Basic Electronics
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Lecture - 03
Useful circuit techniques

Welcome back to Basic Electronics. In this lecture we will look at a very useful theorem called the Thevenin’s theorem; we will first try to understand why it is possible to represent circuit with this thevenin equivalent form. We will then look at how to extract the social credit parameters namely the thevenin resistance and the thevenin voltage. After that we will consider couple of examples to illustrate how was circuit can be represented by is thevenin equivalent. We will also see how the thevenin parameters can be extracted graphically. So, let us get started.

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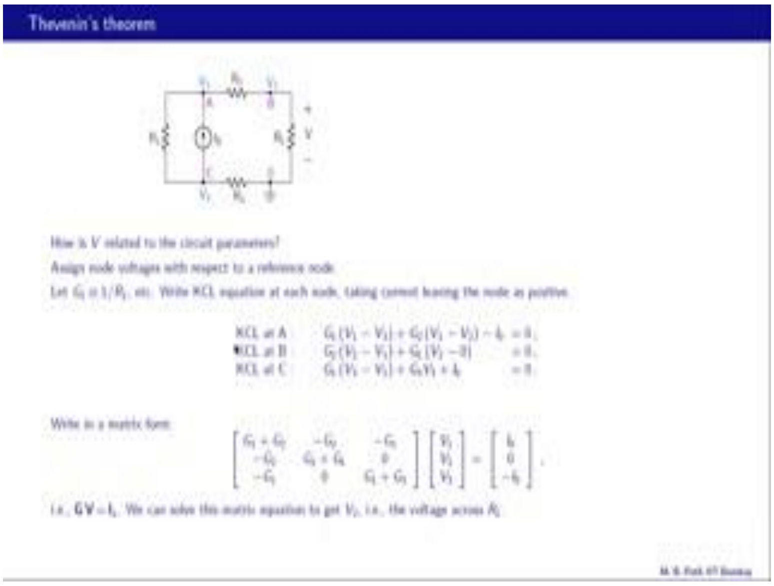

Thevenin’s theorem is a very useful and important theorem and particularly in electronics circuits we will find it very useful, but before will look at the statement of the Thevenin’s theorem let us see briefly where it comes from, and we will do that with the help of this example. Here is the circuit to which we have connected load register R L, and we are interested in this voltage across R L and in particular we want to ask this question, how is V related to the circuit parameters.

We begin by assigning node voltages to this various nodes in the circuits with respect to a reference node. So, let us first choose reference node, we have taken that as the reference node. So, the node voltage there is 0 volts, and with expect to that reference node this voltage at node a a is V 1, the node voltage at b is V 2 and the voltage at node c is V 3. Now what we will do is write KCL equation in terms of V 1 V 2 and V 3 at this nodes; nodes A B and C and in doing that let us define G 1 a conduction as 1 over R 1, G 2 equal to 1 over R 2 etcetera.

These are our KCL equation, let us look at KCL at node A, that is our node A we have 3 currents entering or living node A, these one current going like that and that is V 1 minus V 3 divided by R 1. So, that is G 1 times V 1 minus V 3 and since that current is living this nodes we will take it as positive. Then we have this current V1 minus V 2 divided by R 2; that means G 2 times V 1 minus V 2 and then we have this current I 0 entering the nodes. Since this current is entering the node we take that as negative and that is why it appears as minus I 0 in this equation, and all these currents must add up to 0 and that gives us KCL at node A.

In a similar manner we can write KCL at node B and KCL at node C, in terms of the node voltages V 1, V 2 and V 3 and we can now write these equations in a matrix form like that let us look at one of these equations, let us say the first one; this first equation comes from KCL at node A and let us verify that, what is the coefficient of V 1 in this equation it is G 1 plus G 2. So, that goes over there that is going to multiply V 1 what about V 2 the coefficient is minus G 2. So, that goes there. And that multiplies V 2 and the coefficient of V 3 is minus G 1, so that goes over there. In addition we have minus I 0 over here and since that does not depend on V 1, V 2 or V 3; we take that to the right hand side. So, it becomes plus I 0 and that is how it appears here.

So, this matrix equation is of the form G times V equal to I s where G is this matrix here 3 by 3, V is this column vector consisting of V 1, V 2, V 3 the node voltages in our circuit and I s is the source current vector this one all right. Now if you recall our objective is to find the relationship between V and the rest of the circuit and in order to do that we want to solve this matrix equation and obtain V 2. Since V 2 is the same as b here because this node voltage is 0.

And V 2 can be obtained using Cramers rule, and what does Cramers rule say it says that V 2 is given by d 1 divided by d 2, where d 1 is the determinant of this matrix with the second column replaced with this R H S vector, and d 2 is the determinant of this matrix as it is.

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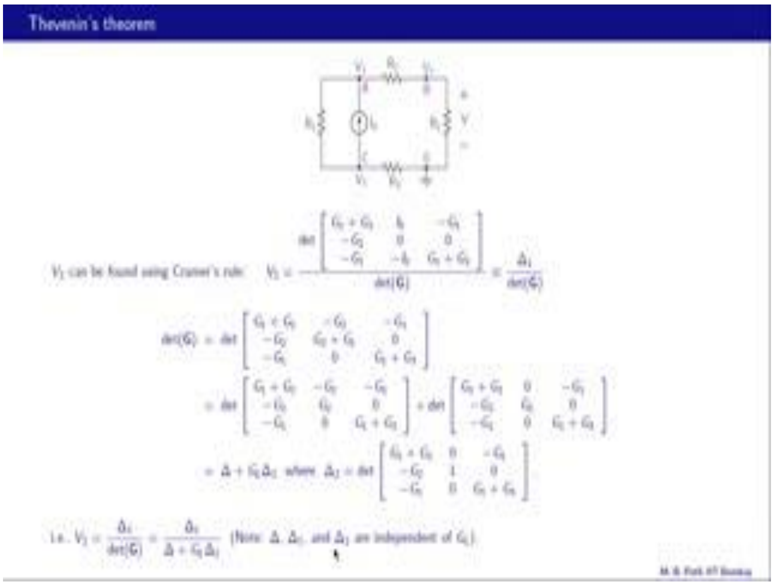

So, let us do that and simplify things after that; so this is what we have for V 2 the ratio of 2 determinants determinant of this original G matrix with the second column replaced with the R H S vector like we mentioned in the last slide and the determinant of the G matrix itself all right. Now let us denote this determinant by delta 1, and what we should note about delta 1 is that it does not have R L or G l in it. So, this value is independent of the load resistance value. Let us look at the denominator now that is determinant of G, this is our G matrix and this determinant does depend on G l, we have G l over here and what we will do now is write this column as the sum of 2 columns; column 1 minus G 2, G 2, 0. Column 2: 0, G l, 0 and if you do that we can write this determinant as the sum of 2 determinants, this determinant as no G l term in it whereas, this one does have G l in it right there.

Let us denote this first determinant by delta, and let us write this second determinant as G l times delta 2; where delta 2 is the determinant of this matrix here and notice that we do not have G l here anymore because that has been taken outside all right . So, with these substitutions we can write V 2 as delta 1 divided by determinant of G that is delta

1, divided by delta plus G l delta 2 that term where delta, delta 1, delta 2 are all independent of G l. So, the only dependence of V 2 on G l is right there.

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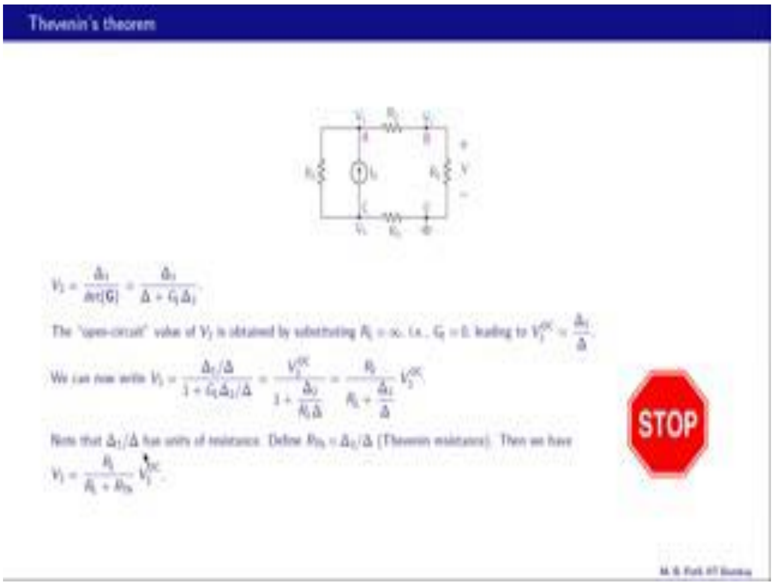

So, here is V 2; once again delta 1 divided by delta plus G l times delta 2, and let us emphasize once again that delta 1, delta, and delta 2 have no dependence on G l or R L and they will only depend on the circuit parameters here all right. Let us now look at the open circuit value of V 2; that means, the value of V 2 when R L is infinite and when R L is infinite, G l which is 1 over R L becomes 0 and therefore, we have V 2 equal to delta 1 divided by delta. So, that is our V 2 in the open circuit condition.

We can now rewrite V 2 by dividing both the numerator and denominator by delta. So, in the numerator we get delta 1 divided by delta, in the denominator we get 1 here. So, 1 plus G l times delta 2 over delta. Now delta 1 by delta is nothing, but V 2 o c, V 2 open circuit. So, we replace that with V 2 o c and that gets divided by 1 plus delta 2 over R L times delta. Here we have used the fact that G l is 1 over R L and now let us multiply both numerator and denominator by R L to get R L divided by R L plus delta 2 over delta times V 2 o c.

Now, it turns out that this quantity delta 2 divided by delta has units of resistance and you should really go back one slide and check that out. So, let us define R thevenin that is thevenin resistance as delta 2 divided by delta, and with that we get V 2 equal to R L divided by R L plus R thevenin times V 2 o c like that.

The stop sign has come and why is it there; that means, there is some unfinished business and that is this delta 2 by delta, you need to really go back and check that it has units of resistance. So, do that, stop the video here and when you complete this assignment start again with the video.

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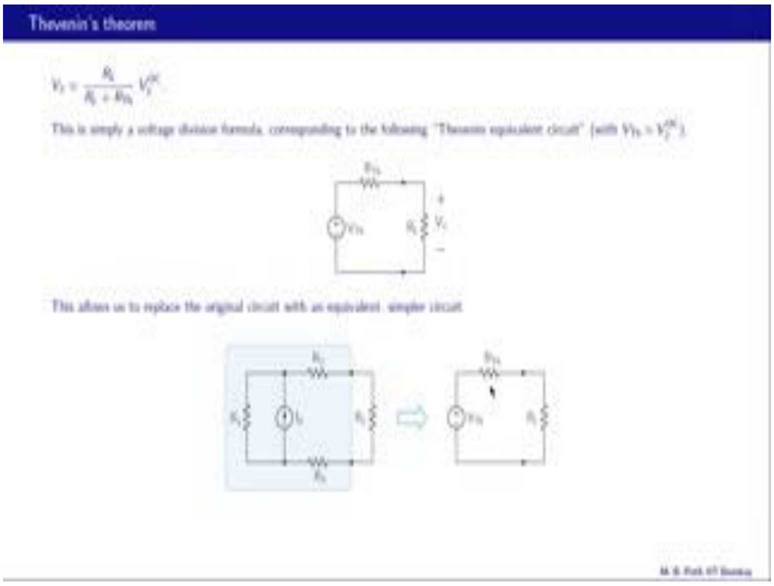

So, this is our expression for V 2, and it is looking very much like voltage division formula. If we have a source which is equal to V 2 o c like that and R thevenin and R L in series then the voltage across R L which is V 2 here would be R L divided by R L plus R T S times V t h. So, this circuit and this expression are compatible with each other, if you have V T S equal to V 2 open circuits.

In other words this expression allows us to replace the original circuit with an equivalent simpler circuit, and that simpler circuit is the thevenin equivalent circuit. So, here is our original circuit and this entire circuit can now be replaced with this much simpler circuit consisting of only 1 voltage source in series with 1 resistance; the voltage is called V T S or thevenin voltage and the resistance is called R thevenin or R T S and later of course, we will look at how we can obtain V T S and R T S from the original network.

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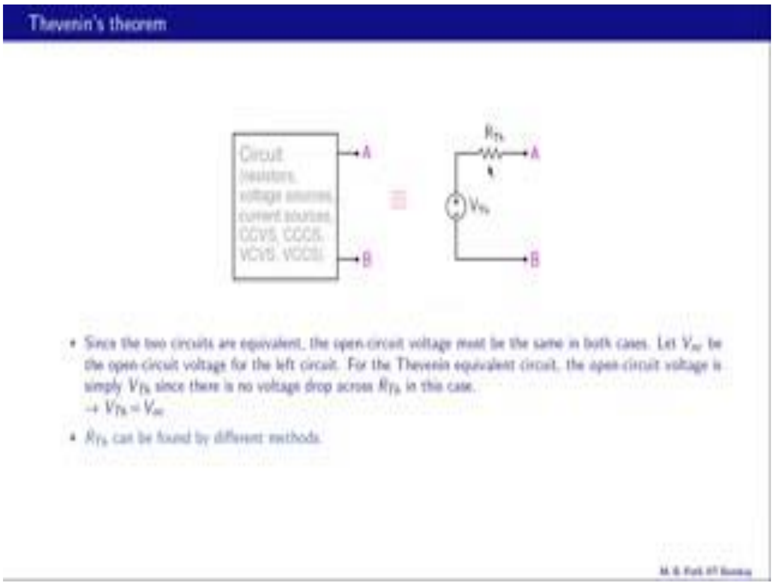

And that brings us to Thevenin's theorem, if we have a circuit consisting of registers independent voltage sources, independent current sources and dependent sources such as current controlled voltage source, current controlled current source etcetera. Then thevenin theorem says that this circuit can be represented by its thevenin equivalent circuit, consisting of a voltage source V T S in series with a resistance R T S.

And now imagine that we have a load resistance R L connected between A and B to the original circuit as well as to the thevenin equivalent circuit; then thevenin theorem says that the load resistance does not see any difference between the original circuit and its thevenin equivalent; that means, the voltage across the load resistance or the current through the load resistance would be identical, in these 2 cases all right.

Now, the next question that comes up is how do we figure out V T S and R T S from the original network. So, let us look at that; first let us look at how to get V T S from the original circuit. Since the 2 circuits are equivalent, the open circuit voltage must be the same in both cases. Now let V o c be the open circuit voltage for the left circuit here; that means, we do not connect anything between A and B and measure this voltage V o c between A and B. For the thevenin equivalent circuit what is the open circuit voltage? If we do not connect anything between A and B there is no current and therefore, there is no voltage drop across R T S and therefore, this open circuit voltage would be the same as this V t h.

And therefore, we conclude that this V T S is the same as the open circuit voltage seen between A and B in the original circuit and that is what this equation tells us.

What about R T S? The thevenin resistances here; that can be found by different methods and let us see what those methods are.

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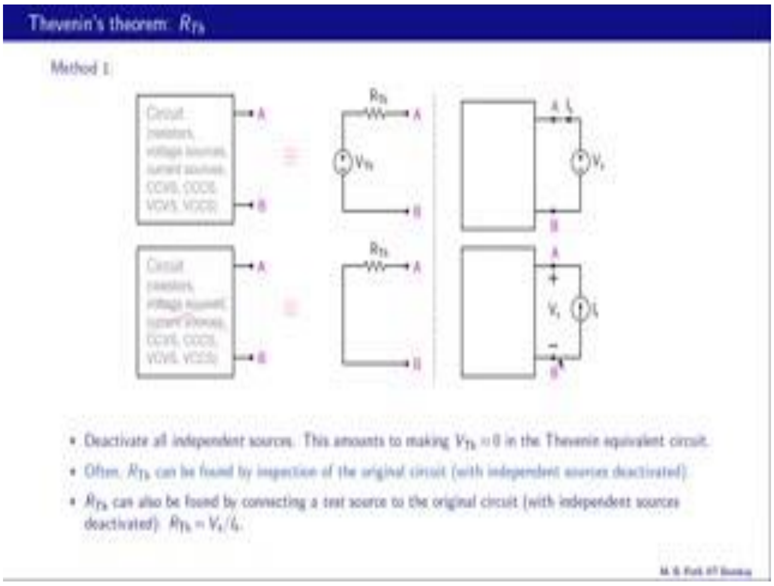

Method one here is our original circuit and that is the thevenin equivalent representation, what we do is deactivate all independent sources in the original circuit, and we do the same thing in the thevenin equivalent circuit as well. And in the thevenin equivalent circuit, the only independent source is this voltage source here and when we deactivate that we replace that with a short circuit like that; and what we see then between A and B is simply the thevenin resistance R T S. And now if we look into the port A B of the original circuit with the voltage sources and current sources deactivated, then whatever resistance we see must be the same as R T S.

Often R T S can be found by inspection of the original circuit with the independent voltage sources and independent current sources deactivated, and if that does not work what we can do is to use a test source. We can use either or test voltage source like that or we can use a test current source like this, and what we do then is for example, if we use a voltage source, then we find this current I s and then R T S is given by V s divided by I s. in this case where we have a test current source then we find this voltage drop between A and B and then R T S is given by V s by I s.

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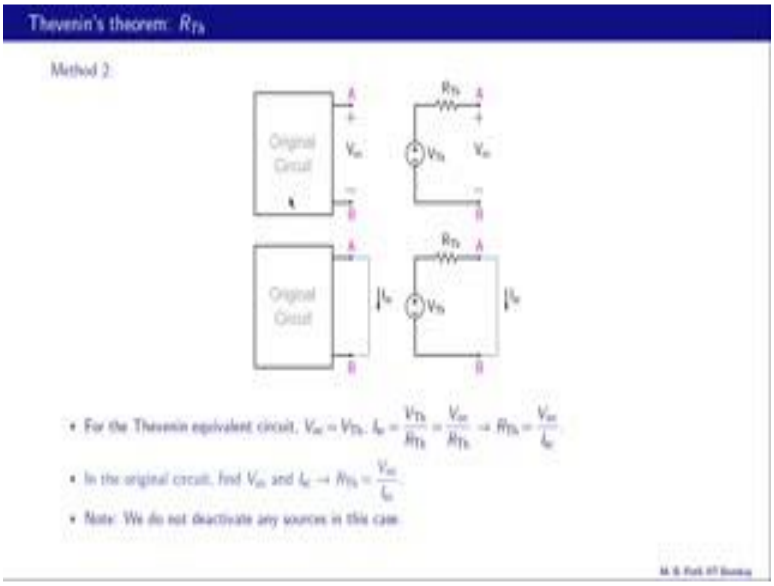

Let us look at our second method now; this is our original circuit and that is the thevenin equivalent. We look at 2 things; 1 the open circuit voltage between A and B in both cases second the short circuit current which flows from A to B, when A and B are shorted and we do that again in both cases and because the original circuit is equivalent to this thevenin representation; V o c here, V o c here must be the same and similarly I s c here and I s c here must be the same. So, we use this fact to obtain R T S for the thevenin representation let us see how.

Let us look at this circuit the open circuit voltage between A and B is the same as V T S here because there is no voltage drop across R T S the current being 0 and therefore, we get V o c equal to V T S, this equation here all right. Let us now look at the short circuit current I s c is simply V T S divided by R T S because there is no other resistance in the circuit. So, that gives us I s c equal to V T S divided by R T S; and V T S is the same as V o c therefore, I s c is equal to V o c divided by R T S and that gives us a formula to obtain R T S that is R T S equal to V o c divided by I s c.

Now, since this V o c; and this V o c are identical because the 2 circuits are equivalent, and also because this I s c and this I s c are the same. What we can do is in the original circuit we find the open circuit voltage and the short circuit current and then R T S is given by that open circuit voltage divided by the short circuit current. And note here that we do not deactivate any sources in this case. So, the independent sources are left as they are in the original circuit.

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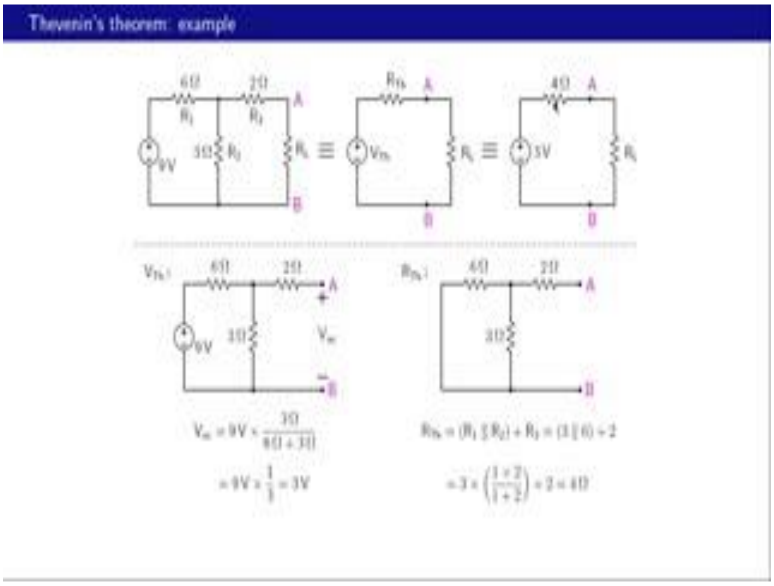

Let us find the thevenin equivalent of this circuit here as seen from R l; so we need to find V T S and R T S. Let us find V T S first, and how do we do that? V T S is the same as V o c the open circuit voltage between A and B. So, we remove R L and then find this voltage and that is the same as V T S. That is the straightforward calculation there is no current through this 2 ohms resistance. So, no voltage drop there and therefore, V o c is the same as this voltage drop, and that can be found by voltage division that is 3 ohms divided by 9 ohms times 9 volts. So, that comes to 3 volts. So, our V T S which is the same as V o c is 3 volts.

Next let us find R T S and how do we do that? In the original circuit we deactivate the independent sources; in this case there is only one independent source that is a voltage source. So, we shot it and then we look from this port A B; and this is what we see from A B, this 6 ohms and 3 ohms come in parallel and that combination comes in series with these 2 works and therefore, our R T S terms out to be 3 parallel 6, plus 2 or 4 ohms. So, over thevenin equivalent circuit as V T S equal to 3 volts and R T S is equal to 4 ohms.

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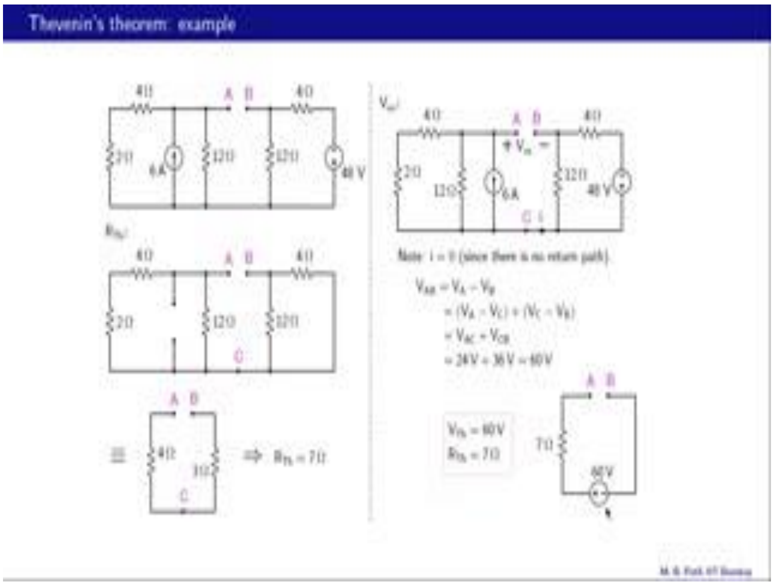

Another example this is a little more complicated than the previous example, we have 2 independent sources. Now our current source 6 amperes and a voltage source 48 volts and we want to find the thevenin equivalent circuit as seen from A B. Let us look at R T S first, how do we find R T S? We deactivate the independent sources that is this current source and this voltage source. The current source is replaced with an open circuit, and the voltage source is replaced with a short circuit; and now we look from A B and find the resistance that resistance is over 48. And that is fairly straightforward to do this for ohms and 2 ohms are in series, so therefore, we have 6 ohms, and that combination comes in parallel with 12 ohms and that entire combination is 4 ohms.

On this side we have 4 ohms in parallel with 12 ohms and that turns out to be 3 ohms. So, altogether between A and B we have 4 ohms plus 3 ohms or 7 ohms; and that is our R thevenin. Let us now look at V thevenin V thevenin is the same as V o c, this voltage here and how do we go about finding that voltage drop? We note that this current I must be 0, because there is no return path for this current there is an open circuit here and if this current were not 0 then that would lead to violation of the charge conservation principle all right. Once we figure that out the rest of the calculation is straightforward, what we do now is find this voltage drop between A and C, this voltage drop between C and B and then V a b is simply V a c plus V c b. So, let us do that.

Since this current is 0 we have 2 independent circuits here; one circuit is here and the other circuit is here and that makes things very easy all right. Let us look at V a c first, what is this resistance as we have already found out that is 4 ohms and therefore, 6 amperes going through 4 ohms gives us 24 volts. So, that is V a c. What about V c b? V c b is given by voltage division of these 48 volts between 12 ohms and 4 ohms. So, it would be 12 divided by 16 times 48 and that terms out to be 36 words. So, altogether between A and B we have 24 volts plus 36 volts and that comes to 60 volts. So, that is our open circuit voltage and that is the same as V T S. So, we have V T S equal to 60 volts R T S equal to 7 ohms and our thevenin equivalent circuit therefore, looks like this.

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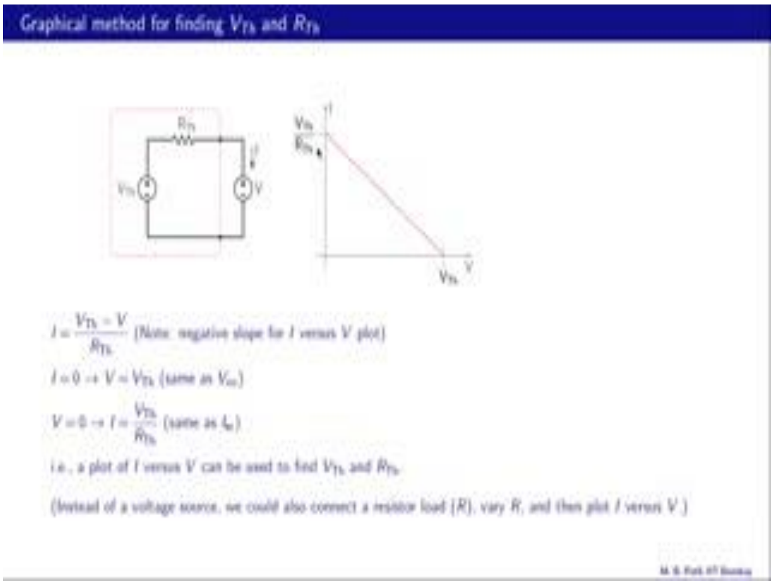

Let us now look at graphical method for finding V T S and R T S; this is thevenin equivalent circuit of some original circuit and what we do is connect a voltage source here and plot this current as a function of this voltage. And the plot that we are going to get is shown over here and let us now tries to understand why the plot looks like this. What is this current? It is V T S minus V divided by R T S this equation and note that it has a negative slope the slope is minus 1 over R T S and that has reflected in this plot here.

What about the intercept of this line on the V axis? To find that we put I equal to 0 and we get V equal to V t h. So, that gives us the x intercept and V T S remember is the same as V o c. What about the y intercept? To find that what we do is put V equal to 0 that gives us I equal to V T S divided by R T S, this quantity here and if you recall that is the same as the short circuit current; that means, it is the current that would flow if we connect these 2 nodes by a wire just a short circuit all right. So, now, from a plot like this we immediately find V T S as the intercept on the V axis and I s c which is equal to V T S by R T S as the intercept on the I axis and from these 2 numbers we can find our thevenin voltage and thevenin resistance; and as a remark let us mention that instead of a voltage source here, we could have also connected a load resistor say R varied that resistance and then plotted I versus V again we would get the same plot once again.

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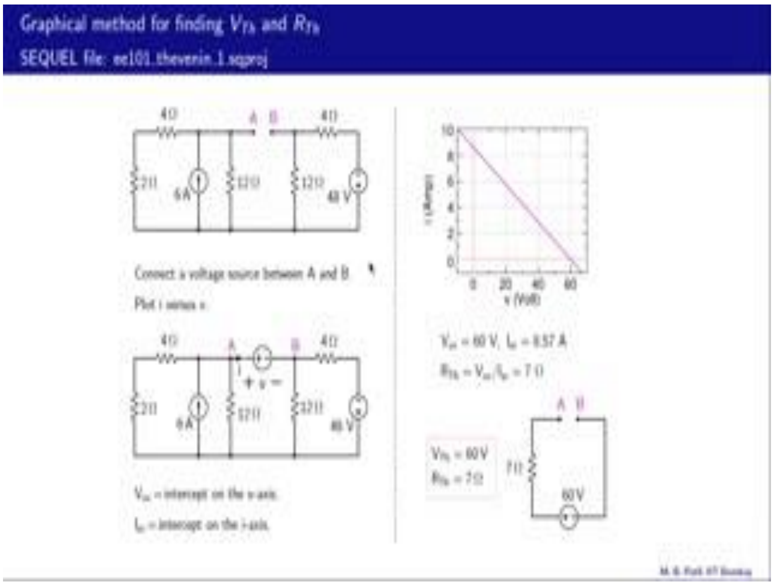

Let us apply the graphical method that we just discussed for this circuit, which we have already considered earlier and here is the sequel circuit file. So, you can run this simulation and obtain the plot that we are going to look at very soon. So, how do we go about it? We connect a voltage source between A and B like that and then plot this current as a function of this voltage and when we do that V o c is given by the intercept on the V axis, and I s c the short circuit current is given by the intercept on the I or the current axis.

And here is the result I as a function of V, the intercept on the V axis is 60 volts and that gives us V o c and the intercept on the current axis is 8.57 amperes and that gives us I s c and V T S is the same as V o c, 60 volts and R T S is the same as V o c divided by I s c and that turns out to be 60 divided by 8.57 or 7 ohms and these numbers of course, are the same as what we got earlier when we analyzed this circuit using different methods.

To summarize we have become familiar with Thevenin's theorem, we have also seen how do I represent a given circuit with this thevenin's equivalent form, this background will be useful to understand R c circuits amplifiers etcetera that is all for now.

See you next time.