Basic Electronics
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Lecture – 36
Difference amplifier

Welcome back to Basic Electronics. In this lecture we will discuss a very important issue in electronics namely; common mode and differential mode voltages. We will then look at a related figure of merit for amplifiers namely the common mode rejection ratio or CMRR. We will look at op-amp circuits called the difference amplifier which can be used to amplify the difference between two input voltages V i 1 and V i 2. So, let us begin.

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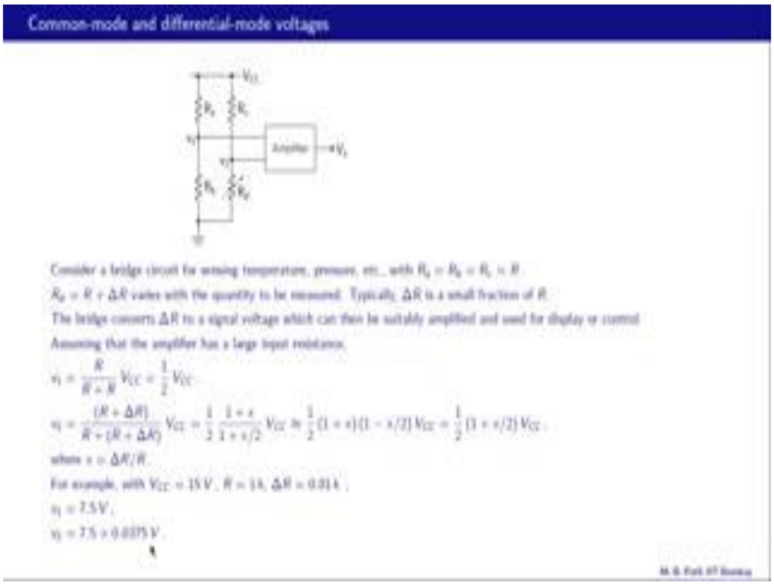

Common-mode and differential-mode voltages

Consider a bridge circuit for sensing temperature, pressure, etc., with $R_a = R_b = R_c = R$.

$R_d = R + \Delta R$ varies with the quantity to be measured. Typically, ΔR is a small fraction of R.

The bridge converts ΔR to a signal voltage which can then be suitably amplified and used for display or control.

Assuming that the amplifier has a large input resistance,

$v_1 = \dfrac{R}{R+R} V_{CC} = \dfrac{1}{2} V_{CC}$,

$v_2 = \dfrac{(R+\Delta R)}{R+(R+\Delta R)} V_{CC} = \dfrac{1}{2}\dfrac{1+x}{1+x/2} V_{CC} \approx \dfrac{1}{2}(1+x)(1-x/2) V_{CC} = \dfrac{1}{2}(1+x/2) V_{CC}$,

where $x = \Delta R/R$.

For example, with $V_{CC} = 15\,V$, $R = 1k$, $\Delta R = 0.01\,k$,

$v_1 = 7.5\,V$,

$v_2 = 7.5 + 0.0375\,V$.

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Let us now begin our discussion about a very important topic in electronics and that is common mode and differential mode voltages or signals. And we will do that with respect to this example: here is a bridge circuits and it is used for sensing things like temperature, pressure, etcetera and we assume that this amplifier has a very large input resistance; that means these two currents are 0.

The bridge circuit is made up a four resistances; R a, R b, R c and R d. Out of these R a, R b and R c are equal constant and that is denoted by R. This resistance R d varies with the quantity being measured such as temperature or pressure. So, essentially this R d is the resistance presented by the transducer that we are using to sense the quantity of interest such as temperature.

So, R d is given by R that is its nominal value which is the same as R a, R b and R c plus delta R, now this delta R varies with the quantity being measured. And typically delta R is small with respect to R such as 1 percent 2 percent and so on. So, delta R is a small fraction of R. Now the whole purpose of this bridge is to convert this delta R into a signal voltage, which can then be suitably amplified and used for display or control purposes as shown in this figure.

So, this v 2 minus v 1 is going to be proportional in some way to the resistance change delta R which is proportional to the quantity being measured. And therefore, the output voltages would be a measure of the temperature or pressure. So, that is the broad idea. Let us now see what the signal is going to be: assuming the amplifier to have a large input resistance as we said earlier v 1 is R divided by R plus R just voltage division times V cc because there is no current going there, so that is half V cc. What about v 2? V 2 is R d divided by R c plus R d times V cc. R d is R plus delta R r plus delta R this is the R c.

So, let us simplify this now knowing the fact that delta R is a small fraction of R. So, we rewrite this expression as this here: half 1 plus x divided by 1 plus x by 2 where x is delta R by R. So, what we have done is we have divided both numerator and denominator by R, and now we want to simplify this further knowing that x is small such as 0.01, 0.02 so on.

So, 1 over 1 plus x by 2 if x is small is 1 minus x by 2; and that is what we have written here. So, this expression is the same as half 1 plus x times 1 minus x by 2 times V cc. And now when we multiply these two terms we get 1 plus x minus x by 2 minus x divide by 2. So, x divide by 2 is much smaller than x if x is small and therefore we can ignore it, and then we end up with half 1 plus x by 2 times V cc; where x is delta R over R.

Let us take an example say our V cc is 15, volts R is 1 k; that means, all of these resistance R a, R b, R c, R 1 k each delta. R is 0.01 k; that means R d is R plus delta R, so 1 k plus 0.01 k that is 1.01 k. And now let us see what v 1 and v 2 are; v 1 is V cc by 2, so that is 15 by 2 or 7.5 volts. v 2 as two terms the first term is V cc by 2, so that is 7.5

again; second term is x by 4 times V cc. And what is x? X is delta R by R, so delta R is 0.01 k R is 1 k so delta R by R is 0.01; x dimensionless of course.

So, the second term then is x by 4 or 0.01 by 4 times 15 so that turns out to be 0.0375 volts or 37.5 millivolts.

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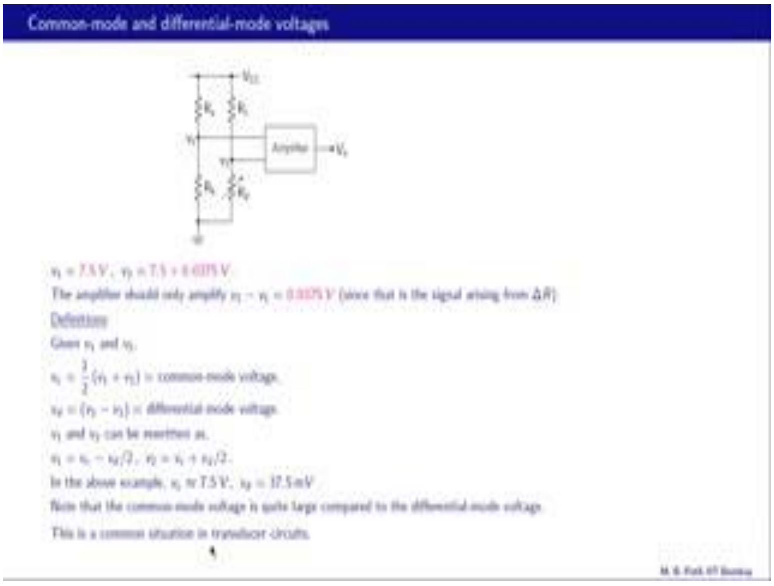

Common-mode and differential-mode voltages

$v_1 = 7.5\,V$, $v_2 = 7.5 + 0.0375\,V$

The amplifier should only amplify $v_2 - v_1 = 0.0375\,V$ (since that is the signal arising from ΔR)

Definitions

Given v_1 and v_2,

$v_c = \frac{1}{2}(v_1 + v_2)$ = common-mode voltage,

$v_d = (v_2 - v_1)$ = differential-mode voltage.

v_1 and v_2 can be rewritten as,

$v_1 = v_c - v_d/2$, $v_2 = v_c + v_d/2$.

In the above example, $v_c \approx 7.5\,V$, $v_d = 37.5\,mV$

Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.

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So, here are the v 1 and v 2 values again; for the example we considered v 1 is 7.5 volts their v 2 is 7.5 plus a small voltage namely 0.0375 volts that is v 2.

Now, the amplifier should only amplify v 2 minus v 1 which is this quantity here a small voltage, because that is the signal arising from delta R. This 7.5 as nothing to do with the signal or the temperature or pressure being measured, and the signal information is embedded only in this term here. So therefore, the amplifier should amplify v 2 minus v 1 and nothing else. So, that is the purpose of this own setup here.

Let us learn some definitions. If we have two voltages v 1 and v 2 the common mode voltage is defined as the average of the two of v 1 plus v 2, and the differential mode voltage is defined as v 2 minus v 1. The common mode voltage is denoted by v sub c and the differential mode voltage by v sub d. We can also do the reverse transformation that is v 1 v c and v d we can obtain v 1 and v 2; v 1 is v c minus v d by 2 v 2 is v c plus v d by 2 and you should verify that these equations are identical to these equations here, the definitions that we considered.

In our example v c is v 1 plus v 2 by 2, so 7.5 plus 7.5 plus this small voltage divided by 2 and that is approximately going to be just 7.5 volts because that is a small number. So, the common mode voltage is 7.5 volts. The differential mode voltage is v 2 minus v 1 so that 7.5 get cancelled and we end up with 37.5 millivolts.

Note that the common mode voltage is quite large 7.5 volts compare to the differential mode voltage 0.0375 volts. And this is in (Refer Time: 09:26) a very common situation in transducer circuits.

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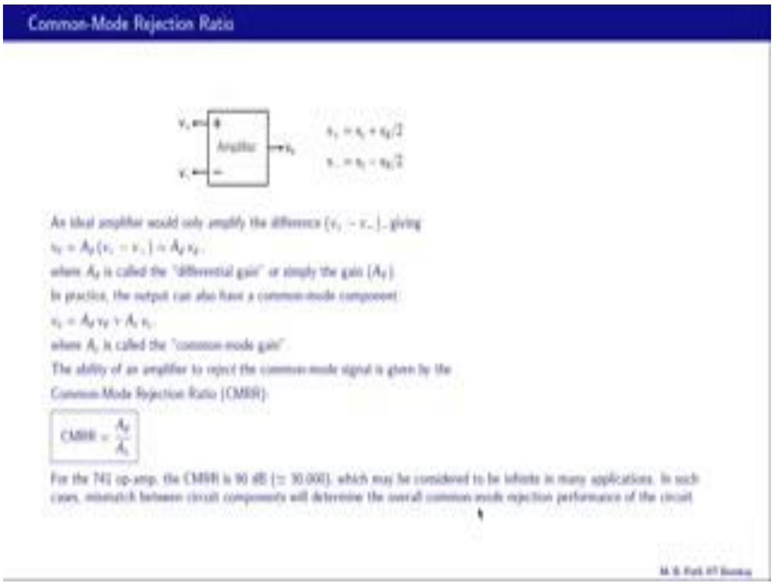

Common-Mode Rejection Ratio

$v_+ = v_c + v_d/2$

$v_- = v_c - v_d/2$

An ideal amplifier would only amplify the difference $(v_+ - v_-)$, giving

$v_o = A_d(v_+ - v_-) = A_d\,v_d$,

where A_d is called the "differential gain" or simply the gain (A_V).

In practice, the output can also have a common-mode component:

$v_o = A_d\,v_d + A_c\,v_c$,

where A_c is called the "common-mode gain".

The ability of an amplifier to reject the common-mode signal is given by the Common-Mode Rejection Ratio (CMRR):

$$\boxed{\text{CMRR} = \frac{A_d}{A_c}}$$

For the 741 op-amp, the CMRR is 90 dB (≈ 30,000), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.

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So, it is clear that we are looking for an amplifier which will only amplify the difference between two voltages; v 1 and v 2. And here is a black box representation of such an amplifier. It has two input terminals plus and minus and the two voltages are denoted by v plus and v minus. Such an ideal amplifier would only amplifier that difference between these two voltages that is v plus minus v minus. Given: the output voltage v o as A d times v plus minus v minus that is A d times v d; where v d is the differential mode voltage.

Now, this A d is a gain, it is a voltage gain and that is called the differential gain of the amplifier. And we have been simply calling it the voltage gain when we talked about the op-amp. Op-amp also has the same configuration, it as a non-inverting input and an inverting input and it also as this relationship when it is working in the linear region. But so far we have only called the gain as simply A v, but strictly speaking we should be calling it the differential gain or A sub d.

In practise the output can also have a common mode component. This component we have already seen that is the differential gain times the differential input voltage. In addition there is this common mode component; this is the common mode gain A sub c and v sub c is the common mode input voltage. So, an amplifier in practise will amplify not only the differential mode input voltage, but also the common mode input voltage, which is of course undesirable like we have seen in the last bridge circuit example. So, A c is called the common mode gain and we would like it to be small ideally 0, because we want to through away the common mode input voltage. So, do not want to have any effect of the common mode input voltage on the output voltage.

And that is where we define figure of merit called the CMRR or common mode rejection ratio. The ability of an amplifier to reject the common mode input signal is given by the common mode rejection ratio, CMRR and that is the ratio of the differential mode gain A d and the common mode gain A c. And of course, ideally we would like this to be infinite because we would like A c to be 0. In practise it may be large, but not quite infinite.

For the 741 op-amp the common mode rejection ratio is 90 dB or 30000 that is a large CMRR. And for all practical purposes we might consider that to be infinite for several applications. And in such cases the circuit as a whole may still have a much smaller CMRR and that is because there could be mismatch between circuit components. For example resistance values; some circuits will require two resistances as to be perfectly matched, but they may not be because of manufacturing limitations. And in such cases the mismatch between circuit components will determine the overall common mode rejection performance of the circuit. And we will look at an example of this situation soon.

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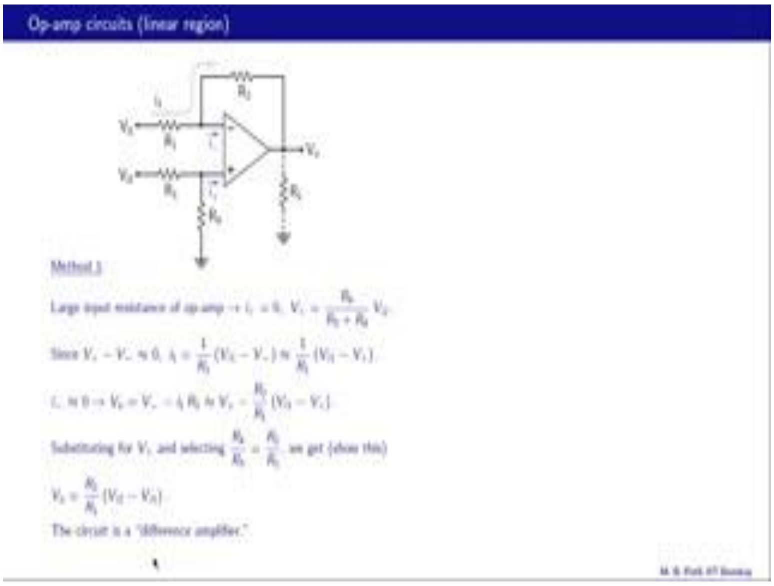

Op-amp circuits (linear region)

Method 1

Large input resistance of op-amp → $i_+ = 0$, $V_+ = \frac{R_4}{R_3 + R_4} V_{i2}$.

Since $V_+ - V_- \approx 0$, $i_1 = \frac{1}{R_1}(V_{i1} - V_-) \approx \frac{1}{R_1}(V_{i1} - V_+)$.

$i_- \approx 0 \rightarrow V_o = V_- - i_1 R_2 \approx V_+ - \frac{R_2}{R_1}(V_{i1} - V_+)$.

Substituting for V_+ and selecting $\frac{R_4}{R_3} = \frac{R_2}{R_1}$, we get (show this)

$V_o = \frac{R_2}{R_1}(V_{i2} - V_{i1})$.

The circuit is a "difference amplifier."

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Here is a circuit which serves as A difference amplifier; that means it amplifies the difference between the two voltages and produces an output which is proportional to that difference. We will look at the operation of this circuit, but before we do that let us ask this question; can we not use the op-amp itself as difference amplifier, after all it does produce an output voltage which is the open loop gain times v plus minus v minus as we have seen earlier as long as the op-amp is operating in the linear region.

So, what is stopping us from using that as a difference amplifier? There are two issues: one as we have already seen the gain of the op-amp is extremely large something like 100000, and that means only very very small voltages can be applied here; a very small differential voltages can be applied at the inputs otherwise the op-amp is going to get given into saturation either positive saturation or negative saturation and we do not want that. That is one, and the second issue as to do with offset voltage of the op-amp and we will discuss that little later.

So, for these two reasons and there could be more, the op-amp itself is not a good practical difference amplifier. Let us now try to analyse this circuit and we will assume that the op-amp is operating in the linear region. There are two ways of doing it: method one the input resistance of the op-amp is very large, so therefore i plus is 0 and therefore we can write v plus using voltage division. So, v plus would be R 4 divided by R 3 plus R 4 times V i 2, like that.

Since the op-amp is operating in the linear region v plus and v minus R nearly equal, and therefore this current i 1 is V i 1 minus v minus divided by R 1 and we can write that as V i 1 minus v plus divided by R 1, because v minus and v plus are nearly equal. Now i minus is 0 that is an op-amp input current, and what does it mean? That means, this i 1 also goes through R 2 and that gives us an expression for v o. So, v o is v minus this i 1 times R 2. So, that is equal to v plus because v minus and v plus are nearly equal minus R 2 times i 1; we already have an expression for i 1 right there. So, we substitute that and get this result.

So, we are now got v o in terms of v plus, the next step is to substitute for v plus this expression and that will give us v o in terms of V i 1 and V i 2. And in doing that if we select R 4 by R 3 to be equal to R 2 by R 1 then we end up with this very simple relationship namely; v o is R 2 by R 1 times V i 2 minus V i 1. So, the circuit is serving as A difference amplifier, it is amplifying only the difference between the input voltages. And we can adjust the gain simply by choosing appropriate values of R 2 and R 1, and of course also making sure that R 4 by R 3 is equal to R 2 by R 1.

So, there is some algebra involved here. So, you are definitely encouraged to start with this equation use this condition and then arrive at our final result. So, this was our method 1, and what did we do in this method? We use the golden rules that is the op-amp input currents are 0 and v plus and v minus are nearly equal when the op-amp operates in the linear region.

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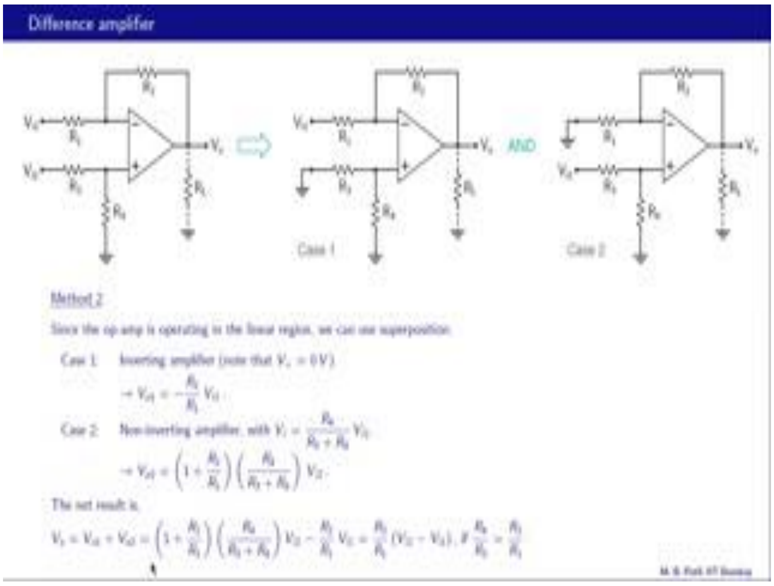

Difference amplifier

Method 2

Since the op-amp is operating in the linear region, we can use superposition.

Case 1: Inverting amplifier (note that $V_+ = 0\,V$).

$$\rightarrow V_{o1} = -\frac{R_2}{R_1} V_{i1}$$

Case 2: Non-inverting amplifier, with $V_+ = \frac{R_4}{R_3 + R_4} V_{i2}$

$$\rightarrow V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2}$$

The net result is

$$V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1} V_{i1} = \frac{R_2}{R_1}(V_{i2} - V_{i1}), \text{ if } \frac{R_4}{R_3} = \frac{R_2}{R_1}$$

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There is another method to arrive at the same result and let us look at that now. Let us now look at method two which is based on the idea that we should not really reinvent the wheel when it is not called for. And let us see what we mean by that. Since the op-amp is operating in the linear region we can use superposition. And the sound strange are first, because when we think of op-amp we think of very large gain we think of saturation and so on, but when the op-amp is operating in the linear region its equivalent circuit is indeed a linear circuit it has got an input resistance it has got v c, v s and it has got an output resistance; and all of these elements make up a linear circuit.

So therefore, our entire circuit is actually a linear circuit and therefore we can use superposition. So, let us do that now. We have to independent voltage sources V i 1 and V i 2 and we can take them one by one. So, we keep V i 1 deactivate V i 2 and we keep V i 2 and deactivate V i 1. So, this we will call as case 1 and that as case 2. And what we can do now is find v o in case 1, find v o in case 2 and then simply add the two.

What is case 1? Case 1 is actually an inverting amplifier and we can figure that out when we look at this voltage here v plus. V plus is simply 0 here and the circuit is exactly the same as the inverting amplifier circuit, and therefore v o then is v i times minus R 2 over R 1. So, we are not relived reinventing the wheel here we are using existing or known results. So, that is our case 1.

What about case 2? Case 2 is very similar to a non-inverting amplifier, provided we get this voltage and that is easy to do since this current is 0 this v plus is given simply by voltage division between R 3 and R 4. So therefore, v plus which is the input for the non-inverting amplifier is R 4 divided by R 3 plus R 4 times V i 2. So, v plus times 1 plus R 2 by R 1 is v o because we already know the gain for the non-inverting amplifier that is 1 plus R 2 by R 1. So, that is our v o 2 1 plus R 2 by R 1 times R 4 by R 3 plus R 4 times V i 2. This is the gain of the non-inverting amplifier. And the input voltage for the non-inverting amplifier is as we have seen this quantity here.

And now we can put this two together, and the net result is exactly what we had in the last slide: v o is v o 1 plus v o 2 and that is our v o 2 that is our v o 1, and therefore again it based down to R 2 by R 1 times V i 2 minus V i 1 provided R 4 by R 3 is selected to be the same as R 2 by R 1. And it is easy to see that this will happen. Suppose we take R 4 out and R 3 out then we see that this 1 plus R 2 over R 1 and 1 over 1 plus R 4 by R 3 will cancel out and then we get this result here.

So, superposition is very handy at least in this case and you can do things almost by inspection, much simpler than what we did in the last slide.

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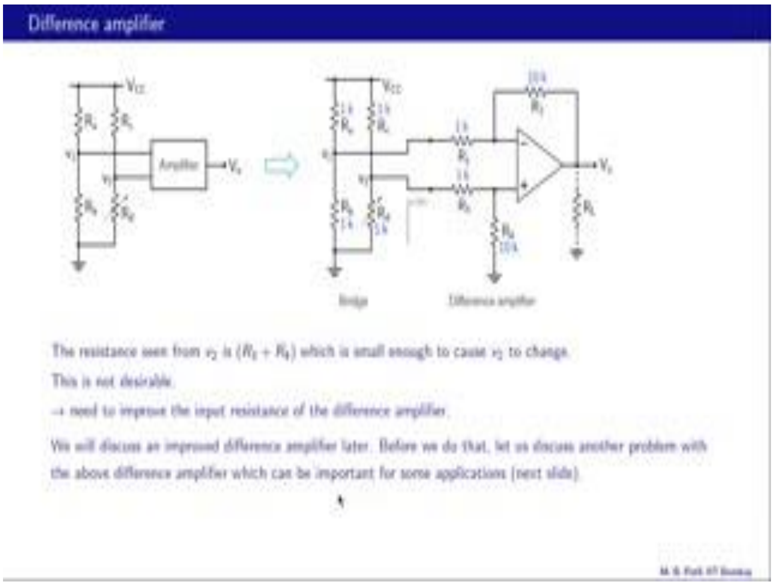

Difference amplifier

The resistance seen from v_2 is $(R_3 + R_4)$ which is small enough to cause v_2 to change.

This is not desirable.

→ need to improve the input resistance of the difference amplifier.

We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).

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Let us now come back to the bridge circuit. If you recall we wanted an amplifier which will give an output which is an amplified version of this difference v 2 minus v 1. And now we have got that amplifier our difference amplifier. And in this example we have R

2 equal to 10 k R 1 equal to 1 k and R 3 the same as R 1 R 4 the same as R 2. So, the gain of this difference amplifier is R 2 by R 1 which is 10 k by 1 k or 10.

So, v o then is v 2 minus v 1 times 10. So, we seem to a got the solution we were looking for but there is a problem, and let us see what that problem is. When we connect the amplifier to the bridge circuit we except that v 1 and v 2 stay the same as before, and when will that happen that will happen if the amplifier does not draw any current from this bridge circuit. Or in other words we except the amplifier to have a very large input resistance compare to all of these resistances.

Now those are not happening in our difference amplifier, and let us see why. The resistance seen from v 2 this resistance here is R 3 plus R 4 because that op-amp input current is 0, so therefore all we see is R 3 and R 4 in series. And in this case it is 10 k plus 1 k or 11 k. Now 11 k is not too large compare to this 1 k or this 1 k, and therefore our v 2 is going to get disturbed and that of course is not desirable. So, we definitely need to improve the input resistance of the difference amplifier.

And we will discuss an improved difference amplifier later, but before we do that let us discuss another problem with the above configuration above difference amplifier which can be important for some applications, and that we will do in the next slide.

In summary we have looked at the meaning of common mode and differential mode voltages, and consider a bridge circuit to illustrate these ideas. We have seen the importance of the common mode rejection ratio or CMRR in short. After that we have looked at an op-amp circuit the difference amplifier which can be used to amplify the difference between two input voltages. In the next lecture we will discuss why the circuit needs to be improved, until then goodbye.