Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 43
Bode plots (continued)

Welcome back to Basic Electronics. In this lecture, we will illustrate how bode plot can be constructed using contributions from the various terms in the transfer function, and then adding them together. We will apply this procedure to an example; finally, we will compare the results of the bode approximation with the actually magnitude and phase plots. Let us begin.

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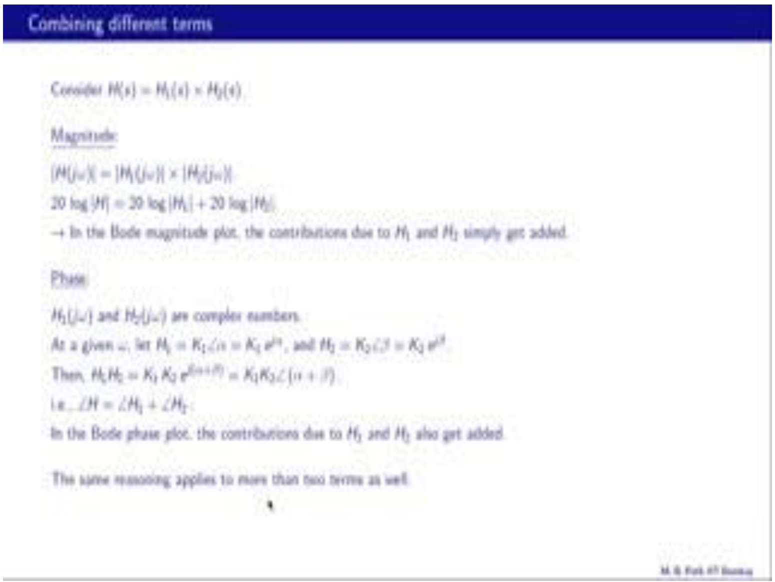

So, we are looked at how to get magnitude or base plots for various terms such as 0 or a pole or a constant s, s squared etcetera. Let us now consider a transfer function, which is a product of two transfer functions, and let us see how to obtain the magnitude and phase plots knowing the behavior of H 1 and H 2 individually, so that is the problem we are looking at. Let us consider the magnitude first. The magnitude of H is mod of H 1 times mod of H 2. Let us now take log of this and multiply by 20. So, we get 20 log H is 20 log H 1 plus 20 log H 2. What is 20 log H 1? It is nothing but H 1 in dB; what is 20 log H 2 is nothing but H 2 in dB. So, at any frequency, to get the net H in dB all we need to do is to add these individual contributions.

So, in the bode magnitude plot for H, the contributions due to H 1 and H 2 simply get added, so that is very simple. Let us look at the phase now. H 1 of j omega and H 2 of j omega are complex numbers as we have seen before. Now, at a given omega, let H 1 b K 1 angle alpha which is the same as K 1 e raise to j alpha; and let H 2 b K 2 angle beta that is K 2 e raise to j beta, here K 1 and K 2 are some real positive numbers. What about the angle of H now, the angle of H is the same as the angle of H 1 times H 2 that is K 1 times K 2 e raise to j alpha plus beta that is what we get by multiplying this expression by this expression here.

In other words, H 1, H 2 has an angle of alpha plus beta that is the angle of H is simply the angle of H 1 which is alpha plus the angle of H 2 which is beta, so that is also very simple. So, in the bode phase plot, the contributions due to H 1 and H 2 at any given frequency also get added. And the same reasoning impact applies to more than two terms as well and that is the nice thing about bode plots. It makes it very easy to plot functions, where we know the individual contributions.

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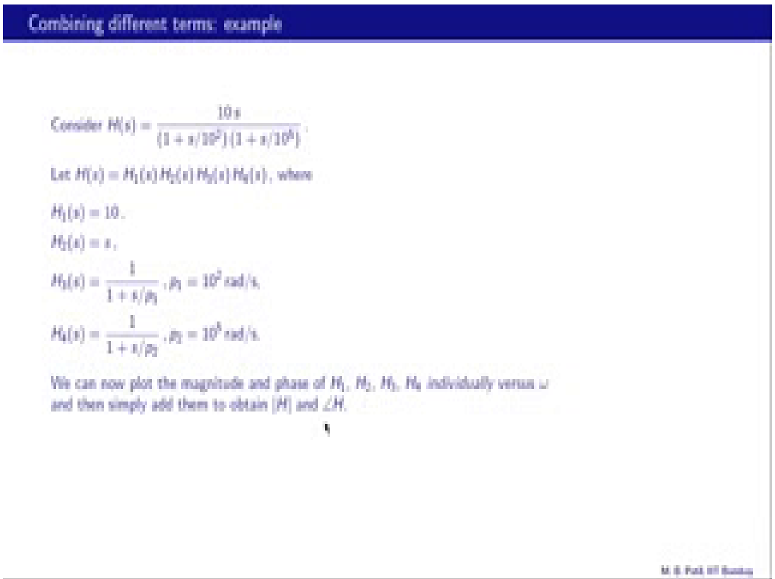

Let us now apply the methods that we have learnt in the last slide to this specific transfer function 10 s divided by 1 plus s by 100 times 1 plus s by 10 raise to 5. Now, this function can be written as a product of 4 functions H 1, H 2, H 3, H 4. H 1 is 10, H 2 is s. H 3 is this pole 10 raise to 2, so it can be written as 1 over 1 plus s by p 1 where p 1 is to 10 raise to 2 radian per second. H 4 is this other pole, 1 over 1 plus s by p 2, where p 2 is 10 raise to 5 radians per second. And we have learnt how to plot the magnitude and phase graphs for each one of these. So, we can now plot the magnitude and phase of H 1, H 2, H 3, and H 4 individually versus omega and then simply add them to obtain the magnitude and phase bode plots.

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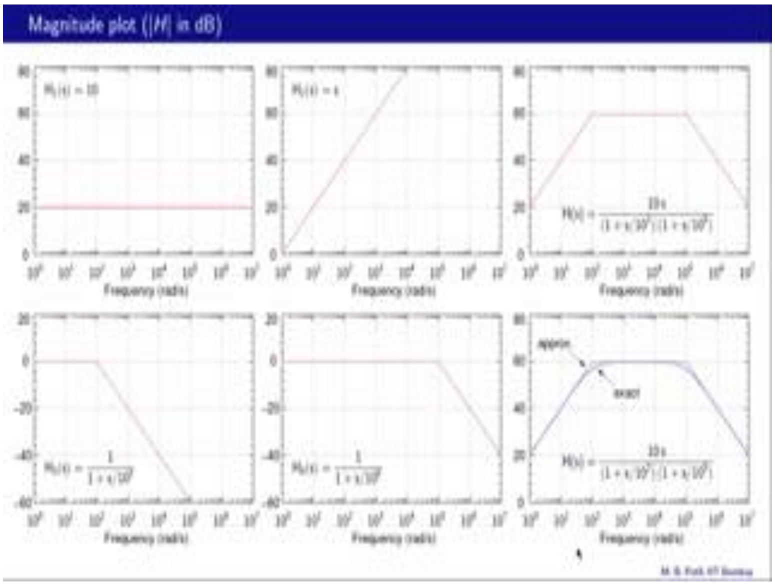

Let us look at the magnitude plot first. So, we are going to plot mod H in dB versus frequency arm a log scale, and the frequency is in radians per second. Let us start with the first function H 1, which is just a constant. So, the magnitude of this is independent of frequency; and in terms of dB, it is 20 times log of 10, so that is 20 times 1 or 20 that is 20 dB constant irrespective of frequency.

Next let us take H 2, which is equal to s. And we have looked at this function before. So, how do we plot this, we know that at omega equal to 1, the magnitude of H 2 is 1, which is 0 dB that is this point here, omega equal to 1 or 10 raise to 0 and H is 0 dB. So, we know that the graph will go through this particular point. And we also know that is H 2 will increase as omega increases, and we know that it will do that with the slope of 20 dB per decade. So, all we need to do now is to draw this straight line with the slope of 20 dB per decade. So, as we go one decade in frequency, we go 20 dB and mod H.

Let us consider this H 3 of H now, which corresponds to a pole. The pole is at minus 10 raise to 2 radians per second and we have seen how to deal with such a term. We have two asymptotes low frequency asymptote, H equal to 0 dB up to omega equal to p, p in this case is 10 raise to 2 radians per second; and then higher frequency asymptote with the slope of minus 20 dB per decade. And both of these asymptotes meet at this common point that is omega equal to p which is 10 raise to 2 radians per second and H equal to 0 dB.

The next function H 4 of s is very similar to this H 3 of s. The only difference is that the pole is now at minus 10 raise to 5 radians per second. So, for frequency is less than 10 raise to 5 radians per second, we have this slow frequency asymptote that is H equal to 0 dB. And for higher frequencies, we have the higher frequency asymptote here which has the slope of minus 20 dB per decade. And now we have all the components of H which if you remember is H 1 times H 2 times H 3 times H 4. And now we can add all of this plots to get our net magnitude versus frequency bode plot.

Where do we begin, let us begin at 10 raise to 0 that is omega equal to 1 radian per second. And see what the various values are. This is 20 dB, this is 0 dB, this is 0 dB this is also 0 dB. So, when we add these four values, we get 20 dB, so that is our starting point omega equal to 10 raise to 0 radians per second mod H equal to 20 dB right there. What happens after this after this our first break point is this one at 10 raise to 2.

So, what is the situation between these two frequencies, this is constant slope 0, this is constant slope 0, this has a slope of 20 dB per decade and this is constant with slope 0. So, the next slope is going to be the sum of all these slopes which is in this case the same as this slope that is 20 dB per decade. So, therefore, up to 10 raise to 2, up to this pole that situation is going to prevail. So, we draw a straight line with the slope of 20 dB per decade and we stop at this point, because now things are going to change because this H 3 is going to change.

What happens beyond tenders to two radians per second? What is our next break point or corner point? This function does not have any break points. This one also does not have any break points after 10 raise to 2. This one also keeps going; it does not have any break point. So, the next break point is supplied by H 4 and that is right here 10 raise to 5 radians per second. So, now, we take the range between our present break point which is 10 raised to 2 and the next break point which is 10 raise to 5.

So, now in this range let us find what the net slope of our magnitude bode plot should be. This one has a constant slope 0 dB per decade, this is minus 20 dB per decade, this is plus 20 dB per decade and this is 0 dB per decade. So, the next slope is 0, minus 20, plus 20 plus 0, so that is next slope of 0 dB per decade and that is why we draw straight line with the slope of 0 dB per decade. And we stop at this break point, which is the same as this pole here.

After this break point, there is no further break point to worry about. So, all we need to do now is to find the next slope for omega greater than 10 raise to 5 radian per second. This is 0 dB per decade, minus 20 dB per decade, plus 20 dB per decade, and minus 20 dB per decade. So, the next slope is 0, minus 20, plus 20, minus 20 that is a next slope of minus 20 dB per decade. So, that is what we do from this point we draw a straight line with the slope of minus 20 dB per decade, and that completes our magnitude bode plot.

Let us make a comment on some practical matters. We note that this y-axis is from 0 dB to 80 dB. And the question you should be asking at this point is; how did we know that it should go from 0 dB to 80 dB. Let say that the frequency ranges is known; given this frequency range how did we know that our H of s is only going to vary between 0 dB and 80 dB. The answer is we did not know, it is in fact not possible to know in advance. And there is some trial and error involved in these matters.

So, we know the starting point, but we do not know what shape H of s is going to take whether it is going to exceed into 80 dB, whether it is going to fall below 0 dB that we do not know in advance. And therefore, it is a very good idea at this point to work out some examples form some textbooks. And there a lot of practice in this matters.

Let us now compare our bode magnitude plot this one with the exact result for H of s. And how do we obtain this exact curve given here in blue. What we do is start at some omega let say this value substitute that in this expression, find mod of H that gives us one data point then we go to another data point and so on. And after we get sufficient number of points, we joint those, so that is of this exact magnitude plot is obtained. One can of course, write a computer program to do that, and you are definitely encouraged to that is fairly straightforward. Write a C program not some higher-level program like MAT log and so on. You will learn much more if you write a C program.

So, how it is the approximation compare with the exact result, it compares pretty well and definitely away from the poles or away from the corner points, the agreement is perfect. And even in between, we have an excellent idea of what our magnitude plot is going to look like.

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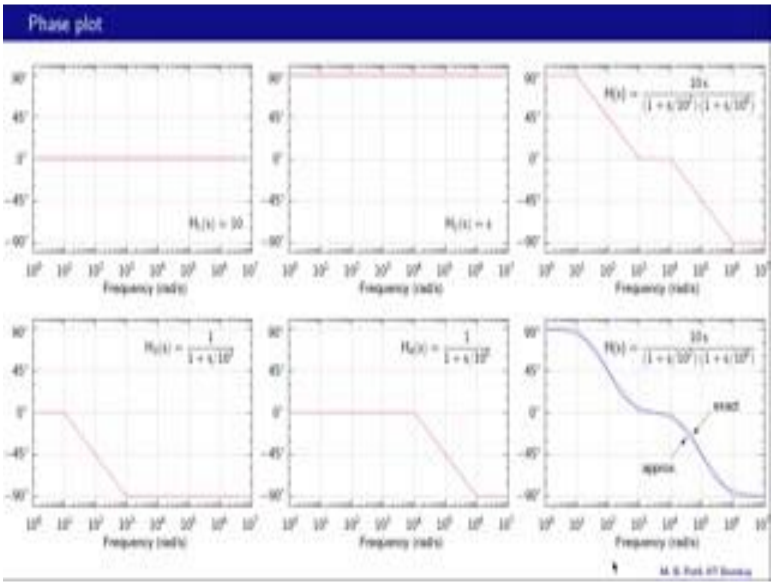

Let us now look at the phase bode plot and we start with H 1, which is equal to 10. This is a real positive number, so the phase is 0 degrees irrespective of omega. Next function is H 2 and that is equal to s; and as we have seen before the phase of H 2 is pi by 2 because s is equal to j omega, and that again is independent of the frequency, so that is 90 degrees or all frequencies. Next, let us take H 3; this represents a pole. And if you remember what we do now is locate the pole that is here 10 raise to 2, go one decade below that frequency. So, we come to 10 raise to 1. And for frequency is less than that we have 0 degrees. Then we go one decade higher than the pole that is 10 raise to 2 that brings us here, and then we draw straight line horizontal straight line at minus 90 degrees and then we just join these two that is how to get the phase of H 3 as a function of frequency.

What about H 4, it is very similar to H 3 only the pole is different 10 raise to 2 here, 10 raise to 5 here. So, the graph is going to look very similar to this graph here except that it will be shifted with 10 raise to 5 radians per second as the pole. We now have the individual contributions H 1, H 2, H 3, and H 4. And now it is a matter of adding them up to get our net phase versus frequency plot. Let us begin at this point 10 raise to 0 radians per second. What is the phase here, 0 degrees, 0 degrees, 90 degrees 0 degrees? So, the overall phase is going to be 90 degrees, so that gives us our first data point right there. So, omega is equal to 10 raise to 0 radians per second phase equal to 90 degrees.

What is our first break point after this data point; let us look at these individual contributions, no break point or corner point here, no corner point here. Here the first corner point is at 10 raise to 1 radians per second and here it is at 10 raise to 4 radians per second. So, this is our first corner point. So, now, we need to focus our attention on this interval between 10 raise to 0 and 10 raise to 1.

So, let us look at each of these plots again. In this interval, this slope is 0, this slope is 0, this slope is 0, this slope is also 0. So, the next slope of our phase plot is going to be 0 and that is what we do here. We draw constant horizontal line with the slope of 0 degrees per decade. What about a next corner point that comes from this plot again 10 raise to 3 radians per second, this is still for away. And now we need to focus our attention in this range between 10 raise to 1 and 10 raise to 3, 0 slope here, 0 slope here, 0 slope here, and this one has a slope of minus 45 degrees per decade. If we go up in frequency by one decade we go down in phase by minus 45 degrees. So, this is the slope that applies to our net phase plot and that is what we do here. We start at this point draw a straight line with the slope of minus 45 degrees per decade up to our next break that is 10 raise to 3 radians per second.

The next break point is this one 10 raise to 4 radians per second. So, let us now calculate the net slope between 10 raise to 3 and 10 raise to 4 radians per second 0 slope, 0 slope, 0 slope, 0 slope, so the next slope is going to be 0. So, we then draw this line segment. What about the next corner frequency; that is 10 raise to 6 radians per second and the slope between these two frequencies needs to be computed now between 10 raise to 4 and 10 raise to 6. And these other three functions have 0 slope in that range. So, therefore, the next slope is going to be the same as this slope which is minus 45 degrees per decade. So, we then get this segment. And beyond this point, what happens the next slope is 0 for all of these transfer functions, and therefore, the overall slope is also 0 and that completes our phase bode plot.

Here is a comparison between bode approximation the red one and the exact phase curve the blue one. And once again we see that the bode approximation is an excellent first step towards understanding the behavior of transfer function.

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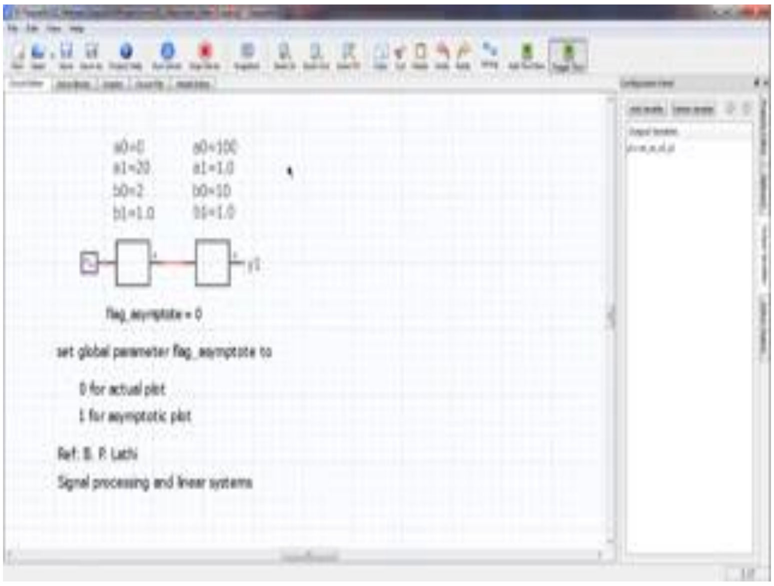

Now, let us now look at a sequel example of computing the transfer function. This element is called filter z 1 p 1; that means, it as 1 0 and 1 pole and this block is also the same filter z 1 p 1. And the net transfer function from the source to the output is the product of this transfer function and this transfer function, so that is two zeros and two poles all together and these parameters here specify what the transfer function is what each of these blocks. And what is a 0, a 1, b 0, b 1 let us see the documentation for this element.

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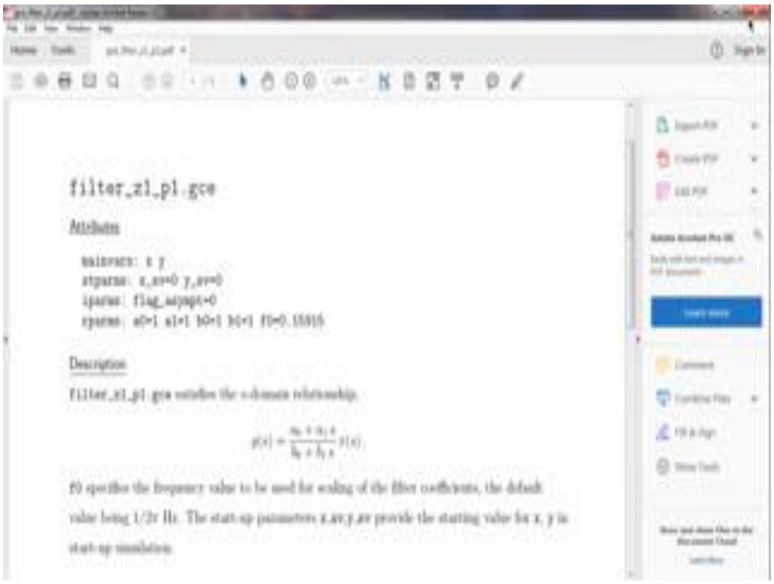

So, the transfer function for each of these blocks looks like this a 0 plus z 1 is divided by b 0 plus b 1 s. So, it is parameters that we saw correspond to these parameters here. Also there is a flag called flag asymptote. If we want bode plots, the magnitude or phase then we set this flag to 1. And if we want the normal plots that is the exact magnitude of phase then we set it to 0. And this specific example it is taken from this book here, Signal Processing and Linear Systems by a Prof. Lathi. And there are several other examples which you should look at and may be work out some of them.

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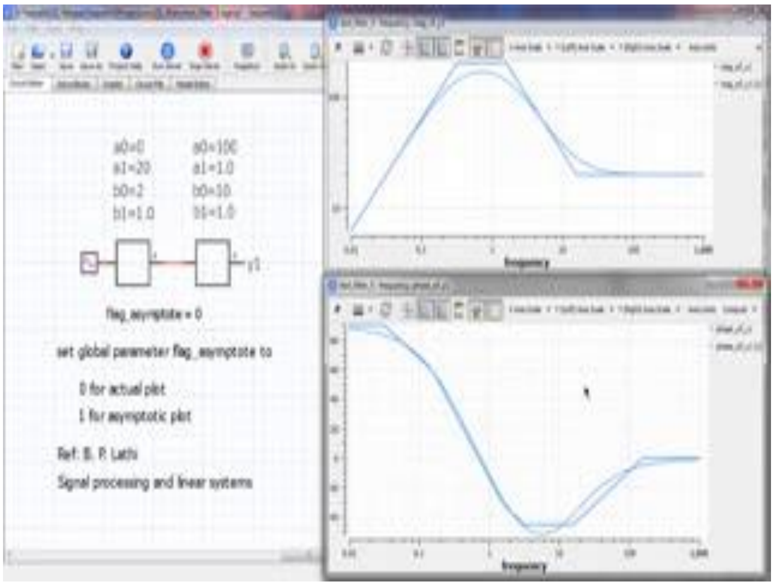

Let us now look at the results. So, here is the magnitude plot. This one is the bode approximation; and the other one - the smoother one is the exact magnitude plot. This of course, is a log axis, this is also logarithmic axis, these are numbers are actual gain and not dB, but since this scale is logarithmic the shape of this plot is what we would expect. If you plot dB versus log f this is the phase plot here the bode approximation is given by this curve here which is a combination of several line segments as we have seen. And the smoother curve, this one is the exact phase curve. And this is a very good example for you to practice. So, take down all these details the a 0, a 1, b 0, b 1 parameters for each of these elements work out, what the magnitude bode approximation should look like also the phase bode approximation and check whether you are actually getting these results are not. The circuit file is right here test underscore filter underscore 5.

To summarize, we have seen how to construct the bode plot of a given transfer function by treating the various terms in the transfer function one at a time. And then adding the individual contributions, bode approximations are useful for understanding transfer functions of filters and also for estimating stability properties of amplifiers, which you may study in a subsequent class on analog circuits. In the next class, we will look at op-amp filters, until then goodbye.