Basic Electronics
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Lecture - 41
Bode plots

Welcome back to Basic Electronics. In this class we will start our discussion of Bode Plots which are convenient way to depict information about transfer function. We will begin with the unit decibel which is used in bode plots, after that we will consider a simple transfer function and see what is the most meaningful way to show its magnitude and phase as a function of frequency. So, let us get started.

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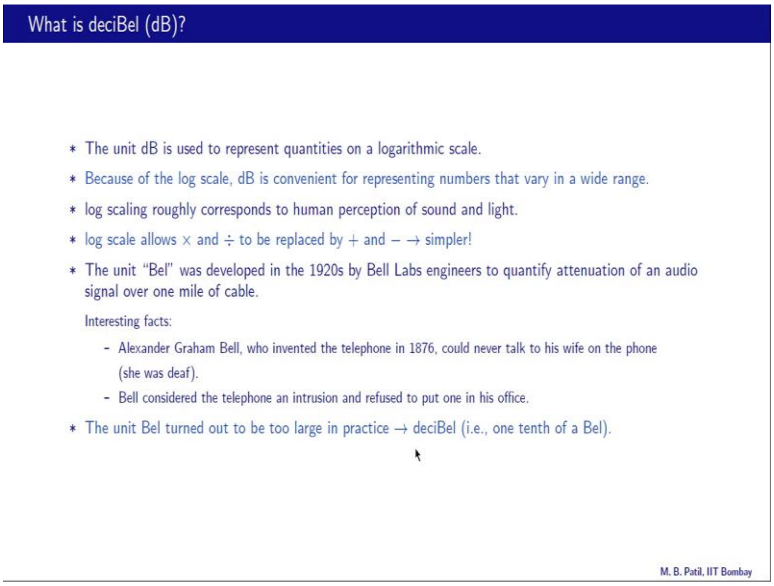

We are now going to look at how to plot things like gain of an amplifier as a function of frequency, and before we do that it is useful to look at this unit called decibel or dB in short The unit dB is used to represent quantities on a logarithmic scale, as suppose to linear scale.

Because of the log scale dB is convenient for representing numbers that very in a wide range for example, the quantity that is varies from 0.12 10 raise to 5, also log scaling roughly corresponds to human perception of sound and light, we respond to sound or light intensities varying over a wide range and dB unit makes things very convenient because it uses logarithmic scale and that allows multiply and divide operations to be replaced by plus and minus, which is much simpler. The unit bell was developed in the 1920s by bell labs engineers, and in those days telephone was the most important technology around, and the bell labs engineers wanted to quantify attenuation of an audio signal over 1 mile of cable.

So, to describe that attenuation, they come up with this unit called bell and of course, it is in horror of Alexander graham bell the inventor of telephone. Here are some curious facts about Alexander graham bell, he invented the telephone in 1876; he could never talk to his wife on the phone because she could not here. Bell considers the telephone to be an intrusion and refuse to put one in his office all right. Now the unit bell trend out to be too large for use in practice and therefore, what is commonly used in practice is 1 tens of a bell that is called decibel or dB.

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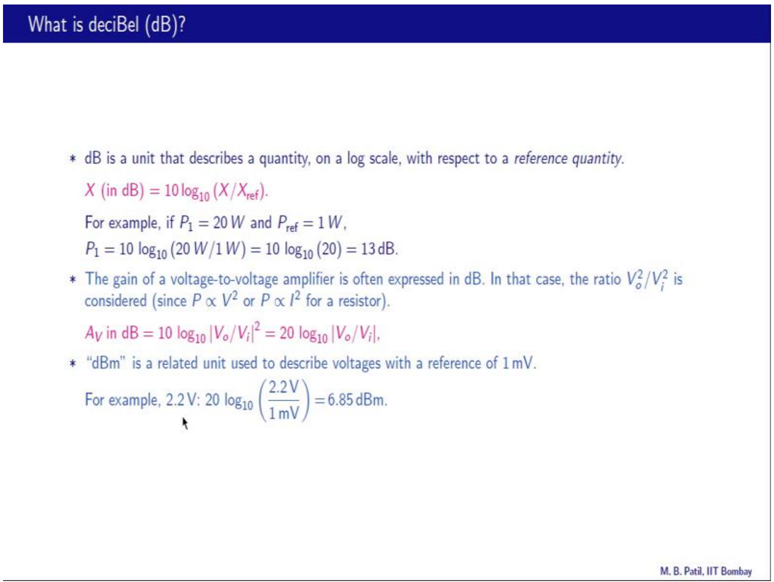

So, the dB is a unit that describes a quantity on a log scale, and it is always with respect to a reference quantity. So, if you have a quantity called X, we will say that X in dB is 10 times log to the base 10, X divided by X reference. Let us take an example let say we have some power called P 1 and that is 20 watts, and we have a reference power called p ref that is 1 watt. So, P 1 in dB would then be 10 log to the base 10, P 1 which is 20 watts divided by P ref which is 1 watt. So, that is 10 log to the base 10, 20 that comes out to be 13 Db. Now the gain of a voltage to voltage amplifier is often expressed in Db, and in that case we do not use V o by V ref, we use V o squared by V ref squared and V ref in that case is the input voltage and why do we use squares? That is because power goes as voltage square or current squared for a register.

So, voltage gain in dB would be 10 log to the base 10, V o by V i squared that is equal to 20 log to the base 10, V o by V i and this of course, is a magnitude. There is another unit called dB m and that is used to describe voltages, when we have a reference voltage of 1 millivolt; so this milli is what does I am stands for. Let us take an example say we have a voltage of 2.2 volts, and we want to express that in dB m that would be 20 log to the base 10, 2.2 volts divided by 1 millivolts. So, that terms out to be 6.85 dB m.

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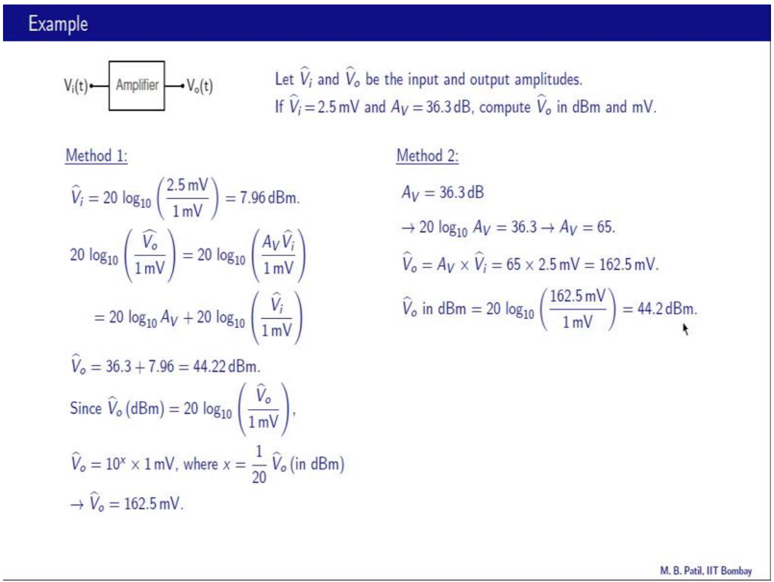

Let us take an example to get use to these dB calculations, here is an amplifier with input V i and output V o, and these are sinusoidal, V i cap and V o cap are the input and output amplitudes.

If V i cap is 2.5 millivolts, and the gain of this amplifier the voltage to voltage gain is given to be 36.3 Db, then we want to compute V o cap both in dB and as well as in millivolts, so let us you have to do that. There are 2 ways of doing it, one we start with V i cap, convert that into dB m, how do we do that? 20 log to the base 10 V i cap, which is 2.5 millivolts divided by 1 millivolt and that turns out to be 7.96 dB m. Now we know that V o cap is A v times V i cap, you can divide both of these by 1 millivolt, then we can take log to the base 10 and then we can multiply both sides by 20; and then this right hand side can be written as 20 log to the base 10 A v, plus 20 log to the base 10 V i cap divided by 1 millivolt.

Basically this is a product of 2 numbers; one is A v, the other one is V i cap by 1 millivolt and that is how we get this relationship. And therefore, we can calculate V o cap na, this number would be V o cap in dB m. We already have this number given to us thirty 6.3 Db, this we have computed that is V i cap in dB m. So, 36.3 plus 7.96 that is 44.22 dB m. So, that is V o cap in the unit of dB m; now we want to get it in millivolts. We know that V o cap in dB m is nothing, but 20 log to the base 10, V o cap divided by 1 millivolt, so from this we can get this inverse relationship, that is V o cap equal to 10 raise to X times 1 millivolt, where X is 1 by 20 times V o cap in dB m .You should verify that this indeed follows from this definition, and then we are all set we can now find V o cap in millivolts.

So, the answer turns out to be 162.5 millivolts; let us look at method 2 now, we know that A v is 36.3 Db, we can convert that in to a real number which is the ratio of V o and V i the magnitude of the ratio. So, what we know is 20 log to the base 10 of A v is 36.3 and from that we get A v equal to 65 and now we know V i cap 2.5 millivolts, we know A v 65 therefore, V o cap is 162.5 volts and once we have V o cap in millivolts, we can find V o cap in dB m using 20 log to the base 10, V o cap divided by 1 millivolt and that comes to 44.2 dB m.

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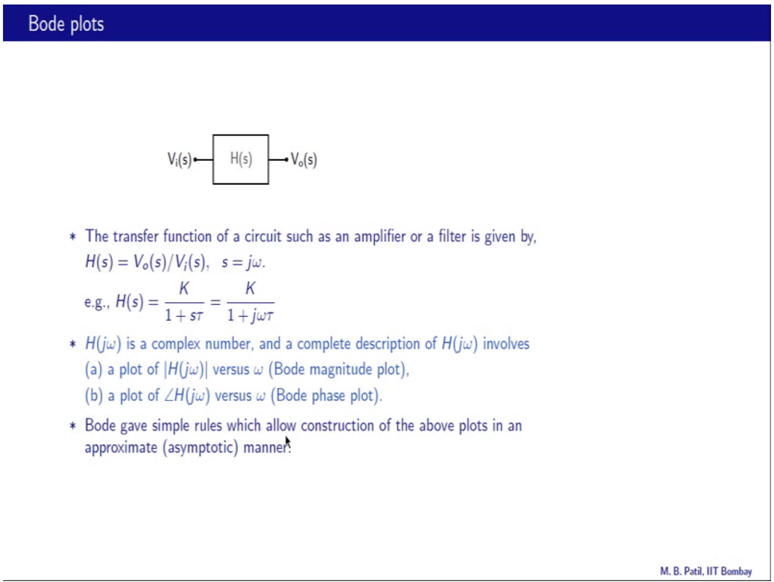

With that languish introduction let us now come to bode plots, the topic of our interest. Here is black box with a transfer function H of s and what is H of s? H of s is V o of s divided by V i of s, s here is the Laplace variable and for our case it is j times omega and here is an example of H of s say it can be k divided by 1 plus s tau, where k is some constant and tau is some time constant and that is the same as k divided by 1 plus j omega tau, because s is equal to j omega. Now H of j omega is a complex number as shown in this example and a complete description of H of j omega therefore, we will involve 2 plots; 1 is a plot of magnitude of H of j omega versus omega an second a plot of the phase of H of j omega versus omega.

This first plot is called the Bode magnitude plot and the second one is called bode phase plot. Bode gave simple rules which allow construction of the above plots in an approximate or asymptotic manner, and we will see several examples of these plots.

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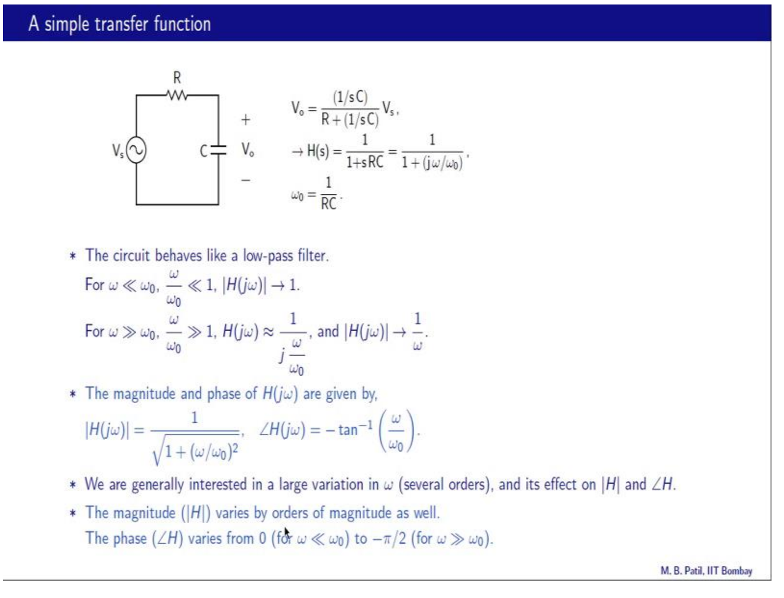

Let us start with the simple transfer function of a series RC circuit, the capacitance presents an impedance which is 1 over j omega C, and in terms of the Laplace variable it is 1 over s c. So, V o in this case is this impedance divided by the some of these 2 my voltage division times V s. I of course, we are talking about sinusoidal study state. So, that is 1 over a c divided by R, plus 1 over a C times V s and these now are phases both V o and V s. So, the transfer function which is defined as V o divided by V s is 1 divided by 1 plus s RC this simply follows from this expression we multiply both numerator and denominator by SC.

In the numerator we get 1, in the denominator we get 1 plus R times s c. So, that is this expression and we can now substitute for s j times omega. So, we get 1, divided by 1 plus j omega RC and that we can rewrite as 1 plus j omega by omega 0, where omega 0 is 1 over RC. So, that is the transfer function for this series RC circuit; let us make a few comments about this transfer function, first the circuit behaves like a low pass filter what does it mean? That means, it will pass input voltages with low frequencies without any attenuation or reduction in the magnitude, where as higher frequencies will get blocked; that means, if the input voltage has a higher frequency, it will appear attenuated or reduced at the output side.

And why does it behave like a low pass filter let us take a look. For omega much smaller then omega 0, this is the low frequency region law, the transfer function is approximately 1 because this component is very small compare to 1. So, therefore, there is no reduction in V o. So, V o and V s are nearly equal, what about omega much larger than omega 0 then this 1 is small compare to this term, and I have transfer function then is approximately 1 divided by j omega over omega 0 like that. And omega 0 is a constant and we take magnitude of this H of j omega we get H of j omega proportional to 1 over omega, what is it mean? That means, if we go on increasing omega, the magnitude of H of j omega goes on deceasing so; that means the output voltage is much smaller than the input voltage in magnitude.

Let us note that this H of j omega could also be written as simply H of omega, but this is a more commonly used convention in text books especially in control systems. So, we will stay with this notation. Next point let us now look at the magnitude and phase of H of j omega, what is H of j omega? It is here it is of the form numerator divided by denominator, and the magnitude is the magnitude of the numerator which is 1 divided by the magnitude of the denominator, and that is the square root of the square of the real part that is 1, plus the square of the imaginary part that is omega by omega 0. So, that is what we get for the magnitude; what about the phase? The phase of H of j omega is the phase of the numerator in this case 0 degrees, minus the phase of the denominator in this case tan inverse omega by omega 0.

So, the net result is the phase or angle of H of j omega is minus tan inverse omega by omega 0. Next point we are generally interested in a large variation in omega, and its effect on the magnitude of H and the angle of H, and what do we mean by large over here? By large we mean several orders of magnitude for example we may be interested in omega varying from 0.1 radiance per second to 10 raise to 6 radiance per second. As omega varies over a wide range, the magnitude of H that is mod H will also vary by orders of magnitude and here is an example we have mod H proportional to 1 over omega here, and if omega changes by several orders of magnitude mod H will also change by several orders.

On the other hand the phase that is angle of H varies in a limited range, let us look at this example; we have angle H given by minus tan inverse omega by omega 0, if omega is much smaller then omega 0, then this number is close to 0 and then the angle H is about 0 degrees or 0 radiance. What if omega is much larger then omega 0? Then omega by omega 0 is a large number, and in the limiting case it could be infinity and then the angle will tend to minus 90 degrees or minus pi by 2 and these are important observations that we should keep in mind, they will help us when we want to plot mod of H and angle of H as a function of frequency.

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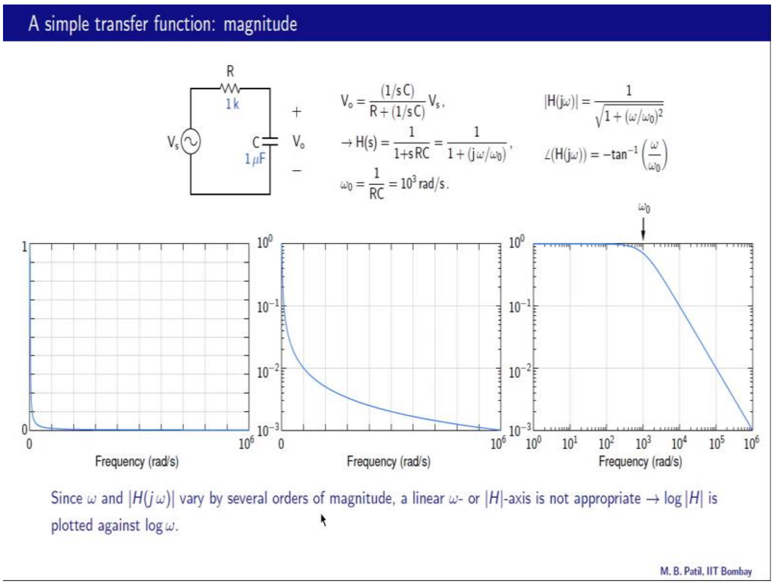

Let us now look at how H varies versus omega and that is given by this relationship we have talking about the magnitude of H right now.

Here is a plot the X axis is omega and radiant per second, this is omega equal to 0, this is omega equal to 10 raise to 6, and the y axis is mod of H and as we have seen in the last slide mod of H approaches 1 if omega is small; because this term is negligible compare to 1, that is what we see over here. As omega increases mod of H goes us 1 over omega. So, as omega goes on increasing mod of H goes on decreasing, and it appears like 0 here in this plot. Now this kind of plot is not very useful because we cannot distinguish between the mod of H values at different frequencies, and clearly some improvement is required so let us see what that is. Here is an improved plot, what is the difference between these 2 plots? Here we have a log axis.

So mod of H is 10 is to 0 here, 10 is to minus 1 here, 10 is to minus 2 and so on

and definitely we can resolve mod of H better now with this log scale for example, let us take omega equal to 2 times 10 raise to 5 radiance per second that is here and now we can read the value of mod of H from this curve, so some where there what is that value? So this is 2 10 raise to minus 3, 3 4 and let say is 5. So something like 5 times 10 raise to minus 3. This was not possible to do in this older plot, because the y scale had also got comprised here all right. So definitely some improvement but we do need further improvement, because our lower frequencies have still got crowded here for example, we cannot tell the difference between omega equal to 10 raise to 2 and omega equal to 10 raise to 3 radiance per second.

So, that calls for some more improvement, let us see what that is. What can be done is to replace the linear X axis with a logarithmic X axis as shown in this figure, this is omega equal to 10 raise to 0, 10 raise to 1, 10 raise to 2 and so on and now we can very clearly easily distinguish between different frequencies, and now it is easy to read the mod of H value for a given frequency for example, 10 raise to 2 it is 1, 10 raise to 3 it is something like may be 7 times 10 raise to minus 1 etcetera. Apart from that when we plot mod of H versus omega in this log log fashion, certain trends become very clearly visible let us what those are. At low frequencies we find that mod of H becomes constant equal to 10 raise to 0 or 1, and at higher frequencies we see that mod of H falls as omega increases and in fact, these 2 regions the higher frequency part and the low frequency part meet at omega equal to omega 0.

So, that indicates the boundary between these 2 regions, what is omega 0? 1 over RC, R is 1 k, c is 1 microns. So, RC is 1 mille. So, omega 0 is 1 over 1 mille, which is 10 raise to 3 radiance per second. So that is what omega 0 is. Apart from that we can also make out the dependence of mod of H on omega in this higher frequency region for example, let us say we have increased omega from 10 raise to 4 to 10 raise to 5 radiance per second, at 10 raise to 4 where here, 10 raise to 5 where here; what does happen to mod of H? Mod of H was 10 raise to minus 1, it is now gone down to 10 raise to minus 2. So, we have increase the frequency by a factor of 10, and mod of H has decreased by the same factor 10 raise to minus 1 to 10 raise to minus 2.

So therefore, we can conclude that mod of H is varying as 1 over omega in this region here. So, this log log plot is very useful and is indeed very commonly used in practice. To summarize since omega and mod H vary by several orders of magnitude each, a linear omega axis or a linear or H axis is not appropriate and therefore, what is done is log of mod H is plotted against log omega. Now what we have done here essentially is the same as plotting log mod H versus log omega, we are used axis here which are logarithmic, but instead of you could actually calculate log of mod H and log omega for each data point, and then plot those on a linear scale that amounts to the same thing as what we have done here.

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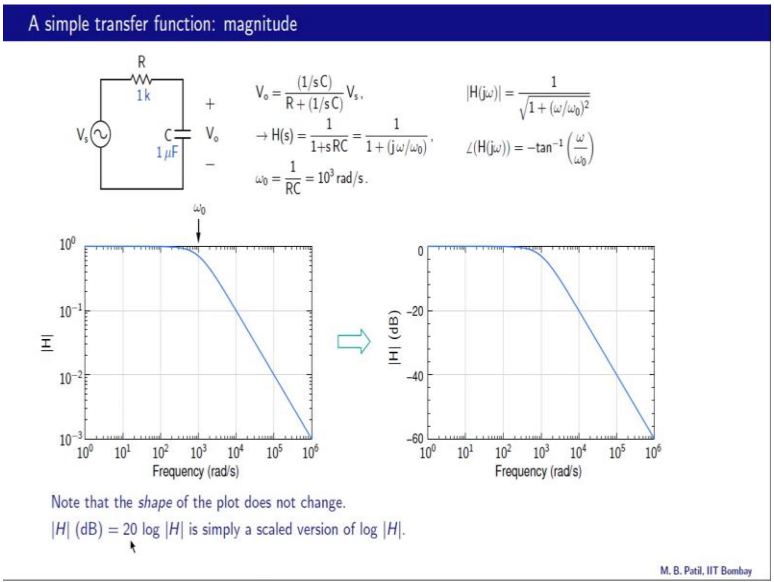

Let us now see what happens if instead of plotting mod H on your log scale like me did before, we plot mod H in dB on a linear scale.

This is a linear scale 0 minus 20, minus 40, minus 60 etcetera and what we see is that the shape of the plot does not change, this shape is identical to that shape and of course, is a very good reason for that, that is mod H in dB is equal to 20 log of mod H, and that is simply scale version of log of mod H scale by this factor of 20. So, there is therefore, one to one correspondence between these 2 graphs for example, mod H equal to 10 raise to minus 1 here, what does it correspond to? 20 times log of 10 raise to minus 1, which is 20 times minus 1 which is minus 20.

So, this point is in fact, over here and so on. So therefore, the shapes or the same, what does changed is only this access here this is a very important point and in practice we might see plots of this kind or of this kind and we should immediately recognize that there in fact identical.

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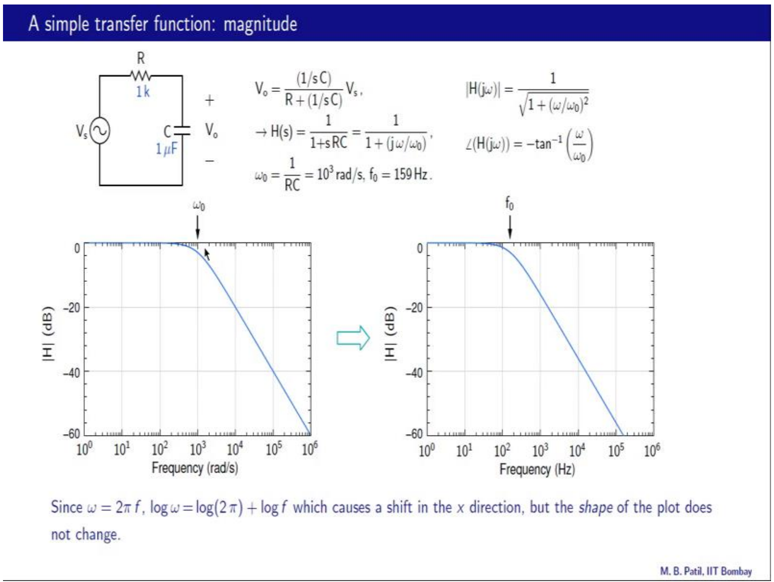

Let us now look at what happens if we plot mod H as a function of frequency in hertz, rather than frequency in radiance per second has been need earlier. In particular we have seen that this omega 0 serves as the boundary between the low frequency part of mod H and the higher frequency part of mod H and so let us focus on this boundary as we change the access from radiance per second 2 hertz. And let say we keep these number the same that is omega is going from 10 raise to 0, to 10 raise to 6 in this plot and the frequency in hertz is going from 10 raise to 0 to 10 raise to 6 here in this plot.

So, what is the frequency that corresponds to omega 0, which is equal to 10 raise to 3 radiance per second that is f 0 and its given by omega 0 by 2 pi, which is about 159 hertz let say 160 hertz. So, what will happen as a result is this omega 0 point will now shift to 160 where is 160 this is 10 this is 100. 160 is somewhere between these 2 that is where it is and this happens to every other frequency point and therefore, the entire mod H versus omega called shifts in this case left what. So, here is the summary since omega is 2 pi times f, log omega is log of 2 pi plus log of f and because of this relationship this is just to constant, changing from radiance per second 2 hertz only causes say shift in the X direction, but the shape of the plot does not change.

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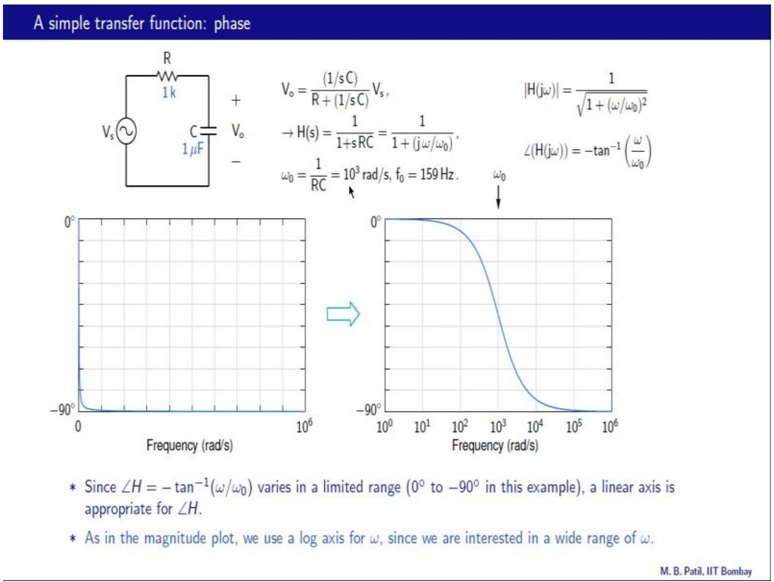

Another very important point to remember; let us now talk about the phase of H as a function of omega, and if you recall this is the relationship, angle of which for the phase of H is minus tan inverse omega by omega 0; and omega 0 for our example is 10 raise to 3 radiance per second. Now the left plot here shows angle of which versus frequency with linear axis and as we saw in the magnitude case, this plot of also not very useful because the low frequency is have got highly compressed here, and we really cannot tell the difference between 10 raise to 2 radiance per second, and 10 raise to 3 radiance per second for example. And to solve the problem what we can do is to use algorithmic frequency axis, and that is what we have done here and that does the trick we can now clearly see the behavior of the angle H as a function of frequency.

And in this case there is no real need to change the y axis or the angle axis to a logarithmic axis, because the angle is only can to change from minus 180 degrees to plus 180 degrees, and it is now going to change by orders of magnitude like in the magnitude case. And as before when we plot the frequency on a log axis, we can clearly see certain trends as well for example, at omega equal to omega 0, which is this point here the phase is minus 45 degrees, and that is something that we have expect from here minus tan inverse omega equal to omega 0. So, this is 1, and that is minus tan inverse of 1 which is minus pi by 4 or minus 45 degrees.

Also as omega become small, this quantity becomes 0 degrees that is this part, as omega becomes large minus tan inverse of infinity approaches minus pi by 2 or minus 90 degrees that is this part. So, from this angle versus log omega plot, we can get a lot of information. To summarize since angle H varies in a limited range in this case from 0 degree is to minus 90 degrees, linear axis is appropriate for the angle and as in the magnitude plot we use a log axis for omega, since we are interested in wide range of omega, in this case from 10 raise to 0 to 10 raise to 6.

To summarize we derived the transfer function H of g omega what a simple circuit, within looked at the variation of the magnitude of H and the phase of H versus frequency plotted in different ways. We concluded that a log log plot for the magnitude and linear log plot for the phase or appropriate. In the next class we will see how to obtain bode plots for a given H of g omega. So, see you next time.