

Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute Technology, Bombay

Lecture - 02
Superposition

Welcome back to Basic Electronics. In this lecture, we will look at the Principle of Superposition. Superposition is very useful in circuit analysis, when there are multiple sources in the circuit. We will consider a few examples to illustrate how superposition is applied in practice. Finally, we will look at why superposition works. So, let us start.

(Refer Slide Time: 00:43)

Superposition

- * Consider a circuit made up of elements of the following types:
 - Resistor ($V = R I$)
 - VCVS ($V = \alpha V_c$)
 - VCCS ($I = G V_c$)
 - CCVS ($V = R I_c$)
 - CCCS ($I = \beta I_c$)
- and independent sources of the following types:
 - Independent DC voltage source ($V = V_0$ (constant))
 - Independent DC current source ($I = I_0$ (constant))
- * Such a circuit is linear, and we can use superposition to obtain its response (currents and voltages) when multiple independent sources are involved.
- * Superposition enables us to consider the independent sources one at a time (with the others deactivated), compute the desired quantity of interest in each case, and get the net result by adding the individual contributions. This procedure is generally simpler than considering all independent sources simultaneously.
- * What do we mean by "deactivating" an independent source?
 - Deactivating an independent current source $\Rightarrow I_0 = 0$, i.e., replace the current source with an open circuit.
 - Deactivating an independent voltage source $\Rightarrow V_0 = 0$, i.e., replace the voltage source with a short circuit.

M. B. Patil, IIT Bombay

Let us consider a circuit made up of the following elements, resistor for which V is R times I , voltage controlled voltage source for which V is α times V_c , V_c is the controlling voltage and α is the dimensionless constant. Voltage controlled current source for which I is G times V_c ; V_c is again a controlling voltage and G has units of conductance. Current controlled voltage source for which V is R times I_c , I_c is a controlling current and R has got units of resistance. And finally, current controlled current source for which I is β times I_c ; I_c is again a controlling current, and β is a dimensionless constant.

What is common in all of these equations? They are all linear that means a quantity on the left such as V or I depends on the quantity on the right such as I here or V_c here

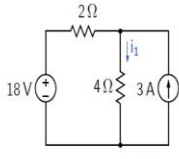
etcetera in the linear portion and that is why when we have a circuit made up of these elements the superposition theorem can be applied. In addition to these elements, we have some independent sources independent DC voltage sources for which V is V_0 just a constant for example, V_0 could be 5 volts or 100 volts and independent DC current sources for which I is I_0 , again a constant I_0 could be 5 milli ampere for example, or 1 ampere.

Now, if we have a circuit of this kind with these elements and these sources such a circuit is a linear, and we can use superposition to obtain its response when multiple independent sources are involved. And what do we mean by the response of the circuit, what we mean is the currents and voltages in that circuit. The next question is how do we use superposition. Superposition enables us to consider the independent sources one at a time with the other sources deactivated. Compute the desired quantity of interest in each case such as a current or voltage, and then get the net result by adding the individual contributions. And the advantage of this procedure is that this procedure is generally simpler and considering all independent sources simultaneously.

Now, let us address this issue of deactivating an independent source. What do we mean by deactivating an independent current source? Let us look at the equation for a current source right there I equal to I_0 . So, to deactivate this current source all we need to do is to make I_0 equal to 0 and what does that mean; that means, no current is passing through that element that is replace the current source with an open circuit. What about deactivating an independent voltage source, here is the equation for voltage source, V equal to V_0 . So, we need to make V_0 equal to 0 that means we replace the DC voltage source with the short circuit, because a short circuit has no voltage drop or zero voltage drop across it. And therefore, we simply need to replace the voltage source with the short circuit.

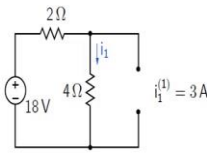
(Refer Slide Time: 04:56)

Example 1



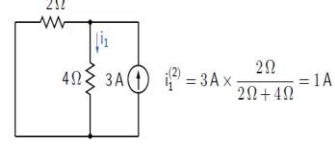
$i_1^{net} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4 \text{ A}$

Case 1: Keep V_s , deactivate I_s .



$i_1^{(1)} = 3 \text{ A}$

Case 2: Keep I_s , deactivate V_s .



$i_1^{(2)} = 3 \text{ A} \times \frac{2 \Omega}{2 \Omega + 4 \Omega} = 1 \text{ A}$

M. B. Patil, IIT Bombay

Let us now apply what we have learnt about superposition to this circuit here. Here we have two independent sources; one is a voltage source 18 volts, and other is a current source 3 amperes, and we want to find this current I_1 here. So, we will take these two sources one at a time. Let us start with the voltage source. So, we keep the voltage source and deactivate the current source and that is our case 1. Keep V_s deactivate I_s . And how do we deactivate the current source as we saw in the last slide, we need to make the current equal to 0 that is replace the current source with an open circuit. And if this case, it is easy to calculate R_1 , it is simply 18 volts divided by 2 plus 4 or 3 amperes, so that is our case 1.

Let us now look at case 2. In case 2, we keep the current source and deactivate the voltage source, so that is our current source. And we have deactivated the voltage source; that means, you have made the voltage here equal to 0 that is replaced the voltage source with a short circuit. And now what is i_1 in this case i_1 can be obtained using current division i_1 would be 3 amperes multiplied by 2 ohms divided by 6 ohms, so that is our i_1 3 amperes times 2 by 6 or 1 ampere. And note that we are used is superscripts this two indicates case 2 and this one indicates case 1. And now this is a final step, we simply add these two quantities i_1 in case 1 and i_1 in case 2 to obtain the net the value of i_1 and that turns to be 3 plus 1 or 4 amperes.

(Refer Slide Time: 07:14)

Example 2

$$v^{net} = v^{(1)} + v^{(2)} = 6 + 9 = 15V$$

Case 1: Keep V_s , deactivate I_s .

KVL: $-12 + 3i + 2i + i = 0$
 $\Rightarrow i = 2A, v^{(1)} = 6V.$

Case 2: Keep I_s , deactivate V_s .

KVL: $i + (6+i)3 + 2i = 0$
 $\Rightarrow i = -3A, v^{(2)} = (-3+6) \times 3 = 9V.$

(SEQUEL file: ee101.superposition.2.sqproj)

M. B. Patil, IIT Bombay

Here is our second example. We have two independent sources here; one is a voltage source 12 volts; and other is a current source 6 amperes. And apart from these two independent sources, we have resistors and we have this dependent source. And what kind of dependent source is this it is a voltage source and the voltage here is controlled by this current; if this current is i then the voltage is 2 times i . So, it is a current controlled voltage source.

Let us now use superposition to obtain this voltage V here, and how do we go about doing that, we have two independent sources the voltage source and the current source. We take these two sources one at a time compute V and then finally add the two values of V . Here is our case 1, we have kept their voltage source and deactivated the current source that means, replaced it with an open circuit like that. And now let us find V in this case. The important point note here is that we have left this dependent source as it is. And now how do we solve this circuit they have a loop here, and we can write the KVL equation for this loop and let us take voltage drop as positive for this purpose. Let us go like this.

What is this voltage drop is 1 ohms times i or just i up to that we come across raise of 12 volts, so that would be minus 12 because this is your raise, then we become across a voltage drop of 3 times i , so that is plus three times i and then another voltage drop of 2 times i . And when we add all of these terms we should get 0 like that. We can solve this

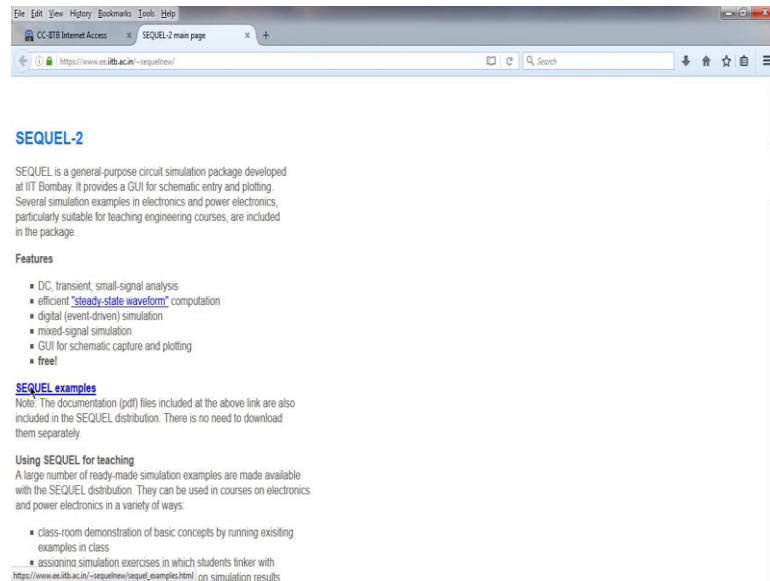
equation for i that gives us i equal to 2 ampere, and therefore, this voltage V is 2 amperes times 3 ohms or 6 volts. And note that we have used this superscript 1 here to indicate case 1.

Let us now consider the second case in which we keep the current source and deactivate the voltage source. So, here is our current source, and we have deactivated this voltage source that means, we have replaced it with the short circuit. Now, how do you obtain i or v in this case. There may be different ways of doing this; one thing that comes to mind is that this current through the three ohm resistance is i plus 6. And once you figure that out, we can write a KVL equation for this loop and then obtain i and therefore v .

Here is the KVL equation, we are going in this loop starting with this voltage drop 1 ohm times i or simply i the first on there, then 6 plus i times 3 that term there and then finally, a voltage drop of 2 times i across the current control voltage source that term there. And all of these terms must add up to 0, and that gives us i equal to minus 3 amperes. This minus sign means that the actual current is flowing in the opposite direction. Once we know this current i we can find the voltage here as 6 plus i times three ohms that is 6 minus 3 times 3 ohms or 9 volts, so that is our v with the super script 2 to indicate case 2. And now the net voltage this voltage here in the original circuit can be found simply by adding v_1 and v_2 and that gives us 6 plus 9 or 15 volts.

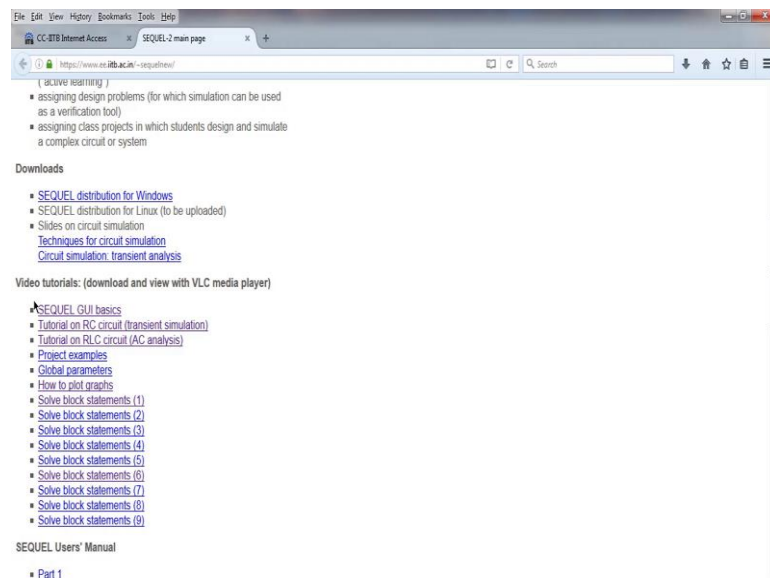
How do we know if our solution is correct? There are two ways of verifying that one we can find the net quantities. The net current here is i equal to 2 minus 3 that is minus 1 ampere. The net voltage we have already found that is 15 volts. So therefore, we know that 15 volts divided by 3 times that is 5 amperes would go like that. And similarly, we can find the voltage drops across each of these elements make sure that KVL is actually satisfied by the net quantities in this loop and also KCL is satisfied at this node. The second approach for verifying whether our solution is correct or not is to use circuit stimulation. Now, in this example and in fact, throughout this course we will use this circuit simulator called sequel which has been developed at IIT Bombay over the past few years. And the sequel circuit file for this particular example is given over here.

(Refer Slide Time: 13:37)



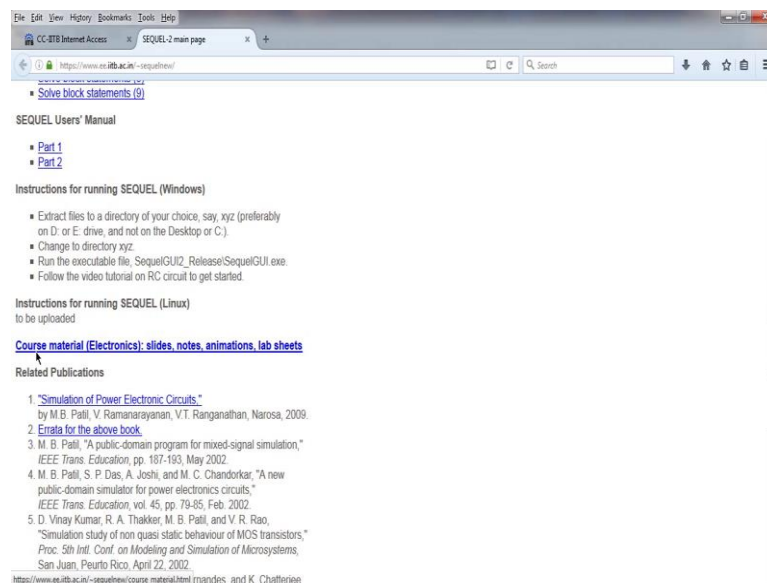
So, we can run this simulation and verify if our results are correct. Here is the URL for the sequel circuit simulator website, and that URL brings us to the sequel main page. There is the description about what sequel is about, what features it offers and this feature of course, is other important for our purposes. It is a free circuit simulator; you can just download and use it. Then this is a link for sequel examples. There is a large number of readymade examples and you can click here and find out more about that.

(Refer Slide Time: 14:21)



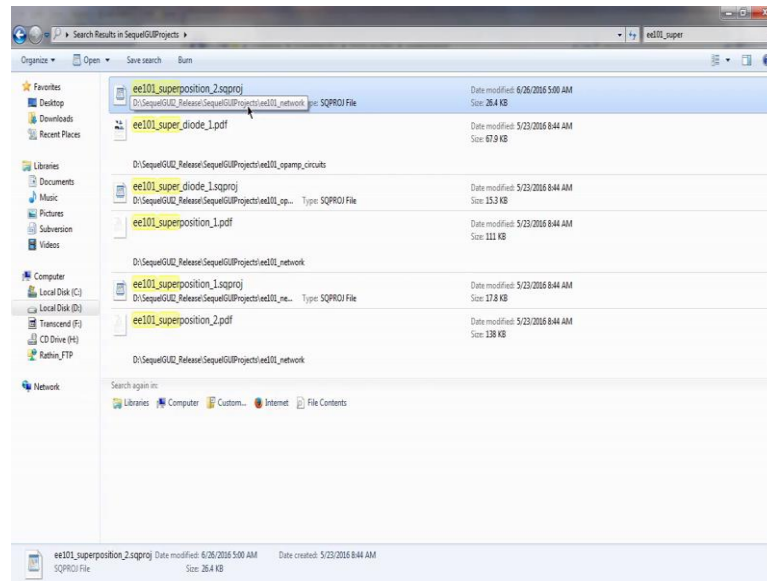
There is a link for downloading the program, and also there are several video tutorials available about how to use the circuit simulator, the basics of the graphics interface tutorials, project examples, how to plot graphs and so on. And these tutorials would be particularly relevant if you are going to make up a new project. On the other hand, if we are going to run an existing project for which the circuit file is already available then we do not really need to go through all of these video tutorials.

(Refer Slide Time: 15:15)



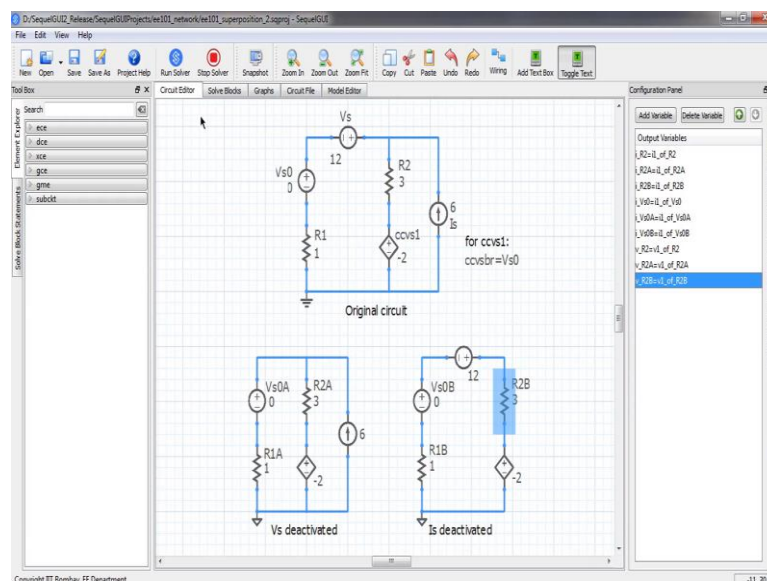
In addition, there is the sequel user's manual part 1 and part 2, and there is some course material as well. To run an existing circuit file, we note the name of that file. So, it is e 1 o 1 underscore superposition underscore 2. And we can look for this specific project and then done it, let us see how to do that. The circuit files are placed in various directories depending on which area that circuit belongs to and so the best way to find were a given circuit file is to search for it.

(Refer Slide Time: 16:10)



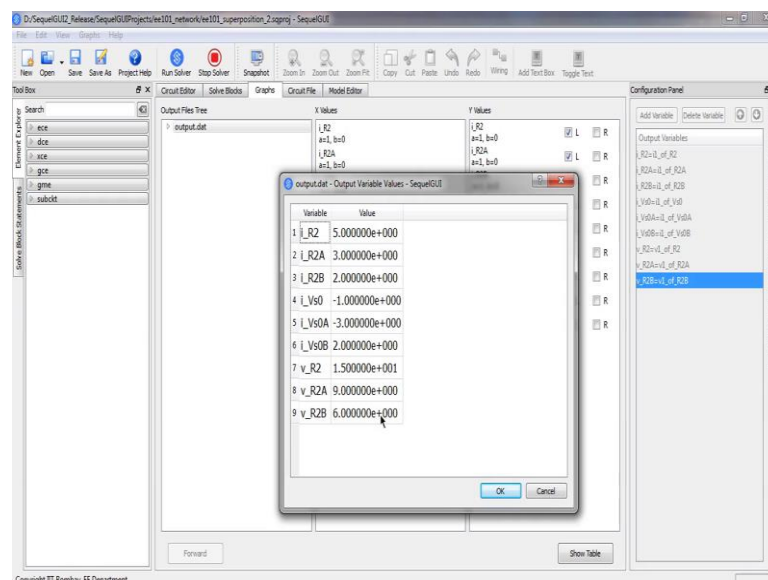
So, let us do that. So, e 1 o 1 superposition 2, this is the file we are looking for. And this file is in a directory called e e 1 o 1 underscore network under sequel GUI projects all right. So, let us remember that and open this particular circuit file. So, this is the executable file for the sequel GUI and that is under sequel GUI 2 release. So, we double click on that and then the simulator starts. And now, we look for the file of our interest; and as we found earlier it is in this directory called e e 1 o 1 network. So, let us open that and that is where that file is.

(Refer Slide Time: 17:21)



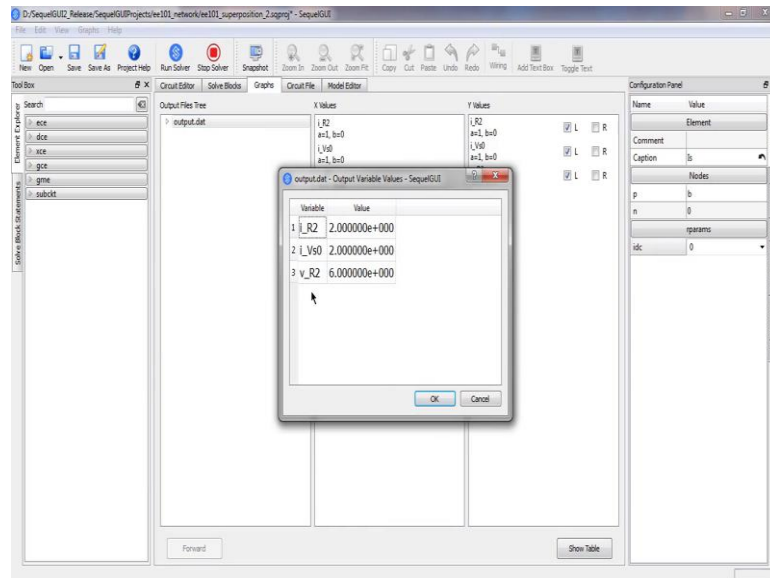
So, let us open that file that is what it looks like. So, this is our original circuit. It has got more the voltage source and the current source this circuit here is the circuit with the voltage source deactivated and this circuit here is the circuit with the current source deactivated. And notice that there is this fictitious voltage source here called $V_s 0$ and that is required because the current through that branch is used by this current controlled voltage source. We have defined some output variables on this right-hand side here. In particular, we are interested in this voltage across R_2 , V_{R2} here is that voltage in the original circuit, V_{R2A} is that is same voltage in this circuit with which V_s deactivated and V_{R2B} to be is the same voltage in this circuit with I_s deactivated.

(Refer Slide Time: 18:43)



Let us now run the simulation and look at these V_{R2} values, so that is the table of the output variables. And the net value of V is 15 volts here as we had computed; and the other two values are 9 volts and 6 volts. Instead of constructing these separate circuits with v is deactivated and i is deactivated we could just change the parameters in the original circuit to verify our calculations.

(Refer Slide Time: 19:26)



Let us do that. So, let us get rid of this part. And first we will keep the voltage source and deactivate the current source by making the current equal to 0. And now let us run the simulation. And now we find that V_{R2} is 6 volts as we had computed. Next, let us bring this back to 6 amperes, and deactivate the voltage source now make it 0. And now run the simulation again. And now we get 9 volts, once again as we expect. And finally, let us include both of these sources in the circuit, so make v_s is equal to 12 volts, i_s is equal to 6 amperes and see what happens. And now we get 15 volts as we would expect.

(Refer Slide Time: 20:38)

Example 3

Find V_1 using superposition.

V_{S1} alone:

$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

V_{S2} alone:

$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

$$V_1^{(net)} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{S1} + \frac{R_1}{R_1 + R_2} V_{S2}$$

M. B. Patil, IIT Bombay

Here is our next example we have two independent voltage sources V_{s1} and V_{s2} . And we want to find this voltage V_1 using superposition. So, let us take these two voltage sources one at a time. So, case 1 - V_{s1} alone with V_{s2} deactivated that is replaced with a short circuit. In this case, we get V_1 while voltage division as the voltage across R_2 which is R_2 divided by R_1 plus R_2 times V_{s1} , so that is our case 1 and we have marked that with this superscript 1 over here.

Here is a second case in which we have kept V_{s2} , but deactivated V_{s1} that is replaced it with a short circuit. And in this case, V_1 is the same as in the voltage across R_1 and once again using voltage division we have V_1 equal to R_1 divided by R_1 plus R_2 times V_{s2} like that. And finally, our net value of V_1 is simply the addition of V_1 in case 1 and V_1 in case 2 and that gives us this expression over here.

(Refer Slide Time: 22:05)

Example 3 (again)

Find V_1 using superposition.

V_{s1} alone:

$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{s1}$$

V_{s2} alone:

$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{s2}$$

Net voltage:

$$V_1^{(net)} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{s1} + \frac{R_1}{R_1 + R_2} V_{s2}$$

M. B. Patil, IIT Bombay

This is our next example and it is essentially the same as our previous example except we are going to treat it a little differently and that is by designating one of the nodes this node as the reference node of the ground node. And this is indeed a common practice in electronic circuits; it makes things much more convenient as we will see. So, this node voltage is a 0 volts, because that is the reference node. With respect to that node voltage, the voltage at this node would be V_{s1} ; the voltage at this node would be V_1 , and the voltage at this node would be V_{s2} . So, we can read all these circuit in which we do not

bother to show explicitly these voltage sources, and we do not even show the ground node it is all implicit now.

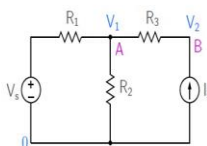
We understand when we draw a circuit diagram like this that there is a reference node which is taken as 0 volts. And with respect to that reference voltage, we have this node voltage as V_s , this as V_1 and this as V_2 . And obviously, this circuit is much simpler to look at as compared to the original circuit and that is why it is more convenient.

Now, let us use superposition to obtain V_1 in this circuit. What we will do is take V_s first take V_2 as 0. And what is the meaning of V_2 equal to 0 that means, this node voltage would be the same as the voltage at the reference node or the ground node that means, it is 0 volts and that is shown using the ground symbol like that. And now in this case, V_1 is given by voltage division essentially it is the voltage between this node and the ground node. And this V_s is the voltage between this node and the ground node and therefore, we can use voltage division. So, V_1 is the voltage across R_2 which is R_2 divided by R_1 plus R_2 times V_s and that is our V_1 in case 1.

And similarly in the second case in which we keep V_2 , but deactivate V_s that means, we connect this node to the reference node we get V_1 as R_1 divided by R_1 plus R_2 times V_2 and that is over V_1 in case two. And finally, we get the net V_1 for the original circuit as V_1 in case 1 plus V_1 in case two and that is this expression which is the same as the one we have seen earlier.

(Refer Slide Time: 25:19)

Superposition: Why does it work?



KCL at nodes A and B (taking current leaving a node as positive):

$$\frac{1}{R_1}(V_1 - V_s) + \frac{1}{R_2}V_1 + \frac{1}{R_3}(V_1 - V_2) = 0,$$

$$-I_s + \frac{1}{R_3}(V_2 - V_1) = 0.$$

Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

i.e., $\mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$

M. B. Patil, IIT Bombay

We have seen that superposition works at least for the examples we have considered so far. Now, let us try to understand why it should be expected to work, and we will do that with the help of this specific example, but our findings are really much more general. So, here we have two independent sources voltage source and a current source. And what we will do is to take this node as the reference node. So, with respect to that this node voltage is V_s this voltage node is V_1 , and this one is V_2 . And now let us write the KCL equations at node A and at node B; and then doing that we will take a current leaving a node as positive.

So, let us look at first equation here. We have three currents at node A, this current and this current. And what is this current it is $P_1 - P_s$ divided by R_1 . And since that current is leaving this node that is comes with the plus sign that is this term over here. What about this current that is $P_1 - 0$ divided by R_2 . The second term and this current is $V_1 - V_2$ divided by R_3 that is the third term. And similarly at node B, we have two currents I_s and this current. Now I_s is entering the node here, and therefore that comes with the negative sign like that and this current which is leaving the node is $V_2 - V_1$ divided by R_3 and that is the second term over here.

So, these are our KCL equations and now let us see what we can do with that. Let us rewrite the KCL equations in a matrix form. And in doing that let us use these definitions the conductance G_1 for example, is $1/R_1$; similarly G_2 would be $1/R_2$ and so on. When we do that we get this matrix equation here. And let us check that this first equation does indeed correspond to the first row over here. What is the coefficient of V_1 in this equation, it is $G_1 + G_2 + G_3$, and that is why we have $G_1 + G_2 + G_3$ here. What is the coefficient of V_2 , it is $-G_3$ and that is why we see $-G_3$ over here. And this term $-V_s \times 1/R_1$ does not depend on V_1 or V_2 . And therefore, we take it to the other side. And in doing that it becomes $+G_1 \times V_s$ and that is why we have $G_1 \times V_s$ over here. And similarly you can verify that the second equation here does correspond to our second KCL equation.

Now, let us write this matrix as matrix A. So, we have an equation of the form $A \times$ equal to B, where x is our column vector consisting of V_1 and V_2 and B is the right hand side vector consisting of $G_1 V_s$ and I_s . And from this matrix equation, we can obtain V_1 V_2 as A^{-1} times our RHS vector.

(Refer Slide Time: 29:28)

Superposition: Why does it work?

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}.$$

We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix}.$$

The first vector is the response due to V_s alone (and I_s deactivated).
 The second vector is the response due to I_s alone (and V_s deactivated).
 All other currents and voltages are linearly related to V_1 and V_2 .
 \Rightarrow Any voltage (node voltage or branch voltage) or current can also be computed using superposition.

M. B. Patil, IIT Bombay

So, here is our V_1, V_2, \mathbf{A} inverse times our RHS vector. Let us now write an inverse as this two by two matrix $m_{11} \ m_{12} \ m_{21} \ m_{22}$. And let us bring this G_1 inside this matrix now and we do that by multiplying m_{11} and m_{21} by G_1 . So, finally, our $V_1 \ V_2$ is given by this matrix multiplied by this column vector which consists of the two source terms V_s and I_s . And now this column vector can be written as a sum of two column vectors $V_s \ 0$ and $0 \ I_s$. And when we do that we get this following equation and now we should be able to see while superposition works at least for this circuit. So, this is our first column vector coming out of this V_s, I_s column vector. And this is the second column vector and this first term is nothing but our V_1, V_2 in case 1 denoted here by this superscript one. And this second term is nothing but our V_1, V_2 in case 2.

In other words, the first vector this one is the response of the circuit due to V_s alone and I_s is deactivated and that is why we have 0 over here. And the second vector, this one is the response due to I_s alone with V_s deactivated, and that is why we have 0 over here. Now turns out that all other currents and voltages in the circuit are linearly related to V_1 and V_2 , because this is the linear circuit. And therefore, any voltage that is any node voltage or any branch voltage or any current in the circuit can also be computed using superposition and this is indeed a very powerful theorem, and we will find many uses of this theorem.

To summarize, we have seen how superposition can be used in circuit analysis. In electronics these technique is very useful as we will discover later in this course that is all for now. See you next time.