Basic Electronics
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Lecture – 42
Bode plots (continued)

Welcome back to Basic Electronics. In this lecture we will figure out the contribution of a pole and 0, to the magnitude and phase bode plots. We will also consider some other terms once. We know these contributions it is a simple matter to add them and obtain the overall bode plot. So let us start.

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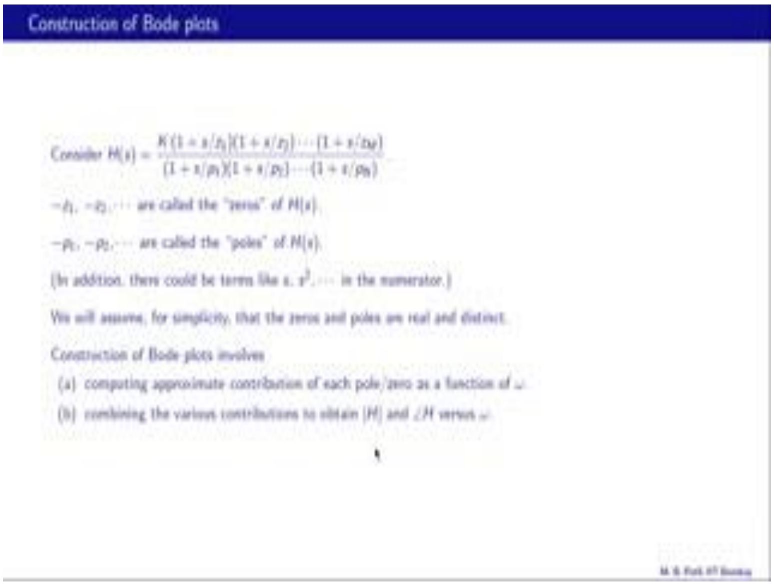

What we have done so for is to look at sample transfer function H of s or H of j omega and look at it is magnitude and phase plots versus omega in radiance per second or f in hertz. Let us now consider generalized transfer function H of s and see how it is bode plots that are the approximate magnitude and phase plots can be constructed.

Here is general expression for H of s in the numerator we have K 1 plus s by z 1, 1 plus s by z 2 and so on. In the denominator we have 1 plus s by p 1 1 plus s by p 2 and so on. Now these quantities minus z 1 minus z 2 etcetera are called the zeros of H of s. For example, if I substitute s equal to minus z 1 this bracket becomes 0 and H of s becomes 0. The quantities minus p 1 minus p 2 etcetera are called the poles of H of s. If we substitute s equal to minus p 1 for example, then this bracket becomes 0. And H of s becomes infinity. In addition, there could be a terms like s and s squared in the numerator and we will see later how to until those as well.

We will assume for simplicity that the zeros and poles are real and distinct. So given 0 for example, z 1 does not appear more than once. Similarly for poles, and they are all real. In reality they actually could be complex, but to make thing simple we will assume that they are all real. Now what is the construction of the bode plots involve, a computing approximate contribution of each pole or 0 as a function of omega; that means, we look at the contribution of this term to H of s we look at the contribution of this term to H of s and so on, and b combining the various contributions to obtain mod H and angle H versus omega.

So once we know the individual contributions like the contribution from this term and this term and so on. Within combine all of those contributions to obtain the net mod H and angle H versus omega plot.

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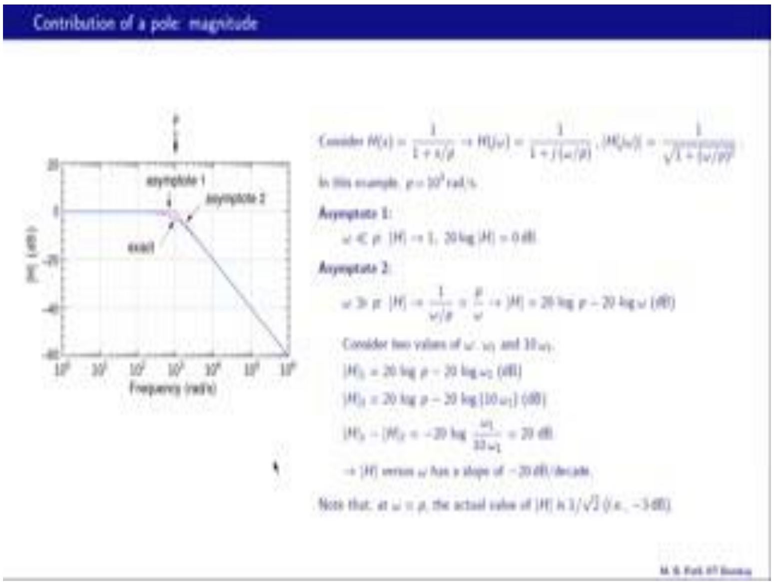

Let us take some examples and see how this can be done. Let us start with a pole; here is a transfer function with a single pole 1 over 1 plus s by p, where p is real and positive number 10 raise to 3 radiances per second. So this system has a pole at minus 10 raise to 3 radiances per second, going by the definition that we saw earlier. Now let us see right H of s as H of j omega, that is 1 over 1 plus j omega by p and the magnitude of H of j omega is 1 over square root of 1 plus omega by p whole squared. And we now want to plot the magnitude of H as a function of frequency. And as we said before we will plot mod of H in dB on a linear scale versus frequency on a logarithmic scale. And for reference let us mark this magnitude of the pole which is given by this positive number p over here and that is 10 raise to 3 radiance per second.

Bode plot is essentially an asymptotic plot. So what do we do we plot the given function in certain limiting cases and that gives us the asymptotes and then we put this asymptote together to obtain the overall picture all right. So let us begin with asymptote 1 that corresponds to omega much smaller than p. And when that happens this term becomes negligibly small and mod H approaches 1. And in terms of dB of course, it is 0 dB because 20 log of 1 is 0. So that is asymptote number 1. And that is what it looks like 0 dB for omega much smaller than p.

Now if you notice we have actually extended this asymptote all the way up to p, because we are only interested in an approximate description of the function; obviously, this is going to be some error at this point because this point is omega equal to p not omega much smaller than p. Over asymptote 2 corresponds to omega much greater than p. And in that case this term is much larger and magnitude then this one.

So we ignore this one here and the magnitude of H then is 1 over omega by p, or p by omega. And H in dB is then 20 log of p by omega that is 20 log of p minus 20 log of omega. This is a straight line in this plane and it has a negative slope so is going to go like that. Further mod is omega is equal to p then this term and this term are equal they cancelation other and H is 0 dB. So for omega equal to p over H is going to be 0 dB. That is precisely this point here.

So, we I am going to have straight line passing through this point with a negative slope. And now let us see what there slope should be. Let us consider 2 values of omega; omega 1 and 10 times omega 1. For example, omega 1 could be 10 raise to 4 and 10 times omega 1 is then 10 raise to 5. For the first omega 1 H 1 is 20 log p minus 20 log omega 1 from this expression. For the second case H 2 is 20 log p minus 20 log of 10 omega 1 because omega 2 is 10 omega 1. Now if you take the difference between these 2 we get H 1 minus H 2, as minus 20 log of omega 1 divided by 10 omega 1. This term cancels. What is omega 1 by 10 omega 1? It is 1 over 10 and log of that is minus 1 so minus 1 times minus 20 is plus 20. So that is 20 dB.

So, what it means is H for omega 1 is going to be higher than H for omega 2 by 20 dB. In other words, if the frequencies are a part by one-decade omega 1 and 10 times omega 1, then H is going to be different by 20 dB. This is the same as saying that H versus omega has a slope of minus 20 dB per decade. This decade refers to this factor of 10 here in omega. And this 20 dB refers to this difference of 20 dB in H. So the slope that we are going to observe for this straight line is going to be minus 20 dB per decade. So that is what our asymptote 2 looks like. It passes through this point omega equal to t and H equal to 0 dB has been discussed. And also it has a slope of minus 20 dB per decade. So if we go up in frequency by one decade become down in H by 20 dB. Let us by this minus sin here.

Now, at omega equal to p if we look at this exact expression, what is the magnitude? This becomes 1 because omega and p r equal. So we have 1 plus 1. So there is 1 over square root 2. So the actual value of H is 1 over root 2 and if we convert that 2 dB it terms out to be about minus 3 dB. So definitely at this point there is going to be that much error what we have done is shown 0 dB here, but the actual value is minus 3 dB like that. So this is the blue one is our exact mod H versus frequency curve. And there is definitely some error at this point as be suspected, but asymptotically it shows excellent agreement as we go away from this pole in both directions. And that is the power of the bode approximation.

We will do that for other functions and then we will see how to put together the different contributions to obtain the bode magnitude plot for more complex function.

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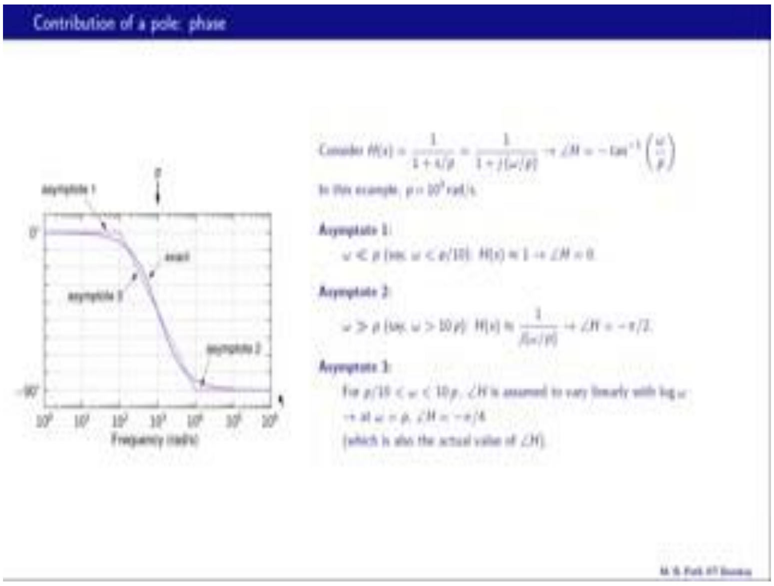

Let us now look at the bode phase plot for this same transfer function, 1 over 1 plus s by p, which corresponds to a single pole. And the pole is at minus 10 raise to 3 radiance per second as in the previous slide. Now as we have seen before we plot the phase on a linear access and frequency on a logarithmic access. And once again for reference we will mod the magnitude of the pole over here that is 10 raise to 3 radiance per second. The angle of H is given by minus tan inverse omega by p and that follows from this equation here. And let us now see how to obtain the asymptotic bode plot for the phase. Let us start with asymptote 1 which corresponds to omega much smaller than p, and by this what we mean is omega less than p by 10. This is our p, p by 10 is here. So we are talking about this range over here.

With this condition omega by p is going to be less than 0.1. And as an approximation you say that angle of H is nearly 0 because this is a small number. So that gives us asymptote 1 so angle of H is 0 in this range here that is omega less than p by 10. And that is what it looks like. Let us look at the second asymptote now which corresponds to omega much larger than p, say omega greater than 10 times p. This is our p; 10 times p is here. So we are talking about this region here all right. Now what happens if omega is much greater than p? This one is small compare to omega by p and can be ignored and therefore, H of j omega is approximately 1 over j omega by p. And the phase of this function is simply minus 90 degrees or minus pi by 2. And that gives us the second asymptote. So for omega in this region the phase is minus 90 degrees and that is the second asymptote.

Let us now look at the third asymptote, which corresponds to p by 10 less than omega less than 10 p. This is our p by 10, and this is 10 times p. So we are talking about this region here all right. Now in this region we assume that the angle of H varies linearly with log omega; that means, we have a straight line connecting these 2 points and that is our third asymptote. Now let us look at what happens at this frequency omega equal to p. What is the phase given by this expression? Omega equal to p; that means, we had minus tan inverse of 1 that is minus 45 degrees. This is our asymptote 3, a straight line connecting these 2 points.

And notice that the asymptote also process through minus 45 degrees at omegas equal to p; let us now compare the bode approximation with the exact result the one given by the blue curve here. At omega equal to p the 2 agree, they both give minus 45 degrees. As we move away from the pole the agreement between the 2 is excellent in that direction as well as in this direction.

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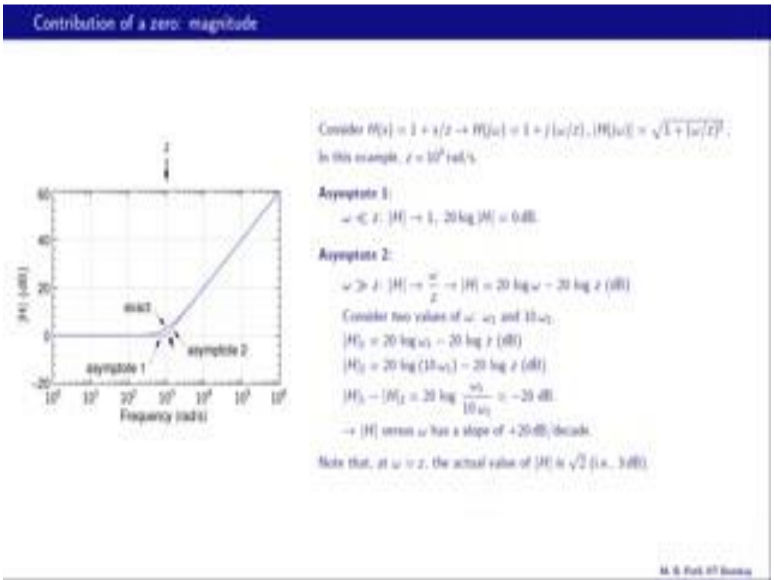

Next let us consider the contribution of a 0. And we will first consider the magnitude bode plot. Here is an example H of s equal to 1 plus H by z, where z is 10 raise to 3 radiance per second. And if you recall this means that our 0 is at minus 10 raise to 3 radiance per second, following the definition that we looked at earlier. Now as before we will plot mod of H in dB on a linear scale versus omega in radiance per second on a logarithmic scale H as shown here. You can rewrite H of s as H of j omega, 1 plus j omega by z, and from that we obtain the magnitude of H as 1 plus omega by z squared. Now as we did in the case of the pole, we will take 2 cases.

Case 1 which gives us the first asymptote omega much less than z and in this case we can ignore this term and our H of j omega then approaches 1, mod of H approaches 1 this term being very small. And what is that mean; that means, mod H in dB is 0 dB. So, here is our first asymptote 0 dB here. And although this applies to omega much smaller than z, we will extend that all the way up to omega equal to z because we have seeking an approximation all right. Now let us take the second case in which omega is much greater than z that gives us the second asymptote.

With this condition we can ignore this one with respect to omega by z squared and then mod of H become simply omega over z like that. And then in terms of dB mod of H is 20 log omega minus 20 log z. Now this is the straight line in this plane here, and let us make a couple of observations about the straight line first if omega is equal to z, then this first of cancels with the second term and we get H equal to 0 dB. That means, this straight line is going to pass through this point that is omega equal to z mod H equal to 0 dB. Point number 1, point number 2 the slope of this line is going to be 20 dB per decade, because we have plotting H in dB versus log of omega and this number then gives us the slope.

We can also figure out the slope by considering 2 values of omega, namely omega 1 and 10 times omega 1 that is 2 values of omega which are one decade apart all right. Now for the first omega value that is omega 1, mod H in dB is 20 log omega 1 minus 20 log z from this equation. For the second value of omega that is 10 times omega 1, mod of H in dB is 20 log 10 omega 1 minus 20 log z. We will call this as mod H in case 1, this 1 as mod H in case 2. Now if we take the difference between these 2 H 1 minus H 2. What we get is 20 log omega 1 divided by 10 omega 1, this term cancels this omega 1 cancels and therefore, we get log of 1 by 10 which is minus 1 and therefore, H 1 minus H 2 turns out to be minus 20 dB. In other words H 2 minus H 1 would be plus 20 dB.

What it means is as we go from some omega 1 to 10 times omega 1, we go up in H by 20 dB and that is why the slope is 20 dB per decade. Let us now look at the asymptote, there is what it looks like, it passes through this point omega equal to z H equal to 0 dB and it as a slope of plus 20 dB per decade. If you change the frequency by a factor of 10 we go up by 20 dB. What about at omega equal to z? Our asymptotes predict that H is 0 dB, but the actual value of course is different and that turns out to be square root 2 from here. All we need to do is to put omega equal to z. So that becomes 1 and we have square root of 1 plus 1 that is square root of 2. 20 log of square root 2 is 3. So therefore, H corresponds to 3 dB at omega equal to z.

So, that is the actual value of H, and therefore the exact curve looks like at this difference is 3 dB. So there is of course, some error, but asymptotically we see that the bode approximation works really well as we move away from the 0 in either direction either that direction or in that direction.

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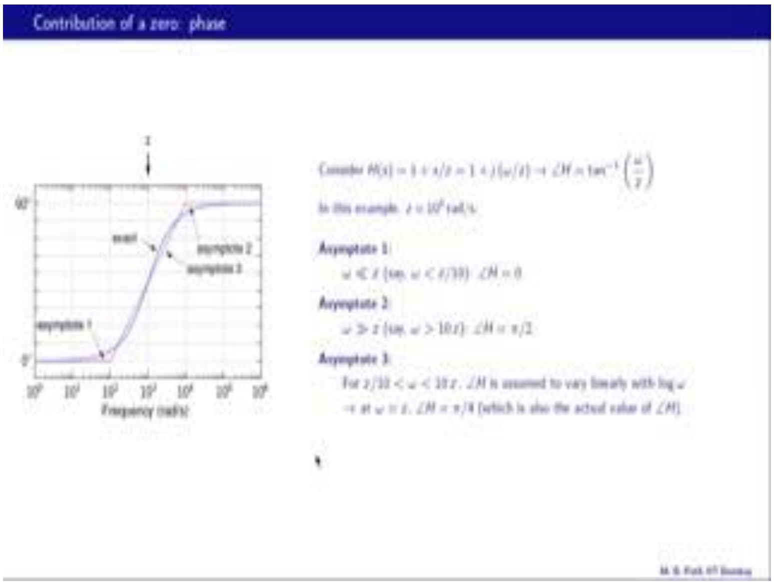

Let us now look at the phase plot for the same H of s 1 plus s by z. That is 1 plus j omega by z which gives us the angle as tan inverse omega by z. And once again we will take the same value of z that is 10 raise to 3 radiance per second. The frequency is on a logarithmic access and phase is on the engineer access. This is our z value all right. Now let us start with asymptote 1 which corresponds to omega much smaller than z say omega 10 times smaller than z. In this case this is our z 10 raise to 3 so z by 10 is here so we are talking about this region here.

What is the angle of H in this case? So this is tan inverse 0.1 or smaller. And we will approximate that as 0. So that is our first asymptote the phase is 0 up to z by 10 that is 10 raise to 2 radiance per second. Asymptote 2 now, which corresponds to omega much larger than z say omega greater than 10 z. This is z, this is 10 z, so we are talking about this region. Now what happens if omega is much larger than z, omega by z becomes infinity, tan inverse of infinities pi by 2, on 90 degrees. And as an approximation we say that that corresponds to this entire region. So that gives us asymptote 2. Asymptote 3 applies to this region in between and again as we did in the case of the pole angle H is assume to very linearly with log omega in this region.

So, what we do is simply connect these 2 points with the straight line. And at omega equal to z that straight line will predict plus 45 degrees, and that is also the exact value of the angle because at omega equal to z, this becomes 1 and tan inverse of 1 gives us 45 degrees. So that is our asymptote 3. And the blue curve is the exact angle H. Once again asymptotically it is in excellent agreement with our bode approximation, in either direction either low frequencies or high frequencies. And at omega equal to z our approximation also predicts the exact value of the angle.

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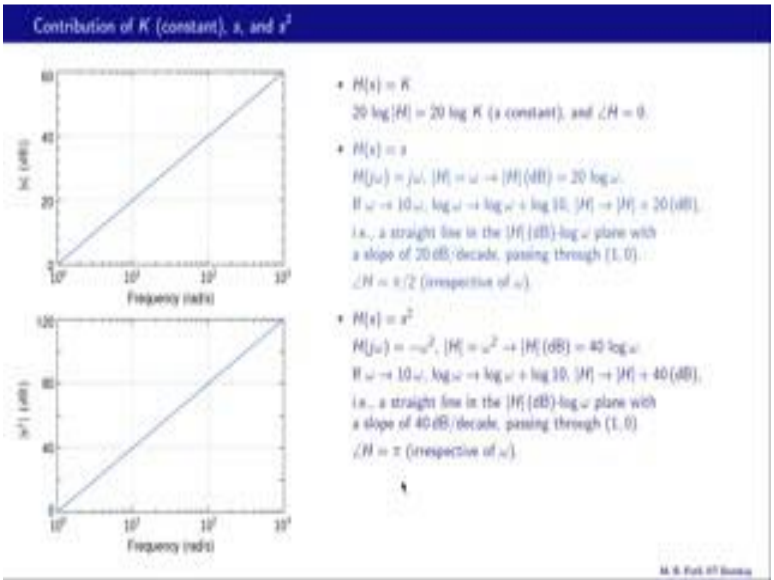

Let us now look at the contribution of term like k, just to constant or s, which is j omega and s squared which is j omega squared. Let us take H equal to K first, just a constant what is 20 log H? It is 20 log K and there is a constant. What about the angle let say K is a positive constant in that case the angle is 0, if it is negative the angle is 180 degrees or minus 180 degrees?

What about H of s equal to s? Rewrite that as H of j omega equal to j omega. The magnitude is simply omega, and H in dB is 20 log of omega. So this is a straight line in our H versus log omega plane, when H is plotted in dB. How do you figure out the slope? That omega go form omega to 10 omega. What happens to log omega? It goes from log omega to log omega plus log ten, so this part will go from 20 log omega to 20 log omega plus 20. That is the H magnitude will go from the old value of H plus 20 dB so; that means, the slope is plus 20 dB per decade. So this is a straight line in the H versus log omega plane with a slope of 20 dB per decade passing through 1 0; omega equal to 1 H equal to 0. If you put omega equal to 1 here, then 20 log of 1 is 0 and that is why we say that this line was pass through; omega equal to 1 H equal to 0 dB.

So, that is the plot at omega equal to 1, which is this point our H in magnitude is 0, that is the same as mod of H here. And as we increase the frequency the function goes up with the slope of plus 20 dB per decade. Is this an approximation? Not really this is actually the exact behavior of mod of H. You are not really made in a approximation. Now let us look at the angle of H. What is H of j omega? It is g omega; omega is real positive number. So the angle of H is simply angle of j which is pi by 2, or 90 degrees. And that does not depend on what omega is. It is a constant all right. Let us now consider H of s equal to s squared or H of j omega equal to j omega squared which is minus omega squared. And the magnitude then is omega squared and H in dB is 40 log omega, 20 log of omega squared so that turns out to be 40 log omega.

Once again we go through this exercise of allowing omega 2 go from omega 2, 10 omega. And then we find that H increases by 40 dB. So therefore, the slope is 40 dB per decade. So it is a straight line in the H versus log omega plane where H is in dB, with the slope of 40 dB per decade and passing through omega equal to 1, H equal to 0 dB once again. Put omega equal to 1 here we get 0 dB, like that. So this is our omega equal to 1. So that passes through 0 dB, and this line as a slope of 40 dB per decade when we go up in frequency by 1 decade or 1 order we go up in mod H by 40 dB, 80 minus 40, 40 dB here. What about the angle H of j omega is minus omega squared is some negative number and therefore, the angle is 180 degrees and that is irrespective of the frequency value? We can write this pi as at the pi or minus pi plus minus 180 degrees.

In summary we have seen how various terms such as pole 0 constant etcetera can be represented with bode plots. In the next lecture we will take an example and construct the overall magnitude and phase bode plots using the individual contributions. See you in the next class.