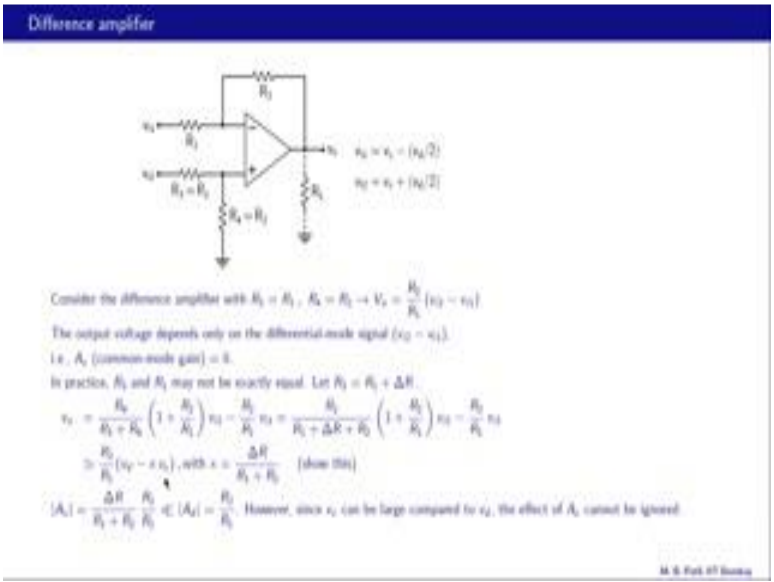
Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 37
Instrumentation amplifier

Welcome to Basic Electronics. In the last class we are seen how difference amplifier works, we have also seen that is input resistance is relatively small and why that is not desirable. In this class we will continue with the difference amplifier and consider the effect of resistance mismatch on this common mode rejection performance. We will first consider a simple case in which only one resistance value deviates from its nominal value, after that we will consider a more general case where all resistances are allowed to very to the extent given by the tolerance specified by the manufacturer. Let us start.

(Refer Slide Time: 01:05)



Let us consider this difference amplifier with R 3 equal to R 1, and R 4 equal to R 2. As we have seen before V o is R 2 by R 1 times V i 2 minus V i 1. So, in this formula the output voltage depends only on the differential mode input signal; that is V i 2 minus V i 1. And in other words the common mode gain is exactly 0, there is no contribution of the common mode input signal here. That is the ideal world, in reality what happens is these resistance values are not exact; for example if you want R 1 equal to 1 k it would not be exactly 1 k. If you use 1 percent resistances then R 1 would be anything between 0.99 k to 1.01 k.

So, in real life these resistance values are not exactly what we intend them to be, and therefore R 3 and R 1 may not be exactly equal. So, let us take this simple case where R 4 is exactly equal to R 2, but R 3 is not exactly equal to R 1; and let R 3 be R 1 plus some small delta R. In this case our v o also will get modified, and let us see what that is. If you recall our final expression came from this expression here and now for R 4 we can use R 2 and for R 3 we can use R 1 plus delta R and then we get this expression here. And now we do some algebra which of course you need to do and arrive at this final result.

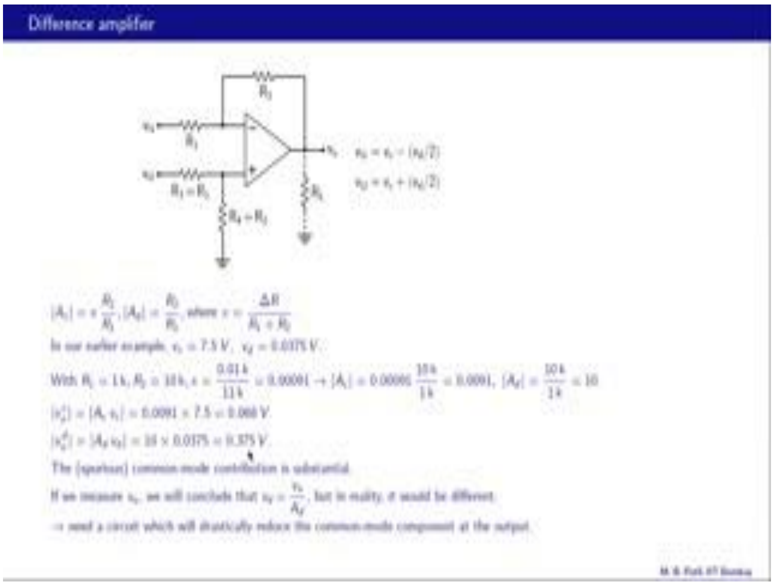
What we do in short is take R 1 plus R 2 common here from the denominator and then we end up with 1 plus x in the denominator; where x is delta R over R 1 plus R 2. Then we use the approximation that one over 1 plus x is approximately 1 minus x. And finally, we use these definitions. After all that algebra we end up with this expression here for v o; R 2 by R 1 v d is the differential input mode signal minus x times v c, v c is the common mode input signal.

Now, this part is the same as before R 2 by R 1 times v d, R 2 by R 1 times v d but now we have added this common mode part and therefore the common mode gain is not 0 anymore. From this result how do we get the differential mode gain and the common mode gain? That is easy. For differential mode gain we put the common mode input equal to 0 and then the output voltage divided by v d gives us the differential mode gain that is A sub d and that in this case is clearly R 2 over R 1.

Similarly, to get the common mode gain what we do is we put the input differential mode voltage to be 0 that is v d equal to 0 and then take the ratio of v o and v c, and then we will get R 2 by R 1 times minus x. So, that is our common mode gain. Now the minus sign is not so important, so we will not only talk about magnitudes. So, the magnitude of A c the common mode gain is R 2 by R 1 times x; x is delta R divided by R 1 plus R 2. This of course is a small number because delta R is small compared to R 1 or R 2 and A d the differential mode gain is R 2 by R 1 and that of course is the same as before.

So, clearly A c is much smaller than A d because we have this small pre multiplier here and that is good news of course, but the problem is the common mode voltage can be much larger than the differential mode input voltage. And we have already seen an example of that in our bridge circuit. If you recall v c was 7.5 volts there and v d was 37.5 millivolts. In that case clearly v c was much larger compared to v d, and therefore the effect of A c may not be small over all, because A c gets multiplied by v c and v c is large.

(Refer Slide Time: 06:28)



Here is the summary: the common mode gain is x times R 2 by R 1 where x is delta R divided by R 1 plus R 2. And remember this delta R came from R 3 which was assume to be R 1 plus delta R. The differential mode gain is R 2 by R 1 as we already seen. And now let us take a numerical example: we will take the same common mode input voltage and differential mode voltage as the bridge circuit example. So, our common mode voltage is 7.5 volts and the differential mode voltage is 0.0375 volts or 37.5 millivolts.

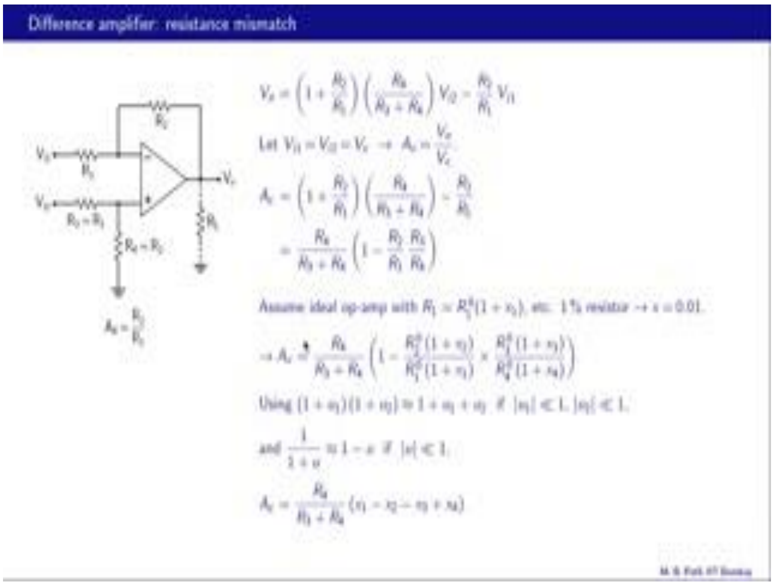
So, these serve as the input to our circuit V i 1 and V i 2, and they are coming from A circuit like the bridge circuit we have seen earlier. Let us take R 1 as 1 k and R 2 as 10 k. So, the gain of our difference amplifier is R 2 by R 1, so it would be 10 k by 1 k that is 10. What is x? X is delta R divided by R 1 plus R 2. Delta R is let us say 1 percent of R 1 which is 0.01 k and R 1 plus R 2 is 1 k plus 10 k or 11 k. So, this x turns out to be 0.00091.

So, the common mode gain A c is x times R 2 by R 1 there, so that multiplied by 10 k by 1 k or 0.0091. A d the differential mode gain is R 2 by R 1 or 10. Clearly this A c is very small compared to A d. Now let us look at the actual output voltage the common mode part of the output voltage and the differential mode part of the output voltage. Here is the common mode part which is A c times v c and that is 0.0091 times 7.5 or 0.068 volts; 68 millivolts.

Let us look at v o d now the differential mode part of the output voltage; that is A d times v d so that is 10 times 0.0375 that is our v d or 0.357 volts. So, we see that this spurious common mode contribution is substantial. So, this is like 0.07 something like one-fifth of the differential mode output voltage, so it can give raise to errors. And if we measure v o let us say we get v o as 100 millivolts, what do we conclude? We will conclude that you are our v d is actually v o by A d or 100 millivolts by 10 or 10 millivolts.

In reality that would not be quite correct because the output also as this contribution from the common mode voltage. And therefore, clearly there will be an error which we definitely do not want. So, in conclusion we need A circuit which will drastically reduce the common mode component at the output.

(Refer Slide Time: 10:32)



Let us now consider the general case in which all of these resistance values are allowed to be inaccurate. We will continue to take R 3 to be nominally equal to R 1 and R 4 to be nominally equal to R 2.

So, let us begin again with our expression for v o and see what happens if we allow all of these resistance values to be inaccurate. In particular our interest is to get the common mode gain. So, let V i 1 be equal to V i 2 equal to v c the common mode voltage. In other words we are removing the effect of the differential mode input voltage on v o. With this condition if we find v o then our common mode gain will simply be that v o divided by v c. And if we substitute V i 1 equal to V i 2 equal to v c here then v o by v c which is equal to a of c the common mode gain turns out to be simply 1 plus R 2 over R 1 times R 4 by R 3 plus R 4 minus R 2 by R 1.

Let us rewrite this expression as R 4 by R 3 plus R 4 times this bracket, and of course you should verify that these two are actually the same. So far we have not really return these resistances in terms of the inaccuracies or the turbulence values and we will do that now. Also we will assume that the op-amp itself is perfect in the sense it as got same R r of infinite. So therefore, the op-amp does not have any effect on the common mode gain.

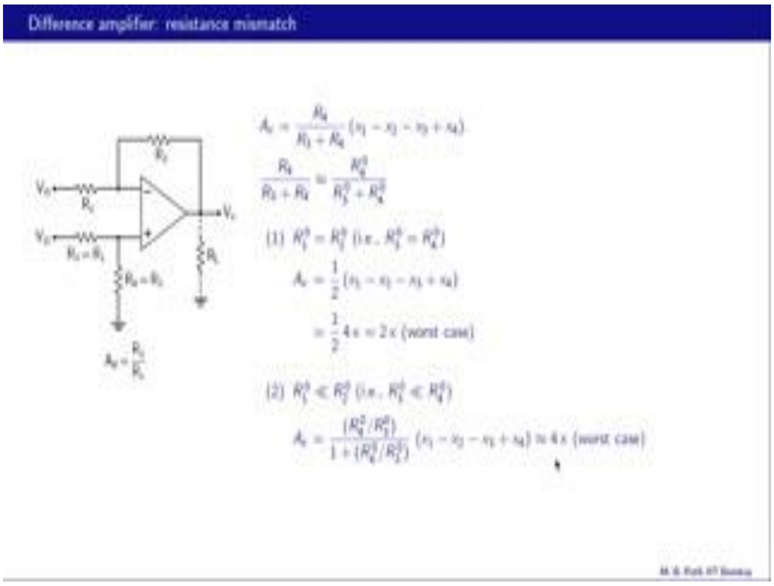
So, how do we incorporate the effect of the tolerance values in the resistances? Using this equation, for example R 1 we can write as the nominal value of R 1 which we will denote by R 1 0 times 1 plus x 1. And this x represents the effect of the tolerance value. For example, if we are using 1 percent resistors then this x 1 can be anywhere between minus 0.01 to plus 0.01. And we do not explicitly write this plus minus sign here because that is understood.

So, with this substitution now our original expression for A c looks like this and note that we have not really changed this pre factor it is the same as before, and we will commit on that later. But in the bracket we have replaced R 2 with R 2 0 times 1 plus x 2 R 3 with R 3 0 times 1 plus x 3 and so on. And the our job now is to simplify this expression and arrive at A c in terms of x 1, x 2, x 3, x 4 and in doing that we will find some approximation very useful.

For example if u 1 and u 2 are small compared to 1, then we can write 1 plus u 1 times 1 plus u 2 as 1 plus u 1 plus u 2. And we can do that because this term u 1 times u 2 is much smaller compared to any of these terms. Similarly one over 1 plus u is approximately 1 minus u if u is small compared to 1, and this is something we have already used earlier if we recall.

So, when we go through all of this algebra we end up with the following relationship A c equal to R 4 by R 3 plus R 4, this factor which is the same as this factor here times x 1 minus x 2 minus x 3 plus x 4. So, this entire bracket reduces to x 1 minus x 2 minus x 3 plus x 4 and it is easy to see why we do not have R 2 0 and R 1 0 etcetera here, because R 2 0 by R 1 0 is exactly the same as R 4 0 by R 3 0. So, all these it is cancels out.

(Refer Slide Time: 15:29)



It turns out that if we are interested in first order terms in x 1, x 2, x 3, x 4 then we do not need to take into account the exact value of R 4 by R 3 plus R 4, we can substitute R 4 0 by R 3 0 plus R 4 0 here without changing the first order expression for A c. So, we will do that. And what we do now is take two special cases just to get an idea of the approximate range of A c.

Case 1: R 1 0 and R 2 0 are equal, so these two resistance values are equal; these two resistance values are also equal. So, all four resistors are the same the nominal values are the same. What do we get in this case? In this case we get A c equal to this is R 4 by R 4 plus R 4 because all resistance is are equal, so that is half and then we get x 1 minus x 2 minus x 3 plus x 4. Now in these circuits we are always interested in the worst case value of A c; that means, the largest possible magnitude for A c.

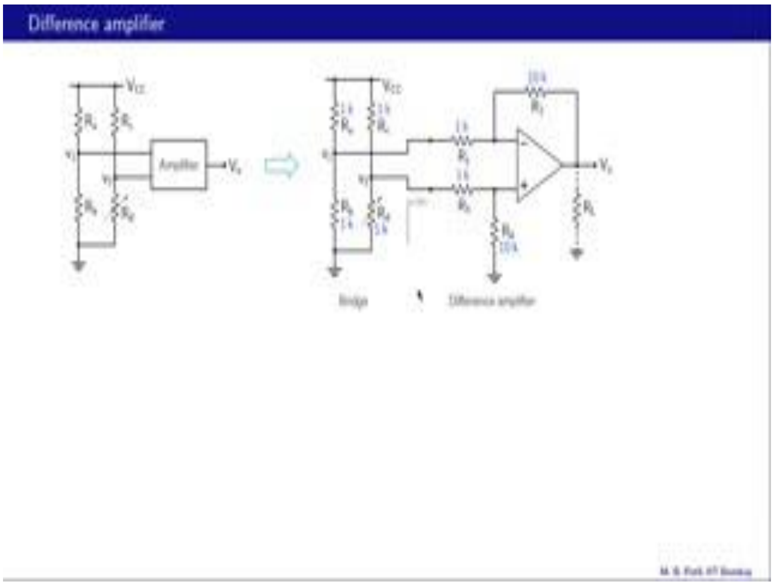
And when will the largest possible magnitude of A c happen? It will happen for example, if x 2 is negative, x 3 is negative, x 1 is positive and x 4 is also positive and all of them have their maximum values in magnitude. So, when we consider that we get half times four x because all of these will then add up and that is 2 x. So for example, if we use 1 percent resistances then this will be 2 times x or 0.2.

Case 2: now in which R 1 0 is much smaller than R 2 0. That means, this ratio R 2 by R 1 is much larger than 1. And this of course, is a more realistic situation because our difference amplifier has a gain of R 2 by R 1 and we are likely to make that R 2 by R 1 large. We continue to use R 3 equal to R 1 and R 4 equal to R 2. And therefore, if R 1 0 is much less than R 2 0 what it means is R 3 0 is also much less than R 4 0. And now let us see what happens to our common mode gain.

The common mode gain the pre factor is R 4 0 by R 3 0 plus R 4 0 we can rewrite that as R 4 0 by R 3 0 divided by 1 plus R 4 0 by R 3 0. Now this number is much larger than 1 because of this condition and therefore this is nearly equal to 1. And once again this bracket as a maximum value of 4 times x that is the worst case, so therefore in this case the common mode gain would be 4 times x in the worst situation.

So, again if you have 1 percent resistances the common mode gain would be 4 times 0.01 or 0.04. So, we say that the common mode gain is not exactly negligible in these circuits and therefore we need to worry about it.

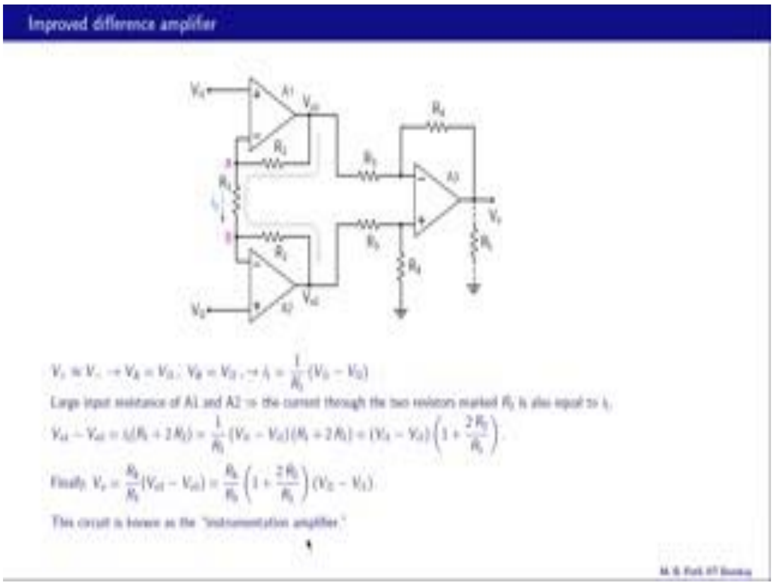
(Refer Slide Time: 19:12)



Before we go further let us do a quick summary. The difference amplifier does give us the desired functionality namely v o equal to k times v 2 minus v 1; that is it amplifies only the difference between the input voltages and that is exactly what we want for situation like this.

But there are two difficulties: one its input resistance is not too large and therefore when we connect the amplifier and the bridge circuit together it is going to disturb these values the input values themselves and that of course is not desirable. Second, its common mode gain is not exactly negligible. What we would like now is to look at a circuit which will address both of these issues namely; the input resistance and CMRR.

(Refer Slide Time: 20:28)



Here is an improved difference amplifier. What we will do is to understand its functionality first, we will check that it is indeed difference amplifier that is it is amplifying only the difference between V i 2 and V i 1. And then we will look at in what way it is an improvement our previous difference amplifier circuit.

So, let us look at the functionality part first, and let us start with this block and we realise that it is already familiar to us it is impact the difference amplifier circuit that we have been discussing. What is the difference? We had R 2 here and R 1 here now we have R 4 here and R 3 here. So, this R 4 and this R 4 is the same, this R 3 and this R 3 is the same. And if you recall the condition for the circuit to work as A difference amplifier was R 2 by R 1 equal to R 4 by R 3.

So, in this case what it requires is R 4 by R 3 is equal to R 4 by R 3 which is of course true. So therefore, this part is simply A difference amplifier and this v o is then given by R 4 by R 3 times v o 2 minus v o 1. And now let us see what this rest of the circuit is doing. Since the op-amps are working in the linear region we can say that v plus and v minus are nearly the same for each of the op-amps and let us see what that implies.

It implies that this voltage v a is the same as V i 1 and also this voltage v b is the same as V i 2. And once we know these v a and v b we can now get i 1 which is v a minus v b divided by R 1, and that is therefore the same as V i 1 minus V i 2 divided by R 1.

Next let us use the fact that these op-amps A 1 and A 2 this have large input resistances and therefore, these currents are 0. And therefore this i 1 must be the same as the current through this R 2 and also this second R 2 here. So, that is the current path that we expect. And once we know this current path we can now find v o 1 minus v o 2. This voltage drop which is i 1 times R 2 plus this voltage drop which is i 1 times R 1 plus this voltage drop which is i 1 times R 2. And we already have i 1 here and we put it all together to get this expression v o 1 minus v o 2 is V i 1 minus V i 2 times this vector here.

Once we know v o 1 minus v o 2 it is easy to get the final output voltage v o, because we have already looked at this and we know the gain of this stage is R 4 by R 3. So, then v o is R 4 by R 3 times v o 2 minus v o 1 and now we substitute for v o 2 minus v o 1 from here and get R 4 by R 3 times 1 plus 2 R 2 by R 1 times V i 2 minus V i 1. So, this circuit is indeed difference amplifier it is amplifying only the difference between V i 2 and V i 1, and it turns out to be a famous and very commonly used circuit it is called the Instrumentation Amplifier.

To conclude we have seen that the difference amplifier needs to be improved both in terms of input resistance and common mode rejection capability. We have started looking at another circuit the instrumentation amplifier. We will continue with this circuit in the next class and see how it provides better performance as compared to the difference amplifier. See you next time.