Basic Electronics

Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 10
RC/RL circuits in time domain (continued)

Welcome back to Basic Electronics. In this lecture we will continue with the RL circuit example from the previous lecture; we will then look at an RC circuit with a switch and learn how to obtain currents and voltages in the circuit, before and after the switch position changes. We will then look at an RC circuit in the periodic steady state with a square wave input let us get started.

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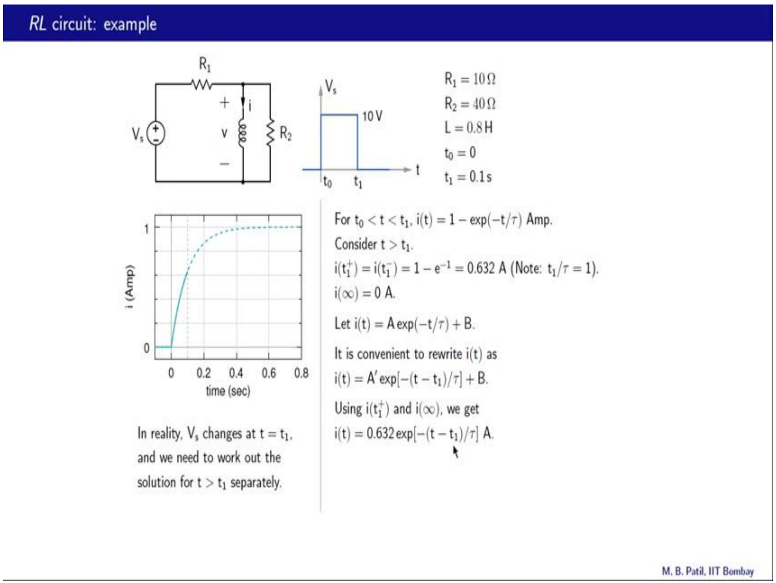

So, this is the solution that we obtained using the conditions that we found for i of t at 0 plus and infinity, at 0 plus i is 0 and as t tends to infinity i is 1 ampere. Now this of course, is only part of the story why? Because in reality the source voltage V s changes at t equal to t 1, so therefore, this solution is really valid up to t 1 that is up to 0.1 second, and we need to work out the rest of the solution that is the solution for t greater than t 1 this interval here and how do we go about that? We know that as we go from this interval t 0 to t 1 to this next interval t 1 to infinity, the inductor current must remain continuous; that means, the value of the inductor current i at t 1 minus that is just before t 1 must be the same as the value of i at t 1 plus that is just after t 1.

For this interval t 0 less than t less than t 1 this interval here, i of t turns out to be 1 minus e raised to minus t by tau ampere; and this expression describes this part of the solution and you are of course, encouraged to derive this expression, how do you do that? Let i of t be k 1 e raised to minus t by tau plus k 2, where k 1 and k 2 are constants to be determined and now use the conditions that we derived in the last slide namely i at 0 plus equal to 0, and i at infinity equal to 1 ampere.

In those conditions you can find k 1 and k 2 and then you will end up with this expression here and once we have this expression we can find i at t 1 minus; that means, i at 0.1 second, all we need to do is replace this t with 0.1 second, this tau as we saw in the last slide is also point once again. So, we end up with i of t equal to 1 minus e raised to minus 0.1 by 0.1 that is e raised to minus 1.

So, that turns out to be 0.632 amperes, and i at t 1 plus as we just argued is the same as i at t 1 minus. So, therefore, i at t 1 plus is 0.632 amperes. So, that is one condition that we have for i of t in this last interval here. What is the second condition? The second condition is i at infinity. So, let us see what that should be; what is the situation at t equal to infinity? V s is 0 and it has been 0 for a long time, the circuit is in steady state all quantities voltages and currents would have become constant, this current would have become constant so therefore, this V which is l di dt would be 0 this voltage is therefore, 0 no current flows through R 2, and i is simply V s divided by R 1. Since V s is 0 at t equal to infinity, the current is 0 amperes like that.

So, now we have 2 conditions i at t 1 plus and i at infinity, and now we can get the complete solution for this last interval here. Let i of t be equal to A e raised to minus t by tau plus B in this interval, that is t greater than t 1 and now let us find A and B using these 2 conditions. It is convenient to rewrite i of t as A prime e raised to minus t minus t 1 by tau plus B, we are not really changing this equation all we are doing is shifting the origin from t equal to 0 to t equal to t 1

Let us now use these 2 conditions i at t 1 plus is 0.632 amperes, and i at infinity equal to 0 ampere. At t 1 plus we have i equal to 0.632 amperes. So, we put 0.632 here, t equal to t 1 plus here. So, this argument becomes 0 and e raised to 0 is equal to 1. So, therefore, we get 0.632 equal to a prime plus B at infinity i is 0 so we put 0 here, and e raised to minus infinity is 0 therefore, this first term goes away and we get 0 equal to B; and putting these together we get our final expression for i of t in this last interval, t greater than t 1, and that is i of t equal to 0.632 e raised to minus t minus t 1 divided by tau amperes.

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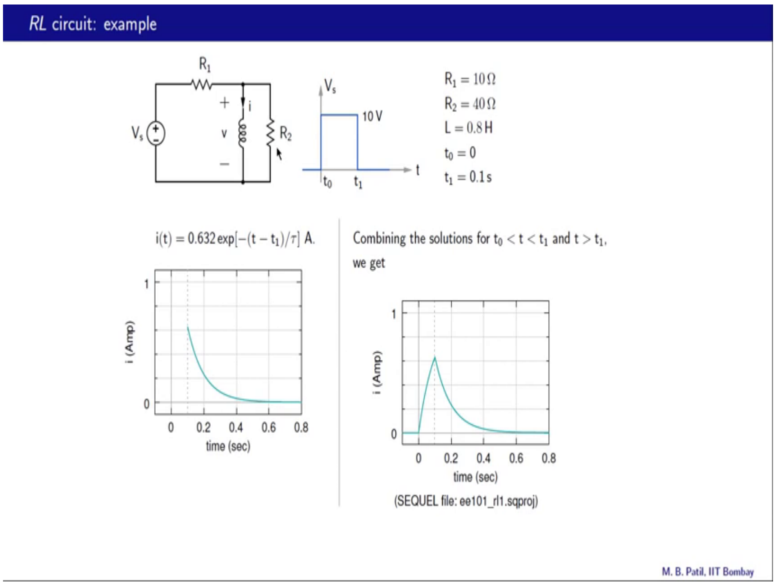

And this is what the solution looks like in the final interval t greater than t 1, at t 1 i is 0.632 amperes, and at t equal to infinity it is 0 ampere.

So, now we have different pieces of the solution available to us, one piece in this interval and one piece and this interval, all we need to do now is to put these together like that. So, this is our complete solution for i of t; the sequel file for this particular simulation is available to you given here and you can play with it for example, you can change R 2 from 40 ohms to 20 ohms, and predict first of all what should happen and then run the simulation and check that your prediction is correct.

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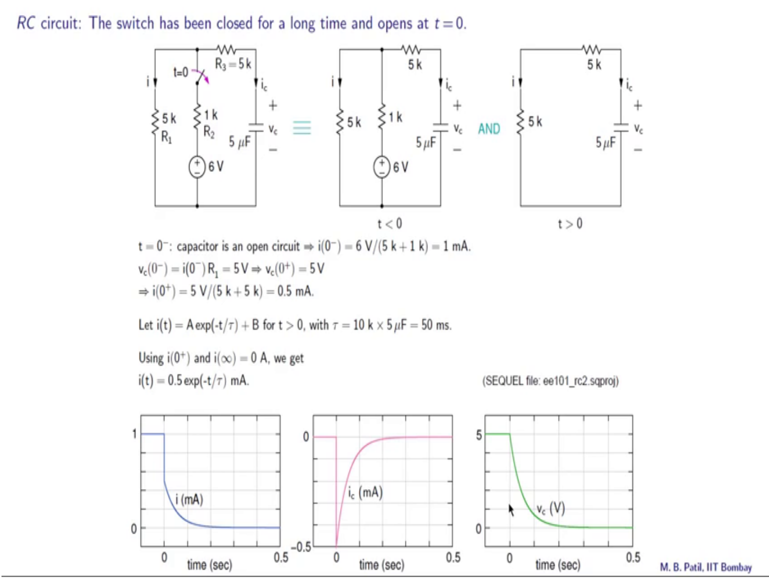

Here is our next example it is an RC circuit, with a switch that opens at t equal to 0. So, the switch has been closed for a long time and opens at t equal to 0, and we are interested in finding this current i as a function of time. Now this situation is very different than what we have been looking at so far, because when the switch opens our circuit change. Before t equal to 0 we have 1 circuit and after t equal to 0 we have another circuit. So, let us see how to handle that.

Here is the situation for t less than 0 that is when the switch is closed; when the switch is closed we have a short circuit here assuming the switch is ideal and therefore, this circuit data applies. After the switch opens that is when t is greater than 0, this switch is an open circuit and therefore, this entire branch is not in the circuit anymore and the circuit reduces to this circuit shown over here.

Now, this situation for t less than 0 has been there for a long time. So therefore, this circuit is in steady state, you can use that fact to find V c over here, and that capacitor voltage value serves as the bridge between these 2 situations, because as we know the capacitor voltage must be continuous. So, we see at 0 plus must be equal to V c at 0 minus.

To begin with let us look at the situation at t equal to 0 minus; that means, we are looking at this circuit, and at 0 minus the switch has been closed for a long time, so this circuit is in steady state; what is the meaning of steady state? That means, all currents and voltages have become constant, so therefore, this voltage we see has also become constant and therefore, i c which is C d V c dt is equal to 0. So, this is an open circuit and the current path is given by this path here and that tells us what this current i should be it is simply 6 volts, divided by the total resistance in the circuit that is 1 k plus 5 k, so 6 volts divided by 6 k that is 1 milliamp.

So, we have i at 0 minus equal to 1 milliamp, what about V c at 0 minus? We have seen that this current is 0 at 0 minus, and therefore there is no voltage drop across this resistance and then this voltage V c is the same as this voltage here, that is i multiplied by 5 k, and i is 1 milliamp so therefore, we have 1 milliamp times 5 k or 5 volts. So, V c at 0 minus is 5 volts and because the capacitor voltage must be continuous, V c at 0 plus in this circuit must also be 5 volts.

So, therefore, we have V c 0 minus equal to 5 volts and V c 0 plus also equal to 5 volts; and that tells us what i at 0 plus should be let us look at this circuit because now we are talking about t greater than 0, this voltage is 5 volts and i therefore, is 5 volts divided by 5 k plus 5 k that is 0.5 milliamps.

Now, let us find the current i as a function of time for t greater than 0, and let us begin by assuming that i of t has this form A e raised to minus t by tau plus B. So, we need to determine 3 things tau the time constant and then these constants A and B, what is the time constant for the circuit? It is R t h times C, where R t h is the terminal resistance as seen by the capacitor and in this case the terminal resistance is simply these 2 resistors in series so therefore, 5 k plus 5 k or 10 k. So the time constant; then is 10 k times 5 micro farad that is 50 milliseconds.

What about A and B? To find A and B we need 2 conditions on i, we already have 1 condition here i at 0 plus equal to 0.5 milliamp. Let us now find i at infinity and that give us the second condition. To find i at infinity let us look at this circuit what is i here i is V c divided by 10 k; now as this current flows the capacitor gets discharged so that will keep happening until the entire charge on the capacitor there is exhausted. So, finally, we have no charge on the capacitor, V c becomes equal to 0 and therefore, i becomes equal to 0.

Another way to find i at infinity is to use the fact that the circuit would be in steady state as t tends to infinity, and all quantities currents and voltages would have become constant at that time, so therefore, this V c would have become constant, and the current i c which is c d V dt would be 0 if i c is 0 then of course, i is also equal to 0.

So, now we have 2 conditions i at 0 plus equal to 0.5 milliamp, and i at infinity equal to 0; and using these 2 conditions we can find A and B and we get this expression, finally i of t equal to 0.5 e raised to minus t by tau milliamps.

Here is a plot of i as a function of time, and the current i is in milliamps here this is 0 and that is 1 milliamp. At 0 minus i is equal to 1 milliamp and in fact, it has been 1 milliamp for a long time, because the switch has been closed for a long time, so therefore the current i has been constant. When the switch opens the current changes and at 0 plus; that means, just after t equal to 0, where i is equal to 0.5 milliamp, 0.5 milliamp is write here.

So, therefore, at t equal to 0 there is a discontinuity, before 0 we have 1 milliamp and after 0 we have 0.5 milliamp; and subsequently i of t is given by 0.5 e raised to minus t by tau milliamps and that part of the curve is shown over here eventually of course, as t tends to infinity, this term becomes 0 and i tends to 0 as we see here.

Now, how long does the transient last we expected to last for 5 time constants, what is the time constant? The time constant is 50milliseconds 5 times that is 250 milliseconds that is 0.25 seconds. Here is 0 here is 0.1 second, 0.2 second and so on. So, 0.25 seconds is here and we do observe that after 0.25 seconds the current does not change it becomes constant.

Here is the capacitor current as a function of time, and that is also plotted in milliamps this is 0, and this is minus 0.5 milliamps, what is the situation at 0 minus? At 0 minus we have this circuit and it is in steady state V c is constant and therefore, the capacitor current is 0 and that is what we see over here, and it has been 0 for a long time because the switch has been closed for a long time. So, that explains this part of the plot. What about 0 plus? After t equal to 0 this circuit comes into picture, and then we have i c equal to minus i.

And since i starts off at 0.5 milliamps and goes to 0 eventually, i c starts off at minus 0.5 milliamps and then goes to 0 finally; and notice that there is a discontinuity at t equal to 0. Let us now look at the capacitor voltage, what is V c at 0 minus we already looked at that V c at 0 minus is 5 volts, and it has been 5 volts for a long time and that explains this part of the plot. What about t greater than 0 for t greater than 0, we have this circuit and V c now is equal to i multiplied by 5 k plus 5 k, that is 10 k multiplied by i. Now i is 0.5 milliamps at 0 plus and then eventually it goes to 0. So therefore, V c would go from 0.5 milliamps times 10 k that is 5 to 0 like that.

And as we would expect the capacitor voltage is of course continuous, there is no discontinuity at t equal to 0. The sequel file for this particular simulation is given over here and you can run this simulation and look at all of these variables, and also some other variables that you may be interested here.

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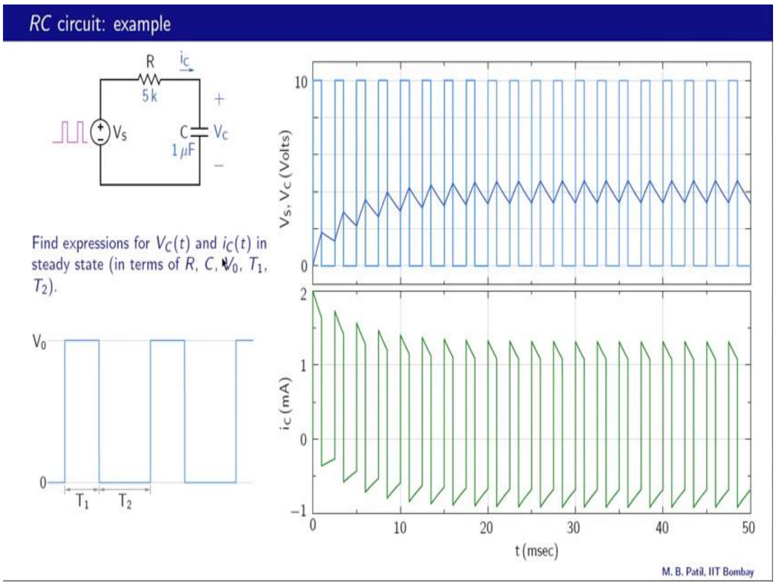

Here is our next example it is a series RC circuit with R equal to 5 k, and c equal to 1 micro farad and the input voltage is a square wave, in this interval the input voltage is high that is 10 volts, and in this interval it is low that is 0 volts and then it repeats it is periodic.

Let us say that our capacitor is initially uncharged; that means, V c is equal to 0 volts at t equal to 0 as shown over here; and at t equal to 0 the input voltage goes high to 10 volts as a result of that the capacitor is going to start charging like that toward 10 volts, but that does not happen of course, because at this point the input voltage goes back to 0 volts and now the capacitor starts discharging towards 0. And that discharging process is also not completed because the input voltage once again becomes high. So, therefore, the capacitor starts charging again, and this process repeats until finally we have a steady state situation.

This kind of steady state is called periodic steady state as suppose to the DC steady state, that we have looked at earlier what happens in the periodic steady state? Let us take this V c as an example, V c starts off at this point it does change within one period, but then after one period is over it comes back to the same value where it started. So, that is the meaning of periodic steady state.

Let us now look at the current waveform; the current is denoted by i c here the capacitor current, in this first interval when the input voltage is high, the current is positive and in fact, that is responsible for this increase in the capacitor voltage. In the second interval the input voltage has gone back to 0, and the current becomes negative as shown over here, and this negative current is responsible for this decrease in V c. So, this keeps happening the current is positive here, negative here, positive again, negative again and so on and finally, the capacitor current waveform also reaches the periodic steady state; that means, within one period there is a change in i c, but after one period is over, the i c value comes back to where it started all right and our problem has to do with finding V c of t, and i c of t in the periodic steady state.

Let us look at the problem statement now, this is our input voltage waveform, it is a square wave, wave from 0 to V 0 and then back to 0.

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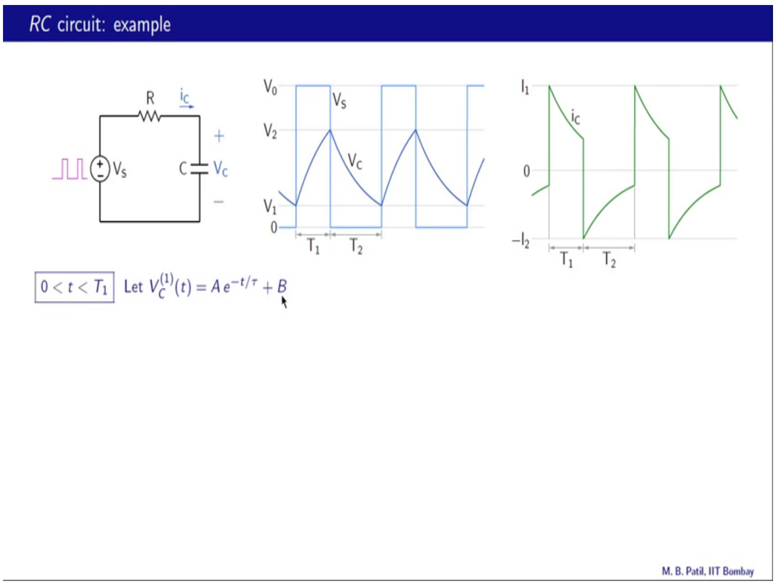

T 1 is the interval in which the input voltage is high that is V 0 and T 2 is the interval in which the input voltage is low, that is 0 volts. So, the problem is to find expressions for V c of t that is the capacitor voltage, and i c of t the capacitor current, in the periodic steady state and these expressions will be in terms of R, C, V 0 this value T 1 and T 2.

Here is a schematic diagram showing the capacitor voltage and the capacitor current as a function of time in the periodic steady state. The light blue graph is the input voltage V s during this interval marked by t 1, the input voltage is high equal to V 0 and during this interval marked by T 2 here the input voltage is low that is 0. During the t 1 phase the capacitor voltage rises from V 1 to V 2 and during the T 2 phase the capacitor voltage decreases from V 2 to V 1. So, within one period, the capacitor voltage comes back to where it started off namely V 1.

In the t 1 phase when the capacitor voltage is increasing we have a positive capacitor current, and in the T 2 phase when the capacitor voltage is decreasing; that means, when the capacitor is discharging, we have a negative capacitor current and the capacitor current is also periodic.

Now, the maximum value of i c is denoted by i 1 over here, and the minimum value is denoted by minus i 2. Now this minus sign is introduced to make our algebra easier, so note that this i 2 is a positive number, so minus i 2 is negative. Our objective is to find expressions for V c of t and i c of t; let us begin with V c and we should note first that V c will be represented by 2 expressions, one expression in this t 1 phase and another expression in this T 2 phase. So, let us begin with the t 1 phase and in that phase let V c be A e raised to minus t by tau plus B and we will call that as V c 1 where the superscript 1 indicates this t 1 phase. So, we now need to find A and B the time constant of course, in this case is simply R times c.

To summarize we learned how to treat an RC circuit with a switch, we also looked at the salient features of the current and voltage plots for that circuit, we then started with an RC circuit with a square wave input, we looked at the meaning of the term periodic steady state and started analyzing the circuit in that condition. In the next class we will complete the analysis, and also compare the results with plots obtained by circuit simulation until then goodbye.