Computational Electromagentics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Exercise No 41
Algebraic Topological Method (ATM-III)

So we have covered quite a lot of things in the case of algebraic topology so far we have looked into quite a lot of new concepts using the new tools all the new language of algebraic topology what we going to do now is we are going to proceed further with the Maxwell equation so when we go forward we have to look into one more aspect that is missing in order to model the Maxwell equation and aspect is called as orientation the idea of orientation has been pretty much discussed in the literature what I mean by literature the engineering literature of Physics literature quite Naroli so what they talk about orientation is mostly about the orientation related to the inner space when I say enough space this might be something confusing to you what is the mean by inner space if there is enough space are there other spaces and how do we make sense of other spaces and these are the things that we are going to look into in today's lecture so with that as a preamble let's look at the topics of today's lecture.

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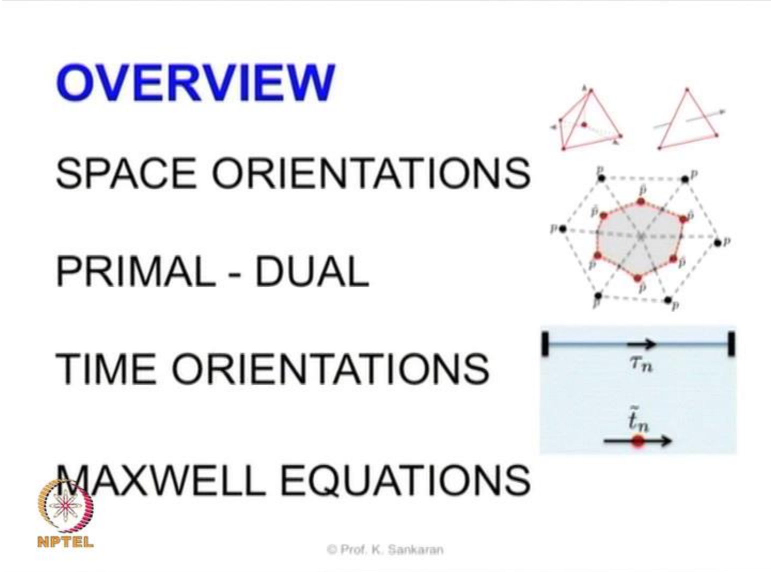

So what we have got is we have got the idea of spatial orientation the second one is we are going to discuss about the Primal and dual complexes and the third one will be on the topic of time orientations and finally will come to the aspect of modelling Maxwell equations using all the things that you have learnt so far so with that being the motivation I think we should be going quite directly into the topic of spatial orientation so let's look At spatial orientation.

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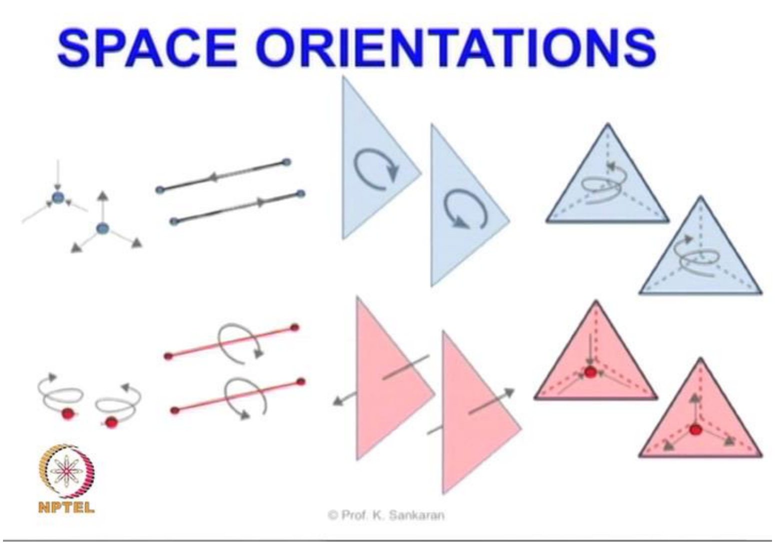

So what you mean by special orientation is something quite broader then what is being discussed in Literature so far that's what I said before that we qualify what I said so when I talk about let's say the orientation of a line.

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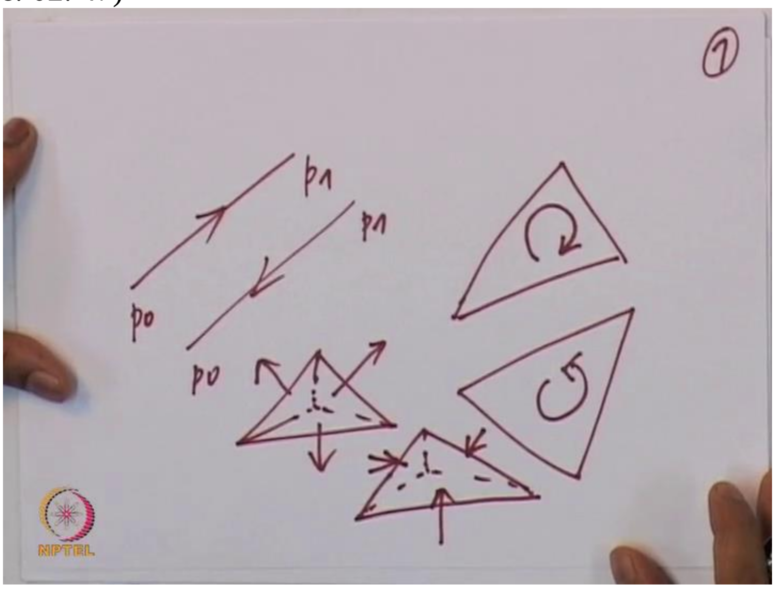

I might think about the orientation that is going from point p 0 to P1 I might talk about an opposite orientation that is going from P1 to P0 likewise when I am talking let's say an area I can talk about the orientation that is going clockwise likewise I can also talk about the orientation that is going anticlockwise the same way we can talk about the kind of a volume let's say this is a tetrahedron I can talk about out appointing normal likewise I can talk about the other side which is the inward Pointing normal so this is the way we define orientation mostly in the literature.

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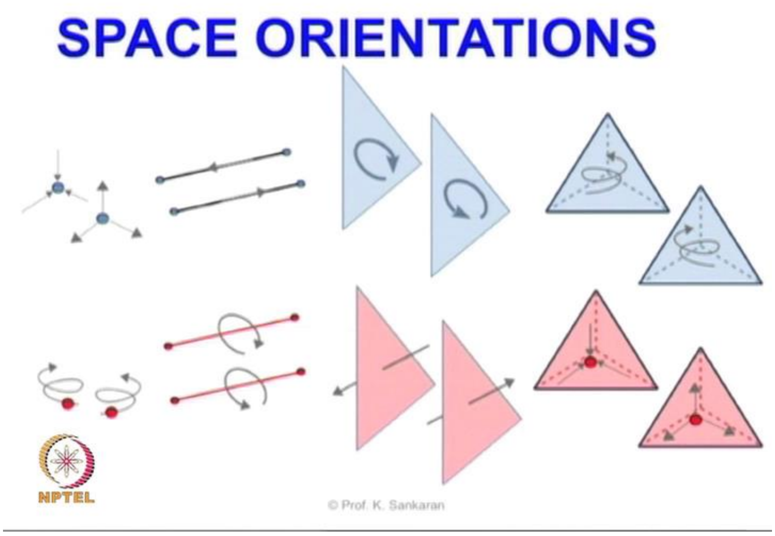

So if somebody talks about orientation they might talk about either right hand oriented or left hand oriented or you know clockwise or anticlockwise a to b or b to a things of that sort. This is a very very narrow idea of orientation and there is something more to it then what we know of and this is something that applied physicist or engineering community don't pay attention to but it is known to theoretical physicist and also to a larger body of mathematician so we will look into the other aspect of orientation which we call it as outer orientation.

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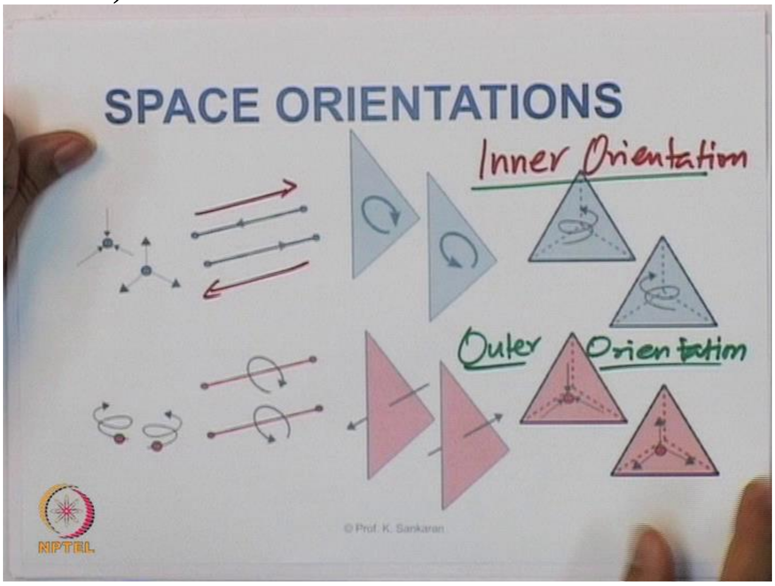

So what we have discussed so far whether the point is coming internally or outward which we discussed whether it is going from A to B or B to a we have also discussed it so clockwise or anticlockwise whether it is going out or coming in whatever it is so these are all the kind of orientations we talked about so far we call them as inner orientation so inner orientation to a larger extent can be understood because it is almost on the same dimension or the same space of the body of the object itself so when we talk about the line it is along the line when we are talking about surface its along the surface, when we are talking about the volume its internally in the volume the volume we talked about the clockwise rotation or anticlockwise rotation so this is a curl or divergence things of short we are talking about whatever is happening in internally to the object but this is very clear in the case of a volume when we talked about divergence we are talking about the outer orientation where is when we are talking about the rotation on the surface we are talking about the inner orientation similarly for a surface the outer orientation is the direction in which it actually you know the flux is going in are going out weather is it is going in a going out so it's the same surface but its outer orientation is outside the surface body itself so in other words in the case of the inner orientation you are along the surface where as in the outer orientation you are outside of that embedding space so I have to tell you something that you have to pay attention to before making a conclusion of inner and Outer orientation that is something called as the embedding space itself .

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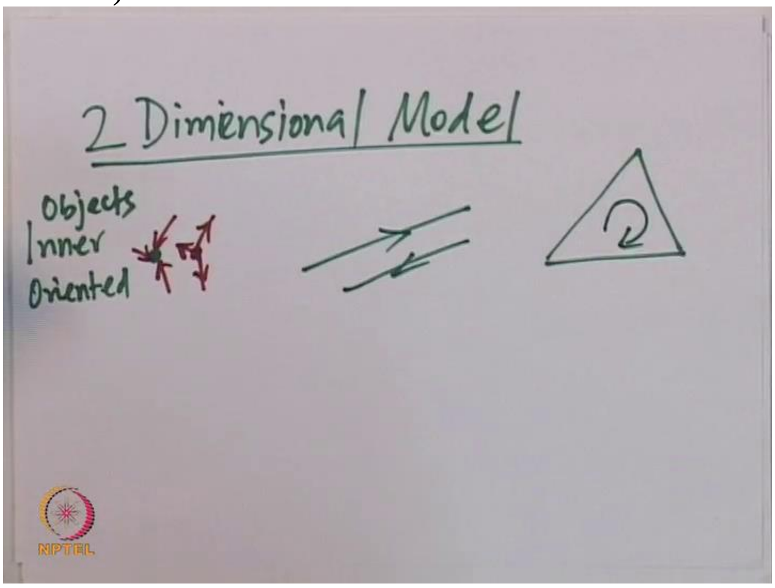

So what I mean by embedding space is nothing but if I am going to be in a two dimensional model if I am in a two dimensional model all I have got is I have got the. I have got a line and I have got the surface and these are the inner oriented objects with inner orientation so here I can talk about both points coming internally or a point where it's going outward so it could be in word pointing Arrow or outward pointing have both are allowed similarly in the case of a line it could be going in this direction or going in opposite direction so for the sake of simplicity I am going to only consider one direction likewise I am going to consider only the clockwise direction.

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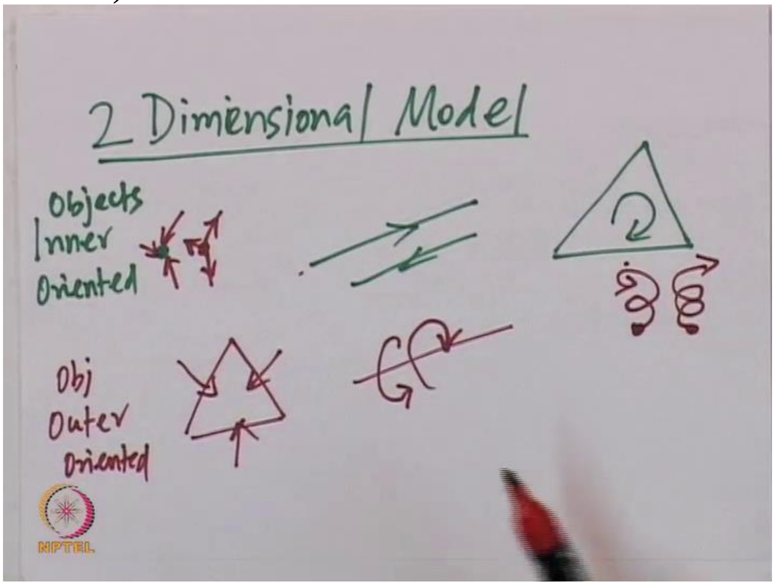

So if that being the case what are the objects that are in the outer oriented space a point It will be a surface for a line it will be align itself for a surface it will be a point but this line and this line they have two different things hear the line has orientation that is along the line where is here the orientation will be above the line so the line will provide me the access and the orientation could be clockwise or anticlockwise depending on the option which we are using. Similarly in the case of a surface it could be I think that is diverging also it could be something coming in or going out for the point. It could be the rotation about the surface in this direction or in the other direction so when you see in the two dimensional space the inner oriented object has a counterpart in a two dimensional case the counterpart of a point is going to be a surface the counterpart of a line is going to be a line itself the counter part of a surface is going to be a point.

So the point will become surface the line will become line itself and the surface will become a point of course we are talking about orientation that are outer that doesn't fall into the object itself in the case of the surface we are not talking about orientation along the surface but going outside or inside so that's why we call it outer orientation. So it's important to know in two dimension that there is a counterpart for each of these objects.

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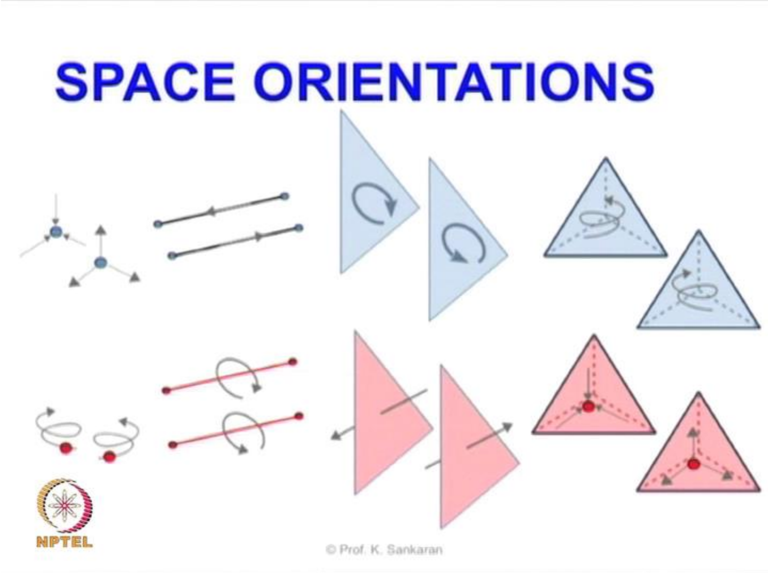

So let’s take the case of 3 dimension so what is important to know is in the two dimensional case we had a point associated to the surface align associated to a line in the dual space or the outer oriented space and the surface has a point. So let’s look what happens in a three dimensional case.

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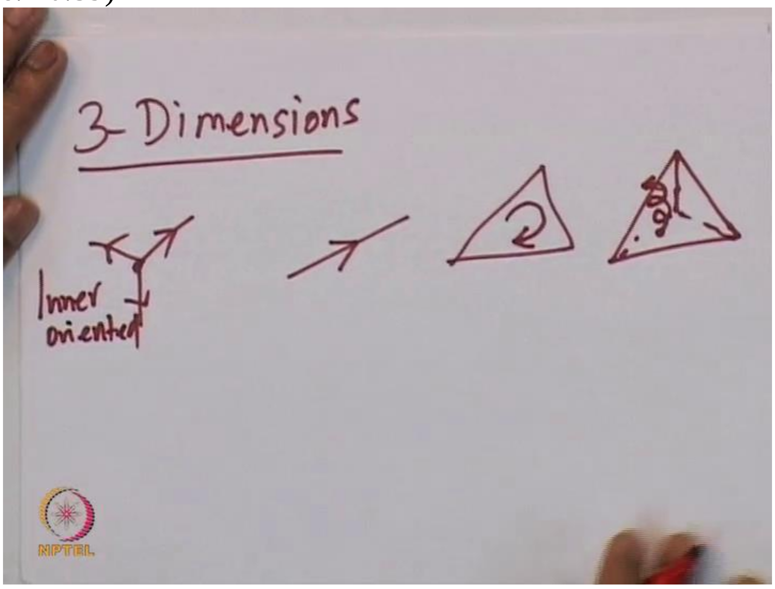

So likewise we have a point we have a line we have a triangle surface and we have a volume for the sake of simplicity I am assuming only one of those two options like the point is the lines are going away from the point likewise I am choosing a direction that is going from left to right likewise I am choosing the orientation that is clockwise likewise I am thinking about a curl that is internally oriented so its on the inside of the volume and the curl direction is let’s say in the clockwise direction so now these are the inner oriented.

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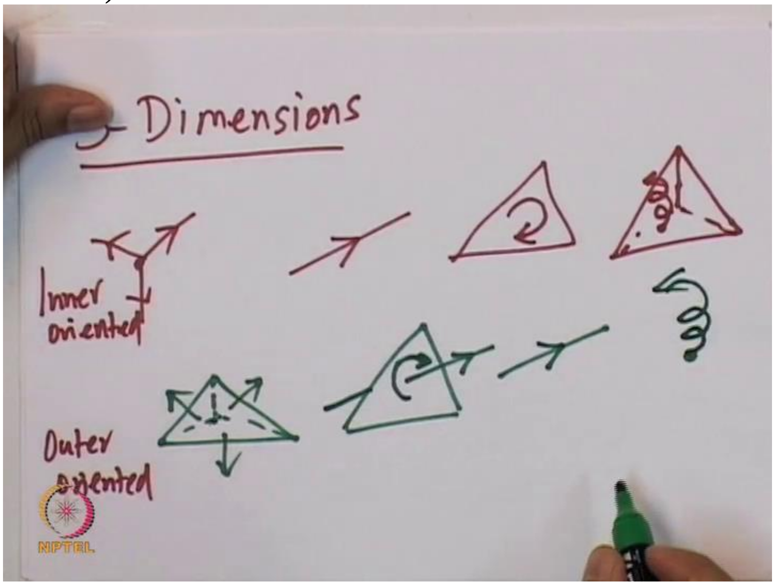

So what we're going to do is we are going to see outer oriented space for these objects what we are going to see his for the point It will be the volume father line it will be a surface for the surface is going to be a line and for volume it's going to be a. What you are going to notices in the case of the point here this is going outside and now what you are going to have here is divergence that is going to happen along this volume and in the case of the surface is going to be a line that is passing through the surface and we are going to talk about the orientation along this line that sits on the surface and in the case of the surface is going to be a line where the orientation of the line is given in One Direction so what we are interested in the direction that is pointing here and for the. It's going to be the curl.

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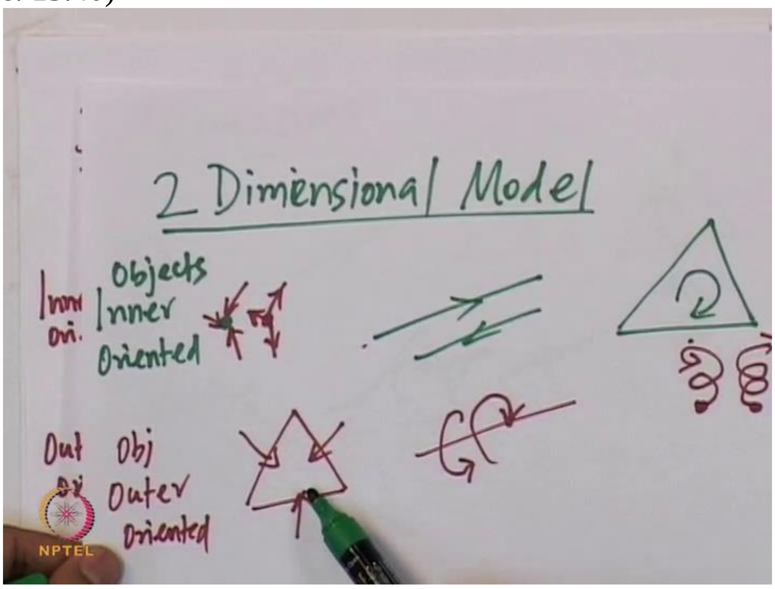

So as you compare the two dimensional case here the points dual is surface.

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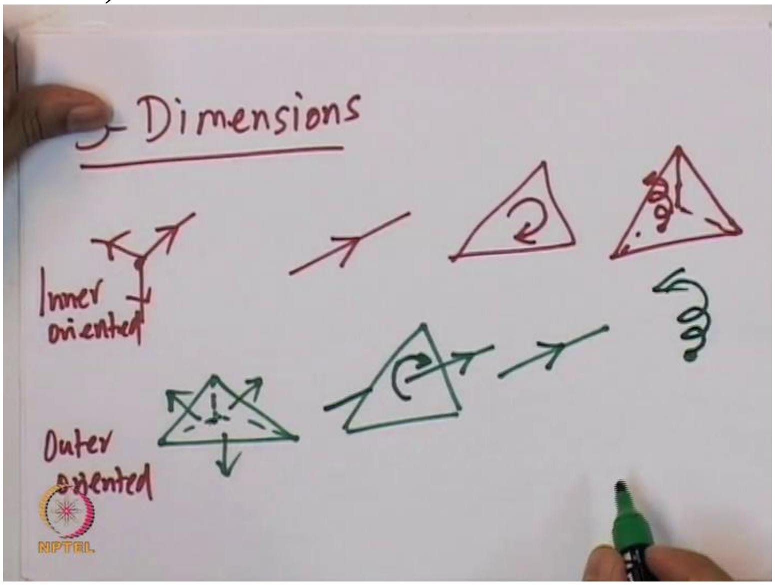

Where is in the case of three dimension the points dual is a volume not surface so depending on the dimension the dual space is going to very. So if you are in a two dimensional space the dual space of the point is going to be a surface where is in the three dimensional space it is going to be the volume in other words what is important to know is the Primal space is fixed a point will always be a point a line will always be a line surface be always be a surface a volume will be always a volume in the Primal space in that oriented space I use the word primal and inner oriented space in a manner that is similar it can be also do a space but what I am talking about is a Primal space is a space where the definition of the field variables are associated but just look at it at a later stage but what is important for you know to know is the inner space is going to be constant whether you are in a two dimension or three dimension or any dimension for most of the practical applications you are going to be within the three dimensional space so regardless of whether we are in the two dimensional or three dimension the point is going to be a point.

The line is going to be the line and the surface is going to be a surface and volume is going to be a volume and the orientation are also going to be along the object itself that's why we call it inner orientation the outer orientation is going to be dependent on the embedded space if you are going to be in a two dimensional problem is going to be like the way we discussed if you are going to be in a three dimensional problem it's going to be the way we discussed.

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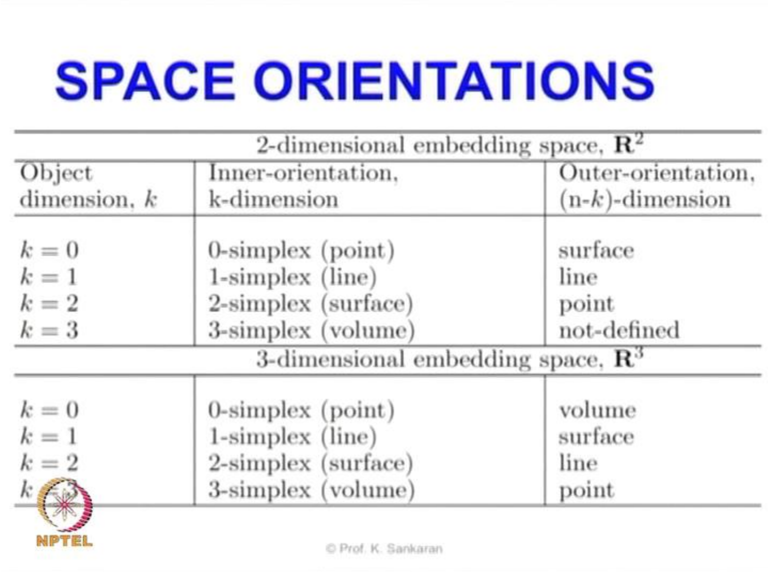

SPACE ORIENTATIONS

2-dimensional embedding space, $\mathbf{R}^2$		
Object dimension, k	Inner-orientation, k-dimension	Outer-orientation, (n-k)-dimension
$k=0$	0-simplex (point)	surface
$k=1$	1-simplex (line)	line
$k=2$	2-simplex (surface)	point
$k=3$	3-simplex (volume)	not-defined
3-dimensional embedding space, $\mathbf{R}^3$		
$k=0$	0-simplex (point)	volume
$k=1$	1-simplex (line)	surface
$k=2$	2-simplex (surface)	line
$k=3$	3-simplex (volume)	point

So with that as an introduction let’s look at let’s look at what we have got for the summary for all the spaces I think we have seen the case of two dimension which is here the point will be a surface will be the outer orientation line will be given by the lines surface will be given by the point Volume will be given not defined here because in two dimension there is no concept of a volume where has in the 3 dimension you have got all the full set the point will be the outer orientation for the point will be volume the order orientation of a line will be a surface will be a line and volume will be a point. So far we have differentiated between the two different kinds of spaces now we are going to call them either Primal or dual I have already used the word primal or dual while describing the spaces but let’s see in the detail What this means for the more topological point of view.

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So we have got the Primal dual complex.

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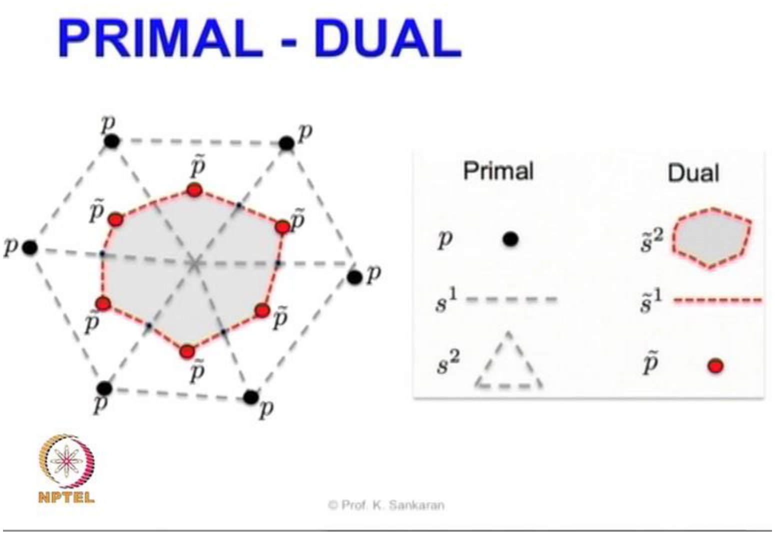

The Primal and dual complex are the two types of oriented complexes if you say my orientation sir going to be within the object in other words inner orientation I call it primal.

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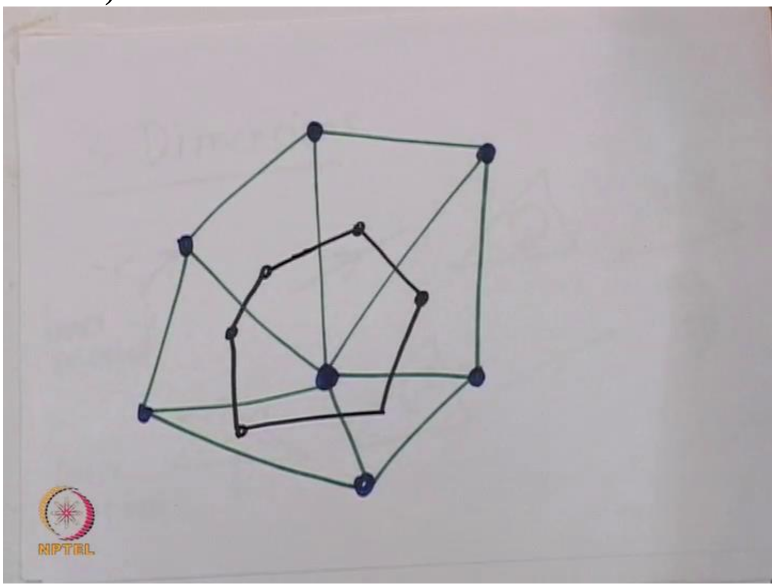

So look at this example where you have got let's say this is a kind of a these are the points we have got another words the zero simplex what we are interested to know is if we call this one as our Primal complex what is going to be the dual Complex that will complex is going to be as shown here let me try it in black so it will have the points which are these points could be either the circum centre of the triangle or it could be the Bari centre so if it is a Bari Centre it will look more symmetrical.

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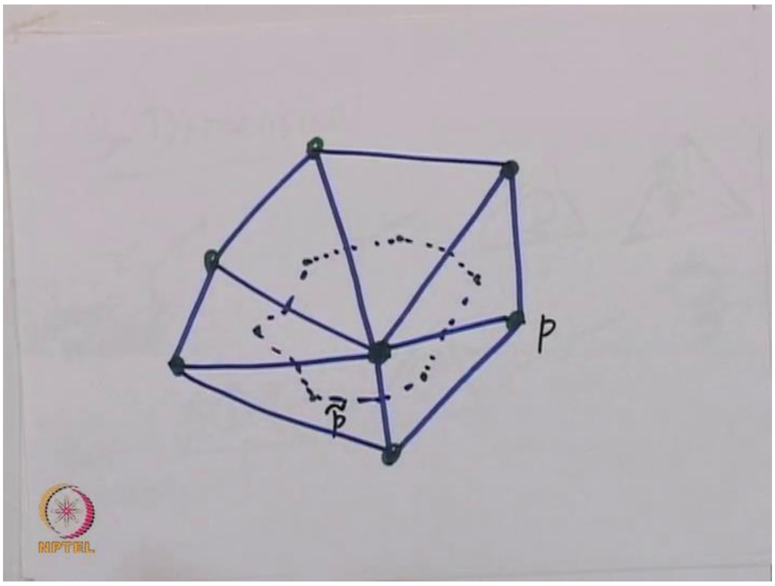

Let me try it using the Bari centre and assuming that these are the note points before and they are interested in the Bari centres let's say these are the centres so what will happen is we are talking about Bari Centre to the face Centre so it will be the line that is connecting the bari Centre to the face centre or the edge centre in two dimensions we are talking about earth centre so we have got now if you call this one has p these points will be P tilde. The tilde is just to show that they are the dual co-ordinate and that's what you have got in this slide.

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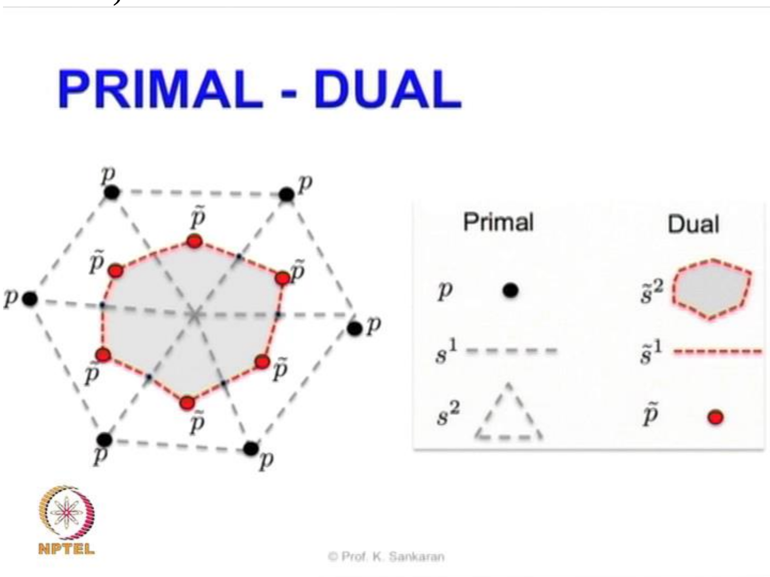

If you can see this slide what you have got is these are the bary centric points which are P tilde they are the dual coordinates of the surface remember we set the dual of the surface in 2 dimension will be a. So we have the surface here the dual of the surface will be this point. Remember we also set the dual of the line will be align itself so the dual of this line this age will be this age remember that we had the dual of the point will be a surface so if you can see this Central point. The central is not as a point. The dual of that point will be the entire thing in other words we have now got simplex that will be more formerly called as dual and Primal simplex.

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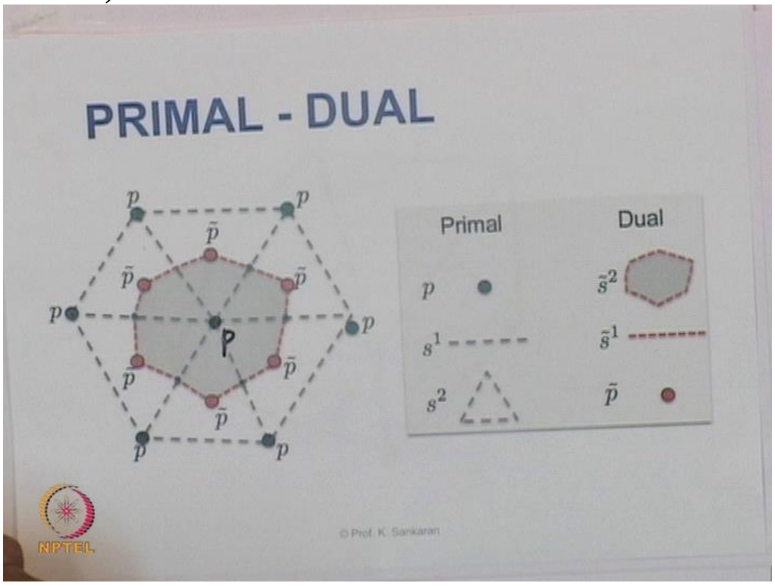

So let me make this more clear this is also a point. So you have a point P so dual of this point is the surface the surface here is what we call here the surface is is this thing

So with this we have covered pretty much the concept of spatial orientation in the next module we will cover what we mean by time orientation and then directly take you to the Maxwell equation thank you.