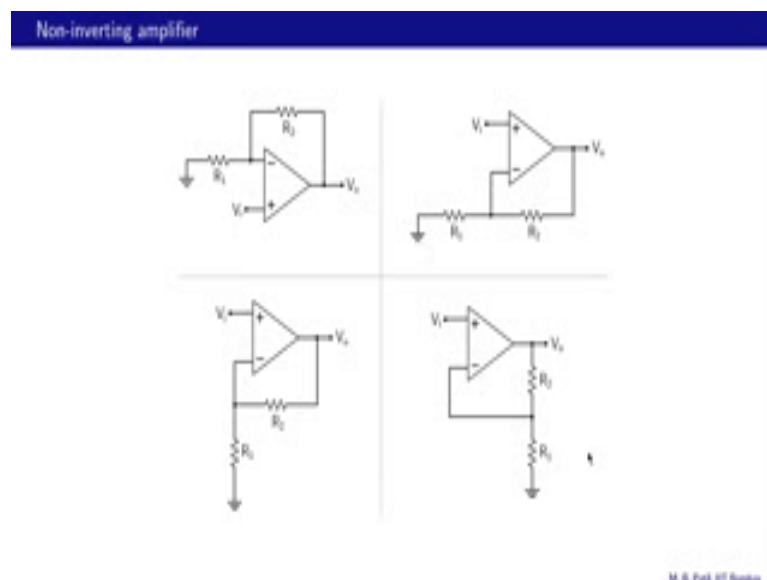


Basic Electronics
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Lecture - 34
Op-amp circuits (continued)

Welcome back to Basic Electronics. In this lecture, we will look at the input and output resistances of the non-inverting amplifier. We will also look at a special case of the non-inverting amplifier that is the op-amp buffer. So, let us begin.

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Let us look at the non-inverting amplifier once again. Here is the circuit. The input voltage goes to the non-inverting input of the op-amp. From the inverting input of the op-amp, we have R_1 going to ground and R_2 going to the output node of the op-amp. And the load resistance is not shown here, because that is not really a part of the amplifier circuit. Let us redraw this non-inverting amplifier circuit as shown here; it looks different, but it is the same circuit same connections. So, V_i connected to the non-inverting input once again. From the inverting input of the op-amp, we have R_1 going to ground and R_2 going to the output node of the op-amp.

Here is another way of drawing a circuit. Again V_i connected to the non-inverting input; and from the inverting input we have R_1 going to ground, R_2 going to the output node. Yet another way V_i going to the non-inverting input again; and from the inverting input,

we have R 1 going to ground and R 2 connected between the inverting input of the op-amp and a output node of the op-amp. So, there are the seemingly different ways of trying the same circuit and we might come across these in some real examples. So, we should familiar with in this different ways of trying the same circuit.

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Non-inverting amplifier

Consider $R_1 \rightarrow \infty$, $R_2 \rightarrow 0$.

$$\frac{V_o}{V_i} \rightarrow 1 + \frac{R_2}{R_1} \rightarrow 1, \text{ i.e., } V_o = V_i.$$

This circuit is known as unity-gain amplifier/voltage follower/buffer.

What has been achieved?

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We now consider a special case of the non-inverting amplifier with R 1 very large and R 2 very small. If R 1 is very large say infinite, we can remove this resistor; if R 2 is very small say 0 ohms, this becomes a short circuit, and we get the circuit shown here on the right side. Now, let us see what V o is in terms of V i. For the non-inverting amplifier as we have already seen V o divided by V i is 1 plus R 2 by R 1. R 2 is 0, R 1 is very large, so this quantity is simply becomes one so that means, V o is equal to V i because the gain is 1. And we can also observe this directly from this circuit, since the op-amp is operating in the linear region V minus and V plus are nearly equal. So, V minus is also equal to V i and since that is connected directly to the output V o is also equal to V i.

And this circuit the one on the right is known as unity gain amplifier, because the gain is 1 or voltage follower because the output voltage simply follows the input voltage or buffer. And why it is known as buffer, we should see very soon. The question is what has been achieved by doing this; we have got an output voltage which is just equal to the input voltage no gain. So, is this circuit useful at all, it turns out that it is very useful and we will see how it can be used.

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Loading effects

Consider an amplifier of gain A_V . We would like to have $V_o = A_V V_i$.
 However, the actual output voltage is

$$V_o = \frac{R_L}{R_o + R_L} A_V V_i = A_V \frac{R_L}{R_o + R_L} \frac{R_i}{R_s + R_i} V_s$$

To obtain the desired V_o , we need $R_i \rightarrow \infty$ and $R_o \rightarrow 0$.
 The buffer (voltage follower) provides these features.

M. S. Fall 07 Boston

Before we look at the buffer circuit, let us discuss something called loading effects. Here is the signal source, which is represented by a voltage source in series with resistance R_s . And we want to amplify this signal voltage and apply the amplified voltage across the load resistor R_L . And we do that using this amplifier, the equivalent circuit of the amplifier is represented by the input resistance R_i , the voltage gain A_V and the output resistance R_o . So, what we would really like to have is V_o to be equal to A_V times V_s . Let us say we have V_s equal to 1 millivolt, and the gain of the amplifier is 100 then we would like to have an output voltage of 100 times 1 or 100 millivolts.

Now, in reality the actual output voltages are slightly different, and let us see how. This V_o is not quite a V times V_i , because there is a voltage division here. So, it is a V times V_i multiplied by R_L divided by R_L plus R_o that is this expression here. Also this V_i the voltage across the amplifier input terminals is not quite V_s , and again there is a voltage division here, so it is V_s times R_i divided by R_s plus R_i . So, the net V_o is given by this expression here. This is less than one this is also less than one. So, this whole gain is not A_V , but something less than A_V and that is not desirable what we would really like is V_o equals to A_V times V_s .

This reduction in gain from A_V to this number is referred to as the loading effects. It represents the loading of the amplifier by the load resistor R_L , and also loading of the voltage source by the amplifier that is this input resistance of the amplifier. Is there a

way to minimize or eliminate these loading effects let us see. Suppose, we make R_i very large then R_i plus R_s is about R_i , and this factor becomes equal to 1. Similarly, if we make R_o very small then R_o plus R_L is nearly equal to R_L this factor also becomes one and then we have the gain equal to A_V times one times one and that is just A_V as we would like. And the buffer circuit that we have seen in the last slide or also called the voltage follower provides exactly these features and that is why it is very useful.

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Op-amp buffer: input resistance

non-inverting amplifier

$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_i$$

KCL at B: $\frac{V_B}{R_1} + \frac{V_B - A_V V_i}{R_2} + \frac{V_B - V_A}{R_2} = 0$

Source current: $I_s = \frac{V_A}{R_1} + \frac{V_A - V_B}{R_2}$

Using $V_i = I_s R_1$, $V_A = V_B - V_i$, and after some algebra, we get

$$R_{in} = \frac{V_i}{I_s} = \frac{\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_1}\right) R_1 \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1} + A_V}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}}$$

STOP

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We now want to look at the op-amp buffer. Find its input resistance and output resistance. To begin with let us look at the non-inverting amplifier, because the buffer is special case of this circuit with R_1 equal to infinity, and R_2 equal to 0 as we have seen before. Now, for this circuit let us replace the op-amp with its equivalent circuit consisting of the input resistance, gain element and output resistance. And to obtain the input resistance, what we do is apply voltage source V_s here, find this current I_s and then the input resistance of the non-inverting amplifier as seen from here will be V_s divided by I_s .

To get V_s over I_s , there may be different ways of doing is what we will do is to treat V_A the node voltage at A with respect to ground, V_B , V_i - the voltage that appears across the input terminals of the op-amp that one and I_s . So, we will treat these four quantities V_A , V_B , V_i and I_s as our unknowns. We will treat this V_s as known number and then we will obtain equations which will enable us to solve for I_s in terms of V_s .

Let us begin with KCL at node B. What do we have at node B, we have this current plus this current plus this current. And these three current must add up to 0; this one is V_B divided by R_1 this one is V_B minus the node voltage here which is a V times V_i with respect to ground. So, V_B minus $A V$ times V_i divided by R_2 , and this current is V_B minus V_A divided by R_2 . So, these three must add up to 0 that gives us to KCL at node B. What about the source current the source current is the same as the current through this R_1 and that is the sum of two currents - this current here and that current. This current is V_A by divided by R_1 and that one is V_A minus V_B divided by R_2 . So, that is what this equations says I_s is equal to V_A by R_1 plus V_A minus V_B by R_2 .

In addition to these two equations, we have two more conditions one is V_i equal to I_s times R_i ; this V_i is I_s times R_i , and V_A is V_s minus V_i . This node voltage is equal to V_s minus this voltage drop, so it is V_s minus V_i . So, we have four equations 1, 2, 3, 4. And we have four unknowns what are the unknowns V_A , V_B , I_s and V_i . And remember we are treating V_s as unknown quantity, so we have four equations and four unknowns we can solve these and obtain I_s . Once we obtain I_s , we can find the input resistance as seen from there as V_s over I_s and that turns out to be this expression here. It is a bit messy.

And you are of course, encourage to do all the algebra and arrive at this result, but let us look at least the dimensions of this R in whether it is indeed ohms or not. What about the numerator dimensionless, dimensionless; dimensionless R_i and this whole quantity as units of 1 over ohms, so ohms by ohms. So, this entire numerator is dimensionless. What about the denominator 1 over ohms, this is dimensionless, and this also is 1 over ohms ohms by ohms square, so 1 over ohms. So, the denominator has dimensions of 1 over ohms. And therefore, this entire expression is in ohms. The stop sign has come, and it is telling you that you have to work out this relationship and then proceed further. It is a bit of algebra where we have page or so, but it is definitely worth doing.

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Non-inverting amplifier: input resistance (continued)

$$R_i = \frac{V_i}{I_i} = \frac{\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_2}\right) + R_i \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_2}\right) - \frac{R_2}{R_1^2} + \frac{A_V}{R_2} \right]}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \left(1 + \frac{R_2}{R_1} + \frac{R_2}{R_2}\right) - \frac{R_2}{R_1^2}}$$

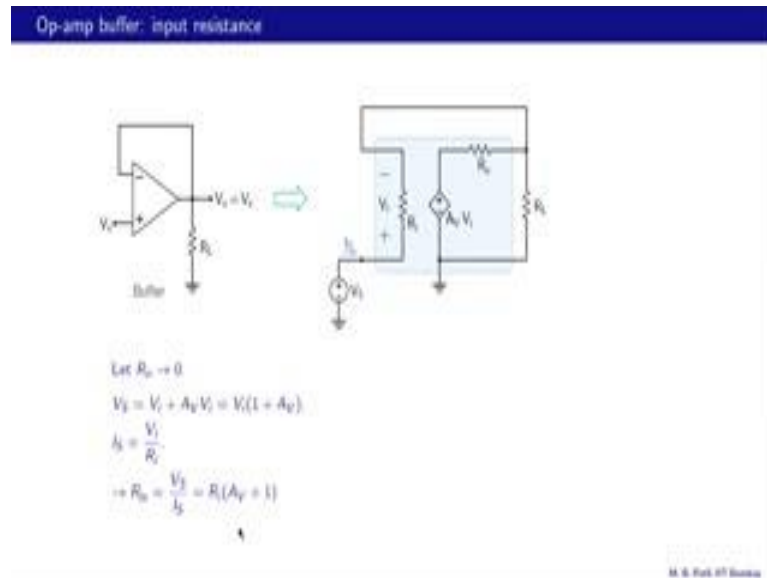
Since R_o is much smaller than R_1 , R_2 , R_L , or R_i ,

$$R_i \approx \frac{1 + R_i \left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{A_V}{R_2} \right]}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{R_i \left[\frac{R_1 + R_2}{R_1 R_2} + \frac{A_V}{R_2} \right]}{\frac{R_1 + R_2}{R_1 R_2}} \approx A_V R_i \frac{R_1}{R_1 + R_2}$$

Let us try to simplify this expression for the input resistance. We know that R_o is much smaller than all of these resistances R_1 , R_2 , R_L would be in the kilo ohms range whereas, R_i would be in the mega ohms range; and R_o is much smaller than all of those, it would be something like 50 ohms. So, therefore, these ratios R_o by R_1 or R_o by R_2 etcetera would be much smaller than 1 and we can ignore those. So, we can ignore all of these terms containing R_o in this expression, and get a simplified expression given here.

Now, in this expression, this one is much smaller than the second term. This represents R_1 parallel R_2 . So, the first term here is R_i divided by R_1 parallel R_2 definitely would be much larger than 1, so therefore, we can ignore this one and then simplify these expression to this quantity, and then after a bit of we get our algebra we get our final result. This R_1 divided by $R_1 + R_2$ is smaller than 1, let say 1 by 10, 1 by 20, 1 by 50 something of that order. Now, A_V times R_i is a very huge resistance A_V is 10 raise to 5 or the 741, R_i is 2 mega ohms. So, altogether this input resistance is very, very large; and therefore, for all practical purposes we see an open circuit from this terminal. And this very large input resistance is definitely a big advantage of using the non-inverting configuration.

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Let us get back to the op-amp buffer circuit, and we are interested in finding its input resistance as seen from here. First, we replace the op-amp with its equivalent circuit shown here consisting of the input resistance, the gain element and the output resistance. We apply a voltage source V_s and find this current, and then the ratio of V_s and I_s gives us the input resistance as seen from the non-inverting terminal of the buffer. As before, we will assume that R_o is very small with respect to other resistances and we will treat it as 0.

First let us look at V_s this voltage. So, what we will do is we will start with 0, we will go like that, we come across the rise of $A V$ times V_i then go like that R_o is 0. So, there is no voltage drop there, and then go like this, so that is a voltage rise of V_i . So, altogether we come across a voltage rise of a V times V_i plus V_i . So, that is a V times V_i plus V_i or V_i times $1 + A V$ that is V_s . What about I_s , this voltage drop is V_i and that is simply I_s times R_i , so that gives us $I_s V_i$ divided by R_i . And now all we need to do is to take the ratio V_s divided by I_s . So, R_{in} as seen from here is then V_s divided by I_s which is R_i times $A V$ plus 1. And this of course, is a very large input resistance R_i is a few mega ohms and $A V$ is 10^5 . So, it is really very huge, so that means, looking from here we see a very, very large input resistance for the op-amp buffer.

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Op-amp buffer: output resistance

Non-inverting amplifier

To find R_{out} ,

- Deactivate the input source.
- Replace R_L with a test source V' .
- Find the current (I') through V' .
- $R_{out} = \frac{V'}{I'}$

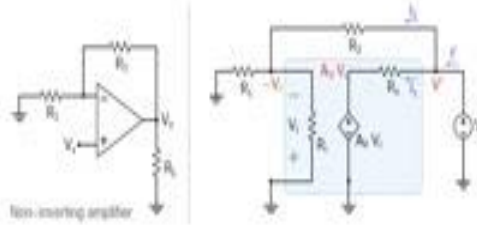
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Let us now look at the output resistance of the op-amp buffer. And let us start with the non-inverting amplifier, because buffer is a special case of this circuit. What do we mean by the output resistance? What we mean is the resistance as seen by the load resistor here? And first what we do is replace this op-amp with its equivalent circuit as shown over here. The rest of the connections of course, are the same, we have R_1 going to ground from the inverting terminal in both cases, and R_2 between the inverting terminal and the output terminal.

Here is our R_L , and R_{out} is defined as the resistance as seen from R_L like that. And how can we find R_{out} , we deactivate the input source first that means connect this node to ground replace R_L with a test source V' . So, we remove R_L and put a voltage source V' over there. Find the current I' through V' that current and then R_{out} is given by V' divided by I' . So, let us do that.

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Op-amp buffer: output resistance (continued)



$$V_1 = -\frac{(R_1 \parallel R_i)}{R_2 + (R_1 \parallel R_i)} V' = -kV'$$

$$I' = I_1 + I_2 = \frac{V' - A_v V_1}{R_o} = \frac{V' - (-kV')}{R_o} = \frac{1}{R_o} (V' + kA_v V') + \frac{k}{R_o} (V' - kV')$$

$$\frac{I'}{V'} = \frac{1}{R_o} (1 + kA_v) + \frac{k}{R_o} (1 - k) \Rightarrow R_{out} = \frac{V'}{I'} = \frac{R_o}{(1 + kA_v) \left[\frac{1}{1 - k} \right]} = \frac{R_o}{1 + kA_v}$$

Special case: Op-amp buffer

$$k = \frac{(R_1 \parallel R_i)}{R_2 + (R_1 \parallel R_i)} \rightarrow 1 \Rightarrow R_{out} = \frac{R_o}{1 + A_v}$$

So, this is what we do, we deactivate the input voltage; that means, connect this node to ground we replace R_L with voltage source as shown over here. And now we find the ratio V' divided by I' and that will give us the output resistance. To begin with let us mark the node voltages. This node is at V' with respect to ground; this node is at kV' with respect to ground again; and this node is at $-kV'$ with respect to ground. Now, this voltage minus V' can be related to V' when we realize that this R_1 and R_i are actually in parallel.

So, from this node, we have two resistances going to ground R_1 and R_i , so they are actually in parallel. So, we have a single resistance $R_1 \parallel R_i$ going to ground from this node. And once we figure that out we can obtain this node voltage, so $-kV'$ would be this $R_1 \parallel R_i$ divided by $R_2 + R_1 \parallel R_i$ times V' and that is simply voltage division. So, this is what we get for V_1 in terms of V' . And notice that R_i is a large resistance is the input resistance of the op-amp. And therefore, $R_i \parallel R_1$ is nearly equal to R_1 . So, what we have over here is essentially R_1 by $R_1 + R_2$ and that fraction has been defined as k over here. As an example let us say R_2 is 9 k, and R_1 is 1 k then this fraction would be 1 k divided by 10 k or 1 by 10.

Let us now look at this current I' . What is I' ? it is $I_1 + I_2$. And let us now get I_1 and I_2 individually. What is I_1 V' minus $A_v V_1$ divided by R_o the first term. What is I_2 V' minus $-kV'$ divided by R_2 - the second term here.

And this V_i we can replace with $-k V_o$, so that gives us I_o equal to $\frac{1}{R_o} V_o + k A V_o$ this term here comes from here plus $\frac{1}{R_2} V_o + V_i$ and V_i is $-k V_o$. So, $\frac{1}{R_2} V_o - k V_o$, so that is our I_o .

So, we now have I_o in terms of V_o all of these are in terms of V_o and we can now obtain I_o divided by V_o what we want is V_o divided by i_o the output resistance. So, let us do that. So, I_o by V_o is $\frac{1}{R_o} + k A$ the first term here plus $\frac{1}{R_2} - k$ the second term here. So, this is like the conductance and it is a sum of two conductances. So, the resistance which is the reciprocal of this quantity which is the output resistance we are looking for is basically two resistors in parallel one of them is R_o divided by $1 + k A$, and the other is R_2 divided by $1 - k$. So, that is what we get for the output resistance as seen from R L.

Now, of these two this resistance is clearly very small because we have R_o here and R_o of an op-amp is a small resistance and therefore, a small resistance in parallel with a larger resistance is essentially that smaller resistance itself. So, then we have R_{out} approximately equal to R_o divided by $1 + k A$. Now, A is a large number 100 thousand for the 741 op-amp. So, therefore, this quantity is actually a very small resistance.

Let us now take the special case of the op-amp buffer. How do we obtain the buffer from the non-inverting amplifier, we make R_1 infinite and R_2 equal to 0. Now this k , which is this fraction here, will become 1 when R_2 is 0. And therefore, this expression for R_{out} will now become R_o divided by $1 + 1 \times A$ or R_o divided by $1 + A$. A of course, is the voltage gain of the op-amp; the open loop gain which is a large number 100 thousand R_o is already very small something like 50 ohms for the 741, 50 or 75. So, this quantity is really, really tiny. So, for all practical purposes we can treat R_{out} as the 0 ohm resistance. So, the op-amp buffer not only provides a very high input resistance, it also provides a very low output resistance. So, let us keep these desirable features of the op-amp buffer in mind.

To summarize, we have found expressions for R_{in} - the input resistance and R_{out} - the output resistance of the non-inverting amplifier. We found that R_{in} is very large and R_{out} is very small and that makes this circuits very useful in certain applications. We also

looked at the op-amp buffer, which is a special case of the non-inverting amplifier. In the next class, we will look at how a buffer can be used to avoid loading effects, until then goodbye.