Computational Electromagentics and Applications
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Exercise No 21
Algebraic Topological Method (ATM-III)

We are now going to look into interesting problems that he also saw in the case of finite element method but you are going to use algebraic topological method solve this problem and in the case of finite element method we have done the problem using a 2D approximation we have not done a 3D problem we have done a 2D problem the case of algebraic topology have to go to a 3D problem because we need certain parameters 3D nature I will explain this as we go but now let's look into the problem itself see how this problem is structured sun and how this problem is going to be replicated using Matlab program that we are going to plan

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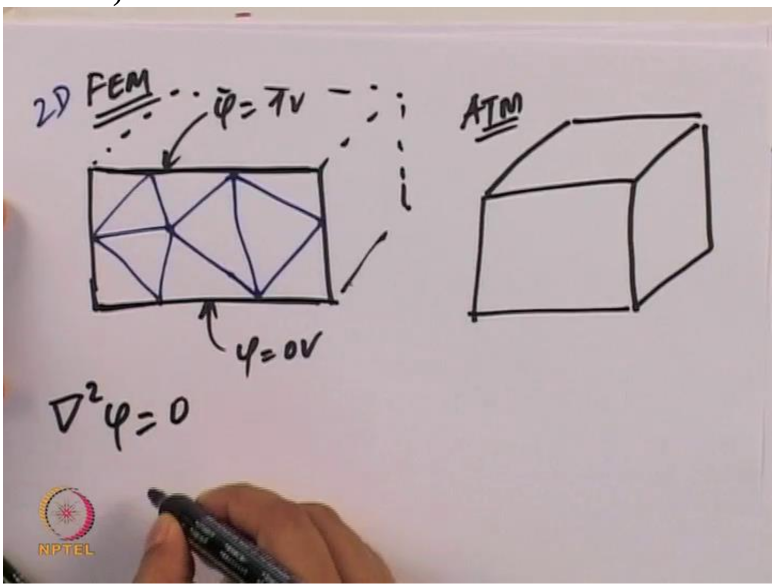

so let's start with the problem so we had this parallel plate capacitor which was in the case of the finite element method basically a 2D approximation and you said that of potential is 1 volte and the bottom potential is going to be zero volt so for us

to this problem in the case of algebraic topology so this is the FEM problem that we saw but in order to do it in the algebraic topological method we need to go to the 3rd dimension the third dimension is let's say this is done in this manner when we are kind of closely discretizing this grid so let's say this is our problem in the case of 2D FEM so this is a 2D problem and in the case of ATM as I said the essential I need to go to the i second dimension because parameters and this parameter will be so if this is a for you to compare this was definitely long but you only took the part of a cross section and we solved it in a FEM method so this was basically Laplace Equation that we solved these are the boundary condition that we have enforced.

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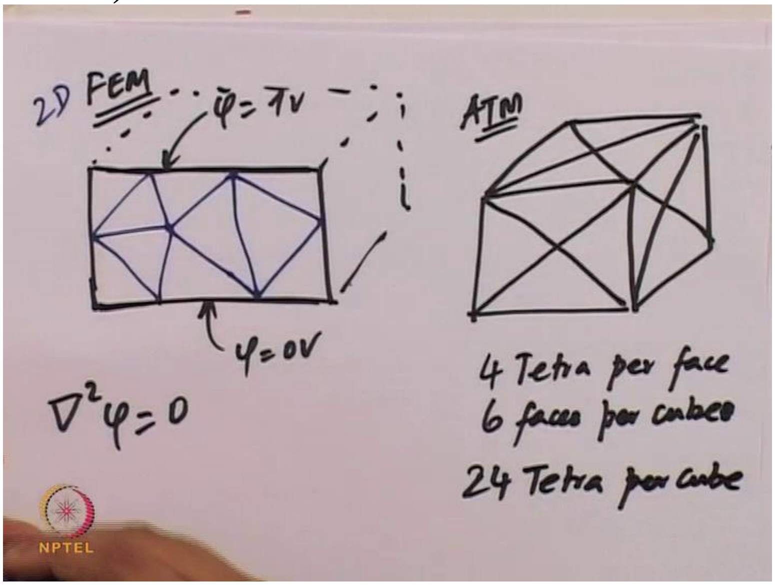

The case of algebraic topological method our discretisation is going to be tetrahedral. So each phase will have certain number of tetrahedral and you can basically see triangle is a side of a tetrahedral so you can imagine there will be 4 triangles per face and there are totally going to be 1,2,3,4,5,6. So there are going to be 4 tetrahedral per face and there are totally 6 faces for cube so we are going to have totally 24 Tetrahedron there is going to be a larger number of elements in the case of the algebraic topological method our elements are the 3D elements they are the tetrahedral we cannot solve the problem only using 2D approximation like the way we did finite element but we need to go to the 3rd dimension and the reason will be obvious when we go into the program itself so remember algebraic topological we set certain parameters

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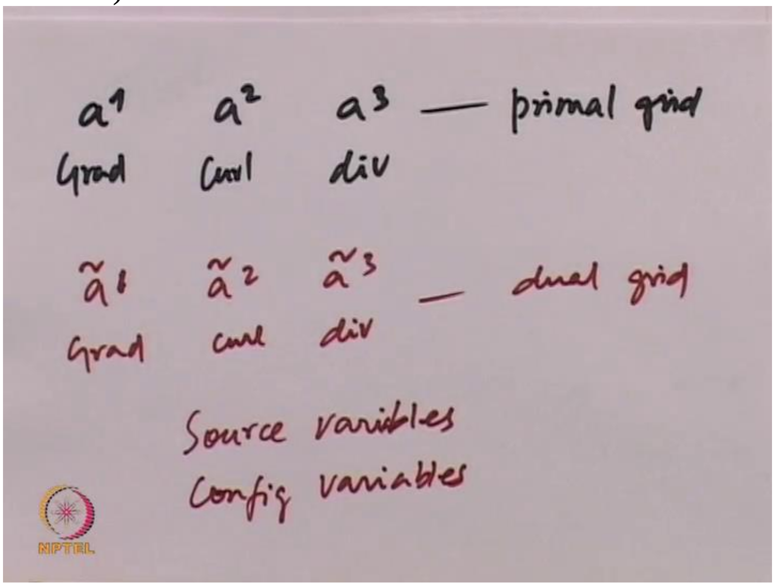

And those parameters are going to be the A 1 a2 a3 these are all in the primal grid the gradient a2 will be the girl 983 will be the divergence these are all in the Primal grade and we will have also the dual grade b a 1 tilde a 2 tilde a 3 tilde and these are the gradient and divergence in the dual grade and what we have done here is we have set two grids when to have types of variables one will be the source variable another will be the configuration variables and the source variable will be on 1 grid send the configuration variables are going to be on the other grid

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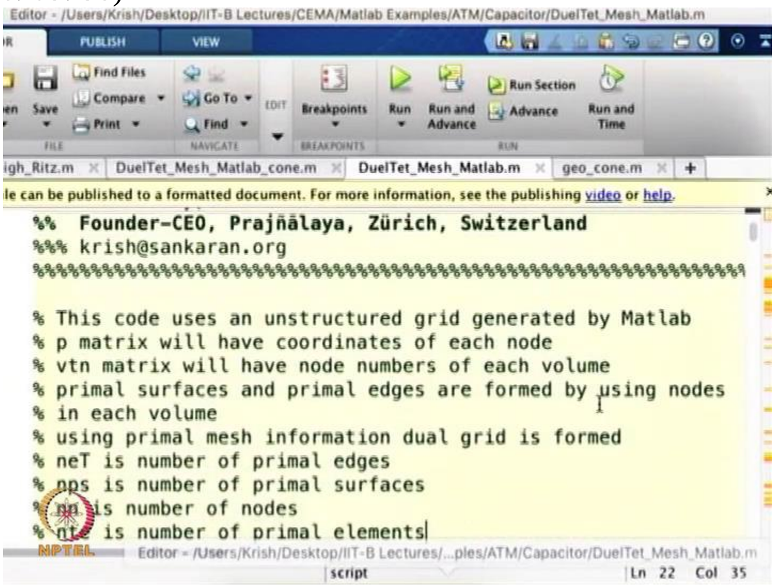

So let’s look into the code itself and try to understand how this code is structured in order for you to get a good appreciation of algebraic topological method and we are solving this for a simple problem of parallel plate capacitor so this code is using unstructured grid and we are generating it directly using Matlab you are not going to use any specific solvers for that and what we are getting what is the Primal surface Primal age and all these kind of things and we are using the Primal match information to generate the dual grid information. So the dual grids are going to be the grades that are going to help us to associate certain parameters where as the Primal grid will be for other kind of parameters and those parameters as I said will be either shows variable or configuration variable.

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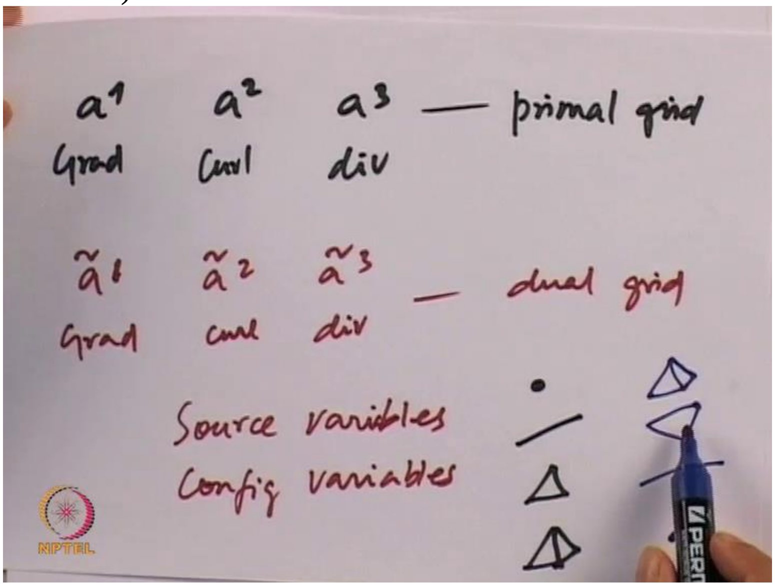

So let's look into the structure of three dimensional problem we have the points we have the lines we have the faces and we have the Tetra the dual grade of the point is going to be the dual volume which is going to be tetra or any other volume so in this case let's say for the sake of simplicity we call this as a Tetra but this could be of any shape any size or any form so it will be a n hedron of the line will be a surface the dual of a surface will be a line the dual of the volume will be a point So the number of lines.

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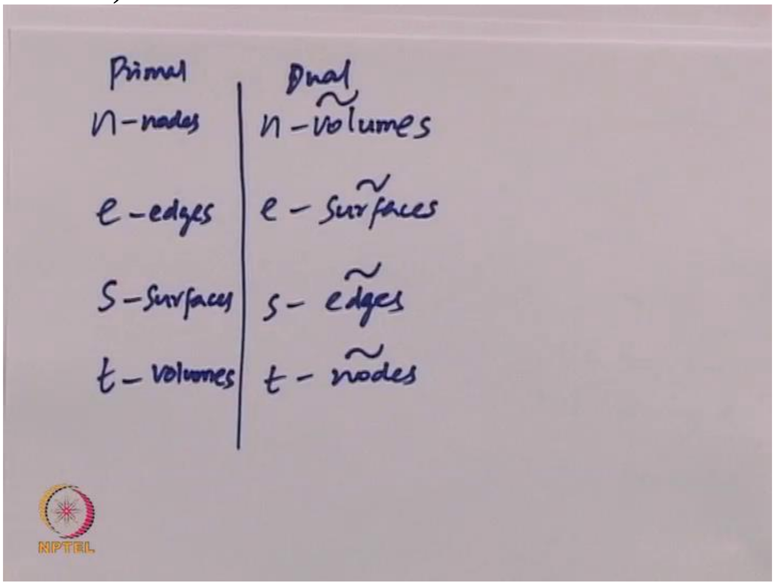

so if you say we are going to have n number of nodes e number of edges and s number of surfaces and t number of tetras in the Primal case the dual case will be number of volumes will be also equal to n number of volumes dual volumes. so I put tilde on the top of it we will have a number of dual surfaces and we will have as number of dual edges. And we will have t number of dual nodes. So this is the number of nodes, number of edges, number of surfaces, number of volume s .so this is the primal and this is the dual.

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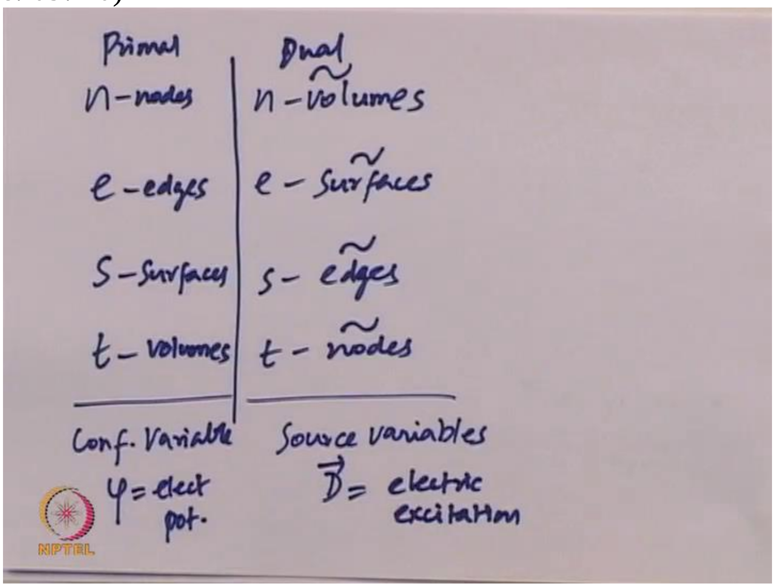

So what we are going to have as the dual grid is going to be the place where we are going to keep the source variables and the primal great is going to be the place where we are going to keep the configuration variables in this case the potential will be in the Primal grade where as they will be in the dual grade B is the electric excitation and v is the electric potential. You also called as electric displacement so the dual grade is going to be the place where we will have the source variables and the configuration variables are in the Primal grade so in our case we will be working only with the configuration variable and source variable in the form that we have described here switching them will not give you the right results so you have to be careful that you don't mix between source variable and configuration variable and we have discussed the way v define source variable and configuration variable during the lecture so for people who have missed it please go back to the lecture module on the algebraic topology and get a sense of what we are doing here so let's going to the code

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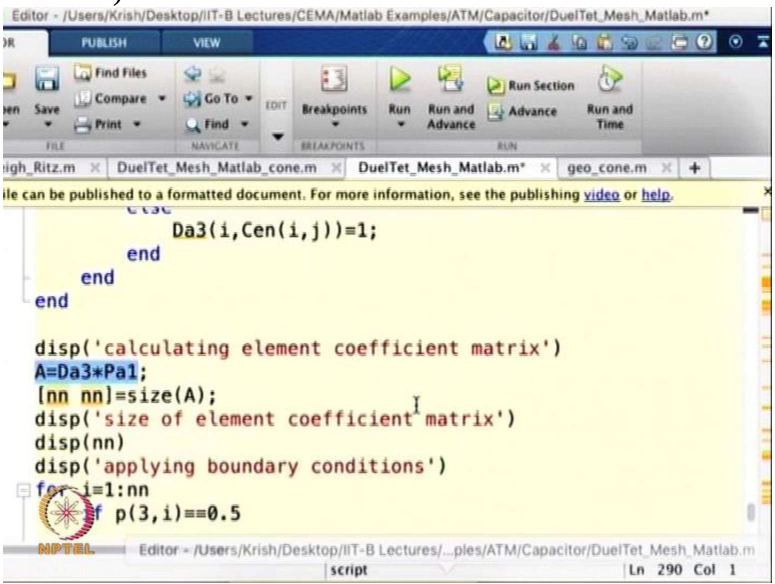

So the code is going to be a simple program for computing the value of the potential in a parallel plate capacitor I said simple because if you followed the logic of algebraic topology you will understand how you are solving the problem.

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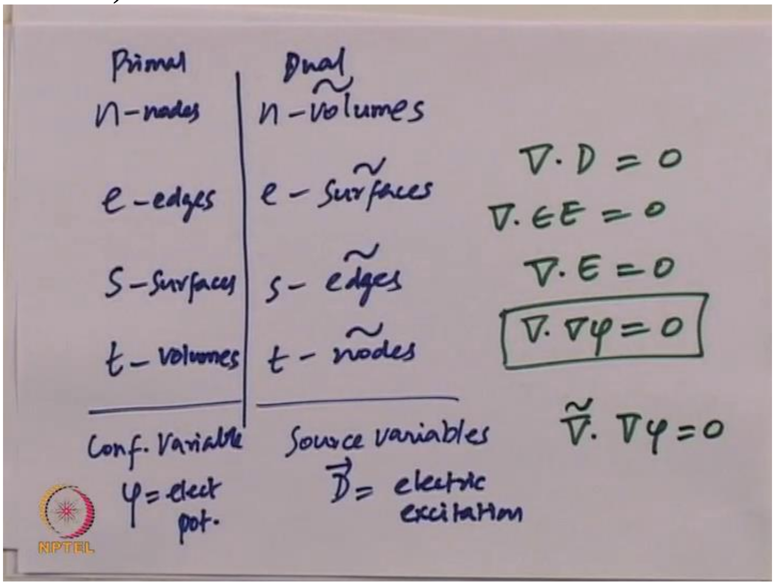

So basically what you are solving for this the divergence of b is equal to zero. But we are going to make this equation look in a different way what you are going to do is we keep the divergence out and remember the divergence of the is in the source variable so we are going to have a dual grid and what we are going to write D as we are going to write it as Epsilon E is equal to zero so we assume that he is positive difference so we will have divergence of equal to zero and we know that he is itself a gradient of Phi k. So we have starting problem so we can write it has minus gradient of Phi and a minus sign will go away and we will have this equation.

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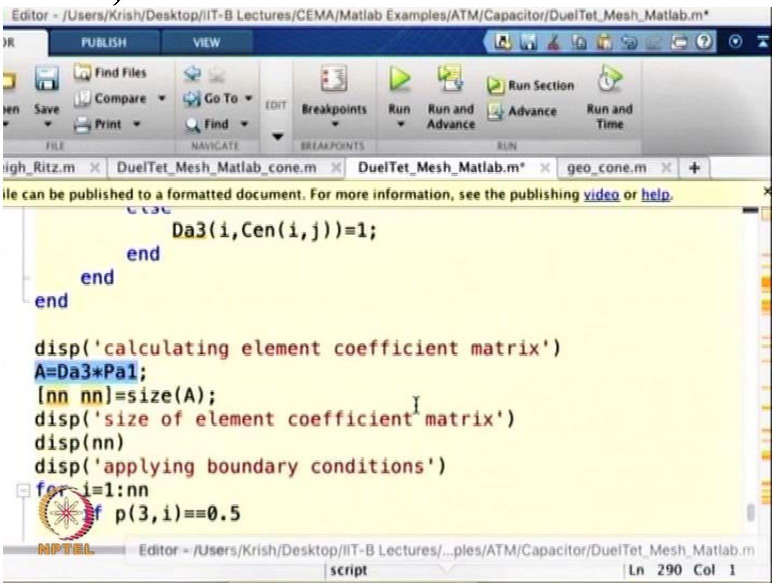

So let’s look at the code what we have done here is we have got a variable as a so basically this is a variable that we are talking about so this is the divergence dot the gradient so this is operator what we are looking at the operator has two components the one is acting on phi and she is on the Primal grade so it will be written with Rho tilde where is the one that is operating on D which is this entire time is going to be on the source variable and it's going to be on the dual grid so we have a tilde. And this is fee is equal to zero what we are going to see is we know that the gradient curl and divergence in the Primal grade and dual grade will have the form A1 A2 and A3 as follows.

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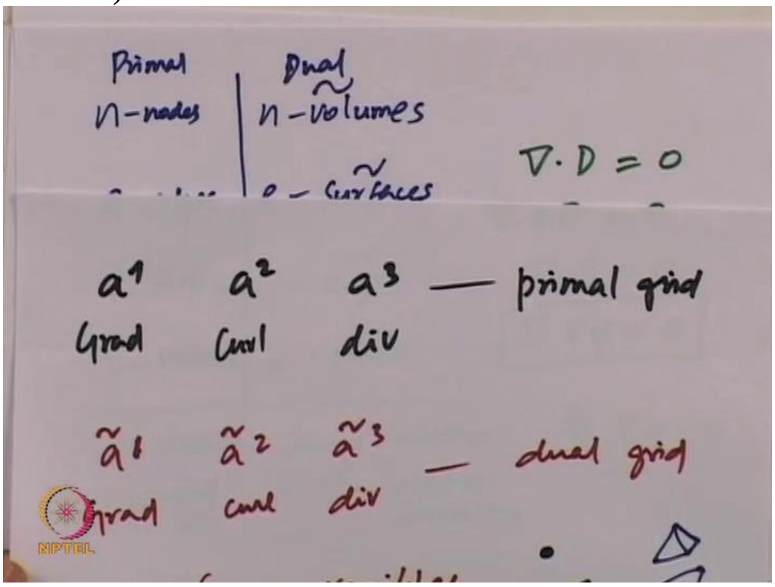

So we can write this particular equation using this logic what we have got so let me put it this way so you can see it below. We are going to substitute the value of divergence and gradient using a A1 A2 and A3 show the value for divergence in the dual grid is A3 tilde.

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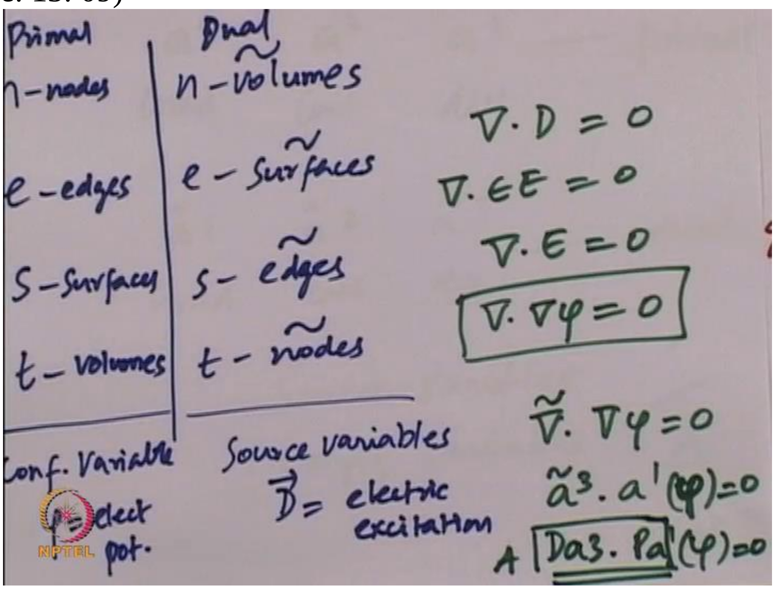

So I am going to put a 3 tilde and the value for the Primal gradient is going to be a one so I am going to put a 1 equal to Phi. So Phi equal to 0. And of course in the coding of the program what we have returned as is this is the dual a3 so we have put it as Da 3 dot this is primal a1 so Phi a 1 multiplied by the Phi is equal to zero. So this value is a value that we are calling as a and this is an operator and this operator is operating on Phi to get the value that we are interested in and let’s go into the code itself my one and see how you are getting there.

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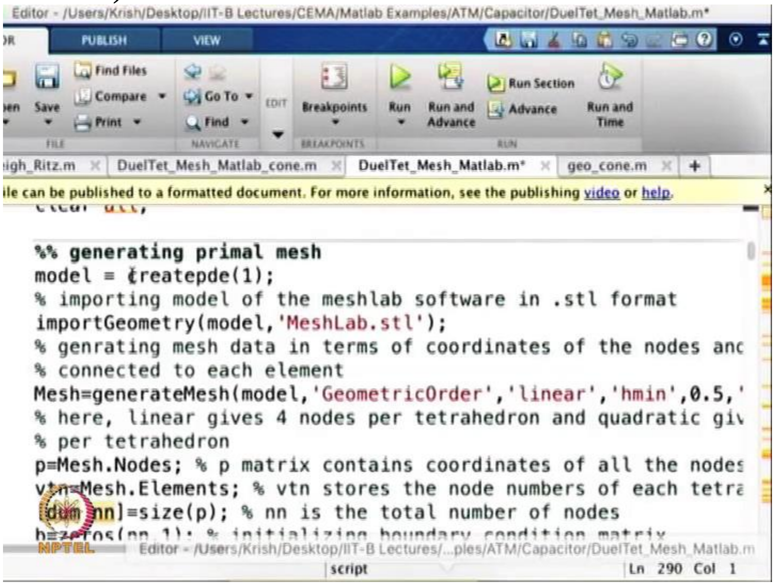

So we are generating the Primal grid using and in build program create PDE.

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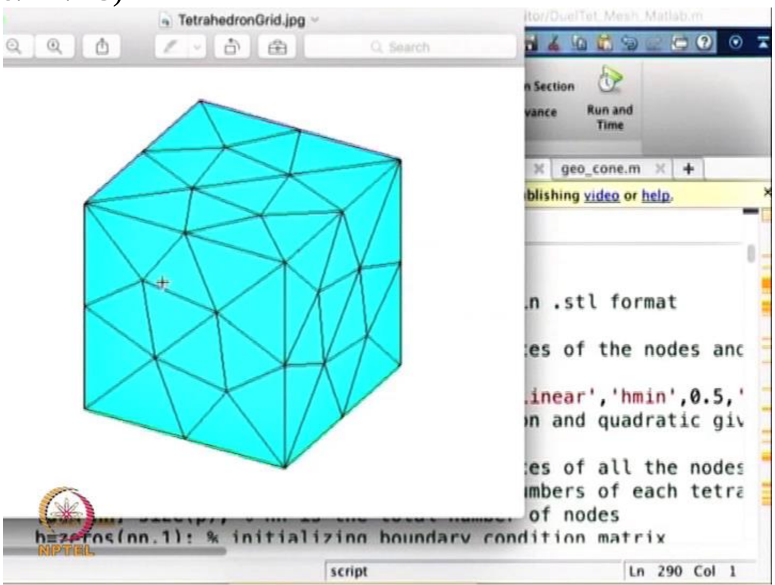

And we are getting the model and the model itself looks in this form it’s a parallel plate capacitor but we are having a three-dimensional KCR so this is a top surface of the top conductor there is a bottom conductor and his other sides and as you can see they are going to be tetrahedral everywhere in our case before I explained it in a simplified manner in a paper where you see that is Q will have several number of tetrahedral.

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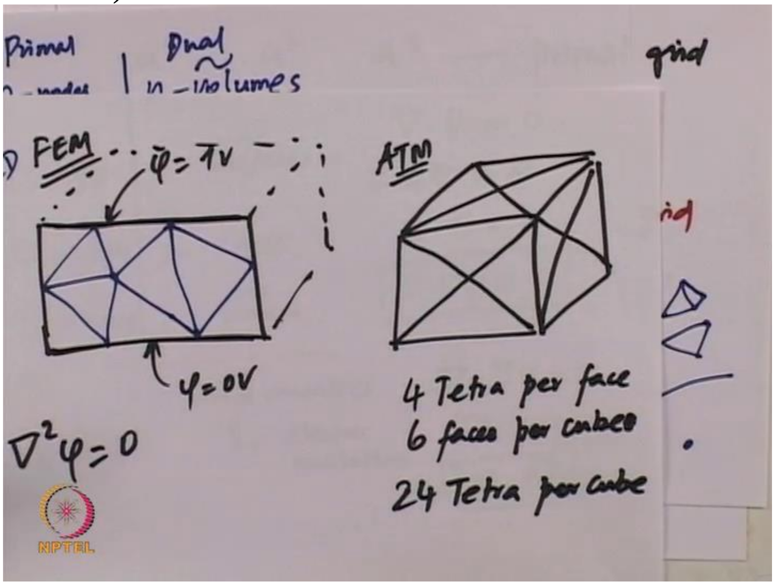

You have unstructured tetrahedral when the tetrahedral structure then you will have certain cemetery like in the case we have shown here.

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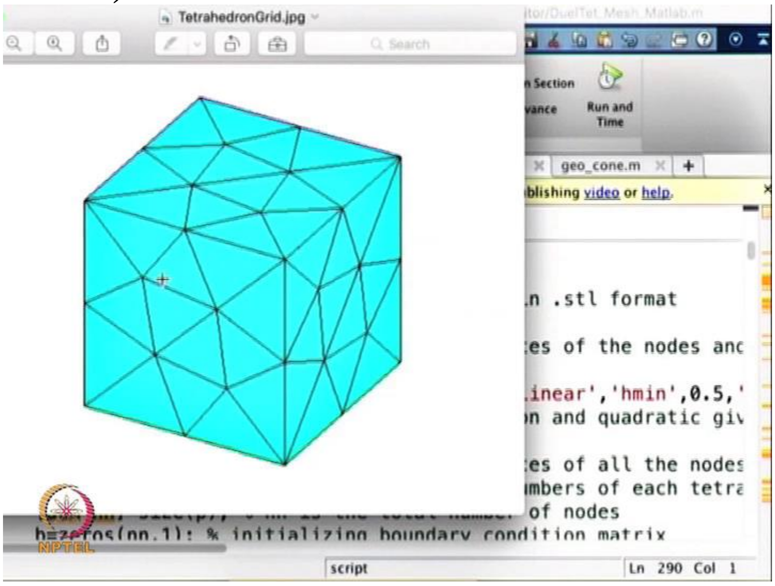

Where is in the case of an unstructured grid you cannot predict number of tetrahedral their they are going to sit on one side of the cube and we have a very very fine discretisation so there are more tetrahedral sitting than a simple case what we have shown.

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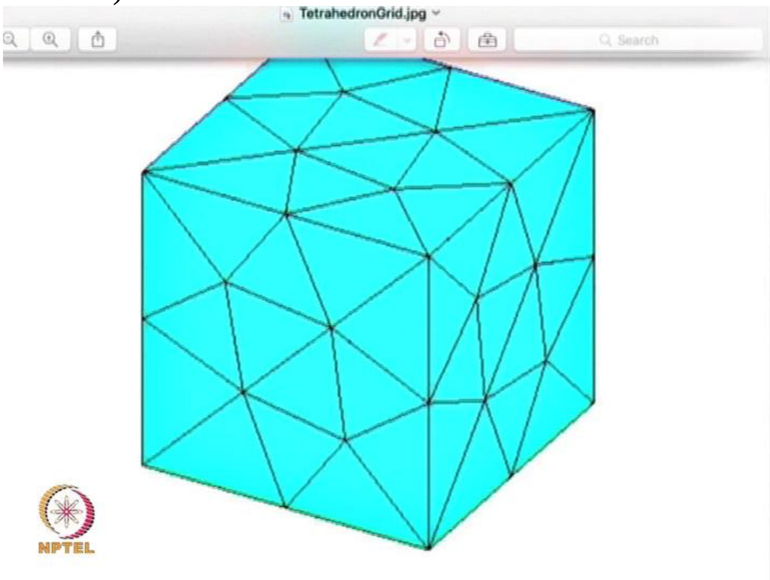

And this is the problem geometry that we are going to use for our simulation.

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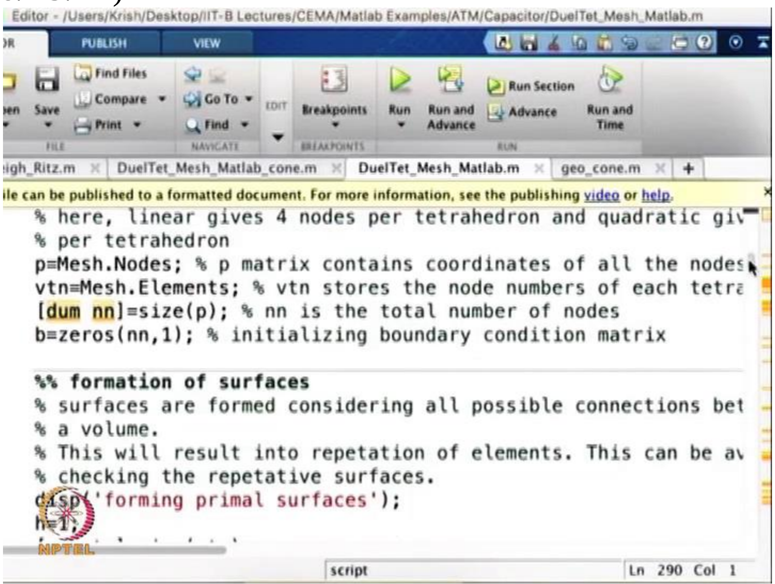

So let’s see what we do with this model once we have the Primal grid we are trying to get the information about it surfaces we are forming the surface is first of all and get certain information about it.

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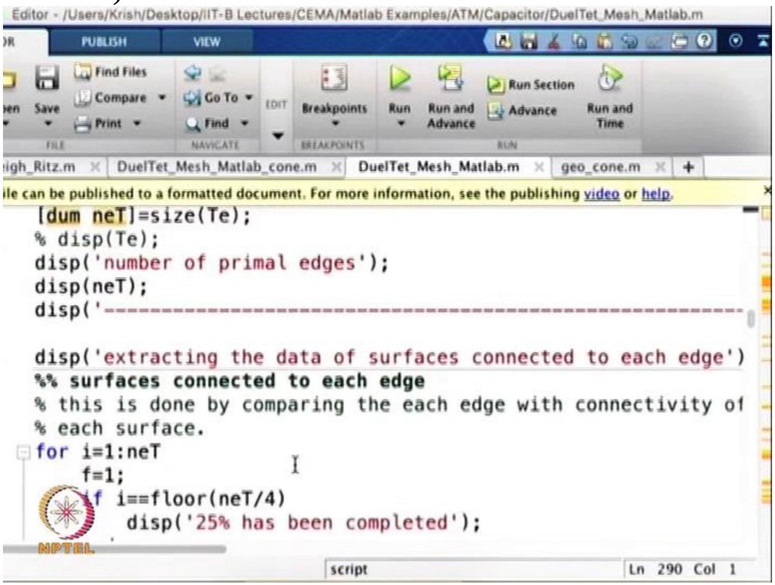

And we are also getting the information about Primal edges and we are going inside each of the surfaces to see the connected edges to the surfaces.

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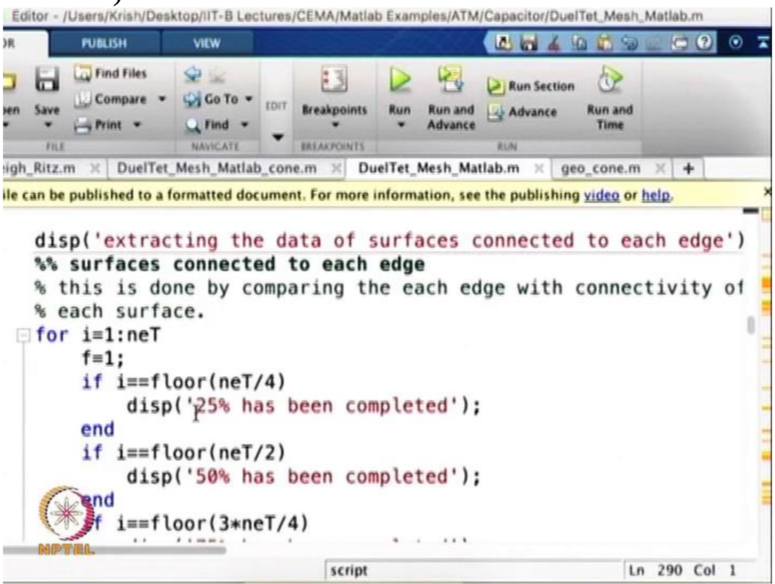

And while we are running the code we always keep track of how much percentage of the work has been done so we say this much amount of work is done when it is 25% 50% so on and so forth because we know the number of tetrahedral and number of edges and number of surfaces.

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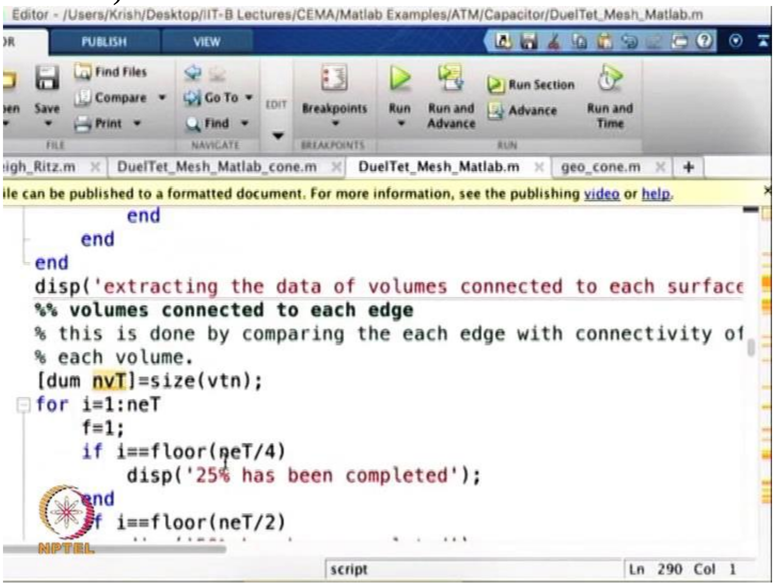

Now we are looking at the volumes and the connected edges. we are doing that to create the A1 A2 and A3 matrices if you remember the way we have to get the A1 A2 and A3 matrices depends on the connectivity and these are connectivity matrices and their connecting Each Other surfaces to the volume so we see volumes at each surfaces once we do that we are basically ready to generate the primal a1 matrix which is the matrix from edges 2 nodes.

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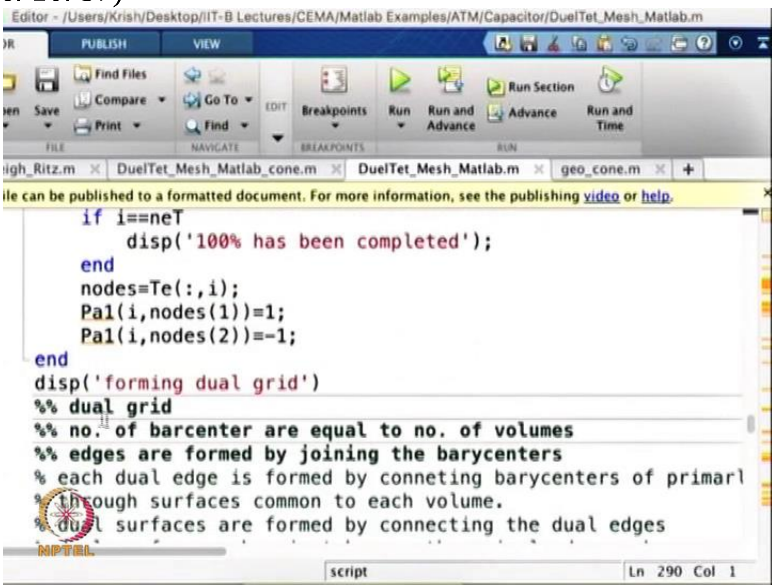

Similarly we will do the same thing to generate the dual grade the dual grade is generated with the information what we have from the primal grid once we have that we basically create the surfaces of each of the volumes.

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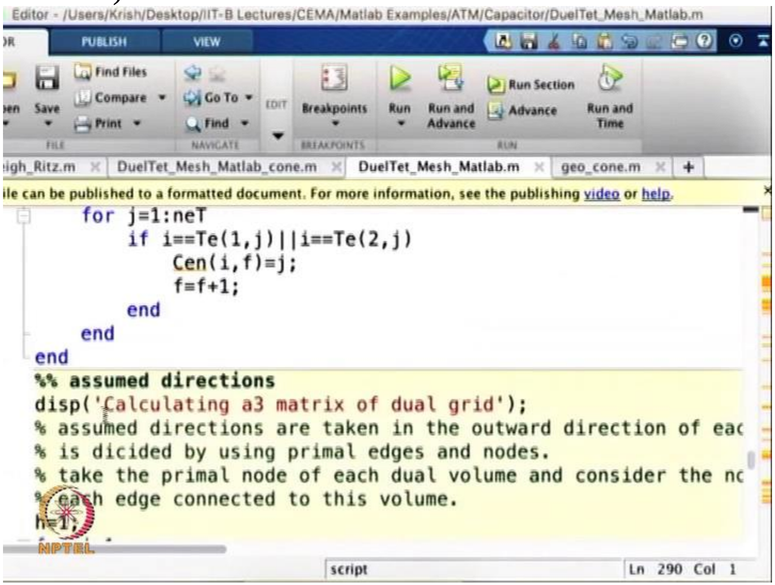

And we see the assume direction this is very important because you need to calculate the a 3 matrix.

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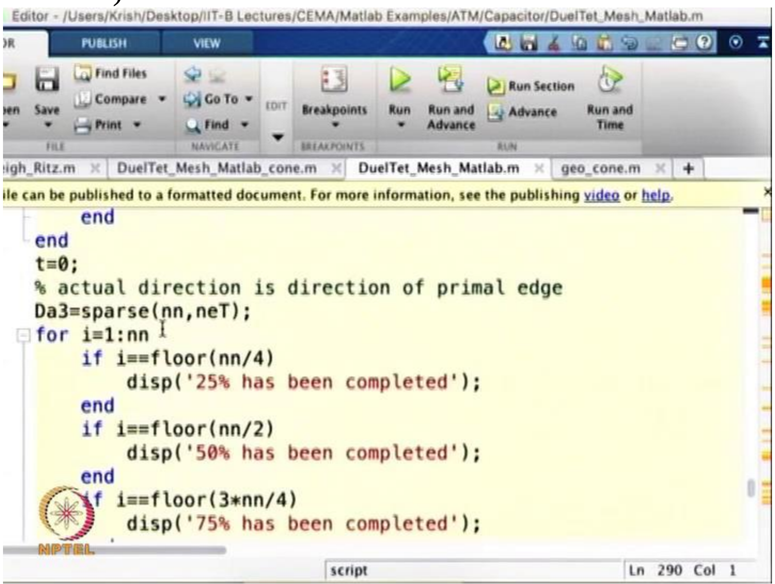

And we generate the dollar 3 matrix in a manner initially we declare it or initialise it as a sparse matrix this is very important step so as to make the code run faster

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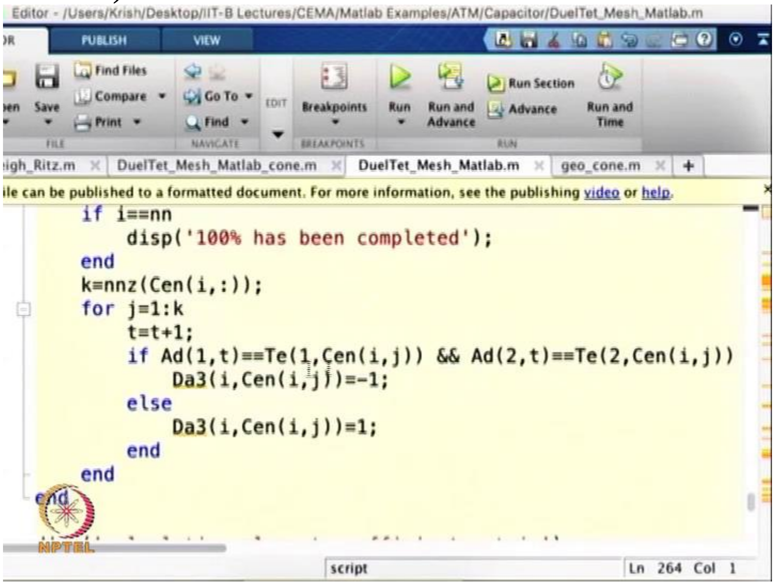

And once we have that we create the connectivity of each of the surfaces and volumes and we are able to create the dual of the A3 matrix.

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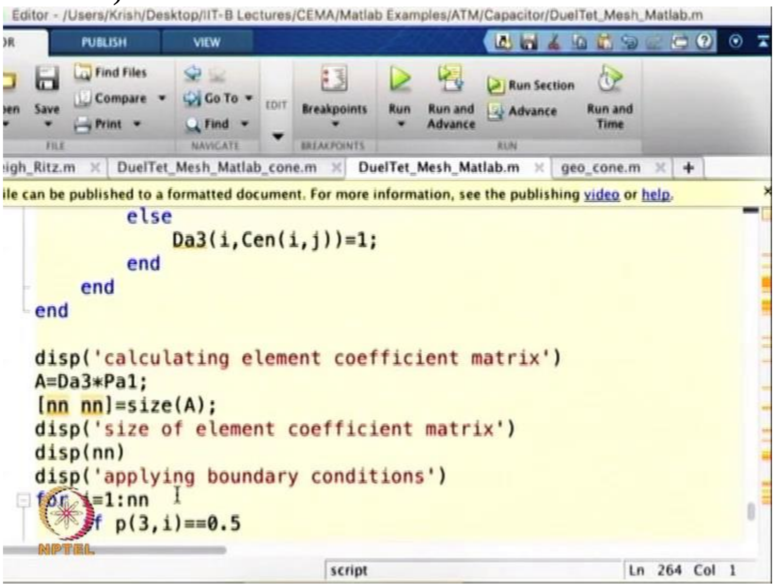

And we are calculating the element Coefficient matrix a as I explained before using this particular formula.

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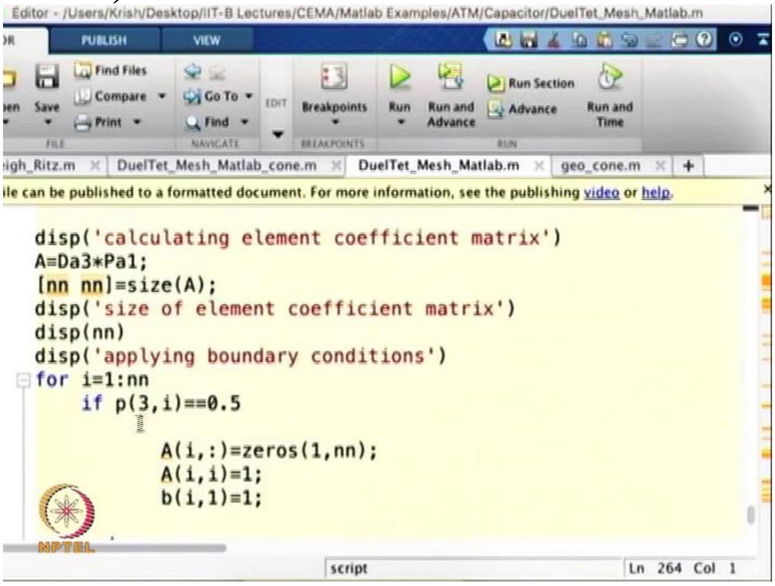

And we are applying the boundary conditions remember the boundary condition are defined in our case.

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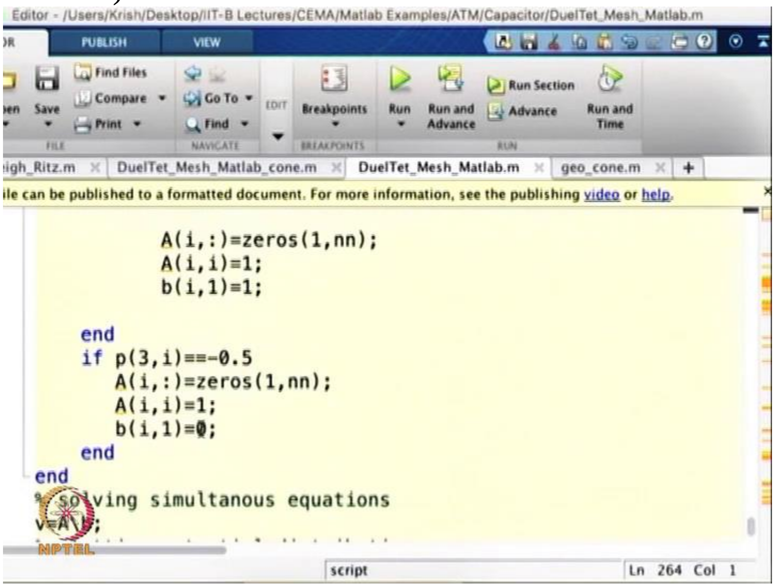

The top plate is going to be at 1 volt and the bottom plate is going to be at o volt this is what we are doing.

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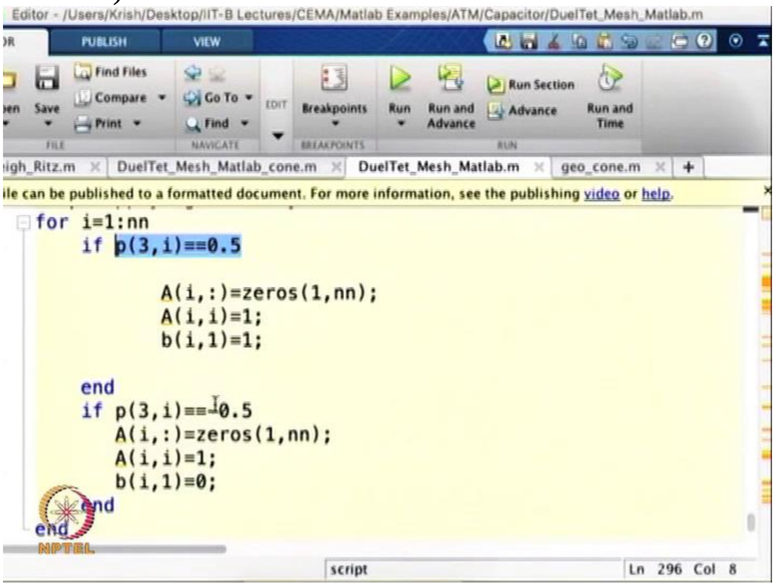

We are knowing that operate and the bottom plate based on the initial indication of the geometry itself you can choose a different geometry this value will change this is particularly for the geometry that we are using.

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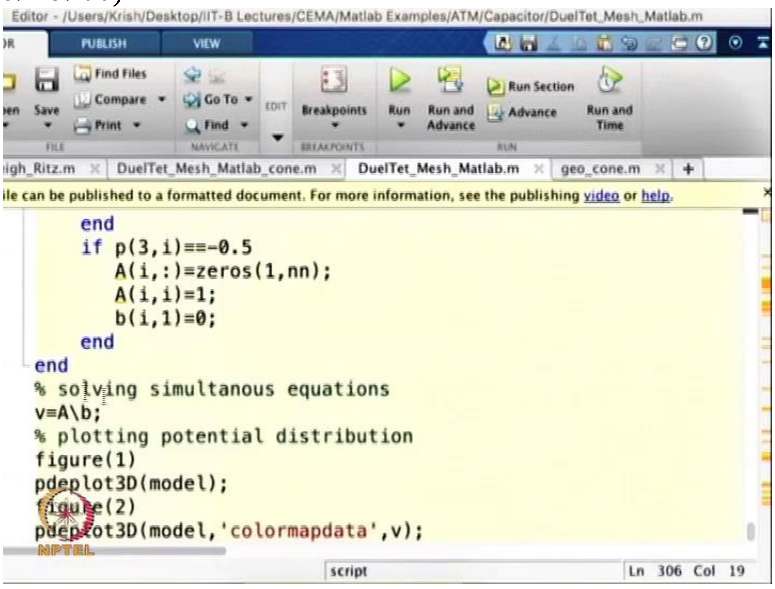

And you are solving the simultaneous equations initially we had we call to be where B is a boundary condition and we are able to solve for this equation by inverting the a and we are plotting the potential distribution for that problem so if you run this particular problem at the path what we will get is several information that we are interested.

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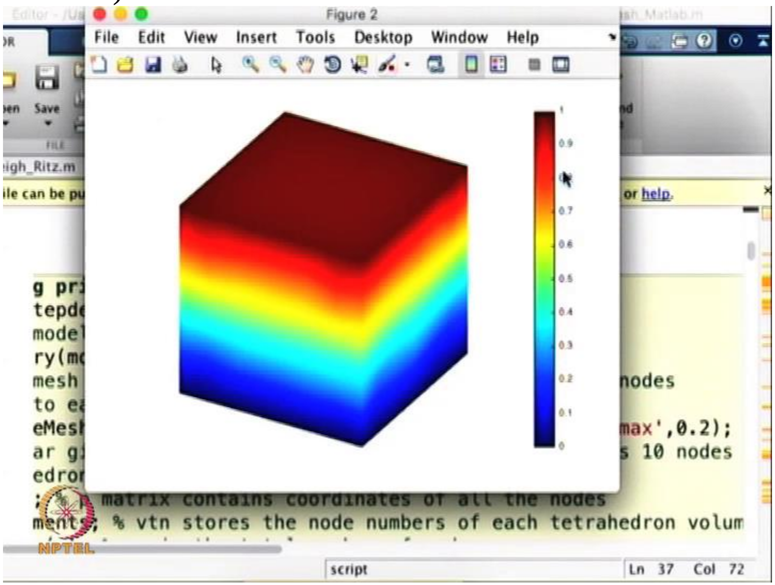

So there are two things one is grid itself that is being generated.

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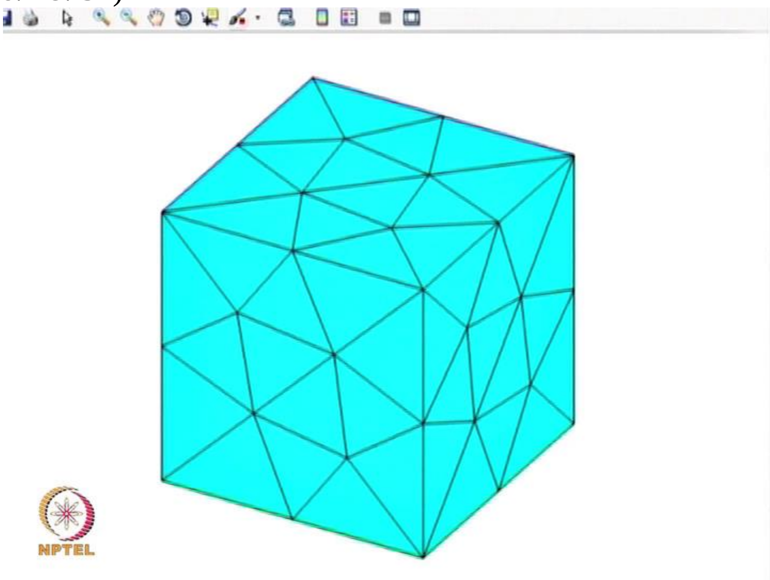

So this is the great that we are going to generate is surface has 1 2 3 4 5 6 7 8 9 10 11 12 13 14 roughly 14 or 15 tetrahedral and we are able to get to the point of seeing the result where we are able to simulate the problem for the potential .

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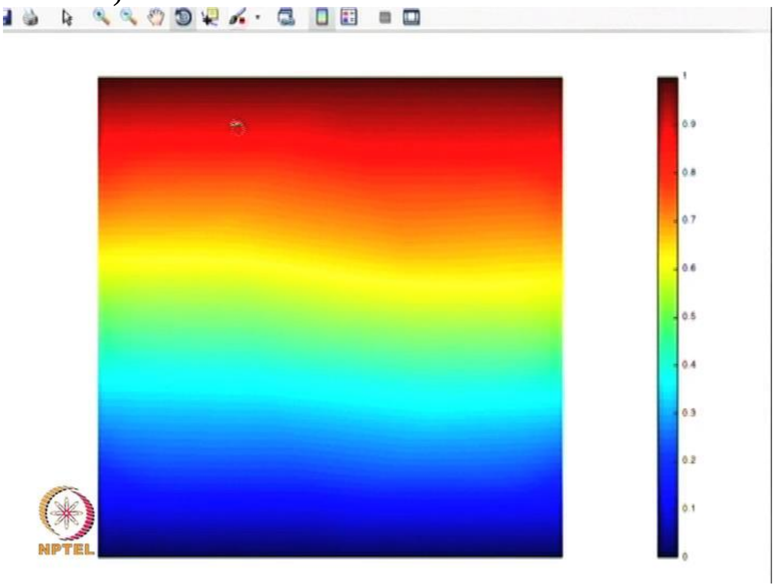

and what you see is the potential this is a three dimensional problem and you will see it is still not a very smooth line the way we want it ideal it should be parallel lines you don’t see the parallel lines because the disc utilisation is very course we see that there are some errors this can be improved if we go into a very very find discretisation and get a better result based on that.

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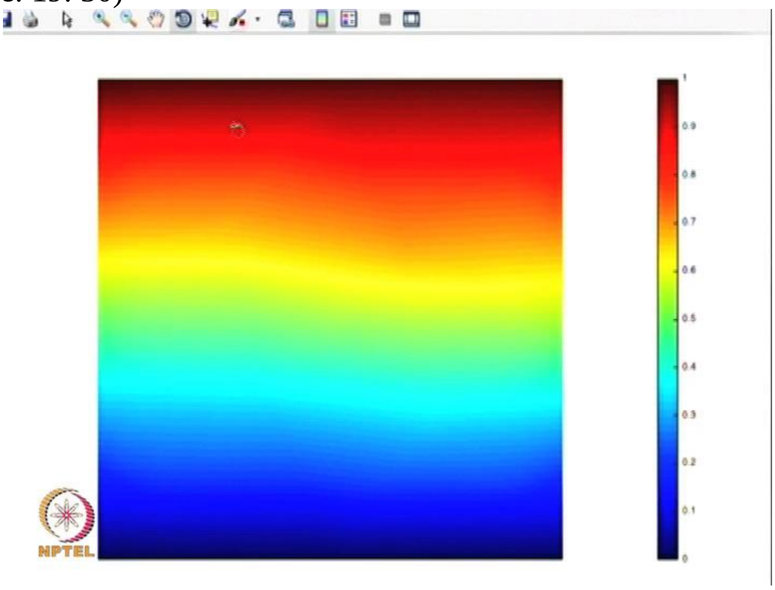

So that’s now get back to the code and reduce the discretisation a bit so as to generate smaller tetrahedral so that our accuracy can be improved the way to do that is in the code we have to set the value for h which is a parameter that we are going to reduce so let’s go back to the code.

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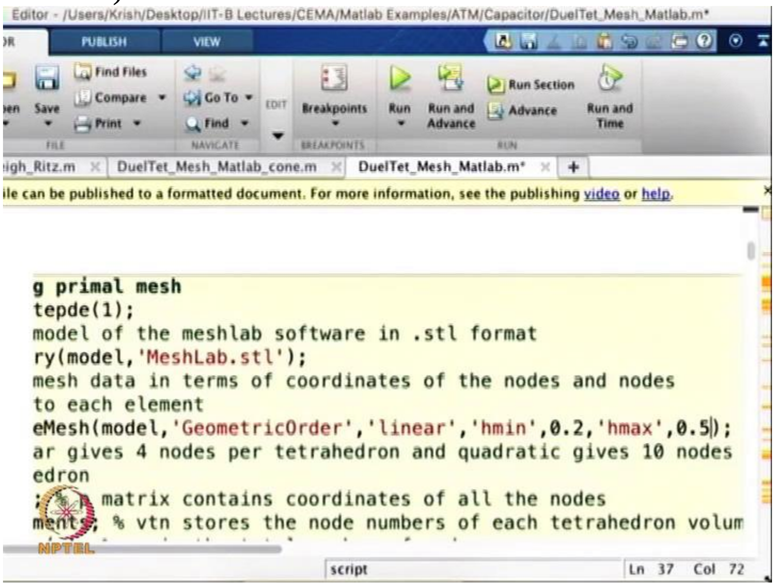

So now we can reduce the discretisation by changing this value here which is the h min value so if you said that its min value to be 0.5 we got the discretisation which we got now so let’s reduce it to 0.2 and as Max is also 0.2.

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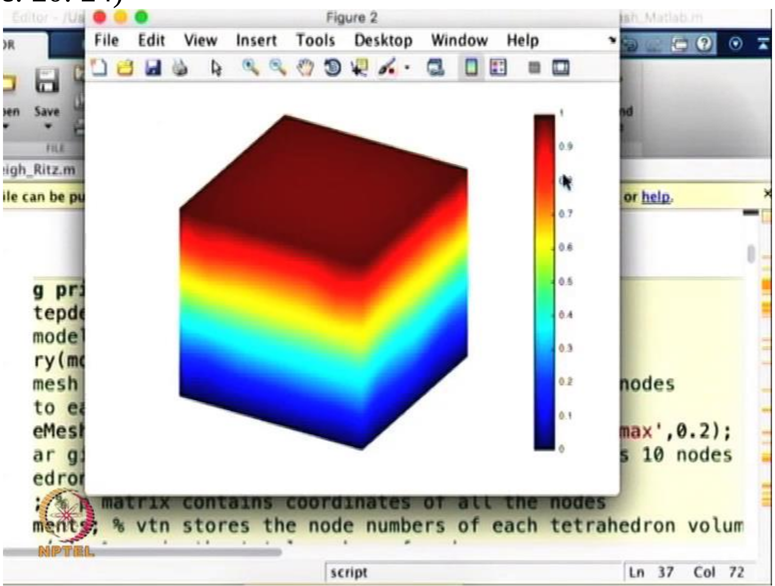

So if you are discretizing this particular problem in the way what we have done now.

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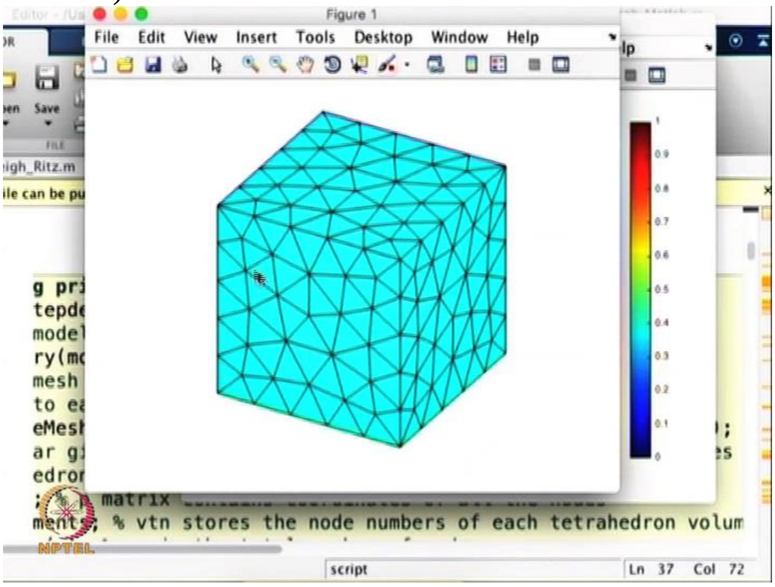

We have reduced the mesh maximum and minimum edge length what we see here is the result.

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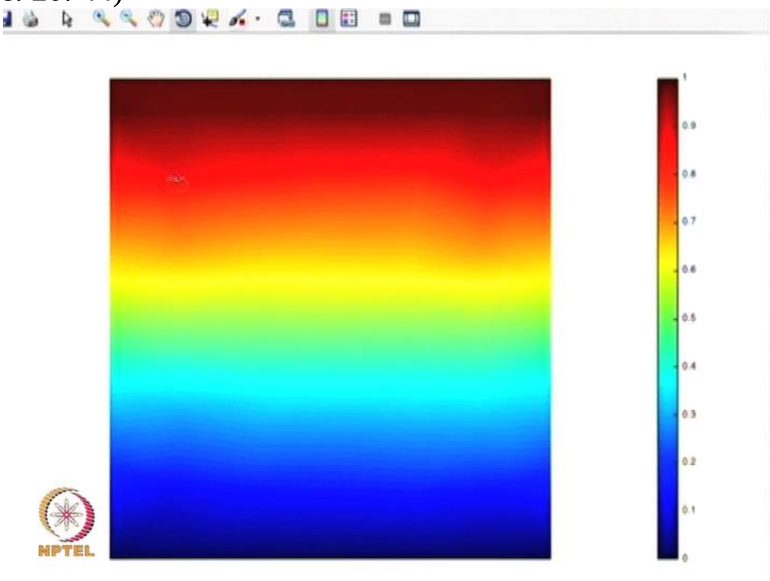

And you can compare that with the earlier result the result are much more finer and you see that you still see certain changes in the equipotential line the reason for this is the grid is still not fine this will change if we go even finer and we can do that by changing the value of h in the main code.

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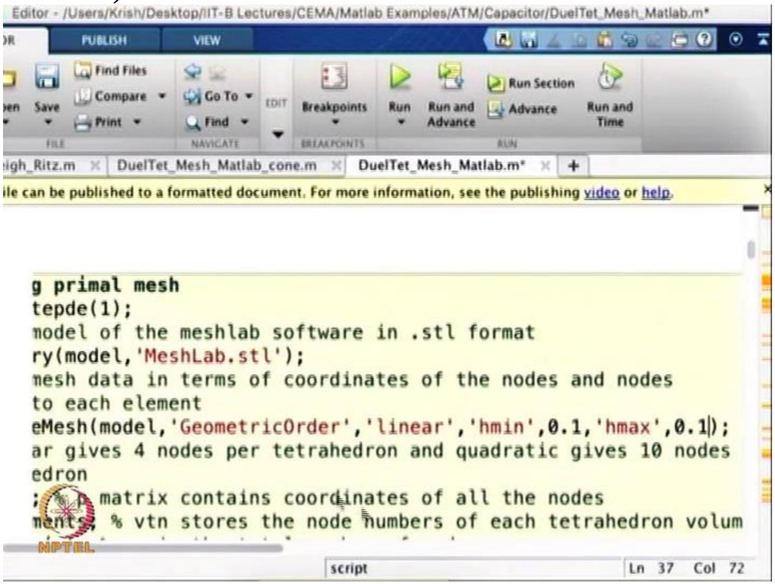

So now we are going to reduce the h min and h Max even find her so as to see what is the impact of discretization on our problem resolution so discretizing it final is going to take us a long time to run the code just to see the proof of concept that we are converging and doing this so you have to make define choice of of what is going to be your discretisation for 0.1 we are already in a very very fine discretisation so I am going to run it.

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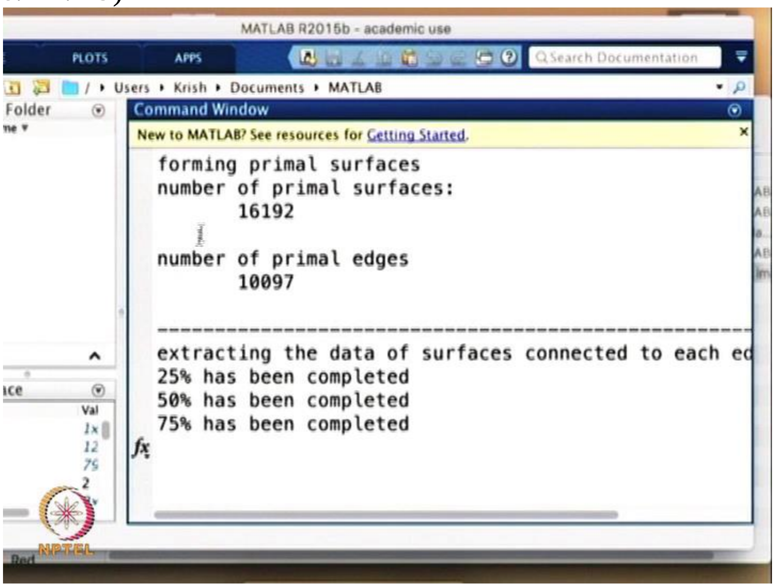

So you will see in the main window that we have 16192 surface 10097 Primal edges and we are extracting the data for the Primal surfaces edges and volumes and with that we will go and do the something for the dual edges dual volumes and dual surfaces and we are extracting the value and we are seeing the result.

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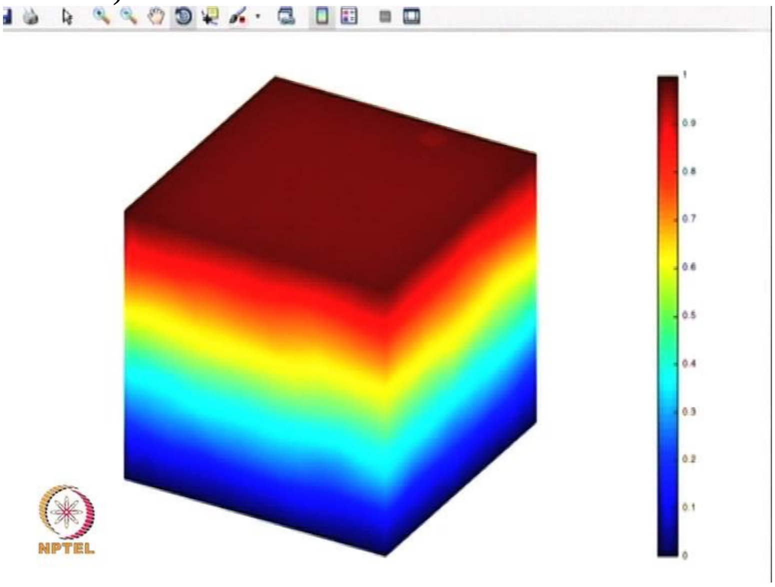

So what you see are two results the first one is the result for the potential itself so before that maybe it’s worth to go and see the problem discretisation itself.

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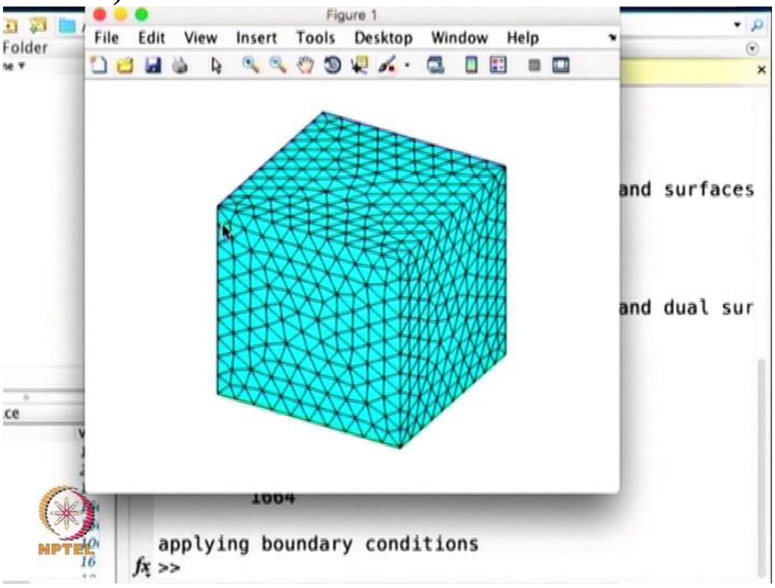
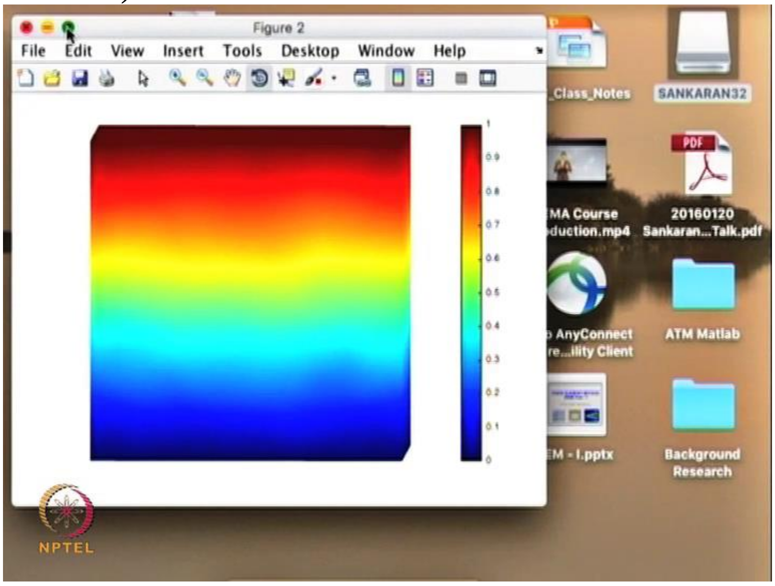


This is a very fine discretisation and as you can see we have gone from the initial problem of 0.5 as the minimum and maximum discretisation we have now 0.1 as the cell size the maximum size of the tetrahedral edges is going to be 0.1 and for this point discretisation we are able to get result which is also much more refined and this week and see in the result of the potential itself what you have computed.

(Refer Slide Time: 23: 30)



You can see that the potential lines are much more finer than before. The equipotential lines are much more refined than before and the way we have computed the dual grade and the Primal grade is very characteristic to the way a normal problem is solved in the case of algebraic topology so what I wanted to do is I wanted to use his opportunity also to do the same problem that we have also so in the case of finite element method the conical capacitor so we are going to do the same problem that we saw before in the finite element method which is a conical capacitor and we are going to run the same problem using algebraic topology and see the result this we will do in the next module we will come back and simulate the problem using algebraic topological method thank you.