Computational Electromagentics and Applications
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Lecture No 38
Algebraic Topological Method (ATM-I)

In the earlier module we have set the basis for terms and have defined our motivation for going forward with algebraic the topological formulation now we will look at some of the topological aspects of this method so we will start looking into a network. That say the network is something define as it is shown in this graph as you can see the arrows are pointing input direction there is no preferred direction in the case of the arrows.

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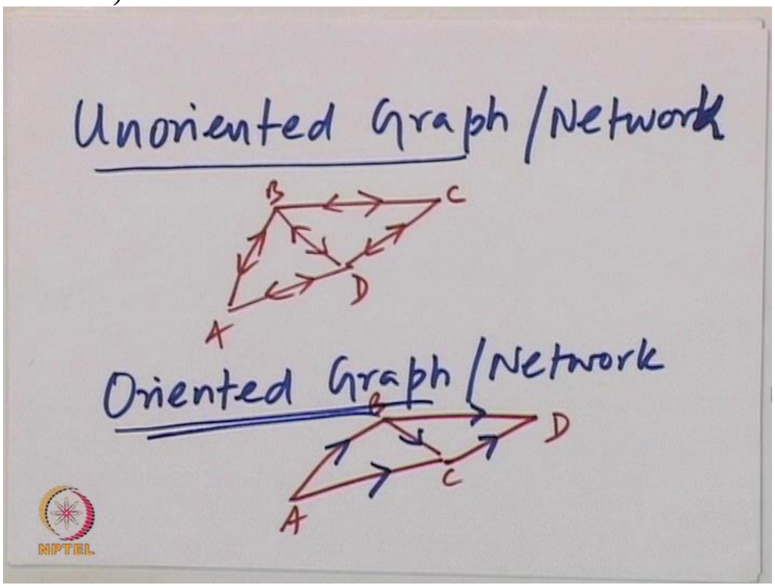

So this is called as un oriented graph or network so an oriented because there is no preferred direction so you can think of it like this it could be like a network of three stations where the trains are running in both directions. So you can go from a to b 2 c2d something of the sort the arrows are nothing but train connections in both directions. So A, B, C, D are the train stations and all the arrows are nothing but the directions in which the train runs in the case of electromagnetics we are not going to talk about an oriented graphs we are going to talk about oriented graph so what are oriented graphs question mark so oriented graphs or networks have certain direction that are define.

So will take the same one we call it a b c and d so we say we are defining A to B as this line so there is nothing from B to A only from A to B and a to see, B2C so on and so forth so these are directions which we define and once we define that we have set certain assumptions and we are going to follow both in the next status so you might as a question why do I have to choose the direction A2B are not B to A you are free to do that once you do that your assumption are fixed so this is the basic starting step of the oriented graph.

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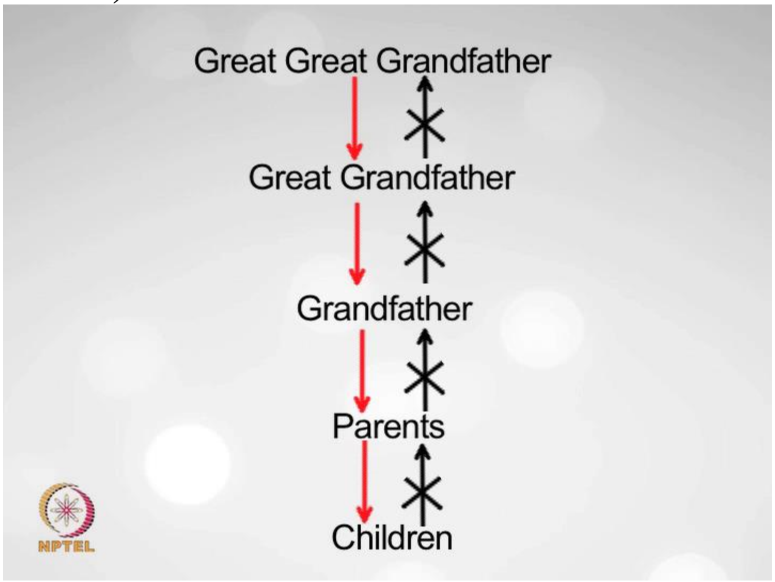

you can think of oriented graph like this so you have let's a starting point as a great great grandfather and then you have great grandfather and then you have grandfather and then parents and then children so there is always transition from one level to another level it doesn't make sense to go on the other side . so in this case network itself a structured in a natural way but in the case of electromagnetic problems there is no natural forcing that has to be made you can define a direction and then you are following in that direction.

So what I mean is you are free to choose a direction there is no need for you to follow certain directional aspect so regardless of whatever it is you have to know that in the case of electromagnetic problem we are going to look into directional graphs are oriented networks

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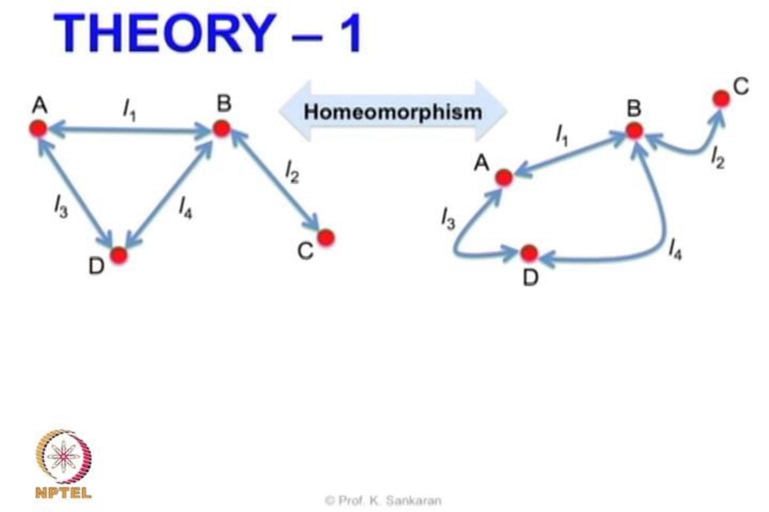

But for the analysis let's say we are using in in this case and an unoriented graph this analysis is equally valid for and oriented graph but for now let's assume that we are given a graph or network. So topologist is a person who cannot make a distinction between let's say a coffee mug and a doughnut. you might are supposed to invite so is he blind actually there is some level of sadal information hidden in this statement topologically a coffee mug is same as a doughnut let me explain you this way.

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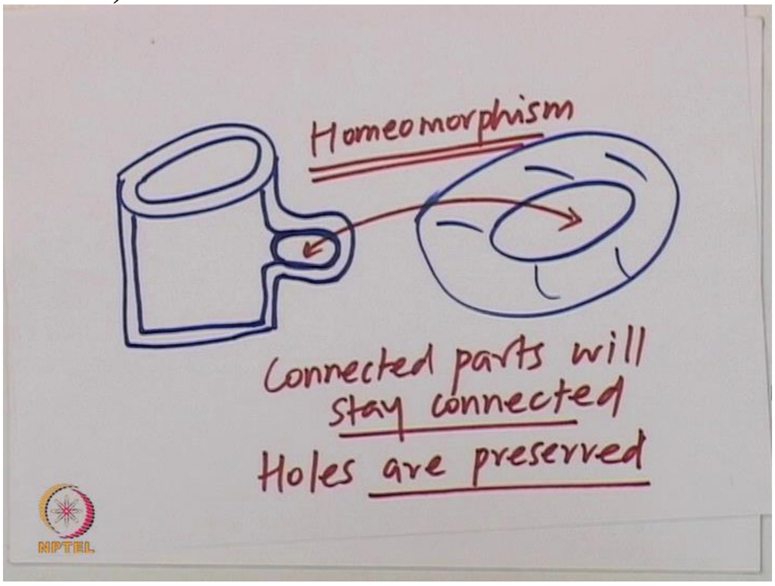

Let say you have a coffee mug so in this coffee mug you have a natural hole that is here so topologist is a person who cannot distinguish between this and let's say you have a doughnut so what's happening here is this whole is preserved in both cases and this coffee mug can be transferred to doughnut and Vice a versa so this is called as home you morphism so what we have in homeomorphism is if there is a hole in one model that hole will be preserved and there are no new holes that are created so in this case there is only one hole and this whole is preserved here you might say there is a hole here but actually it's not a whole if there is hole your coffee will be dripping out so this one is just a shallow there is no whole here but here there is a hole for your finger to go inside and hold it so in a homeomorphism connected parts will stay connected . and then holes are preserved so these are the two important things that you should remember with this understanding let's look at this graph.

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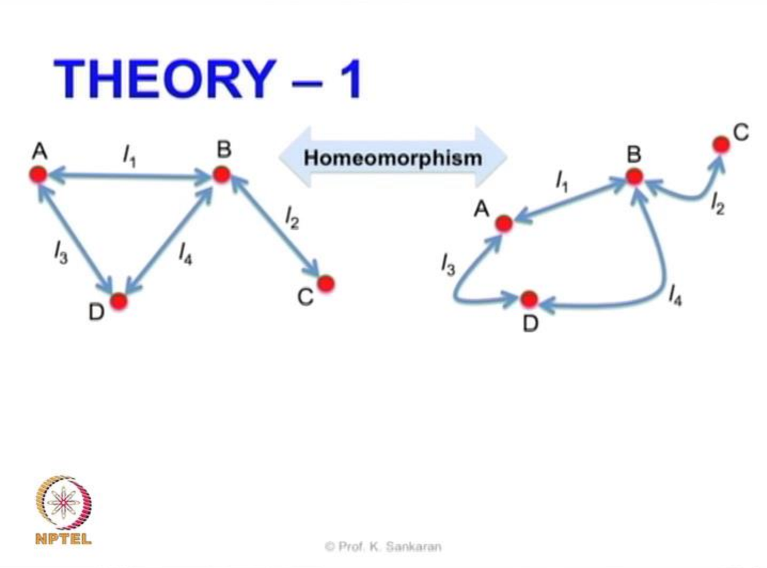

Here what we are doing is we are transforming this network into something else we are stretching it we are contacting it we are bending it but a will always be connected to be and we will always be connected to D we will always be connected to A and there is no new connections that are made and the entire set of stays the same what is not same as the length L1 L2 L3 and L4 they can change and the surface area can change but the points will always be connected and the holes will always be preserved there is no new holes that are created there is no new circles that are created something of that sort so with this understanding we are going to define some of the basic topological objects in this network let's see what they are.

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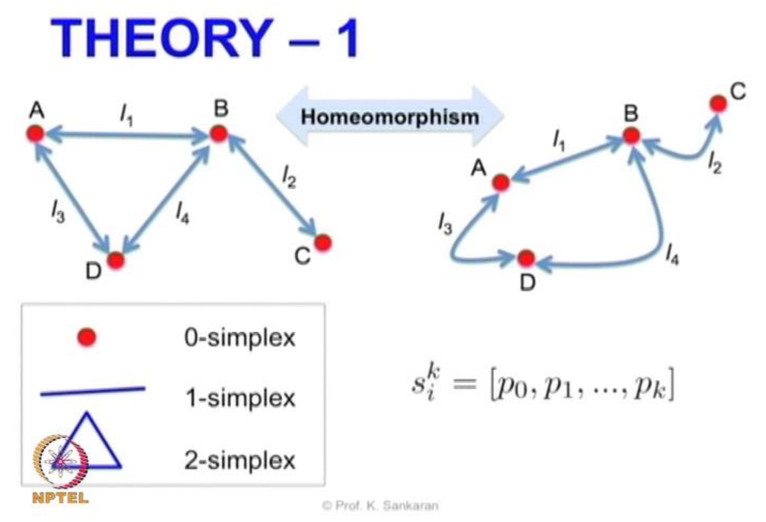

The first one is called as zero simplex we use the simplex instead of points lines and surfaces because once we say.

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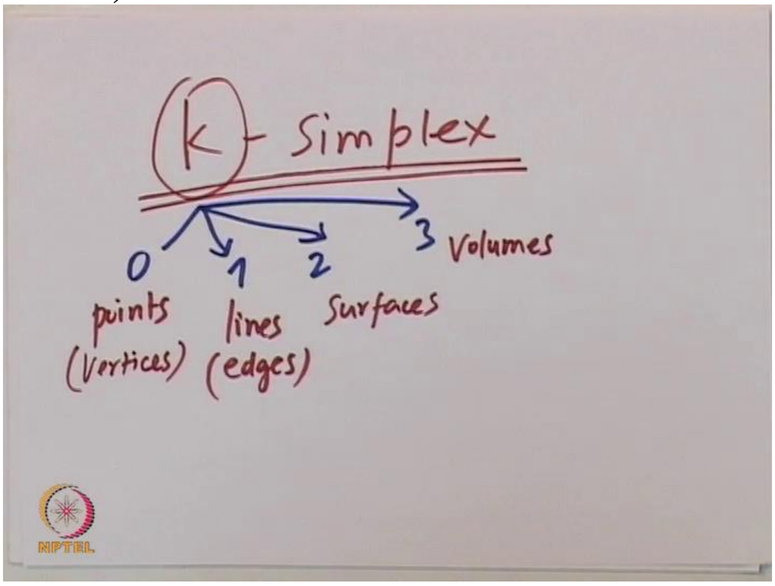

Let's say we have a k simplex K defines the dimension of that particular object so care can be 0 1 2 and 3 in fact k and also go about but most of the practical applications we are interested only upto 3 dimension so if K is zero this is something we know these are points if K is one these are lines or edges. We can also use the word vertices or edges and if k equal to 2 we call them surfaces and then hear these are volumes so with one term which vary in case we are defining all the different topological objects and that is what you are saying here in this particular slide.

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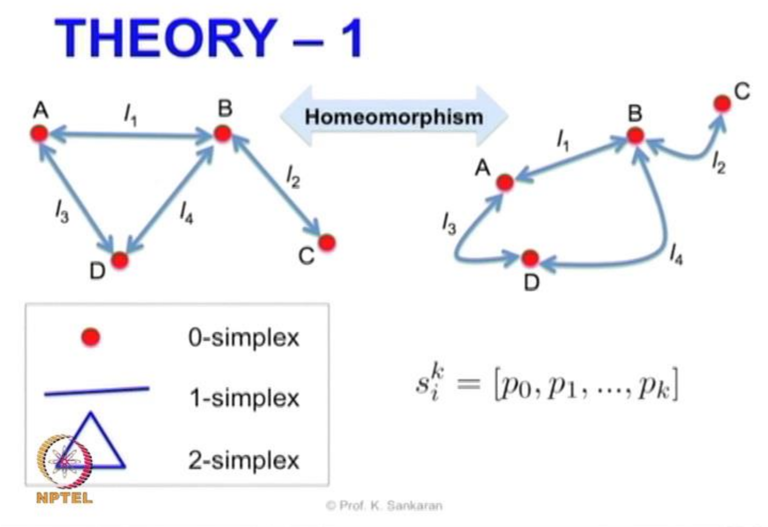

So if K equal to zero you have zero simplex if equal to one you have 1 simplex and equal to two you have to simplex and this case is about a two dimensional problem we are not talking about three simplex but you can imagine a 3 simplex using the analogy what I have given here so regardless of what is a dimension of the simplex we will have the simplex as defined by the set of points so let me explain this here.

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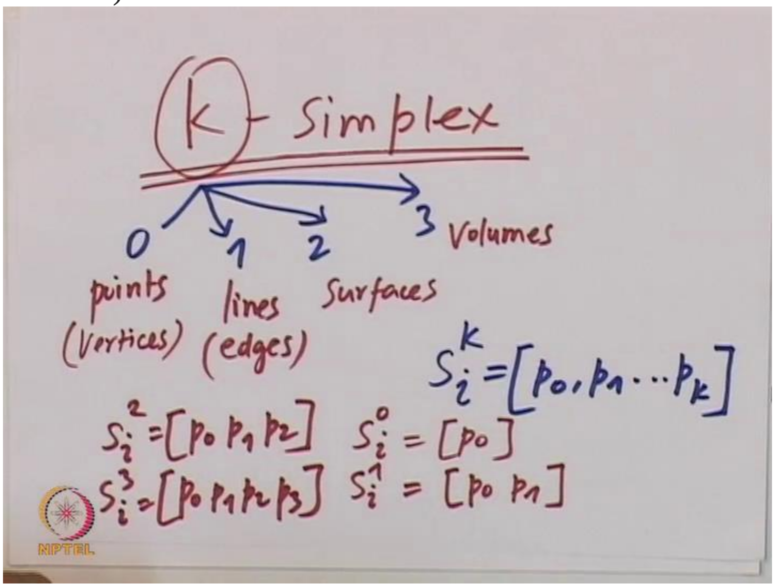

A simplex is defined as a set of points S of I where is the i th simplex and k is the dimension of the simplex is Defined by a set of points so point could be [p 0 p 1 so on and so forth until p k] showcase simplex will have k th dimensions so you can really look at it from a very simple point of view. So if you put k equal to zero simplex will have only one point similarly the i th 1 simplex will have 2 points p 0 and P1 and the i th 2 simplex will have 3 points p 0, P1 and P2. And the eye 3 simplex will have 4 points p0, P1, P2 and P3. So depending on the number for k the number of points are going to change so this is quite easy to understand from the example what we have in our slide.

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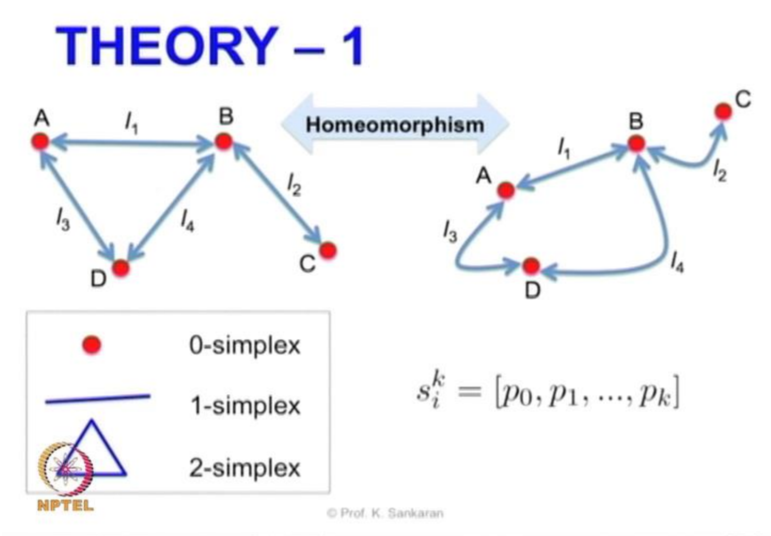

So here we are saying that the k simplex with I representing the original number because there can be a number of Eyes for example in the case of this problem each of the. Let's say A is equal to 1 B equal to 2, C equal to 3, D equal to 4. so we can say the i will be here s i will be equal to s a representing this point; and its 0 will be represented by that particular nodal value p0.

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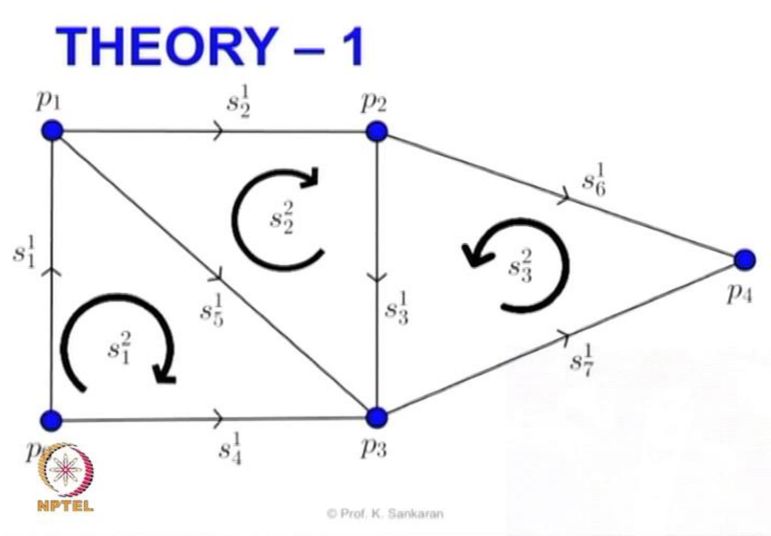

So let's take a very simple example and let's deduce various simplex us what we have so in this case we are having a network represented by a set of modes so what we have got here is we have got the values represented by P 0 comma P1 comma P2 comma P3 and P4 which are here.

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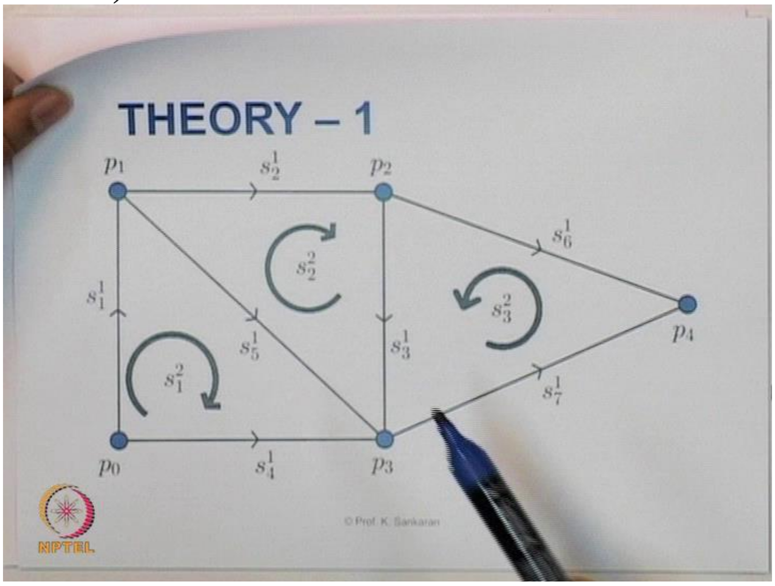

Let's see here in this graph we have P 0 comma P1 comma P2 comma P3 and P4 we have totally 5 nodes 1 2 3 4 5 nodes or vertices and we have totally 7 lines S 1 S 12 Sccccc13 S 14 s 15 s 16 and as 17 totally 7 lines and there are 3 surfaces as 21 as 22 as 23 as you can see I am defining the direction of as 11 as going from P not to P1 I have forced this direction I can also choose the opposite direction if I want and I have chosen it this way because I wanted to take example there is no need for you to choose in one direction you can choose it also in the other direction but once you choose it this is fixed similarly I have chosen the surfaces as 21 as clockwise as 22 also clockwise where is as 23 is in the anticlockwise I have chosen it such that it doesn't really matter which direction I will choose but once I choose it it's going to be fixed and it is going to affect the following development .

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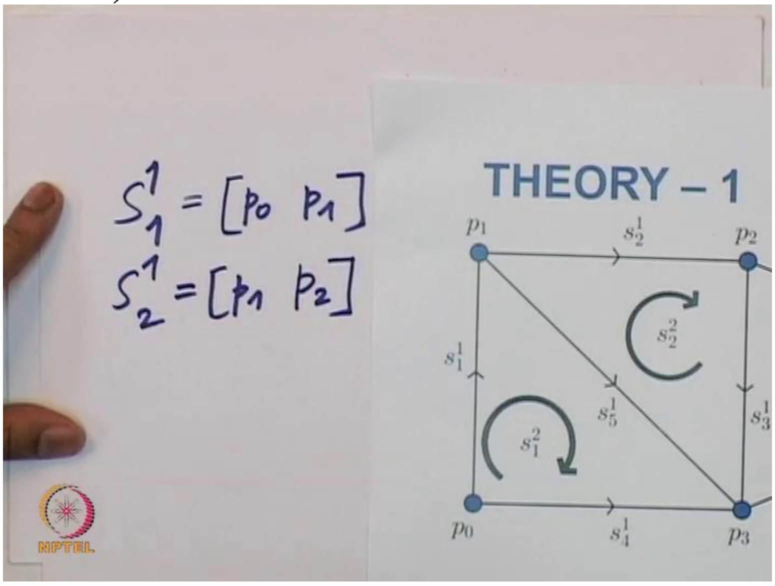

So let's see how we can define the first node S1 of 1 so as one of one is A1 simplex With The Identity original number 1 so what I am talking about is in this case this line so this line is Defined by the starting node which is P 0 and the ending node which is P1 similarly I can do the same thing for other not say for example as 1 of 2 will be the starting node will be P1 and then the ending node will be P2 so we can do that for other nodes that is what we have done in this case.

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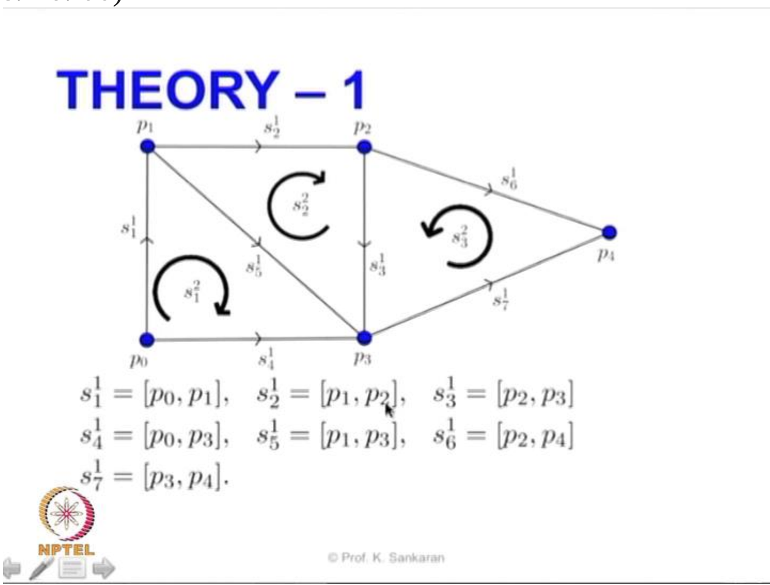

Let's see this in this example so we have define all the 7 nodes accordingly so the first one is this one the second one is here the third one is here and so on and so forth the first node will be the starting point the second node will be the point where it is ending so with this understanding you can also write the same way also the surfaces so let's take the example of the surface as 21.

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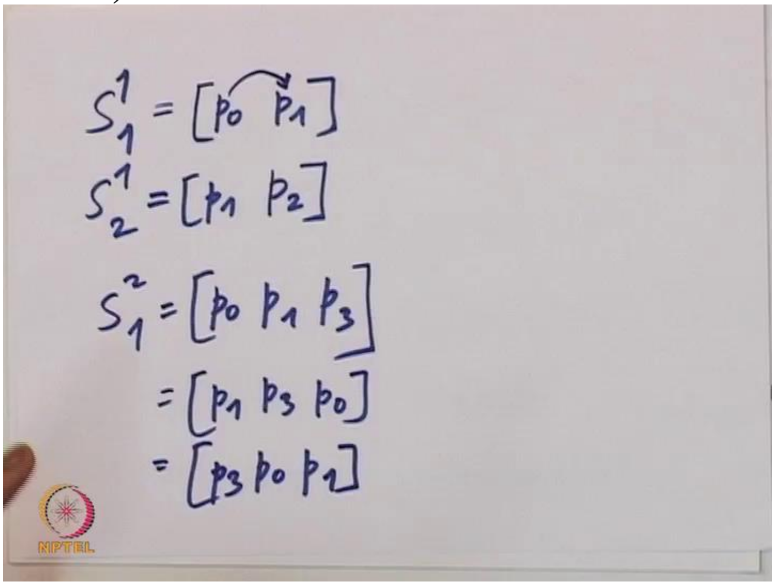

So as 21 is a first surface I have 3 nodes that are going to come and I am going to go in the direction where I have defined p0 P1 and p 3 so let me write it down p 0 P1 and P3 so it is important to notice that why should I start with p 0 I can also start with p 1 so if I start with p 1 again going in the same direction What I will get is P1 P3 and p0 similarly the argument continues that I can also start with p 3. P3 p0 and P1 what is important to see here is there is a cycle here so if I am going to change any of the notes positions the sign will be changing accordingly in other words if I say I am going to change the position of p 0 and p 1 as 11 will be equal to let's say p0 P1.

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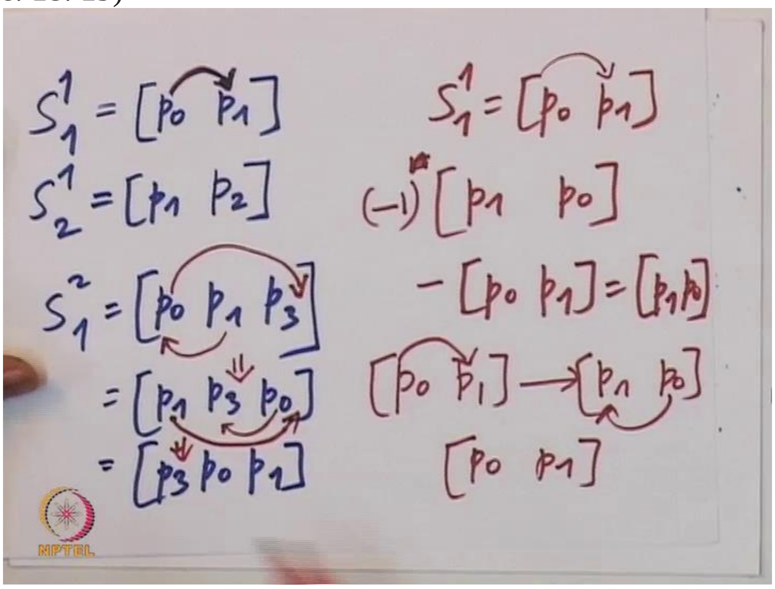

I am going to change the position and I say P1 p0 this is a big difference simplex compared to this one the length of it is going to be the same but the direction is going to be opposite so if I change positions ones what I will get is the value of the thing will change also accordingly

Where n is the number of times I am going to change so if n is one so this will be minus of p0 P1 is equal to P1 p0 so what you are saying is the number of permutations number of changes is here so since I had changed only once so this will be a minus one so if I am going to change it twice for example I am going to put it here and change this one I will come back to the original one so in other words what I am doing is if I say p0 P1 I am transforming this here what I will get s P1 p0 and then I say I transform this again what I will get is p0 P1 so here I am transforming twice once and twice then the number will be an equal to 2 so minus 1 or 2 is again the same simplex so it is a positive term so same thing same way we can see in this case what is happening is I put p0 if I change be zero to this position and I change P1 to the first position I am going to get this one similarly in this case if I change to a second position and I am going to change P1 to the last position I am going to get this one so that is the reason when we go in a cyclic manner regardless of where we start it's going to be the same simplex .

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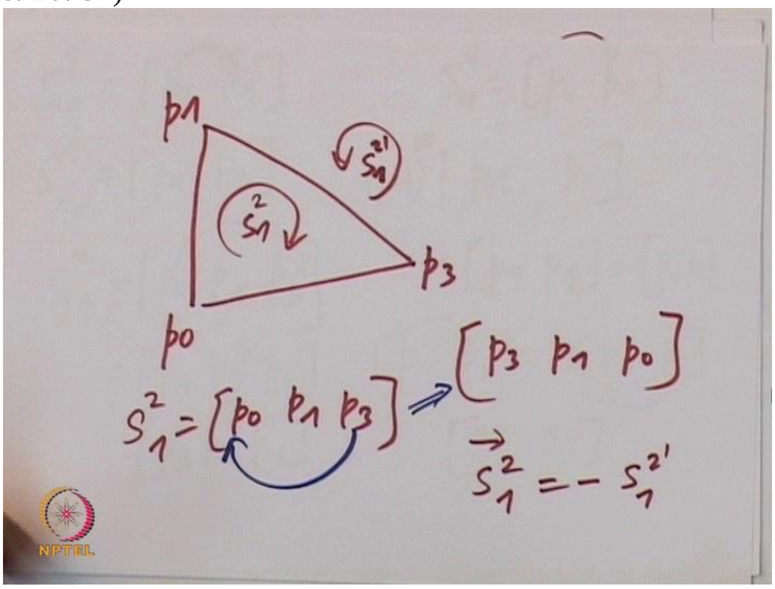

In other words if I start from let's say a value which is given by let's take the same example so this is p 0 this is P1 and this is P3 so I said this is the direction of S 21 I said S 21 is equal to p0 P1 P3 but if I start with p 3 and then I go in the opposite direction so let's say I am going to choose a different direction I will have P3 P1 and p0 the difference between these two is I have changed only once what I am changing here is I put the p 3 in the beginning and this is the reason why I am getting here is the negative one so these two things are negative let's call this one as S 12 Dash what we get is S 21 is minus of s 12 Dash so the transformation is if I go from anticlockwise to clockwise I am basically changing the orientation only once so that's what we have seen in this graph and what we have is we can write down all the expressions as we have shown here in the slide.

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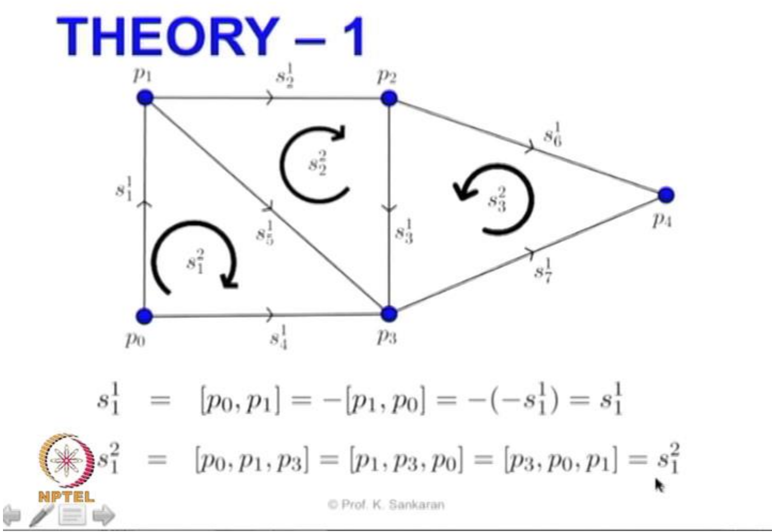

We can write down all the equations for S 21 22 and S 23 you can notice that in the case of S 21 and S 22 they are both in clockwise direction where as S 23 will be in anticlockwise so I start with p 2 p3 and P4 I am starting from p to p 3 and before I can also start from p 4 p 2 and p3 likewise so with this being said we are going to see how you are going to define the transformation so this is something I have already explained so when you change from p0 to p1 and you change it only once you will get the minus sign coming in similarly in the case of the surfaces you can do the same thing when it even number of rotations you will get the same thing .

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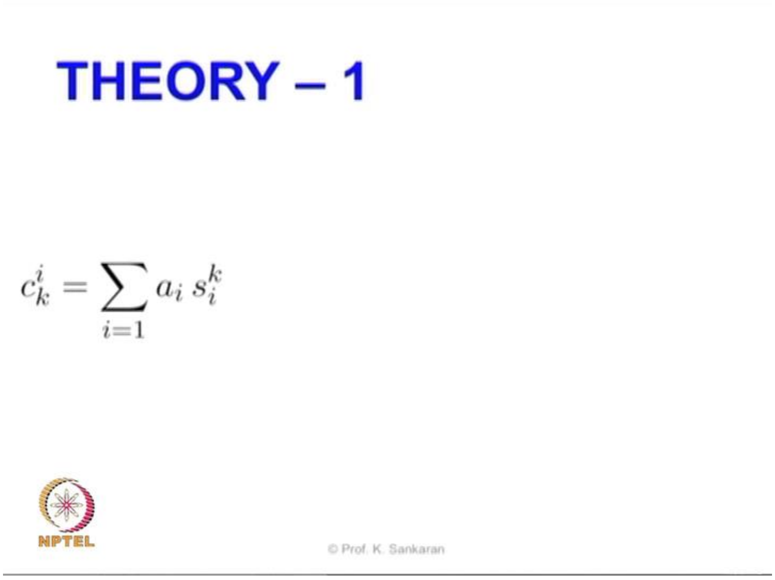

So with this being said let's go and discuss the concept called as chains. So basically so far we have been discussing about simplex S simplexes are topological objects but we are going to talk about a concept called as chain the chain like in the case of simplex s also has a dimension.

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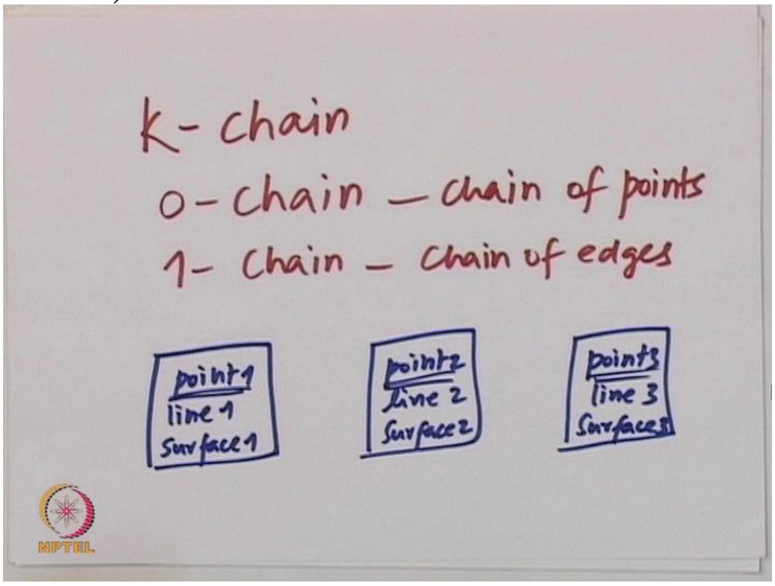

So we can call a k so if you are talking about a chain of points its a zero chain similarly we are talking about a chain of lines this is edges so on and so forth so what is actually chain is nothing but let's say we have you have topological objects this could be either point Or it could be surfaces or lines whatever it is I just put the Black Box assume that this is a black box and what you have US it could have either point; Point 1, point 2, point 3 or you could have Line 1, line 2, line 3 are you could have surface ,1 Surface 2 and surface 3 in other words let’s say we are talking about example where there are 3 points 3 line segments and 3 surfaces hypothetically so if I say I am going to connect amount of.

1 certain amount of. 2 to certain amount of. 3 so I am going to connect them in a way that I have certain weight age for what I am going to connect so if I am talking about zero chain I am only going to connect points if I am going to talk about 1 chain I am only going to connect lines if I am talking about to change only surfaces so we can’t combined points and lines in a chain if it is zero Jain it has only points if it is a one chain it has only edges so similarly in a case of a chain as I told you I said I will take certain amount of point, one certain amount of point 2 to certain amount of point 3 so that certain amount is something called as a weighting function .

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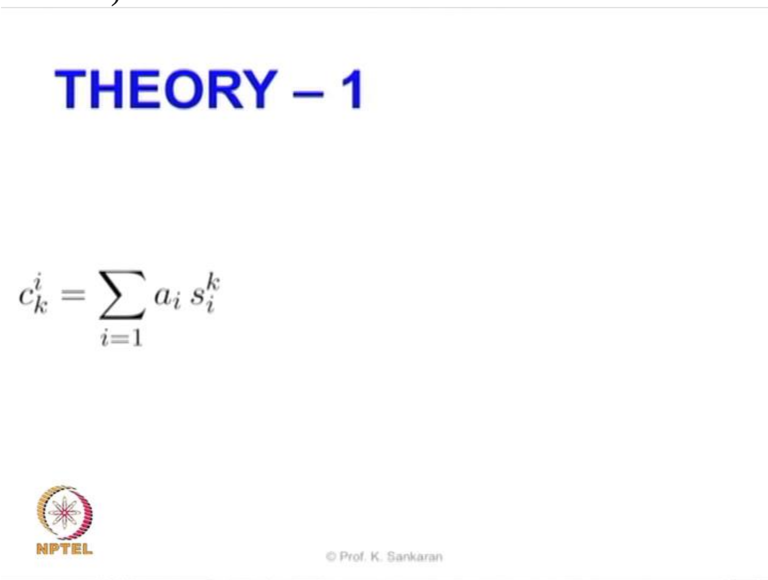

So this is something we have represented in this slide as a 1. So A1 for simplicity could be either zero OR one in other words you can either have a. Connected or not connected.

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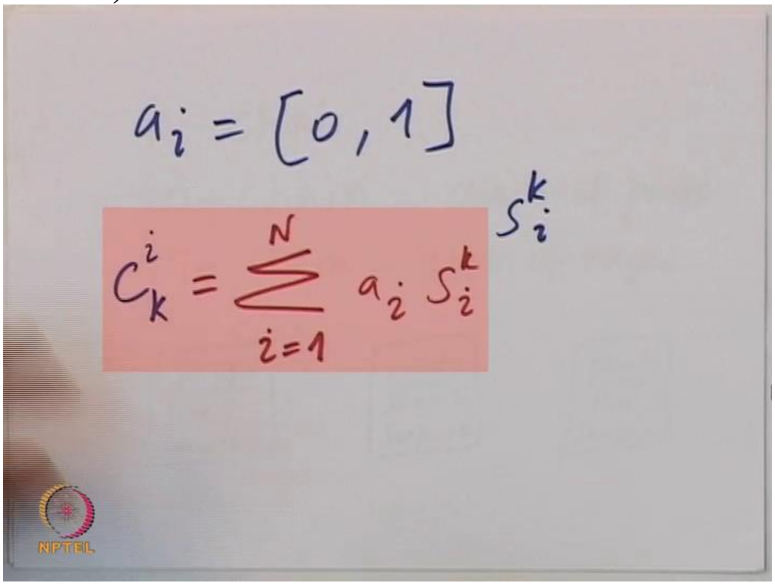

So a of a i could be either zero or one so if it is zero that. Will not be connected or that line will not be connected or that surface will not be connected if it is 1 it will be connected so what you are seeing is chain so in the case of a simplex we use the terminology to define a simplex as SK of eye reset the K is a dimension of the simplex and we put K as a superscript but in case of a chain we will use K as a subscript and the ordinary number as the superscript you will know the reason why I am doing that later on for now let's assume this is the way it is define.

So c i k is equal to sum of all I so I goes from 1 to let say n A of I x s i s k so let's take a simple example consisting of a list of points

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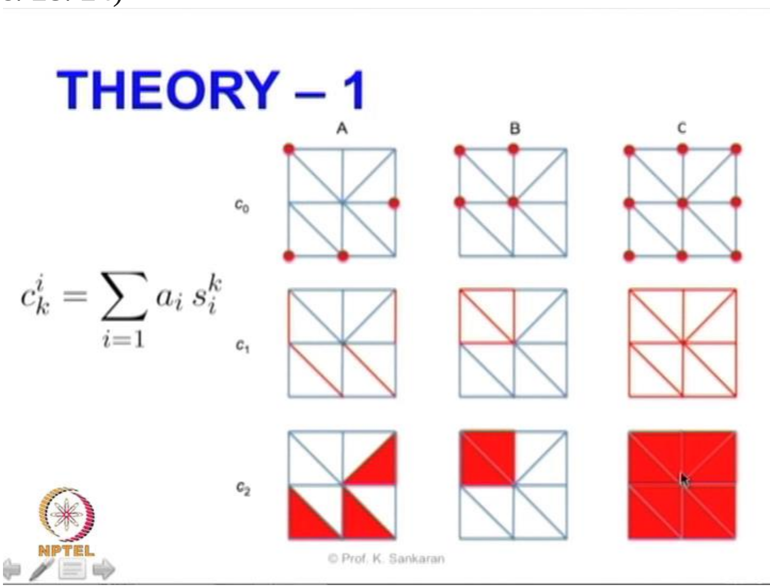

So let's say we're talking about c 0 so in the first case where K is equal to zero this is a not and we have three cases A B and C in the first case a we are talking about points which are scattered so these are the points we are talking about so c0 is a chain of those points which are highlighted in the case be we are only talking about things have scattered certain points which are in two of the cells and III cases we are talking about all the points in the domain so in other words you will see in case a excluding those points which are selected who's weights as 0 where a of that point here at this point will be zero AI of this point will be zero similarly in this case excluding for these points all other points will have zero in the case of sea all points will have words as one similarly in the case here where we have talked about C1 which is a one-dimensional chain where we are talking about lines which are scattered in this case they are together and in this case we just collectively all lines in the domain similarly the two chain here is an example where we have a list of surfaces taken similarly here we have a list of weight closely related surface is taken and hear all the surfaces in the domain are taken.

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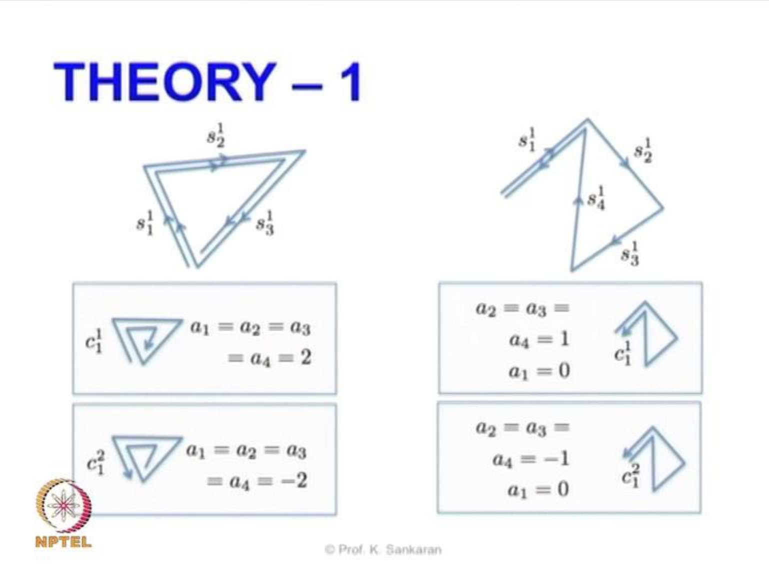

So the idea should be clear for you by now a is A one which are used for the trading what we have now is let’s say we are interested in a domain which has let’s say one simplex.

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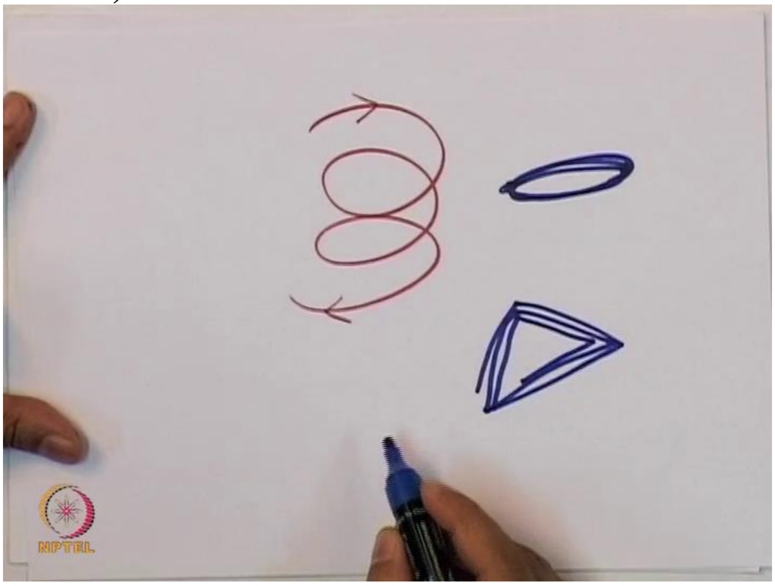

So let’s say we’re talking about a coil and a coil which is bounded in this manner so we have a coil which is bonded in this manner let’s say the current is going in this direction and the current is going to come out in this direction so this coil if it is so closely packed they are very closely connected can be seen as if they are going from one point to another point again and again write if the calls are so closely stacked what we get is a kind of a very closely tightly packed coil and the points here will be defined but they are almost like co-located so this is the example I am going to show you where you have the coils are going to go through points over and over again and the starting point is define and the ending point is defined.

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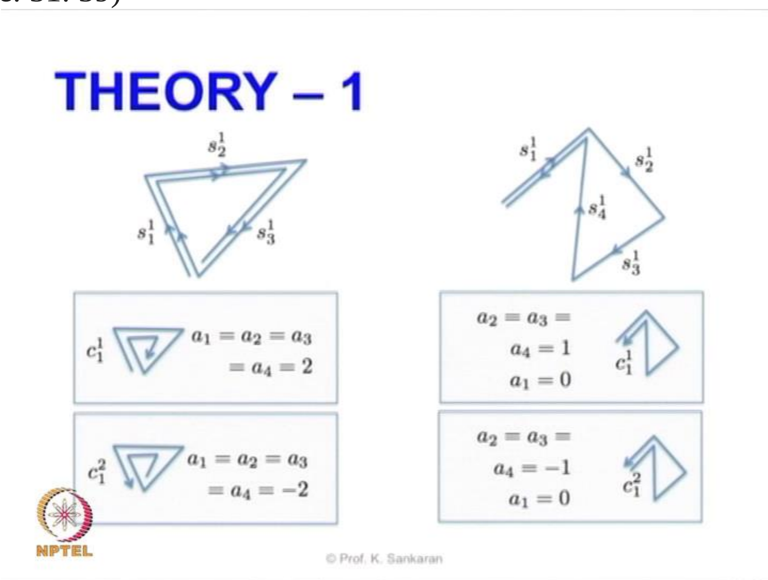

That’s what you are going to see here in the slide when you see the starting point and the ending point are defined and I have shown them to be separate here but they are actually on the top of each other what you see here is there is a direction in which it goes and comes out

So as 11 is defined here as 12 is define here as 13 is defined like this so you can see if I have a chain of C 1 of 1 but I am having his A1 A2 A3 and A4 are all having weights of 2 because they are all in the same direction so let’s say this is a one A2 and A3 so they are all having the same weights similarly in the case of C 12 if I choose a different direction instead of going in this direction I go in the opposite direction of the define path I will have A1 A2 A3 and A4 as minus 2 actually in this case you will not have a 4 you will have only A1 A2 A3 because of points are A1 A2 and A3 so we can say we are only going to talk about A1 A2 and A3 in the case of the first example so let me write it down here.

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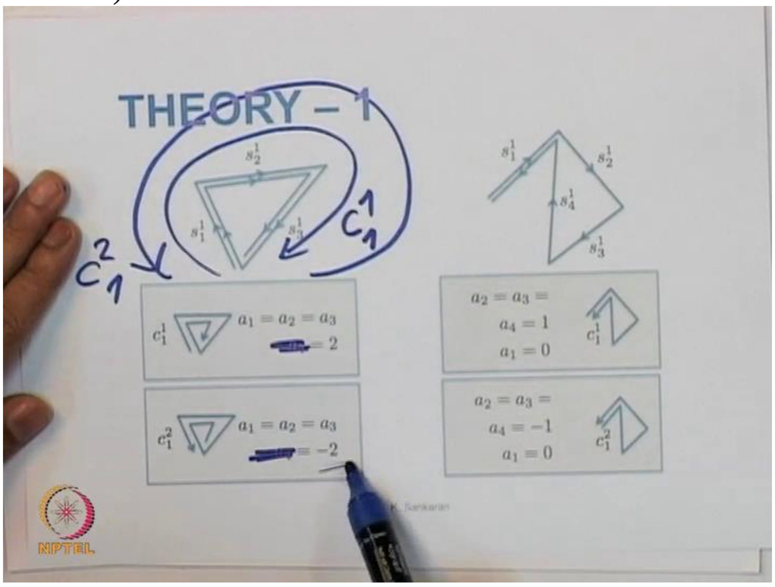

So basically we’re talking about A1 is equal to a 2 equal to a 3 equal to so A 4 is not required so it’s basically a 1 equal to 2 in the first example where I go this way and the direction is same as a direction which I have already chosen so this is C one of one like I have mentioned here and the other one is I go in this direction which is C2 of 1 I am going the opposite direction if I go in the opposite direction I am going to go in the direction opposite to that of what is already defined so I will have a 1 equal to a 2 equal to a 3 equal to minus 2 because there are two times I am crossing the same path but in the opposite direction so I will have a weight of minus 2 in the first case it’s plus 2.

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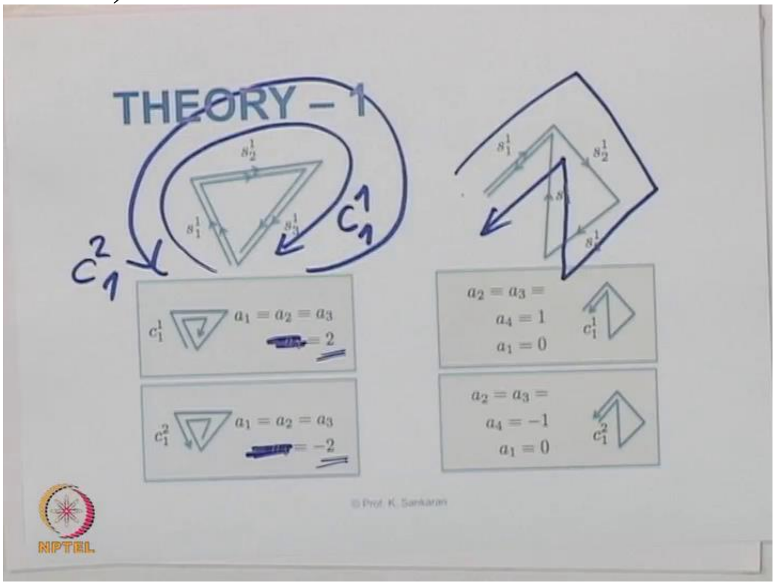

Similarly in this example you will see if I choose the direction as this way what is happening is in the first case in one type in one case I am going in one direction but while coming back I come in d opposite direction so that's why my value of a one will be zero in this case because once I go in this direction while I come out I come in the opposite direction they will call cancel out in the case of s a 2 the value will be same as before because I am going only in one particular direction and the value is equal to 1 only the value of a 1 will be zero because once I go here and once I come back but if I choose opposite direction you will see a one will still be zero because in one way I go in the direction and other way I go in the opposite direction but in the case of A2 A3 and A4 the value will be minus one because I am going in the opposite direction as that of the define direction so with this we have come to the point where we have define the concept of chains and we have defined zero chain2 Chain 3 Chain and so on and so forth.

The next lecture we will start defining some of the Other concepts like coaching and so on and so forth thank you.