Basic Electronics

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Lecture - 04
Useful circuit techniques (continued)

Welcome back to Basic Electronics. In this lecture we will continue with Thevenin's theorem, we will consider a circuit with a dependent source and learn how is thevenin equivalent circuit can be obtained. We will then see how the thevenin form of a circuit can be converted into another equivalent form called the Norton form we will illustrate how this thevenin to Norton transformation can be used in circuit analysis. At the end of the lecture we will look at the maximum power transfer theorem for circuits. Let us get started.

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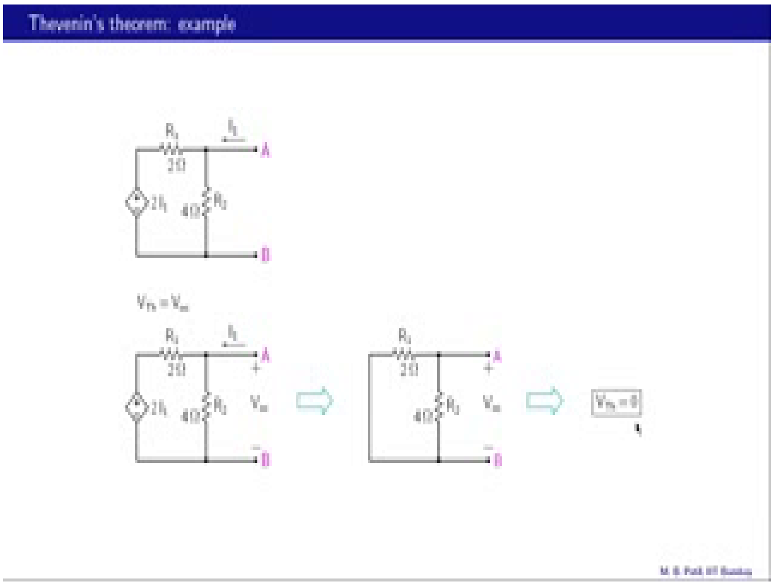

Here is another example it has a dependent source, but no independent current or voltage sources and let us obtains the thevenin equivalent for this circuit as seen from A B. What kind of dependent source is this? It is a voltage source judging from this symbol and the voltage difference between these 2 nodes is 2 times I 1, where I 1 is this current. So, this voltage is controlled by a current and therefore, it is a current controlled voltage source. Let us find the thevenin voltage first which is the same as V o c as shown here, in this situation this current I 1 is 0 and therefore, this voltage drop which is given by 2 times I 1 is also 0.

So, therefore, we can replace this c c b s with a short circuit like that and now we find that V o c is going to be 0 volts and therefore, our V T S the thevenin voltage is going to be 0 volts.

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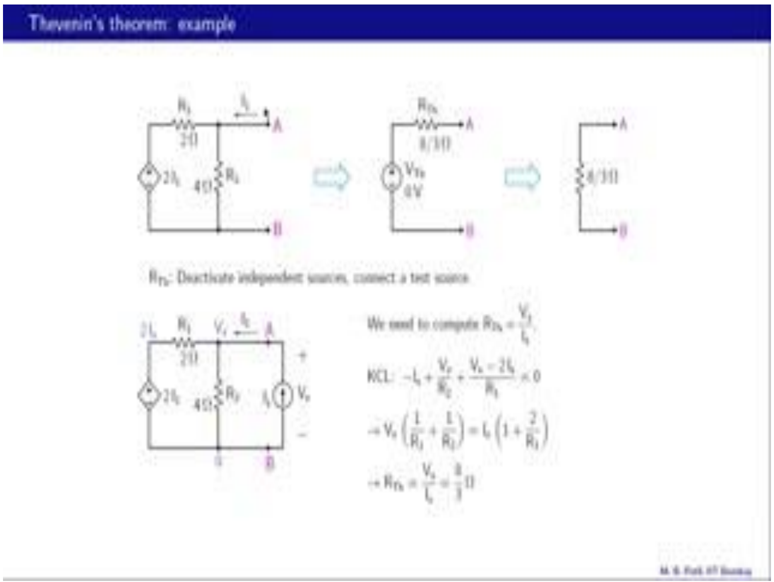

Let us now find the thevenin resistance, and to do that we deactivate the independent sources in the original circuit in this case of course, there are no independent sources. So, nothing needs to be done and then we look from A B. In this case because we have this dependent source, we cannot really figure out what the equivalent resistance as seen from A B is and therefore, let us use another method that is connect test source and then find the ratio of this voltage and that current that gives us R T S.

So, we have connected a test current source here. So, what we will do is find the ratio V s divided by I s. In this case I s and I 1 happen to be the same where do we begin; let us take one of the nodes as the reference node say this one, if that is 0 volts then this voltage is 2 times I 1 and I 1 is the same as I s, so therefore this voltage is 2 times I s. What about this node voltage? That is the same as V s. So, now, we have all the node voltages and we can now write a KCL equation to obtain V s divided by I s and that is our KCL equation at node A you have 3 currents here I s this current and this current.

Now, I s is entering this node and therefore, we will take that as minus I s; this current V s minus 0 divided by R 2 is leaving the node therefore, we take that as a positive quantity plus V s by R 2, what about this current? It is V s minus 2 I s divided by R 1. So, all these 3 currents add up to 0 and now we can get V s divided by I s. So, after simplifying this equation we obtain R T S as V s divided by I s or 8 over 3 ohms. So, our thevenin equivalent circuit has V T S equal to 0 as we saw in the previous slide and R T S equal to 8 by 3 ohms. So, that is what it looks like and since V T S is 0 volts we may as well replace that with a short circuit, and therefore all we have between A and B is this resistance R T S equal to 8 by 3 ohms. So, this circuit is equivalent to this original circuit

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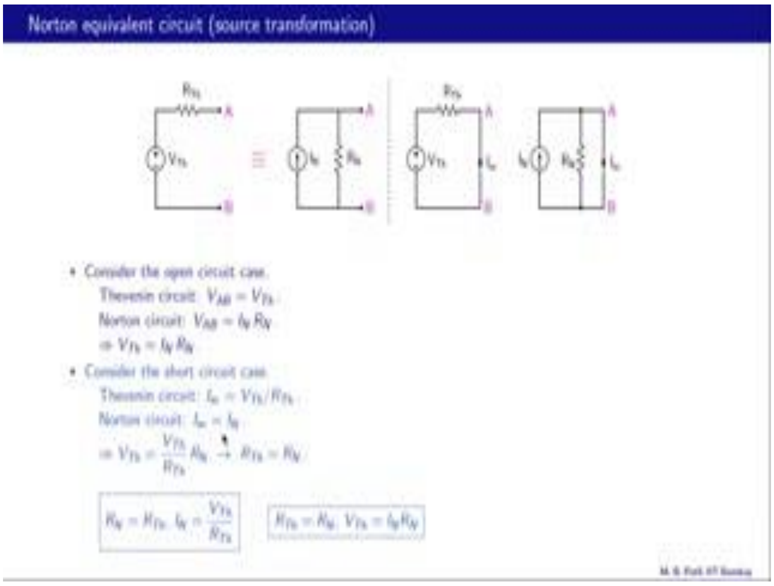

We have seen so far that a linear circuit can be represented by it is thevenin equivalent circuit, consisting of a voltage source V T S in series with a resistance called R T S or thevenin resistance. Now equivalently we can represent the same circuit with the Norton equivalent circuit shown over here, which consists of a current source I n in parallel with a resistance R Norton or R n. For these 2 circuits to be equivalent to each other, we must have some relationship between V T S, R T S and I n, R n and let us now see what those are.

Let us consider the open circuit case; for the thevenin circuit the open circuit voltage between A and B is simply equal to V T S because this voltage drop is 0. So, for the thevenin circuit V A B is V T S. For the Norton circuit this voltage drop between A and

B is I n times R n and that is what this equation says over here. Now for these 2 circuits to be equivalent, we require that this voltage must be the same as this voltage and that gives us the condition that V T S must be equal to I n R n like that.

Next let us consider the short circuit case as shown here on the right, what we do here is connect A and B with a wire and look at this short circuit current called I s c, and we do that for the thevenin equivalent circuit as well as for the Norton equivalent circuit. For the Thevenin circuit I s c is simply V T S divided by R T S this equation here; further Norton circuit the voltage drop between A and B is 0 and therefore, low current flows through R n, and therefore I n must flow like that and that gives us I s c equal to I n; and because the 2 circuits are equivalent, these 2 short circuit currents must be equal and that gives us the second relationship between V T S, R T S, I n and R n.

Let us now use this first relationship and substitute for I n, V T S divided by R T S because these 2 short circuit currents are equal and then we get V T S equal to I n which is V T S by R T S like that times R n. This V T S cancels out and we get R T S equal to R n. So, what this means is if we know the thevenin equivalent circuit, we can obtain the Norton equivalent circuit using these relationships here. R n and R T S are equal, and I n is equal to V T S divided by R T S. So, we can go from this circuit to that circuit. Similarly we can go from the northern circuit to the thevenin circuit, because R T S is equal to R n and V T S is equal to I n times R n which follows from the equality of these 2 short circuit currents.

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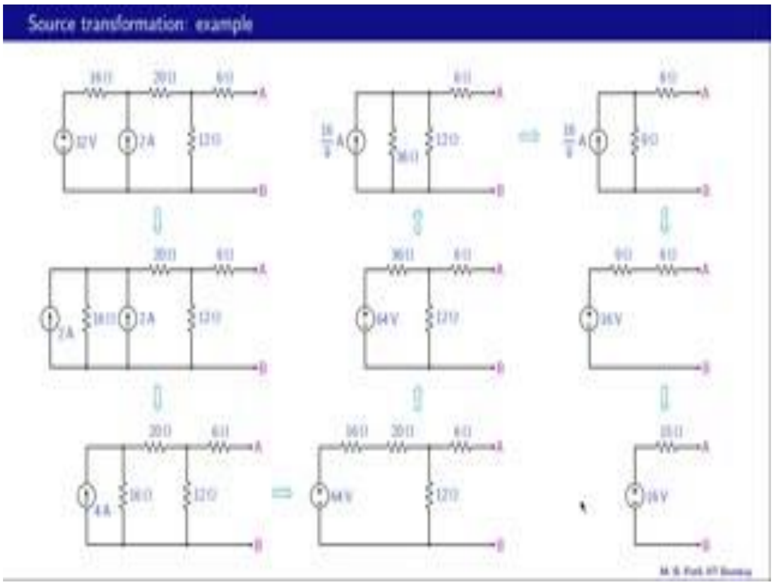

Let us consider this problem in which source transformation will be very useful, that is transformation from a thevenin equivalent to the Norton equivalent and vice versa. So, what is the problem here? We want to find the thevenin equivalent circuit as seen from A B. As step number 1 what we will do is to convert this thevenin form consisting of these 32 volts in series with 16 ohms, into which Norton equivalent, what is over I n? I n is equal to V T S divided by R T S or 32 divided by 16 that is 2 amperes; what about R n? R n is the same as R T S that is 16 ohms.

So, with that transformation we get this equivalent circuit here, this is our Norton equivalent circuit I n is 2 amperes, R n is 16 ohms and the rest of the circuit all of this has been left untouched. Now these 2 current sources, these 2 amperes and these 2 amperes are actually in parallel, they share the same nodes and therefore, they can be combined into one single source or for amperes as shown over here. So, we are 4 amperes in parallel with 16 ohms, this resistance and then we have the rest of the circuit here like that.

Next let us convert this Norton form consisting of this 4 amps current source with 16 ohms in parallel, into a thevenin equivalent; what is V T S? It is I n times R n that is 4 times 16 or 64 volts; what about R T S? R T S is the same as R n that is 16 ohms. So, with that transformation we get this circuit here 64 volts in series with 16 ohms and then we have the rest of the circuit that one.

Now, these 2 resistances R n series and that give us 16 plus 20 that is 36 ohms like that. Next we can convert this thevenin form into it is Norton equivalent as given by this circuit here, what is I n? It is 64 divided by 36 that turns out to be 16 by 9 amperes, and R n is the same as R T S that is 36 ohms. Now these 36 ohms is in parallel with these 12 ohms and that gives us 9 ohms as shown here and now we can convert this Norton form into it is thevenin equivalent, what is V T S? It is I n times R n, so that is 16 by 9 times 9 or 16 volts; what about R T S? That is the same as R n that is 9 ohms.

So, now we get this circuit here and now these 2 can be combined to give us 15 ohms; so that is our final thevenin equivalent circuit as seen from A B. Before leaving this example let us make a couple of important points, one notice that our numbers having rather friendly, as an example take this 36 ohms in parallel with 12 ohms. Now this is a calculation which we can very easily do on paper without using a calculator. In real life of course, the numbers may not be as nice for example, this could be 2.7 k, this could be 2.2 k and in that case we will end up using our calculator. Our second point has to do with the method that we used, in our calculation we used source transformations either from thevenin to Norton or from Norton to thevenin, to go from this original circuit all the way to it is final thevenin equivalent circuit this one.

But that is not the only way to do that. In fact, there may be other ways some of them may even be simpler for example, if we want R T S, you can get that directly from the original circuit; what do we need to do? We deactivate the independent sources; that means, we short this voltage source and open this current source and when we do that, these 16 ohms comes in series with this 20 ohms. So, that gives us 36 ohms; that 36 ohms comes in parallel with 12 ohms and that entire combination comes in series with 6 ohms.

So, the R T S calculation is fairly straightforward and similarly for V T S we do not need to go all the way using source transformations, we can stop somewhere in between for example, when we come to this stage, all we need to do is to find the open circuit voltage between A and B and that is very straightforward. Since there is no current in this 6 ohms resistance, V o c is the same as the voltage drop across this 12 ohm resistance and that can be obtained simply by voltage divisions, that would be 12 ohms divided by 12 plus 36 times 64 and that will give us this answer.

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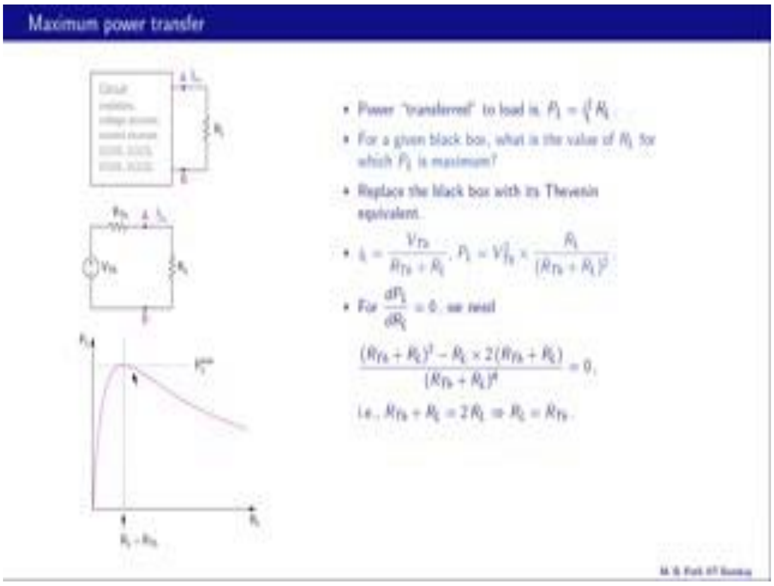

Let us now talk about maximum power transfer which is a very useful concept in electronic circuits; here is a linear circuit consisting of resistors independent voltage sources and we will consider DC voltage sources here, independent DC current sources and dependent sources such as current controlled voltage source, current controlled current source etcetera. To this circuit we connect load resistance R L, now this load resistance is going to draw a current and that we will denote with I l. This load resistance is going to absorb some power and that power is given by P L equal to I L squared times R L.

In other words there is a transfer of power from this circuit to the load resistance and the question that we asked now is for a given circuit, or for this given black box, what is the value of R L for which P L is maximum; to answer that question let us replace the original circuit with this thevenin equivalent; and when we do that we get this circuit here. So, this entire original circuit has been replaced with this thevenin equivalent circuit and now it is straightforward to find I L. I l it simply V T S divided by R T S plus R L like that and once we know I L, you can find P L as I L squared times R L and that gives us this expression over here.

To find the value of R L for which P L is maximum; all we need to do now is to differentiate P L with respect to R L and put that equal to 0. That is d P L d R L should be equal to 0 and when we do that we get this following equation here and notice that this V T S squared is a constant, which is independent of R L and therefore, it does not figure in this equation here.

Now, when we simplify this further we get R T S plus R L equal to 2 R L you should go through this algebra of course, and finally, we get R L equal to R T S. So, if you have a circuit which is linear, we can find it is R T S and then if we have a load resistance which is equal to that R T S, then maximum power transfer will take place from the original circuit to R L. Let us look at a plot of P L as a function of R L, this is where R L is equal to R T S and the function goes through a maximum.

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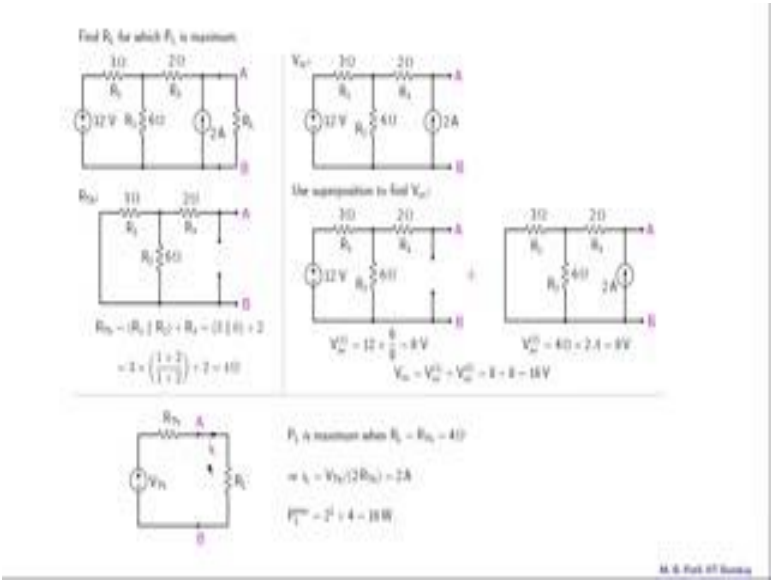

As an example let us consider this circuit here, we have a load resistance between A and B and we want to find it is value for which P L the power transfer to R L is maximum. So, step number 1 is to find the thevenin equivalent circuit for this circuit here and how do we go about doing that? Let us look at R T S first, R T S can be found by deactivating the independent sources; that means, we shot this 12 volt source here and open this 2 amperes source and then we end up with this circuit over here.

Now, R T S is simply the resistance as seen from A B and what do we see? We see that R 1 and R 2 are in parallel and that combination is in series with R 3. So, our R T S is R 1 parallel R 2 plus R 3 that is 3 parallel 6, 3 parallel 6 plus 2 ohms, 2 ohms here and that comes to 4 ohms.

Now, let us proceed with the calculation of V T S; what is V T S? V T S is the same as V o c; that means; we remove R L and find the open circuit voltage between A and B. So, let us do that now. So, this is our circuit for b t h calculation, and since there are 2 independent sources here we can use superposition it turns out to be convenient. So, case 1 we have the 12 volts source as it is and 2 amps deactivated; that means, replaced with an open circuit. Case 2 we have the 2 amp source as it is, and the 12 volt source deactivated; that means, shorted. In the first case B A B is the same as the voltage across R 2, because the current here is 0 and so there is no voltage drop across R 3 and V o c is then given by voltage division that is R 2 divided by R 1 plus R 2 times 12 volts and that turns out to be 8 volts.

In the second case we can first find this equivalent resistance and that. In fact, is the same as the resistance we found here that is 4 ohms and then the open circuit voltage is 2 amperes flowing through 4 ohms that is 2 times 4 or 8 volts like that; and the net open circuit voltage the thevenin voltage that we are looking for is V o c equal to V o c in case 1, plus V o c in case 2 that is 8 plus 8 or 16 volts. So, this is our thevenin equivalent circuit, and we can replace the original circuit as seen by R L with this much simpler thevenin circuit with V T S equal to 16 volts and R T S equal to 4 ohms.

Now, we have seen earlier that the power transfer to R L is maximum, when R L is equal to R T S; that means R L should be 4 ohms. So, we choose R L equal to 4 ohms then the power transferred from this original circuit to R L is maximum. It is now a simple matter to calculate this maximum power, let us find I L first what is I L? It is V T S divided by R T S plus R L and since R L is equal to R T S, this I L is given by V T S divided by 2 times R T S like that and that turns out to be 2 amperes. Once we know I L we can find the maximum power as I L squared that is 2 squared times R L that is 4 ohms. So, that comes to 16 watts.

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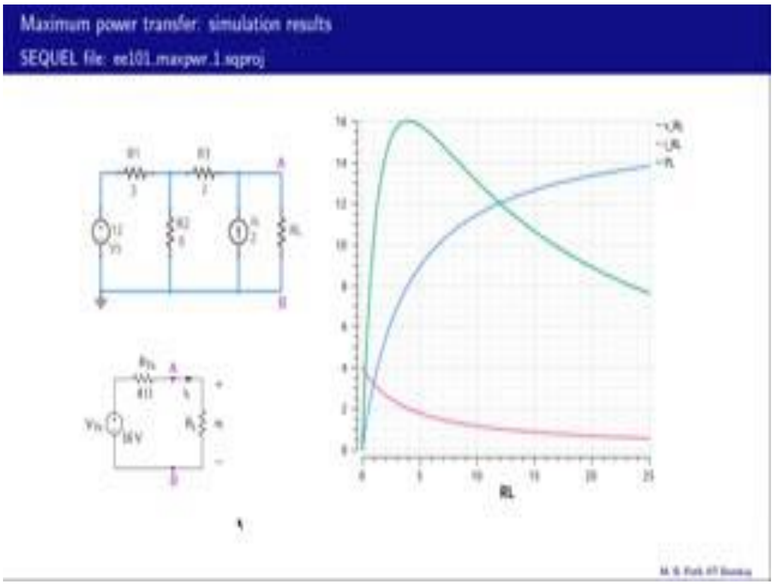

Let us now look at the simulation results for the example we just discussed, and the circuit file is given over here. So, you can also try out the simulation and look at the results. Here is our circuit and what we are going to do is, plot the power absorbed by R L that is the power transferred by this circuit to R L as a function of R L we will also plot the current through R L and the voltage across R L as a function of R L.

First let us look at I R L that is this current as a function of R L and that is given by this red curve over here; when R L is 0 what is the current through R L, we can look at this circuit and find that out. When R L 0 the current reach 16 volts divided by 4 ohms, that is 4 amperes and that is what is seen over here. As R L is increased the resistance in the circuit increases and therefore, the current goes down like that eventually of course, it will reach 0.

Next let us look at this voltage across R L denoted by V L over here as a function of R L, and that is given by this blue graph here. What is V L? It is R L divided by R L plus R T S times 16 volts, if R L is 0 V L is going to be 0 and that is our starting point here. As R L is increased V L also increases and eventually if R L becomes very large, then this entire voltage is going to appear over here. So, this blue graph is going to approach 16 volts as R L tends to infinity.

Now, the power transfer to R L is given by the product of V R L and I R L. V R L increases as R L increases whereas, I R L decreases as R L increases. So, clearly the product of these 2 is going to go through a maximum at some point and that is what is happening over here. And that happens at R L equal to 4 ohms and as we have seen in the last slide that condition is the same as R L equal to R T S that is 4 ohms. And the maximum power is 16 watts, and that agrees with the calculation that we looked at in the last slide.

To summarize we have learnt how to convert the thevenin form into the Norton form and vice versa. We have also seen the maximum power transfer theorem for circuits, and we have seen that maximum power is transferred when the load resistance is equal to the Thevenin resistance.

See you next time.