Basic Electronics
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Lecture - 21
Diode Ratification (continued)

Welcome back to Basic Electronics. In this lecture, we will continue with the half-wave rectifier example from the previous class. We will estimate the maximum current that flows through the diode and also obtain an analytic expression for that current. We will then repeat the calculations; that are ripple voltage, average and peak diode currents for a full-wave rectifier circuit. Finally, we will compare the half-wave and full-wave rectifier circuits with respect to different criteria. So, let us get started.

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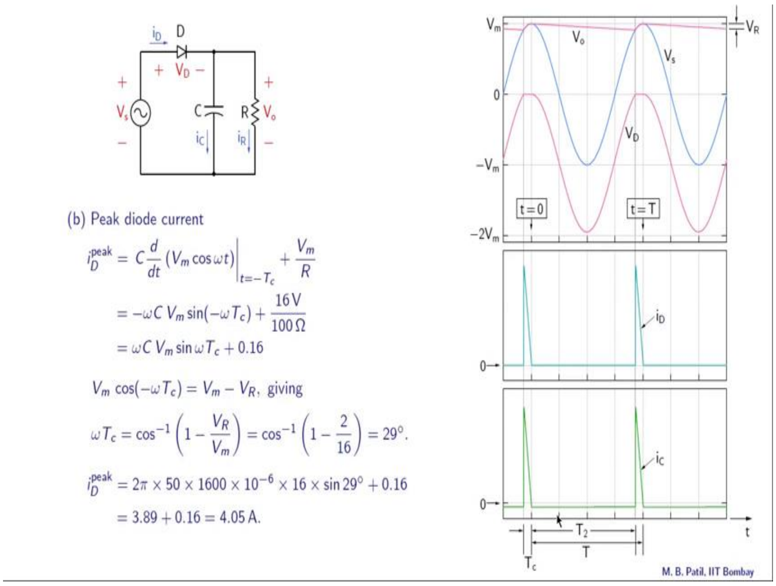

(b) Peak diode current

$$i_D^{\text{peak}} = C\frac{d}{dt}(V_m \cos\omega t)\bigg|_{t=-T_c} + \frac{V_m}{R}$$

$$= -\omega C\, V_m \sin(-\omega T_c) + \frac{16\,\text{V}}{100\,\Omega}$$

$$= \omega C\, V_m \sin\omega T_c + 0.16$$

$V_m \cos(-\omega T_c) = V_m - V_R$, giving

$$\omega T_c = \cos^{-1}\left(1 - \frac{V_R}{V_m}\right) = \cos^{-1}\left(1 - \frac{2}{16}\right) = 29^\circ.$$

$$i_D^{\text{peak}} = 2\pi \times 50 \times 1600 \times 10^{-6} \times 16 \times \sin 29^\circ + 0.16$$

$$= 3.89 + 0.16 = 4.05\,\text{A}.$$

Let us now talk about the peak diode current. And to begin with let us remind ourselves of what i D looks like as a function of time, this plot here i D is nonzero only in this short interval marked as T c here, and otherwise it is 0. Now what is i D given by i D is given by i C plus i R; i R is more or less a constant because b o is constant approximately equal to V m. So, therefore, i R is V m by R and that is already known. What we need to now worry about is this capacitor current. And in particular, we need to worry about i C in the charging interval this interval here, because that is when this diode is conducting. Now, when the diode is conducting this voltage is 0, and therefore V c is the same as V s. So, therefore, the capacitor current i C which is c d V c dt is the same as c d V s dt. And the diode current is maximum when i C is maximum, because this current here is a constant.

So, we need to now answer two questions: one when is this derivative d V s d t maximum because c times that derivative is going to give us the maximum capacitor current and therefore the maximum diode current. And second what is the value of that derivative. And to answer those questions, let us once again look at the interval in which the capacitor is getting charged that is when the diode is conducting that is this interval here. And we are interested in finding the maximum value of d V s dt in this interval. So, this is how V s varies and clearly the maximum slope happens at this point right here and that is T equal to minus T c this is over T equal to 0. So, this is T equal to minus T c. So, what we need to do now is to find the slope d V s dt at this T equal to minus T c, so that is the equation for the peak diode current c d V s dt evaluated at T equal to minus T c plus the register current i R, which is approximately constant V m by R.

V s is V m cos omega t and that is because we have taken our time equal to 0 at this point here. So, when we differentiate, we get this result this 0.16 is coming from i R V m by R. And the first term the capacitor current is omega times C times V m sine omega T c. Now, in this expression we know everything omega c V m except T c. So, T c is something that we need to figure out. And let us see how that can be done approximately and that is from this equation. So, what can you say about the value of V o at this point T equal to minus T c that is nothing but V m minus V R because the difference between these two is V R and therefore at T equal to minus T c V o is V m minus V R. So, we can use that condition to get V m cos of minus omega T c equal to V m minus V R and that gives us an idea of what T c should be. So, omega T c then a is cos inverse of 1 minus V R by V m; V R is 2 volts V m is 16 volts, and omega T c then turns out to be 29 degrees.

When you use your calculator to compute this omega T c, you need to be a little careful because your calculator may give this value in radians, and then you need to convert that into degrees or you can you keep it in radians, but then be aware that it is in radians. Now, we have got omega T c now it is a simple matter of substituting omega T c here and this current then can be evaluated, like that. So, this is the peak diode current the first part here is 2 pi times f or omega f is 50 hertz, C is 1600 microfarads, we have already computed that earlier, V m is 16 volts then followed by sine of 29 degrees, that 29 degrees is coming from omega T c.

So, all of this turns out to be 3.89 to that we have to add the register current 0.16 ampere. So, it is 3.89 plus 0.16 about 4 amperes. One important thing to note here is that the maximum capacitor current is much larger than the average register current and that we can also see from this figure here that is the maximum capacitor current and this value here the one which is going negative is the average register current.

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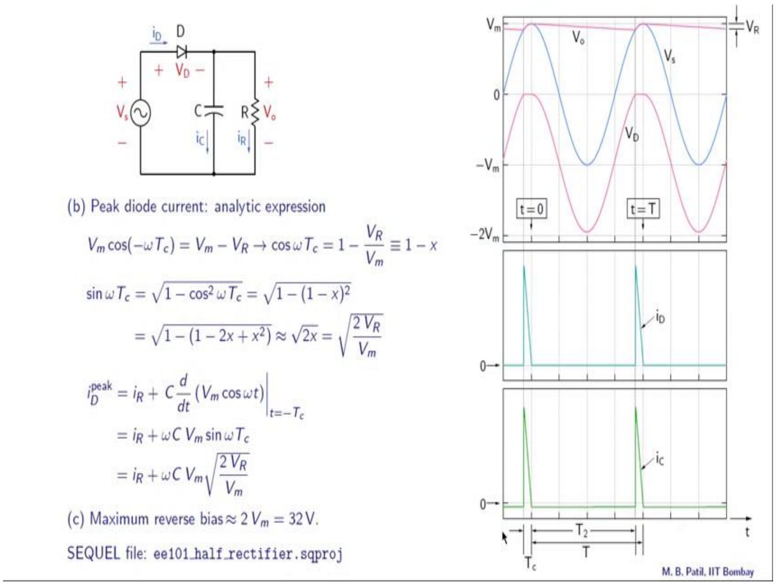

(b) Peak diode current: analytic expression

$$V_m \cos(-\omega T_c) = V_m - V_R \rightarrow \cos\omega T_c = 1 - \frac{V_R}{V_m} \equiv 1 - x$$

$$\sin\omega T_c = \sqrt{1-\cos^2\omega T_c} = \sqrt{1-(1-x)^2}$$

$$= \sqrt{1-(1-2x+x^2)} \approx \sqrt{2x} = \sqrt{\frac{2\,V_R}{V_m}}$$

$$i_D^{\text{peak}} = i_R + C\frac{d}{dt}(V_m\cos\omega t)\Big|_{t=-T_c}$$

$$= i_R + \omega C\,V_m \sin\omega T_c$$

$$= i_R + \omega C\,V_m\sqrt{\frac{2\,V_R}{V_m}}$$

(c) Maximum reverse bias $\approx 2\,V_m = 32\,$V.

SEQUEL file: `ee101_half_rectifier.sqproj`

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Let us now obtain an analytic expression for the peak diode current. We begin with this condition the source voltage at minus T c is equal to V m minus V R at that point we have seen that in the last slide and that gave us an expression for cos omega T c as 1 minus V R by V m. And earlier we calculated omega T c using a calculator got a numerical value for omega T c and then proceeded. Now, we want to do the same thing analytically.

So, let us write V R by V m as x, what is x in this case, V R is 2 volts V m a 16 volts. So, x is 2 by 16 or 1 by 8. So, x is small compared to 1; and we will use that. From cos omega T c, we can write sine omega T c as 1 minus cos squared omega T c square root of that, so that is 1 minus cos omega T c is 1 minus x, so square of that. And now let us expand this we get 1 minus 1 minus 2 x plus x squared this one cancels with that one and we get 2 x and there is a term in x squared over x is 1 by x. So, x squared is going to be 1 by 64 much smaller than x and therefore, we can ignore x squared compared to x. So, we end up with square root of 2 x, so that is square root of 2 times V R by V m.

And now you can use this result to get the peak diode current is i R plus C d V s dt evaluated at t equal to minus T c and as we have seen in the last slide this is the same as omega c V m sine omega T c. And the sine omega C T c can now come from this result and that is what we get finally. What about the maximum reverse bias, we have already seen earlier that it is going to be two times V m, where V m is the peak value of the input voltage. Let us quickly go through that argument once again V D is equal to V s minus V o. Now, V o is nearly constant that is equal to V m and therefore, V D is equal to V s shifted down by V m because V D is V s minus V o and that essentially is this waveform here. And the lowest value is minus 2 V m so that means the maximum reverse bias is 2 V m in this case it is 32 volts.

And we should keep in mind that all these calculations that we are doing are not simply academic curiosity, these numbers that is the average diode current the peak diode current, the maximum reverse bias are very important in practice, because that will decide what diode we are going to pick over here; and consequently the cost of the circuit that we are going to build.

Here is the sequel circuit file. So, you can run the simulation, and check whether you get all of these results given here. You can also change some of these component values for example, you can make this capacitance double of what it is now, predict what the new ripple voltage is going to be, what is the new peak diode current and so on. And then check with the simulation results whether your predictions are correct. In general, it is always good to at least have some idea of what to expect from your simulation, and not just run the simulation blindly and see what you get. Sometimes, your predictions may not be correct and that is even better because then you will figure out what mistake you made and chances are that you will not make the same mistake again.

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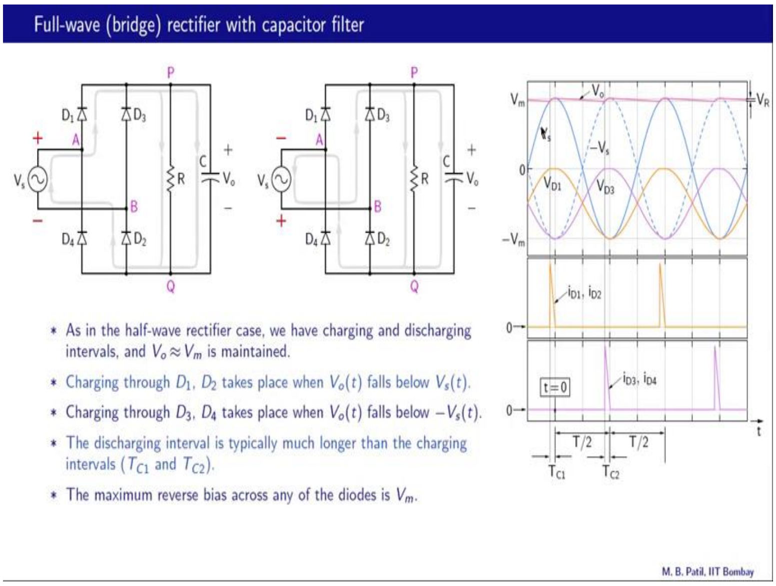

Full-wave (bridge) rectifier with capacitor filter

- As in the half-wave rectifier case, we have charging and discharging intervals, and $V_o \approx V_m$ is maintained.
- Charging through D_1, D_2 takes place when $V_o(t)$ falls below $V_s(t)$.
- Charging through D_3, D_4 takes place when $V_o(t)$ falls below $-V_s(t)$.
- The discharging interval is typically much longer than the charging intervals (T_{C1} and T_{C2}).
- The maximum reverse bias across any of the diodes is V_m.

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We now look at the full-wave or bridge rectifier with capacitor filter. And at this point, it may be a good idea to go back a few slides and revise the case of full-wave rectifier without a filter. What we will do now is make a few observations and then take up a numerical example. First as in the half-wave rectifier case, we have charging and discharging intervals and V o is approximately maintained at V m. So, let us look at the waveforms this is the input voltage V s that dashed curve is the negative of input voltage; V s curve goes from plus V m to minus V m and that is the output voltage, the one shown in the pink graph. And the output voltage is nearly constant, but there is a small ripple the difference between the maximum and minimum output voltage levels that is called V r.

When the input voltage V s goes beyond the output voltage D 1 and D 2 conduct and when minus V s goes beyond the output voltage D 3 and D 4 conduct. And once again the diode currents will turn out to be large compared to the load current. So, those points are listed here. The discharging interval is typically much longer than the charging intervals T c 1 and T c 2; this is T c 1 here. And in the T c 1 interval, D 1 and D 2 conduct and d three and d four are not conducting at that time; in the T c 2 interval, D 3 and D 4 conduct and D 1 and D 2 remain off. The maximum reverse bias across any of the diode is V m here is the diode voltage V D 1 and it is actually the same as V D 2. And we notice that it goes from 0 to minus V m. So, the maximum reverse bias across D

1 or D 2 is V m; this one is V D 3 the same as V D 4. And once again the maximum reverse bias across D 3 or D 4 is V m.

Now, this also that V D 1 or V D 2 is 0 when D 1 and D 2 conduct. Similarly, V D 3 and V D 4 are 0; and D 3 and D 4 conduct. So, we observe that there are many similarities between the half-wave rectifier, and full-wave rectifier with filter. The main difference is in the charging of the capacitor. In the half-wave rectifier, there was only one charging interval; in the full-wave rectifier, there are two charging intervals within one period. For example, if we go from t equal to 0 to t equal to T, this point here then there is one interval in which charging takes place through i D 3 and i D 4 and another interval in which charging takes place through i D 1 and i D 2.

Apart from that things are very similar there is a charging interval in which the output voltage that is the capacitor voltage follows the input voltage, and the charge on the capacitor gets replenished. And when the input voltage goes down, the charging stops and now the capacitor discharges through the load register like that.

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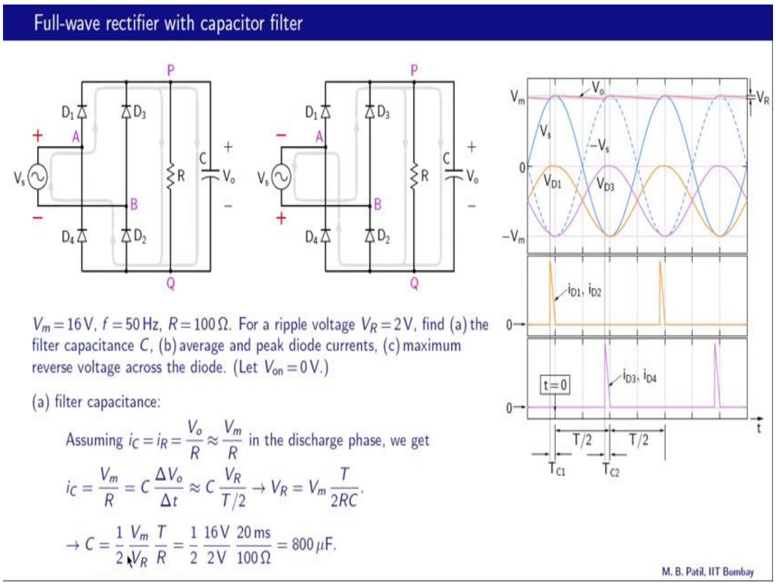

Full-wave rectifier with capacitor filter

$V_m = 16\,\text{V}$, $f = 50\,\text{Hz}$, $R = 100\,\Omega$. For a ripple voltage $V_R = 2\,\text{V}$, find (a) the filter capacitance C, (b) average and peak diode currents, (c) maximum reverse voltage across the diode. (Let $V_{on} = 0\,\text{V}$.)

(a) filter capacitance:

Assuming $i_C = i_R = \frac{V_o}{R} \approx \frac{V_m}{R}$ in the discharge phase, we get

$$i_C = \frac{V_m}{R} = C\,\frac{\Delta V_o}{\Delta t} \approx C\,\frac{V_R}{T/2} \rightarrow V_R = V_m\,\frac{T}{2RC}.$$

$$\rightarrow C = \frac{1}{2}\,\frac{V_m}{V_R}\,\frac{T}{R} = \frac{1}{2}\,\frac{16\,\text{V}}{2\,\text{V}}\,\frac{20\,\text{ms}}{100\,\Omega} = 800\,\mu\text{F}.$$

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Here is a numerical example, and we have kept the numbers the same as the half-wave rectifier circuit. The maximum voltage input voltage is V m equal to 16 volts, frequency is 50 hertz again, the load resistance is 100 ohms and for the same ripple voltage of 2 volts, we want to find the filter capacitance, the average and peak diode currents and the maximum reverse voltage appearing across each diode. Once again, we will assume the voltage drop across the diodes to be 0 volts and when they conduct.

Let us look at the filter capacitance first. When the diodes are not conducting we have R and C, the capacitor discharging through the load resistor, and we are talking about this interval or this interval in these intervals the capacitance current is the same as the resistor current in magnitude. So, i C is equal to i R, and since V o is nearly constant we have i C equal to V m by R the capacitor current is given by C d V dt. And since this variation is once again linear, we can replace d V dt by delta V o by delta t. delta V o is the same as ripple voltage here; and delta t is the duration of the discharge interval. So, let us take this interval for example.

So, that is from t equal to 0 to this point here which is nearly equal to T by 2. So, we substitute for delta t T by 2 and then we get V R equal to V m T by 2 R C. And capacitance is now V m by V R T by T divided by 2, this half is new here, it was not there in the half-wave rectifier. And when we substitute all the numbers, we get 800 microfarads.

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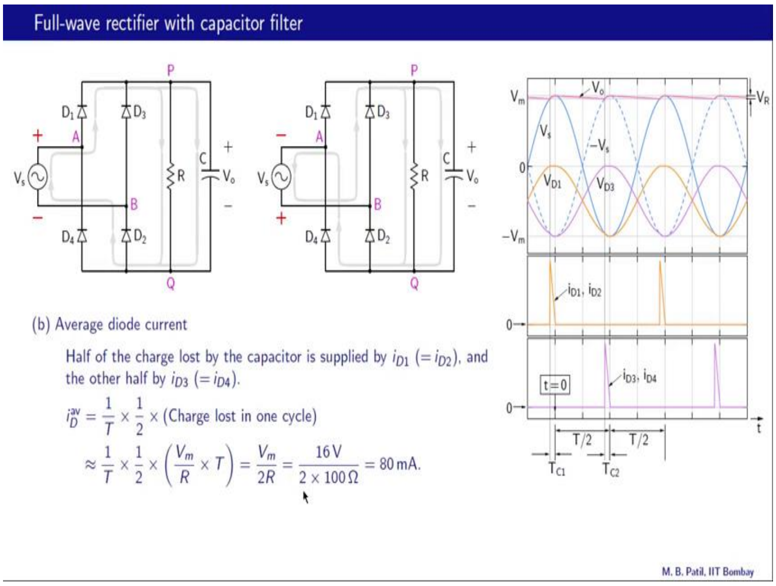

Full-wave rectifier with capacitor filter

(b) Average diode current

Half of the charge lost by the capacitor is supplied by i_{D1} ($= i_{D2}$), and the other half by i_{D3} ($= i_{D4}$).

$$i_D^{av} = \frac{1}{T} \times \frac{1}{2} \times (\text{Charge lost in one cycle})$$

$$\approx \frac{1}{T} \times \frac{1}{2} \times \left(\frac{V_m}{R} \times T\right) = \frac{V_m}{2R} = \frac{16\,\text{V}}{2 \times 100\,\Omega} = 80\,\text{mA}.$$

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What about the average diode current. Now, as we observed earlier half of the charge lost by the capacitor is supplied by i D 1 or i D 2, because D 1 and D 2 conduct at the same time; and the other half is supplied by D 3 and D 4 that is this situation here. And therefore, we can write the average diode current let us say through D 1 is 1 over T times half of the charge lost in one cycle; and the charge lost in one cycle is easy to calculate, it is simply the register current the average current times the time period T. So, this T cancels and we end up with V m by 2 R, so that 16 volts by 2 times 100 ohms which is 80 milliamps.

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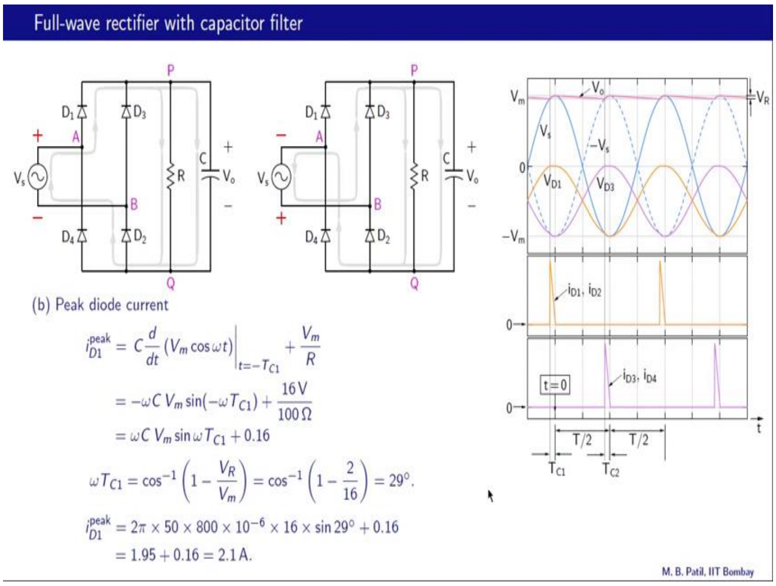

Full-wave rectifier with capacitor filter

(b) Peak diode current

$$i_{D1}^{\text{peak}} = C\frac{d}{dt}(V_m\cos\omega t)\Big|_{t=-T_{C1}} + \frac{V_m}{R}$$

$$= -\omega C\,V_m\sin(-\omega T_{C1}) + \frac{16\,\text{V}}{100\,\Omega}$$

$$= \omega C\,V_m\sin\omega T_{C1} + 0.16$$

$$\omega T_{C1} = \cos^{-1}\left(1-\frac{V_R}{V_m}\right) = \cos^{-1}\left(1-\frac{2}{16}\right) = 29^\circ.$$

$$i_{D1}^{\text{peak}} = 2\pi\times 50\times 800\times 10^{-6}\times 16\times\sin 29^\circ + 0.16$$

$$= 1.95 + 0.16 = 2.1\,\text{A}.$$

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Let us look at the peak diode current and let us say we are talking about i D 1. So, i D 1 is maximum here. And as before this is the sum of i C and i R, because the current it is applying both the resistor and the capacitor. And the resistor current of course, we already know its approximately V m by R, and the capacitance current like in the half-wave case is the slope of this is given by the slope of this input voltage, and it is maximum at t equal to minus T c 1, so that is what we get.

And the next step now is to find T c 1. And we do that following the same procedure that we used for the half-wave rectifier that is we know that at this time, which is minus T c 1. The output voltage is V m minus V R and therefore, we can get omega T c 1 as cos inverse of 1 minus V R by V m, which turns out to be 29 degrees. So, now, we substitute this in the expression for i D peak that is omega 2 pi times 50 C - 800 microfarads V m- 16 and sine of 29 degrees that is omega T c 1, so all that is the capacitor current at minus T c 1 to that we add the resistor current 0.16 amp and we get a total of 2.1 ampere. So, that is the peak diode current for D 1 and by symmetry it would be the same for all the diodes.

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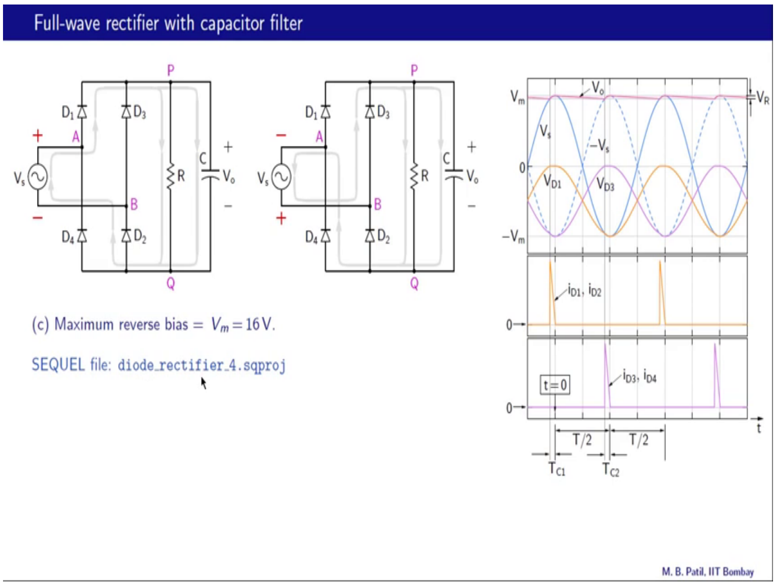

Full-wave rectifier with capacitor filter

(c) Maximum reverse bias $= V_m = 16\,\text{V}$.

SEQUEL file: `diode_rectifier_4.sqproj`

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Let us look at the maximum reverse bias that each diode comes across. Let us take D 1 as an example. Here is V D 1 when D 1 and D 2 conduct of course, V D 1 is 0, and it goes through its maximum negative point when D 3 and D 4 conduct in this interval here. So, let us look at the circuit when D 3 and D 4 conduct this circuit D 3 is conducting. So, no voltage drop there, D 4 is conducting so no voltage drop here. So, the negative end of D 1 is at node P in the positive end is at the same as is at the same potential as node Q.

So, the difference between P and Q which is V m appears across D 1 and that of course, is the reverse bias because V P is higher than V Q, so that is how we get the maximum reverse bias across d one it happens to be V m that is 16 volts. And that is true also for the other diodes. The sequel file for this circuit is available and you can run the simulation.

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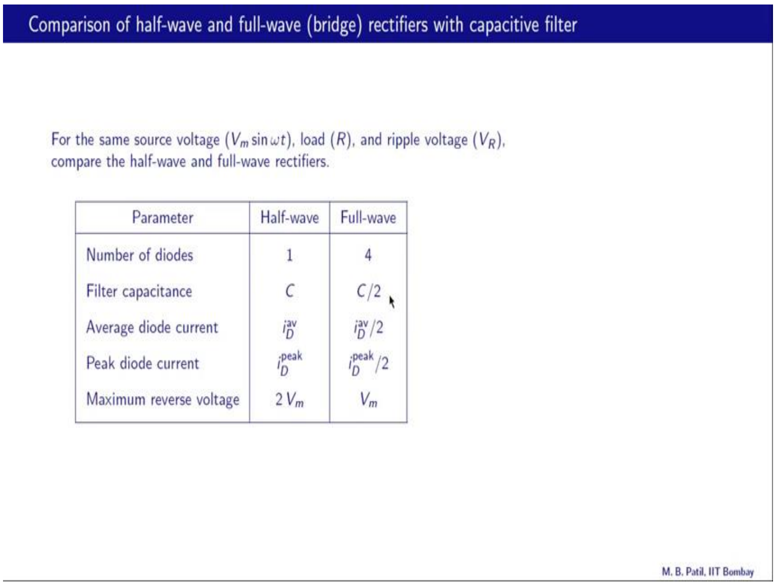

Comparison of half-wave and full-wave (bridge) rectifiers with capacitive filter

For the same source voltage ($V_m \sin \omega t$), load (R), and ripple voltage (V_R), compare the half-wave and full-wave rectifiers.

Parameter	Half-wave	Full-wave
Number of diodes	1	4
Filter capacitance	C	$C/2$
Average diode current	i_D^{av}	$i_D^{av}/2$
Peak diode current	i_D^{peak}	$i_D^{peak}/2$
Maximum reverse voltage	$2\,V_m$	V_m

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Let us now compare the half-wave and full-wave rectifier with the capacitor filter. For the same source voltage that is same V m, same load resistance R, and the same ripple voltage. And if you recall in the numerical examples that we looked at we did follow these conditions that is we kept V m R and V R the same for the half-wave example and full-wave example. So, this table shows the comparison, number of diodes one in the half-wave case, 4 in the full-wave case.

Filter capacitance if this is C then this is C by 2; average diode current if this is i D average then this is i D average by 2. Peak diode current if this is i D peak then this is i D peak divided by 2. Maximum reverse voltage in this case it was 2 times V m in the full wave case it was V m. So, although the full wave rectifier requires four diodes as opposed to one here, it requires a smaller capacitor and also the diode specs are less stringent in the full-wave case and that is an advantage.

To summarize, we have carried out calculation of ripple voltage, average diode current and peak diode current for the full-wave rectifier. We then compared the half-wave and full-wave rectifier circuits with respect to different criteria; and found that the full-wave rectifier requires less stringent specifications for the components namely the filter capacitor and the diodes that is all for now.

See you next time.