saw in the last slide and by changing this slider position what we are doing is controlling the frequency response of that particular filter with the centre frequency of 800 hertz.

And we can do that for each of these to obtain the frequency response the overall frequency response that we desire, for example, we might want to suppress the low frequency components and enhance the high frequency components etcetera.

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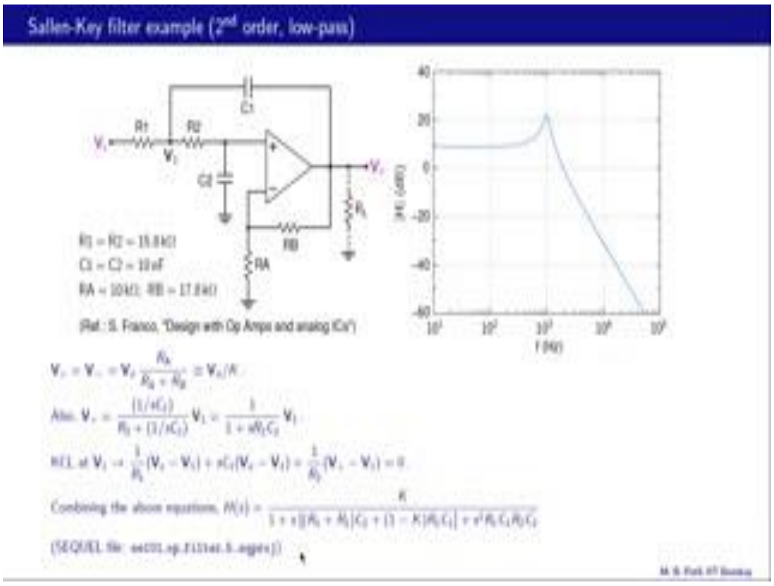

There is a class of filters called Sallen-key filters and here we will only take a look at one of those the second order low-pass filter. The circuit is given here and the component values are also given here and the circuit is taken from this book by Franco again. How do we analyse the circuit, the op-amp is working in the linear region. So, therefore, V plus and V minus are nearly equal, and what is V minus that can be given by voltage division because this current is 0. So, V minus is R A by R A plus R B times V o lets call that V o by K. What about V plus V plus can also be obtained by voltage division from V one voltage division between these two components, because this current is 0 and that gives us this expression here. And finally, we write KCL at this node this current plus this current should add up to 0.

Now, combining all of these equations we get the net transfer function H of s which is given by this expression here. And you are definitely encouraged to derive it yourself starting from these three equations here. Here is the frequency response, this is the pass-band, and we can see that in this case there is some ripple here and that is the high frequency section. What is the order of this filter, we already said that it is second order, but let us also check it with our magnitude plot. Let us change the frequency from there to there, and we see that the difference in H is about 40 dB from there to there. So, this is a second order low-pass filter. Here is the circuit file.

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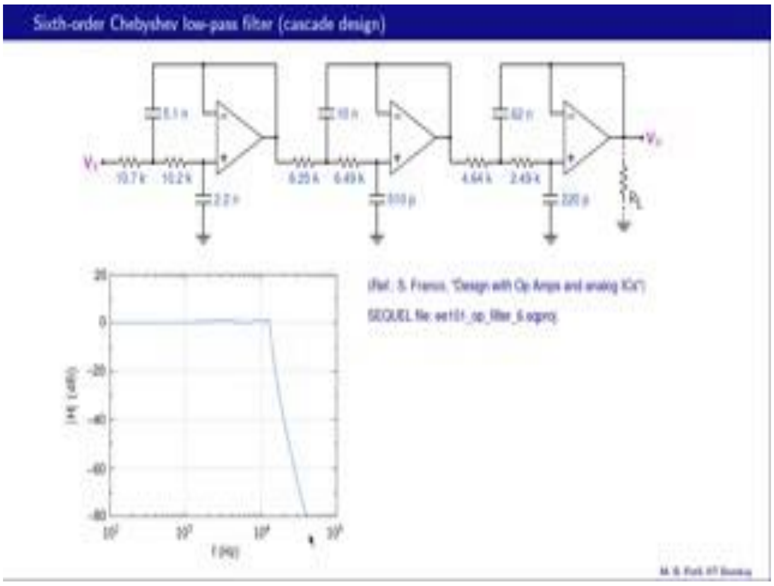

Here is the more complex filter circuit a sixth order Chebyshev low-pass filter and it is taken from Franco's book again. As we have seen earlier when we are talking about Butterworth and Chebyshev filters, Chebyshev filters exhibits this ripple in the pass-band and we can see the ripple right here. And say this is the sixth order filter there is a very sharp drop after the cut-off frequency and this slope would be 6 times minus 20 or minus 120 dB per decade.

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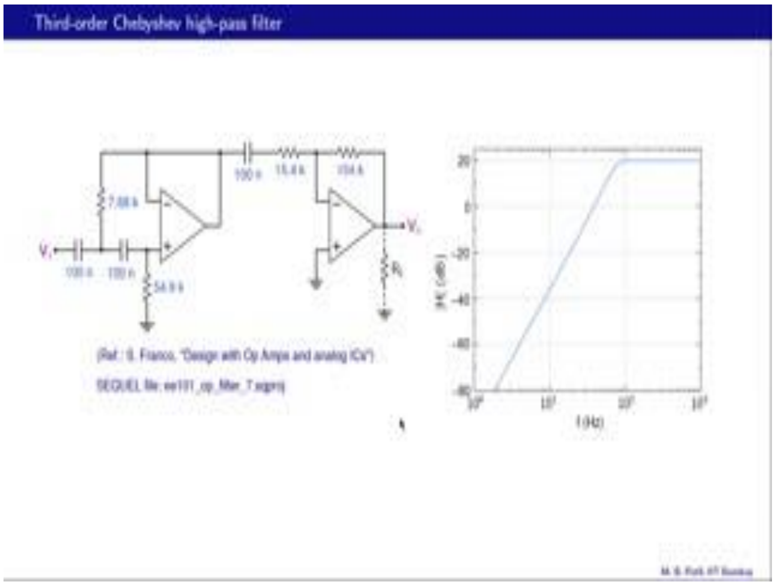

Here is the third order Chebyshev high-pass filter. And in this case, we should be able to find the order from the magnitude plot. Let us see, let us take this section going from there to there, so that is 60 dB and this part is about 1 decade. So, the slope is plus 60 dB per decade that is what we would expect from a third order filter.

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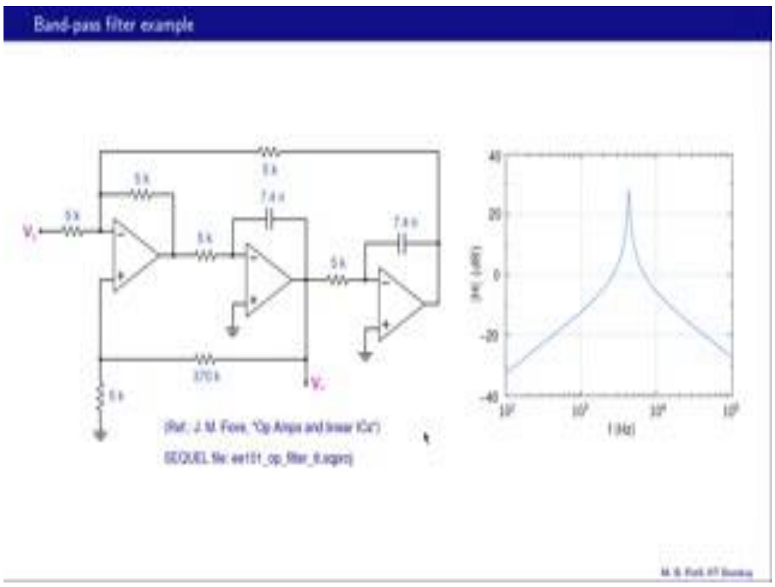

A band-pass filter taken by this book by Fiore; here is a notch filter example and that is the frequency response; what is the meaning of a notch filter it is essentially a band reject filter, but the band is very narrow.

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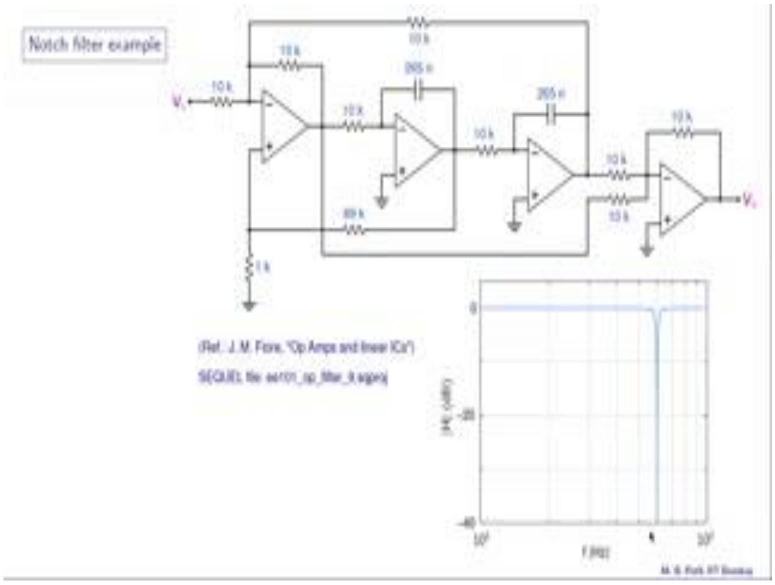

So, it passes most of these frequencies the gain is 0 dB here which means it is actually just one V o equal to V s, but at this frequency which is a very narrow band here the gain is extremely small minus 40 dB or even lower. So, what it means is that it will remove this particular frequency from the input by passing all others. And what is so magical about this particular frequency why would we want to do that. Let us see. This is 10 hertz this is 100 hertz, this is 20, 30, 40, 50, 60. So, this frequency the centre frequency is 60 hertz and that corresponds to the power supplying frequency in the US and may be some other countries.

So, this circuit is removing a 60 hertz component from the input signal and why is that important that is important because electronic circuits often pick up this power frequency noise or disturbance from the surrounding circuits such as SMPS and so on which have a lot of switching activity which happens at the frequency of 60 hertz. And that corrupts our signals and therefore, that 60 hertz component has to be removed. So, that is where this notch filters are very useful.

In summary, we have looked at several op-amp filters. It is a good idea for you to take one of these simple op-amp filters, hook it up in the lab, and check out its functionality that is all for now. See you next time.