Basic Electronics
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Lecture – 11
RC/RL Circuits in Time Domain (Continued)

Welcome back to Basic Electronics. In the last lecture we got started with the series RC circuit with a periodic piecewise constant input namely a square wave. We will now continue with that example and work out the steady state solution; we will also learn about the meaning of charge conservation for a capacitor and look at its implications for this specific example. So, let us begin.

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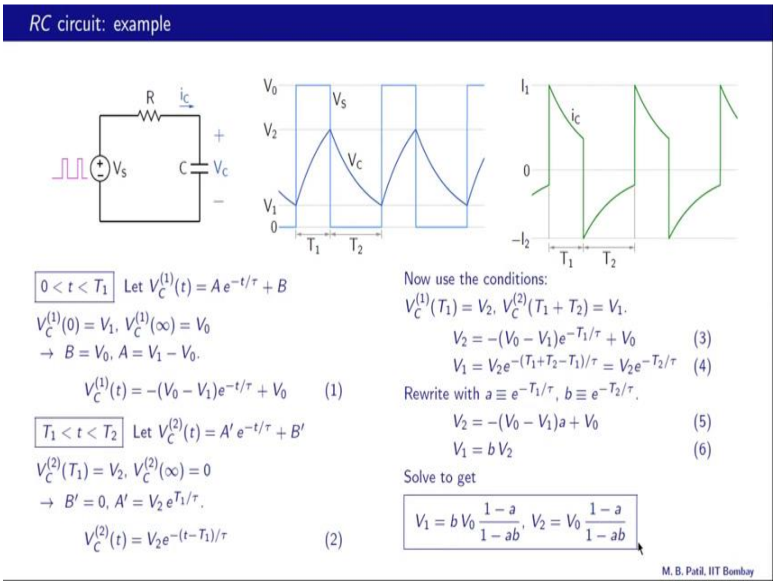

In order to find these constants A and B, we need 2 conditions on V c and phase one; condition number 1 at t equal to 0 this time point here, V c is equal to V 1 what about condition number 2? Let us look at this interval and this interval the input voltage V s is a is equal to V 0, and the capacitor has started charging; now in this phase the circuit is not aware that this transition is going to take place and therefore, its response would be as if this voltage did not change and if that were the case then this V c would finally reach V 0 as t tends to infinity. So, V c at t equal to infinity is equal to V 0, and that gives us the second condition on V c.

So, these are the 2 conditions you have; at t equal to 0 V c is V 1, and t equal to infinity V c is V 0. Now using these conditions we can find A and B; B turns out to be V 0, and a turns out to be V 1 minus V 0. So, putting this back in this equation we get V c in phase 1 equal to minus V 0 minus V 1 times e raised to minus t by tau plus V 0, note that this number a is positive because V 0 is larger than V 1.

Now, let us look at the T 2 phase, and let V c in the T 2 phase denoted by this superscript 2 over here A prime e raise to minus t by tau plus B prime. To find these constants A prime and B prime, we need 2 conditions on V c in phase 2, one of the conditions is that at t equal to T 1, V c is equal to V 2 and the second condition is that as t tends to infinity, this V c would have gone to 0. So, these are the 2 conditions that we can use and with these conditions we get B prime equal to 0, and A prime equal to V 2 e raise to T 1 by tau and when you substitute these back into this equation then you get V c in the T 2 phase equal to V 2, A raised to minus t minus T 1 by tau.

So, these are the 2 equations that describe V c of t, the first equation describes V c in this interval and the second equation describes V c in this interval and notice that our job is not done yet, because we have these unknowns V 1 and V 2 here and we need to figure them out and to find V 1 and V 2 we can use these conditions here, what did they say? V c in phase 1 at t equal to T 1 is equal to V 2; this is V c e in phase 1 and at t equal to T 1 that is this time point here we have V c equal to V 2. So, that is the first condition over here. Second condition V c in the T 2 phase at T 1 plus T 2 is equal to V 1 this is our V c in the T 2 phase, and T 1 plus T 2 is this point here, so what this equation says is that this V c at t equal to T 1 plus T 2 is equal to V 1.

The first condition here gives us this equation 3, let us see how. What is this condition it says that V c in the T 1 phase at t equal to T 1 should be equal to V 2, what is V c in the T 1 phase that is given by this equation 1 here. So, we substitute V 2 over here and we substitute t equal to T 1 and that gives us this equation it is V 2 equal to minus V 0 minus V 1, times e raised to minus T 1 by tau plus V 0. The second condition namely V c in the T 2 phase at T 1 plus T 2 equal to V 1, gives us this equation here let us see how. What is V c in the T 2 phase that is given by equation 2 here. So, what we do is substitute V 1 over here and for t we put T 1 plus T 2 and then we get V 1 equal to V 2, e raised to minus T 1 plus T 2 minus T 1 the whole thing divided by tau. This T 1 cancels out and then we get V 1 equal to V 2 e raised to minus T 2 by tau.

To simplify the algebra let us define 2 quantities; a equal to e raised to minus T 1 by tau and b equal to e raised to minus T 2 by tau; and using these a and b we can rewrite equations 3 and 4 as follows. Equation 3 can be written as V 2 equal to minus V 0 minus V 1, times e raised to minus T 1 by tau now this quantity is the same as a so therefore, a comes here plus V 0. And equation 4 can be written as this one here V 1 is equal to V 2 times e raised to minus T 2 by tau and this quantity is defined as b. So, therefore, we have B 1 equal to b times V 2.

And now we can solve equations 5 and 6 to obtain V 1 and V 2; and the results are written over here. Now if we know r and c we can calculate tau, tau is equal to r times c and if you know T 1 and T 2 then a and b can be calculated. So, therefore, this entire factor is known and if you know V 0 that is the maximum value of the input voltage, then V 1 and V 2 can be computed. Once we know V 1 and V 2 we have the complete picture for V c in the T 1 phase and V c in the T 2 phase.

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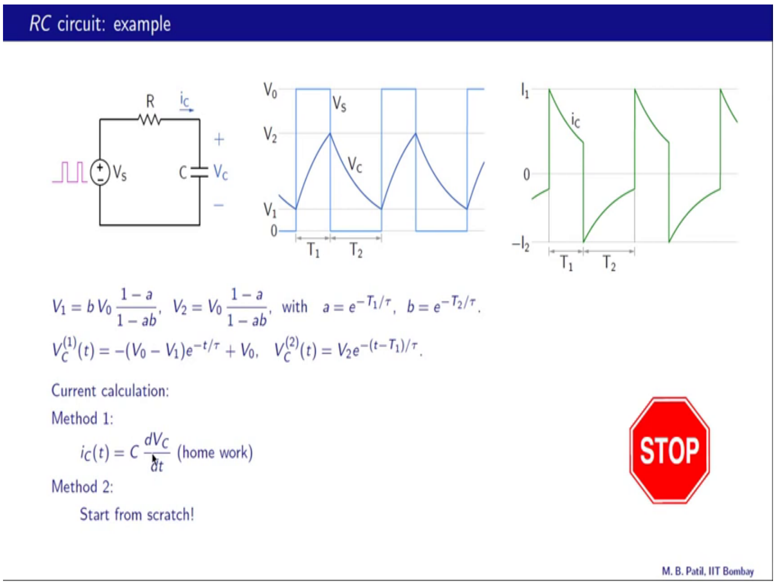

Here the results are summarized for the capacitor voltage; we have V 1 and V 2 here in terms of a and b, where a and b are given over here and once we know V 1 and V 2 we can find V c in the T 1 phase using this expression, and V c in the T 2 phase using this expression.

Let us now look at the current calculation i C of t and once again this will have 2 parts i C in the T 1 phase, and i C in the T 2 phase how do we go about this there are 2 ways of obtaining the current; one we already know the capacitor voltage in the T 1 phase as well as in the T 2 phase, all we need to do is to differentiate that multiply that by C and then we get i C of t now this is something that you must do as homework we will not do that here.

The second method is to start from scratch pretend that we do not have these results and we start from the beginning and see what we get. And in doing that we will definitely learn something more, let us proceed further now the stop sign has come now, what is it saying? It is saying that there is something to be completed and that is this homework over here. So, you really should stop the video at this point compute i C in the T 1 phase and also in the T 2 phase by differentiating the capacitor voltage in the T 1 phase and in the T 2 phase and once you have done with that you can get back to the video.

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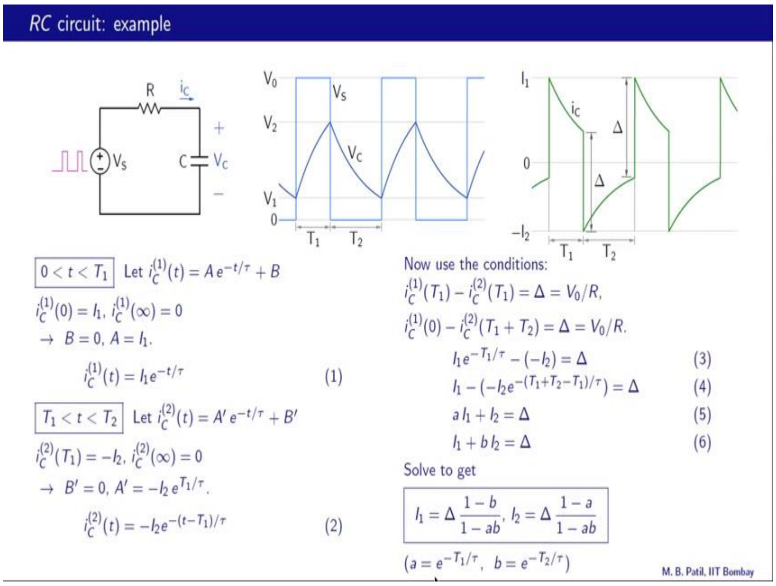

Let us begin with i C in the T 1 phase this part, and let i C be a e raise to minus t by tau plus b and that is i C in the T 1 phase and that is denoted by this superscript 1 over here. Our job is to find A and B and for that we need 2 conditions on the capacitor current. One condition is obvious at t equal to 0 we have i C equal to i 1; what about the second condition? The second condition comes from the fact that if this change did not happen; that means, if this input voltage continued to be b 0 then this i C would have gone eventually to 0. So, that gives us the second condition namely i C at infinity equal to 0. So, these are the 2 conditions we can use to find A and B.

So, B turns out to be 0 and A turns out to be I 1 and putting this back in this equation now we get i C in the T 1 phase equal to I 1 e raise to minus t by tau all right let us now look at the T 2 phase; that means, this part of i C of t and in that phase let i C be A prime e raise to minus t by tau plus B prime and here we put a superscript 2 to denote that we are in the T 2 phase.

Now, to find A prime and B prime we need 2 conditions on i C in the T 2 phase. The first condition is that i C at t equal to T 1 is equal to minus i 2, what about the second conditions? The second condition can be obtained using the fact that this current would have gone to 0 at t equal to infinity, if this change had not happened. So, these are the 2 conditions that we can use namely i C at T 1 equal to minus i 2, and i C at infinity equal to 0 and with these conditions we get V prime equal to 0 and a prime equal to minus i 2 e raise to T 1 by tau, and putting these back in this equation now we get i C in the T 2 phase equal to minus i 2 times e raise to minus t minus T 1 divided by tau.

Our next step is to find these unknowns I 1 and I 2 in equations 1 and 2, and let us see how to do that. Let us look at this picture this is our i C in the T 1 phase and that is over i C in the T 2 phase; at t equal to T 1 i C in the T 1 phase is this value here and at the same time i C in the T 2 phase is this value over here, which is minus i 2 and the difference between these 2 is denoted by delta over here. So, let us first figure out what this delta should be? At this point t equal to T 1, the input voltage is changing from V 0 to 0 the capacitor voltage is not really changing because V c at T 1 minus is the same as V c at T 1 plus and therefore, coming to the circuit now we observe that there is a change in V s, there is no change in p c and therefore, this change in V s must appear across this resistance otherwise K V L would not get satisfied.

V s is changing by V 0 this quantity here and therefore, the voltage across the resistor must also change by V 0 and what does that mean? That means, there must be a corresponding change in the current i C here, which is V 0 divided by R and therefore, we conclude that this delta must be equal to V 0 by R, and that is what we write over here; i C in phase 1 at T 1 that is this value here minus i C in phase 2 at T 1 this value over here must be equal to V 0 divided by R. And by the same logic this difference must also be V 0 by R; now what is this value? This i C value is the same as this i C value and that is i C in the T 1 phase at t equal to 0, what about this value? That is i C in the T 2 phase at t equal to T 1 plus T 2 and the difference between these 2 values must be equal to V 0 by R and that is what this equation says over here.

The rest is a straightforward what is i C in the T 1 phase at t equal to T 1? This is our equation for i C in the T 1 phase. So, let us put t equal to T 1 over here and then we get this force term over here. What about i C in the T 2 phase at t equal to T 1? This is over i C in the T 2 phase, so we put t equal to T 1 here then we get e raise to 0 that is 1 and therefore, this i C at T 1 boils down to minus I 2 like that, and the difference between these 2 terms must be equal to delta.

Let us use this second condition now, what is i C in the T 1 phase at t equal to 0? We put t equal to 0 here and that gives us I 1 the first term here, what about i C in the T 2 phase? At T 1 plus T 2 we put t equal to T 1 plus T 2 over here, and then we obtain this expression over here and the difference between these 2 must be equal to delta. Note that, T 1 cancels out over here and we get e raise to minus T 2 by tau. Now using the constants a and b that we defined in the previous slide we can rewrite these equations 3 and 4 as; a I 1 plus I 2 equal to delta that is our equation 3 and I 1 plus b times I 2 equal to delta that is our equation 4 and these 2 equations can now be solved for I 1 and I 2 I 1 turns out to be this expression here, delta times 1 minus b by 1 minus a b and I 2 is delta times 1 minus a by 1 minus a b, where a is e raise to minus T 1 by tau and b is e raise to minus T 2 by tau as we saw in the last slide.

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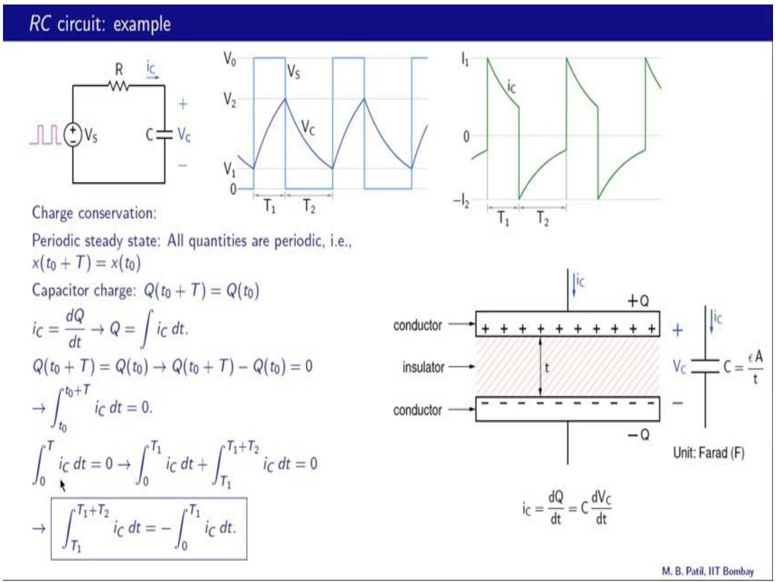

So, now we have obtained expressions for the capacitor voltage V c in the steady state both in the T 1 and T 2 phases, and also we have got expressions for the capacitor current i C in the T 1 phase as well as in the T 2 phase. But let us go a little bit beyond that and look at the implications of charge conservation and in the process we will learn a lot more about these waveforms. Here is a schematic diagram showing a capacitor there is 1 metal plate over here with charge plus Q and there is another metal plate here with a charge minus Q. If this is plus Q this has to be always minus Q these 2 metal plates are separated by a dielectric with a thickness t and a dielectric constant epsilon.

Now, in real life these may not be quite plates, these may be metal foils and the insulator may also be in the form of a dielectric film and this whole thing may be rolled in a cylindrical shape for example, in any case the cross section does look like this and the capacitance is given by epsilon A by t, where epsilon is the dielectric constant of this insulator and t is the thickness A here is the area of the capacitor. Let us now look at the charge here plus Q on this plate and minus Q on this plate; this charge can change only if there is a non 0 current i C either entering this terminal or leaving this terminal, and the relationship between i C and Q is given by i C equal to d Q d t and since Q is c times V c, we get i C equal to c times d V c d t and this of course, is the equation that we have been using all along.

Let us now look at the implications of this equation in our context and in particular we will look at the implications in the periodic steady state. What is the meaning of periodic steady state? All quantities are periodic that is x at t 0 plus T must be equal to x at t 0; where x could be a current or a voltage or even the charge on the capacitor. Let us take x equal to Q the capacitor is charged and then we have q at t 0 plus t equal to Q at t 0, this T is the period in our case it is T 1 plus T 2. Now since i C is d Q d t we can write Q as integral i C d t and let us see what this implies. Now this equation can be rewritten as Q at t 0 plus T minus Q at t 0 equal to 0 and because we have this relationship between Q and i C, this equation here can be rewritten as integral i C d t from t 0 to t 0 plus T equal to 0.

let us take a specific case let us take t 0 equal to 0, this time point here and then this becomes integral i C d t from 0 to T equal to 0 and now we can split this into 2 integrals one from 0 to T 1 and the other from T 1 to T 1 plus T 2. So, we can write integral i C d t from 0 to T 1 plus integral i C d t from T 1 to T 1 plus T 2 equal to 0; and this equation tells us that integral i C d t from T 1 to T 1 plus T 2 must be negative of integral i C d t from 0 to T 1. Now let us see what this implies in terms of our graph here, what is integral i C d t? It is simply the area under the i C curve. So, this term here the integral from T 1 to T 1 plus T 2 is this area here and this is a negative area because our i C is negative in this region, and this integral from 0 to T 1 is this area and that area of course, is positive because i C is positive. So, what this equation is saying is that these 2 areas must be equal in magnitude.

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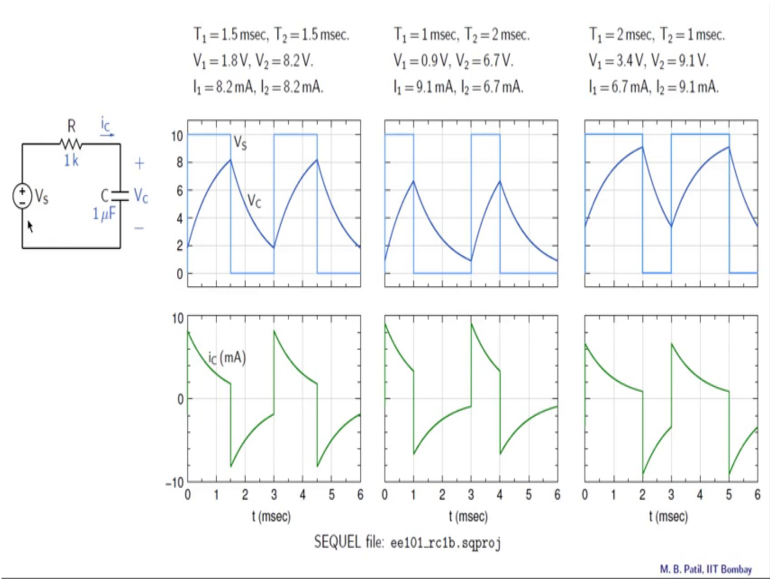

Let us look at some simulation results now, in particular let us take T 1 equal to T 2 1.5 milliseconds each, R equal to 1 k, C equal to 1 micro farad and V 0 equal to 10 volts with these numbers using the formulas that we derived earlier V 1 turns out to be 1.8 volts, V 2 is 8.2 volts, I 1 is 8.2 milliamps and I 2 is also 8.2 milliamps. So, our current the capacitor current is going to vary between plus 8.2 milliamps and minus 8.2 milliamps. The simulation results are shown in these plots this is our input voltage that is the capacitor voltage and that is the capacitor current a milliamps.

Now, V c is going from 1.8 volts to 8.2 volts and that agrees with these results here and I 1 goes from plus 8.2 milliamps to minus 8.2 milliamps. What about delta this discontinuity? If you recall that is expected to be V 0 divided by R, where V 0 is this voltage ten volts and R is 1 k. So, delta should be 10 volts by 1 k or 10 milliamps; and this is indeed ten milliamps the same height as this height here and this is also equal to delta that is 10 milliamps.

So, all is well and our analysis seems to be correct. Let us now look at the implications of charge conservation, if you recall we expect the area under the i C curve in the T 1 phase and in the T 2 phase to be equal in magnitude, and that does in to be the case this area is equal to this area here. Here is our second case what we have done is we have kept T 1 plus T 2 the same, that is 3 milliseconds, but we have decreased T 1 and increased T 2. This is our T 1 it is now 1 millisecond, this is over T 2 it is now 2 milliseconds with these changes we get V 1, V 2, I 1, I 2 as given by these numbers here and you can check whether these numbers do actually correspond to the plots which have been obtained by a simulation.

What about charge conservation? We expect this area to be equal to this area here in magnitude, and that does appear to be the case. This is the third case we have once again we have kept T 1 plus T 2 equal to 3 milliseconds, we have increased T 1 to 2 milliseconds and decreased T 2 to 1 millisecond. So, this is our T 1, 2 milliseconds and this is over T 2, 1 millisecond and these are the results obtained using our formulas and you should check that these do agree with their simulation results given here. What about charge conservation? This area is expected to be equal in magnitude to this area and that does appear to be the case.

So, there is really quite a lot we can learn from this simple example, here is the sequel circuit file and you are strongly encouraged to run this simulation make sure that you do get these results that we have shown here, and apart from that you can make up your own product choose some different values of R, C, T 1, T 2 calculate V 1, V 2, I 1, I 2 using our expressions and then turn the simulation to make sure that your values are correct; and in fact you do not need to stop at simulation, you can go to your electronics lab look up this circuit and verify that the results that you get on your oscilloscope do only with your calculations, and in that process you will learn how to use function generator to generate a square wave with different duty cycles, you will learn how to pick components such as 1 k register, or 1 micro farad capacitor and you will also learn how to use an oscilloscope.

So, take this as a learning opportunity and it is unlikely that you will regret it.

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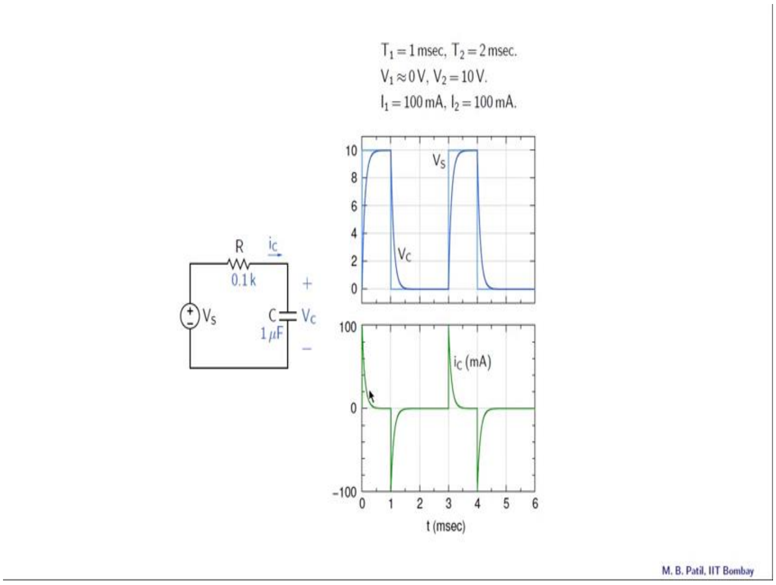

Let us consider 1 more case before we leave this example and that is T 1 equal to 1 millisecond, T 2 equal to 2 milliseconds, and we have seen this case in the last slide; but what is different now is the value of r it was 1 k earlier and now it is 0.1 k, and as a result the time constant which was 1 millisecond in our previous example, is now 0.1 k times 1 microfarad which is 0.1 millisecond and therefore, 5 tau is 0.5 millisecond, and that turns out to be smaller than T 1 or T 2. And what does it mean? That means, we expect the capacitor voltage to reach its final destination10 volts here, or 0 volts here; and in fact we do not even need to use the formulas that we have derived earlier, because we know that the capacitor is going to get charged all the way to 10 volts during the T 1 phase, and we know that get it is going to discharge all the way to 0 volts in the T 2 phase.

What about the current? The jump in the current which we are called delta is now different because although V 0 is the same as 10, r have changed. So, the new delta would be 10 divided by point 1 k or 100 milliamps and that is what we see over here. So, the jump here or there is 100 milliamps. And as in the voltage waveform we observe that the current also does manage to reach its final destination, that is 0 ampere. Both in the T 1 phase as well as in the T 2 phase.

That brings us to the end of RC circuits with a piecewise constant source. We will find this background very useful in understanding some of the applications to be covered later in this course. See you next time.