Computational Electromagentics and Applications
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Lecture No 37
Algebraic Topological Method (ATM-I)

We have set our motivation we have got into the introduction about algebraic topology now let's get back to business so in the beginning I told you that I am going to redefine or relearn some of the aspects of electromagnetics particularly the parameters using the new set of tools so we will start looking into the basic definitions so let's start with the theoretical framework

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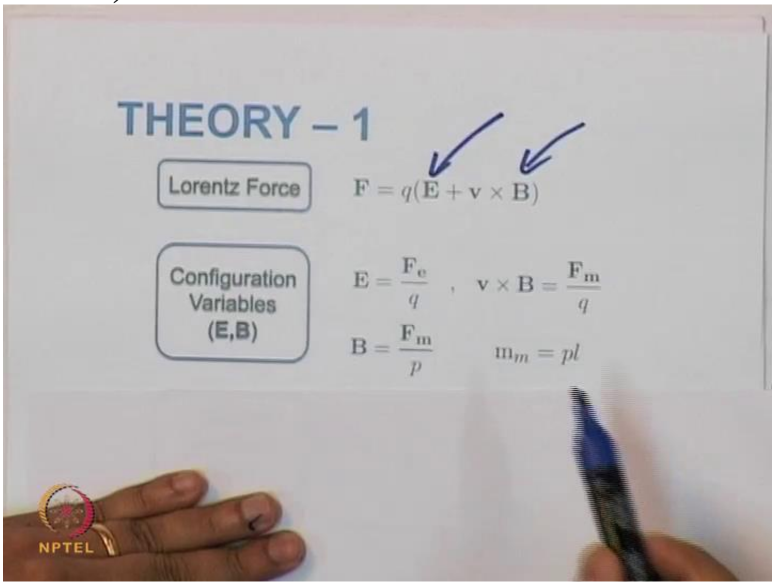

So what we have any initially is the idea of source and configuration variables so what I mean by that so let us take the simple example of Lorentz Force let's look at the slide here this we know from our basic electromagnetics lectures that if it is purely static field the force is purely in electrical force if it is a moving charge then we also have a magnetic force in the case where we have both.

So the total force will be given by the electric force plus the magnetic force but there is something specific about this equation what we call as electric field and magnetic field the normal way in which we used to talk about electric field so there is something we need to look into much for the so when we look at electric field we always a sin electric field to e but when we look at magnetic field we use the letter h to define magnetic field strength to be specific but the physicist community they might always use the letter B to use magnetic field there is a reason for that the reason is very simple whatever field that is related to force they are called as intensity fields normally.

So in the case of electric field it's the and in the case of magnetic field it's b so that is the reason why we have E and B as the configuration variables so we call them configuration variables because they are related to the charges in other words if you can call atom which is magnetic moment so magnetic moment is given by the product of pole strength and distance of separation between the Pole.

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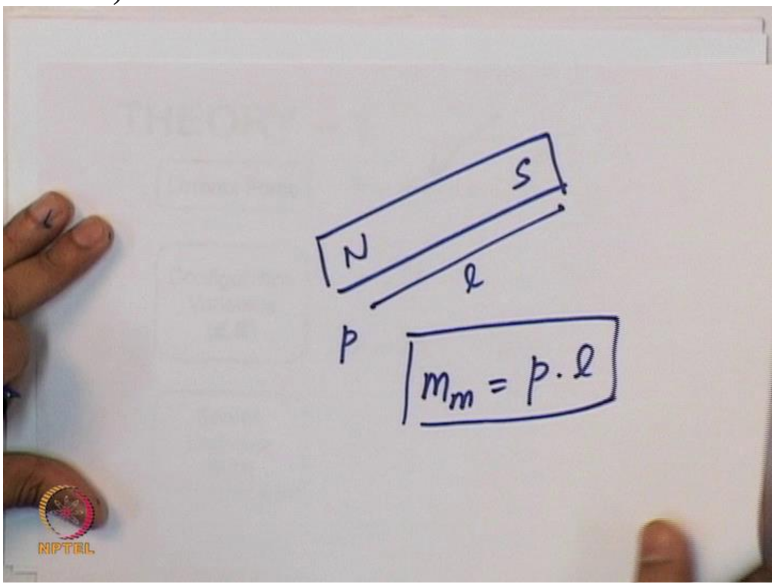

Let me explain this further so assume that we have a magnet where there is a North Pole and South Pole so let's say the distance between these two things is given by l and the pole strength individual pole strength is given by P. So we can write the magnetic moment as the product of the pole strength X the length of the separation so this is the basic definition of the magnetic moment so accordingly you can get the value of the poles strength how much strong or what is the strength of a pole so that is what we are using here.

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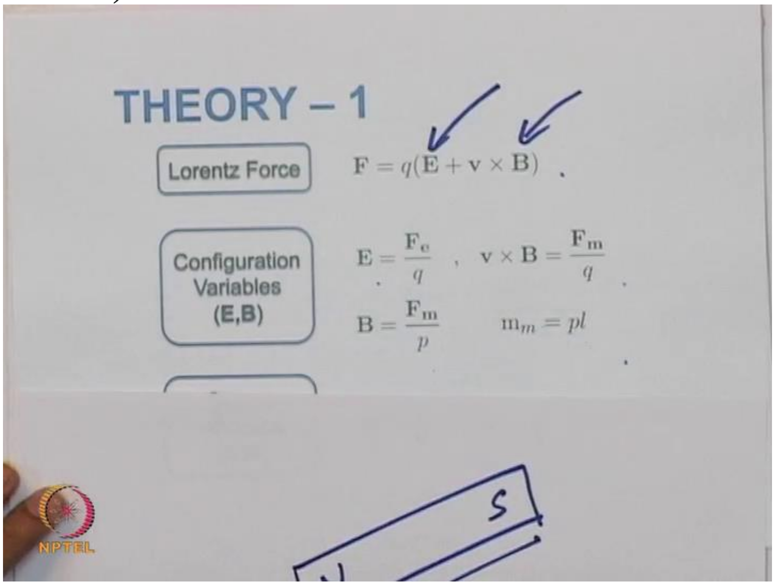

So what we are using here is pole strength to define the magnetic field obviously if you look into literature they want define magnetic field particularly the b they will call B gas magnetic flux density and they will define it in a different way but in the case of algebraic topology we are going to define B as the force acting on a unit pole assuming like that similarly the electric field will be defined as the electric force acting on a unit charge and this is the definition for the configuration variable the configuration variables as I have told you are variables that are related to the charges of electric or magnetic field so that being said let's go father a bit.

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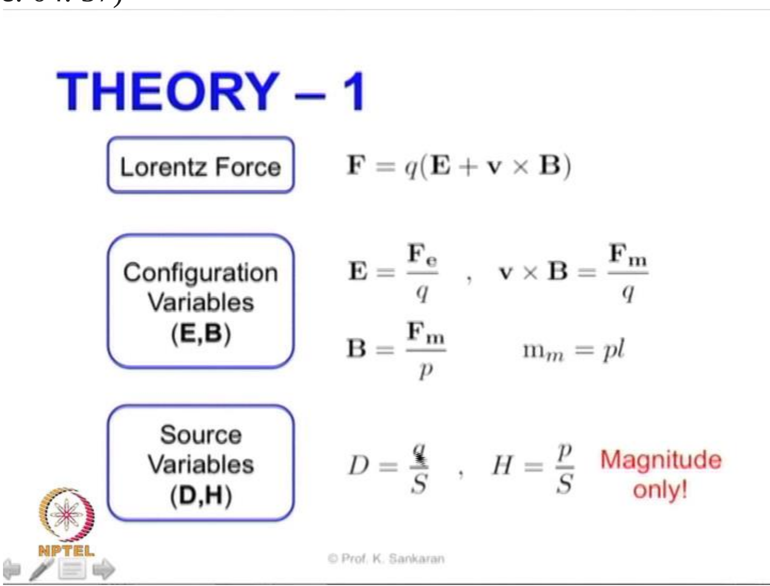

We can have also so this we have looked into it so we can also have something called as source variables again here I am using only the magnitude this equation is not valid if you are using also the direction so we are only interested in the magnitude sign in the case of D we know that it is a charge contained divided by the surface area.

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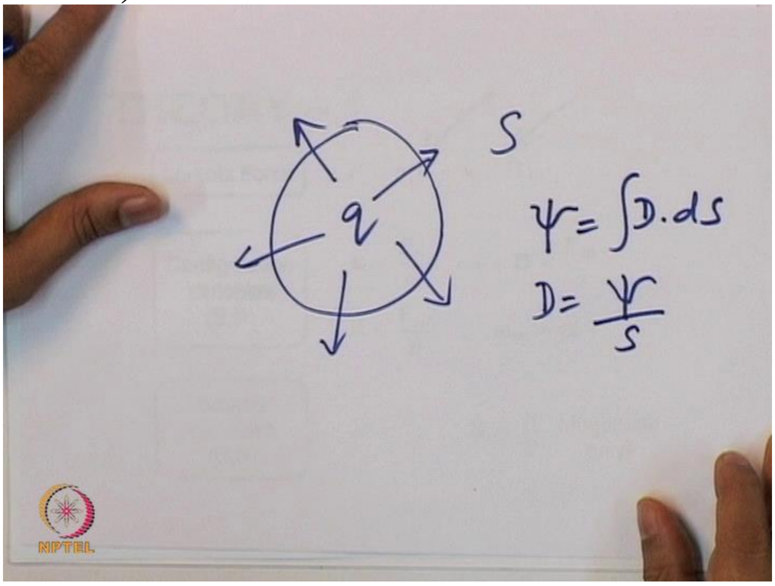

In other words what we are doing here is assuming that there is a surface and there is a charge inside and surface area is and the surface area of this closed surface is given by as we know that the electric flux is equal to so will call the electric flux as Phi is equal to integral D.ds so what we get is under the condition that we can go into a very small surface the value of D will be the flux density where us is the surface area. So this is the way we have defined we have been defining the fluxes and this is nothing but the charge contained inside the closed surface.

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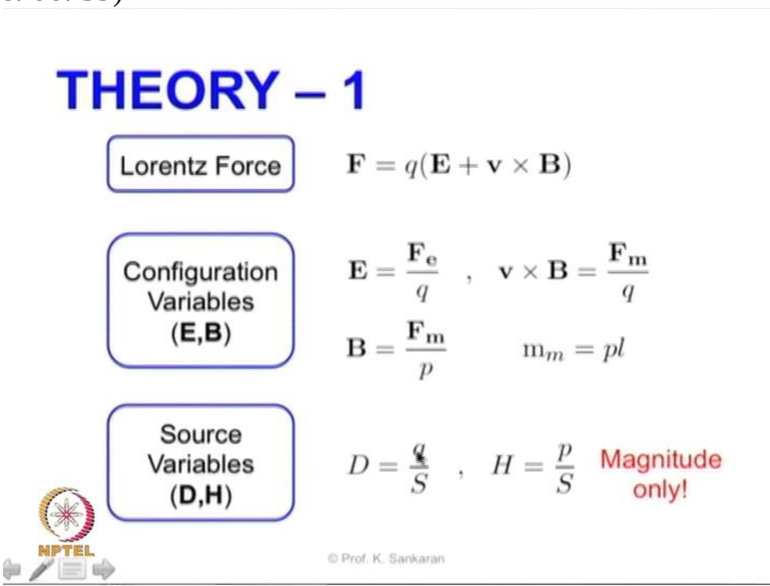

So you can see here I have used the tour magnitude only because I want to associate D and h similarly as can be related to the magnetic pole strength what we have defined here divided by the surface the word of caution to be exercised here as I told you the engineering community is not used to be as magnetic field strength and as as source or configuration variable we have been taught to use it as a magnetic field strength and be as magnetic field density I have tried to use a different terminology obviously is not something new if you look into people like Arnold Somerfield Richard Feynman our people who are physicist they have always argue in favour of calling be as the magnetic field strength so in either case it doesn't really matter to us what we are doing now a setting the basis for using the right topological association for those field quantities in fact I will not even need those field quantities I told you before we have the centre right Foundation to associate them and you will see the reason why we are doing that in our next slides.

so we have used the word source variable and configuration variable source variables are directly related to the sources and configuration variables are force that are created due to those electric and magnetic forces so in the case of the source variable there is no force Association but in the case of configuration variable there is always a Force Association so that is the starting point where we are going to begin our topological understanding of those field quantities.

So I'll be using the word topological several times what is the difference between geometrical and topological geometrical means there is some amount of or in fact A metric associated with anything that we model so if you say we are geometrical e modelling the metric is always important whether we are in millimetre or micrometre Pico metre or metre always there is a metrical Association but in the case of topological we don't have any Matrix attached to it it's purely the manifold itself.

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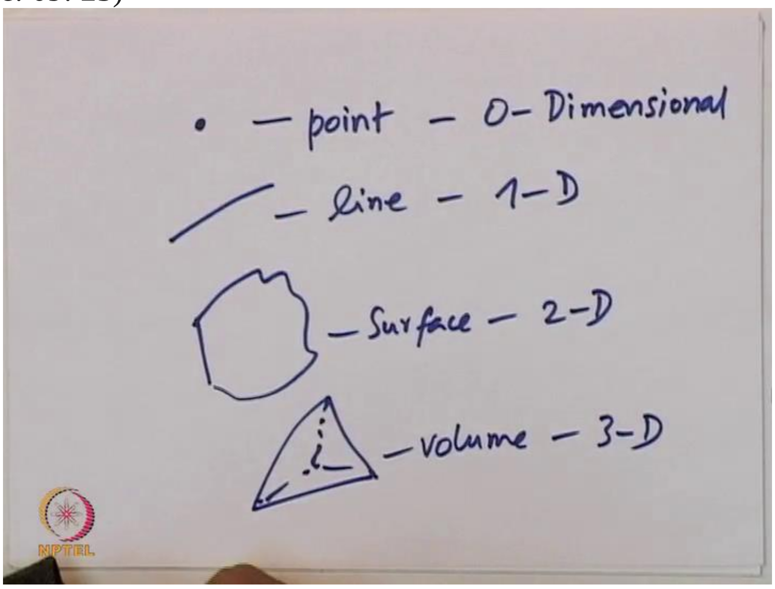

so when we are talking about a one dimensional or two dimensional three dimensional space for example a point is zero dimensional object align is a 1 dimensional object any surface is a 2 dimensional object and let's say you have a volume it need not be tetrahedral any volume is a three dimensional object so depending on where we are in the dimension so they are going to have certain association and there is also not only that the electric field and magnetic field and also other quantities are going to be associated with geometrical object that is what we are going to see in our next slides.

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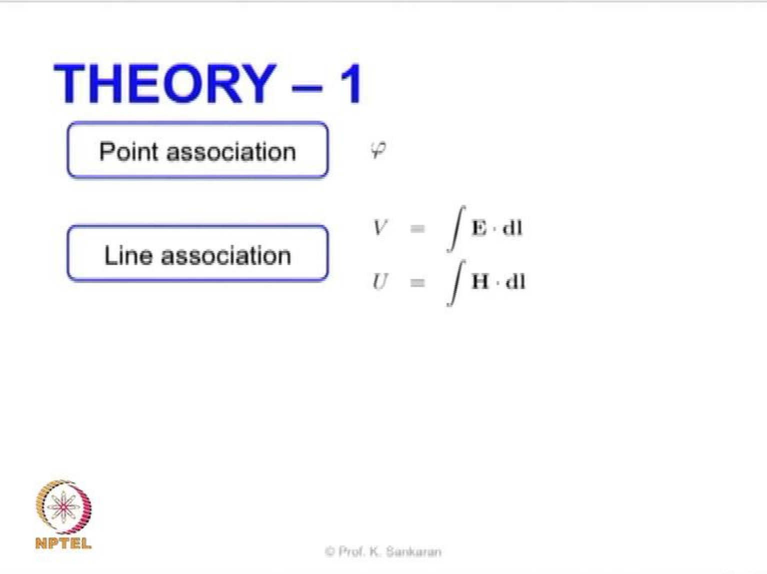

So when you see a point association what we call here phi nothing but electric scalar potential so the electric scalar potential is defined on a particular point. It doesn't make sense to define a potential along a surface along a volume because it doesn't make physically sense the potential is defined at a point similarly we are talking about v and u as I told you we are going to use different in fact a right turn to define we and you in most of the engineering literature we use the word electromagnetic force which is questionably not the right way of putting it because this is not a first I'm there is no and the unit of this is not Newton but we kept using it because in the historical time we normally used the word force because force is a basic quantity letter on cam energy and letter on cam field quantities.

So they are talking about force at the distance they don't know what is that. So when they associated something they said it's a force at a distance similarly there been using the term electromagnetic force because they thought it is a force term which is not the case but still be kept using it because we got used to it sometimes for people who find it difficult to accept as force they still kept using the term EMF because they don't need to say the word electromagnetic force but again this fundamentally wrong so when we are using algebraic topological method we are going to use the term electro Motance.

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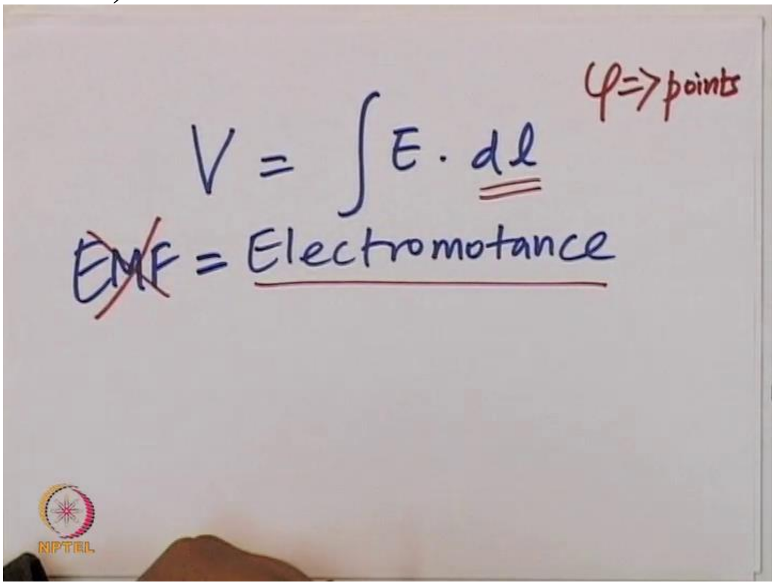

So what we call as electromotance is nothing but the line integral. So V is equal to the line integral e dot DL so this is the electromotance this is a term that is well known but not used because we have been used to use the word EMF. So this is actually the same but we will not be using the word EMF and he will by using the word EMF electromotance is associated with the line like the case where we had the case of potential which is associated with points so in the case of electromotance the association with respect to line.

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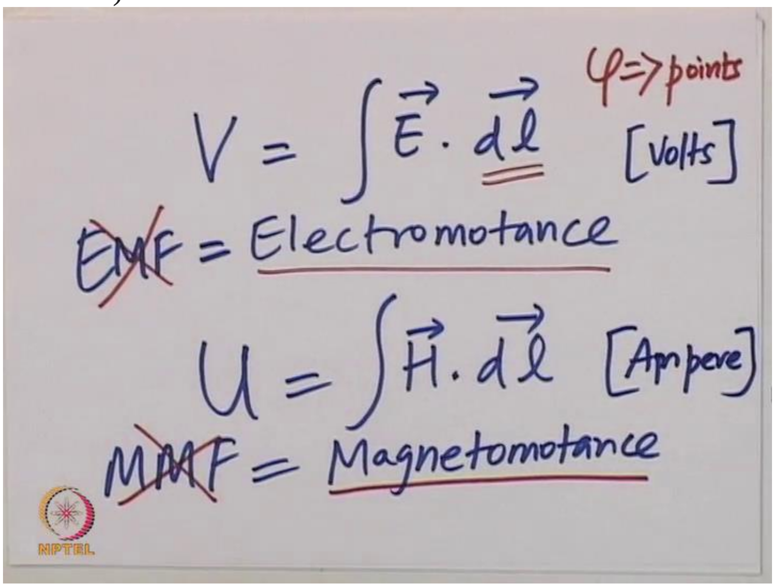

And the other one is called as magnetomotance which is the counterpart basically the counterpart is you equal to integral HCL solution dot products because they are all vectors. So here the normal time what will be using is the magnetomotive force but which is actually the right term that should be used is Magnetomotance. So magnet importance and you will not use the word EMF or MMF and we will stick to the terms electromotance and Magneto motance obviously we didn’t do this in the earlier lectures because we didn't have the need to associate with the topological dimensions to each of these quantities but in the case of algebraic topology it is fundamental to begin with the right definition show the unit of electro Motance is volts and unit of magnetic motance is ampere.

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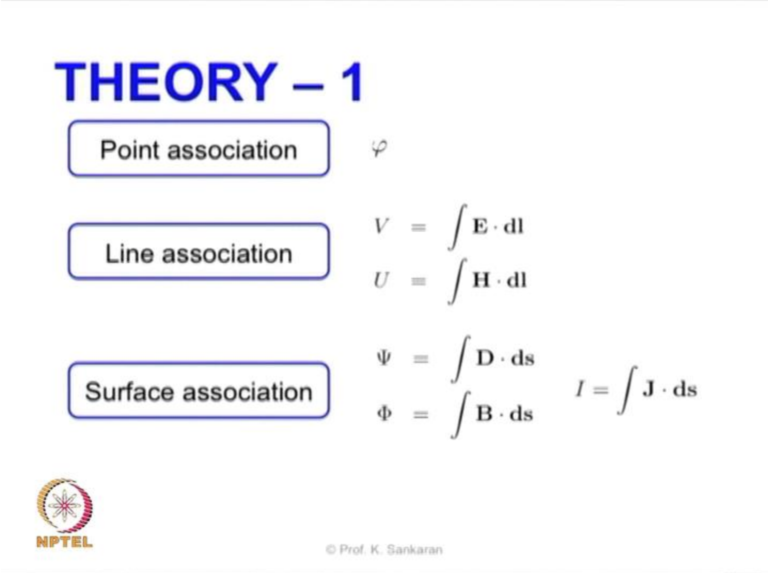

So that being said let's go to the next one so we have covered the point association we have covered the line association let's look at the next one which is the surface association.

The surface association also has quantities which are in most cases properly represented we already make mistakes normally in the case of V and u but we have the right terms for different fluxes so the first flux.

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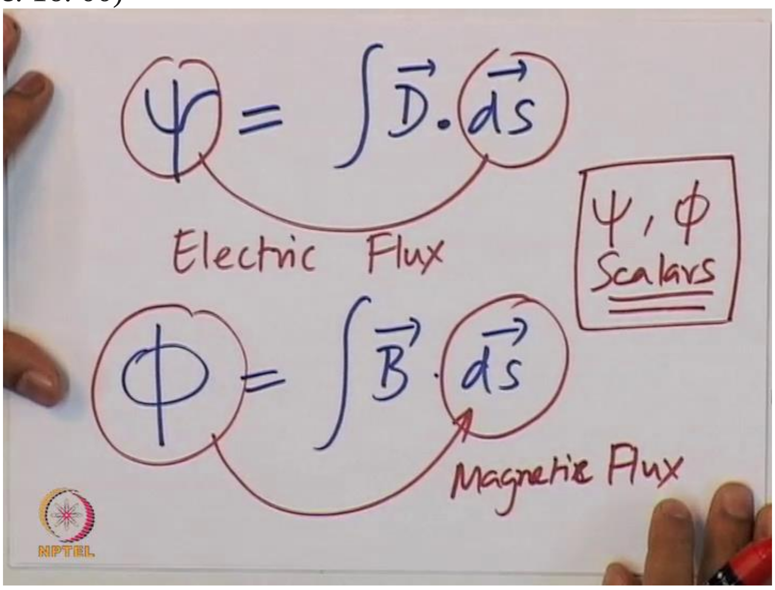

We are going to talk about is something that we already saw is the electric flux which is equal to integral d.ps again it's a dot product they are all vectors and this is related to the surface so this is the electric flux the next one is magnetic flux which is written as capital Phi integral here you can see values are also associated with the surface and this is a magnetic flux so normally we use the term electric flux and magnetic flux in the right contacts show we are not going to worry about it and the important thing is these are all scalars. So phi is a scalar

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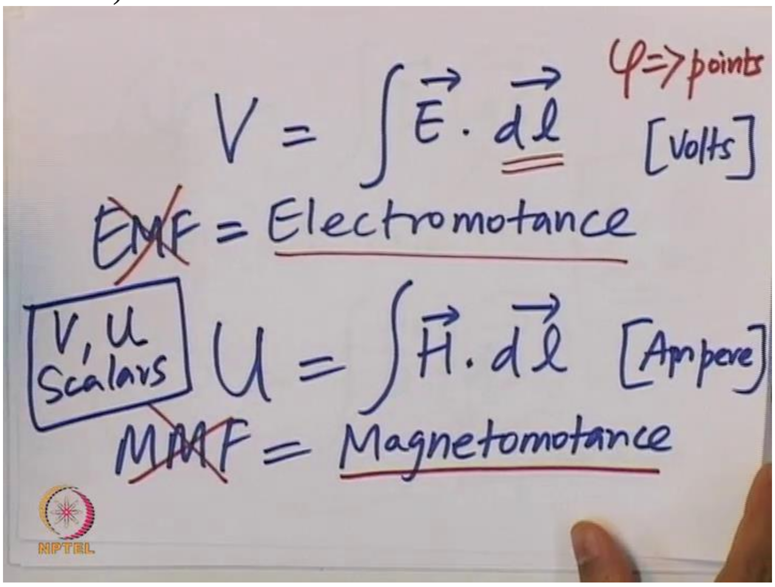

And also in the earlier case where we had V and u they are also scalars so we have covered electromotance, Magneto motance, electric flux and magnetic flux.

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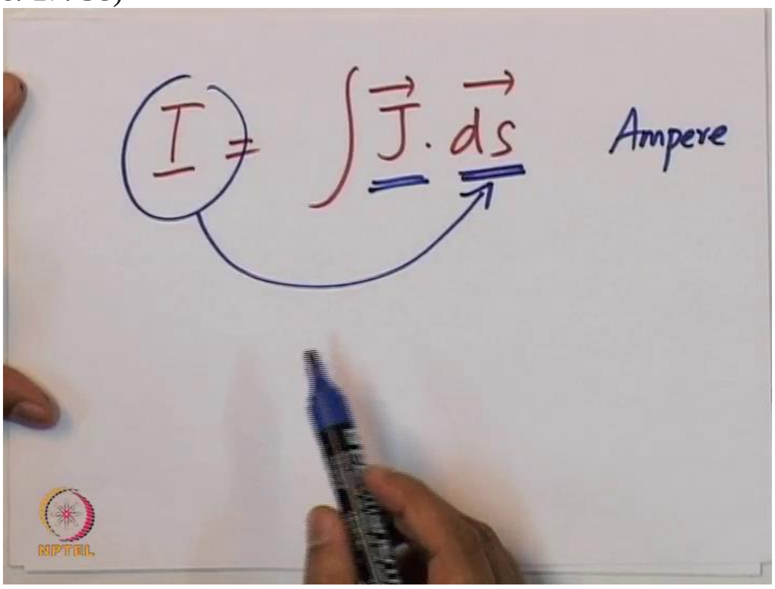

so there is another thing which is associated with the surface which is something that we all know the most familiar one is the current so we call it I is equal to integral j.ds J is the current density and I is the current so the unit here is ampere and here we have J that is ampere per square metre and here we have the value of the surface so it is associated with the surface. So that being said we have covered all the surface association so there is something missing that is the volume association that we are going to see in the next slide.

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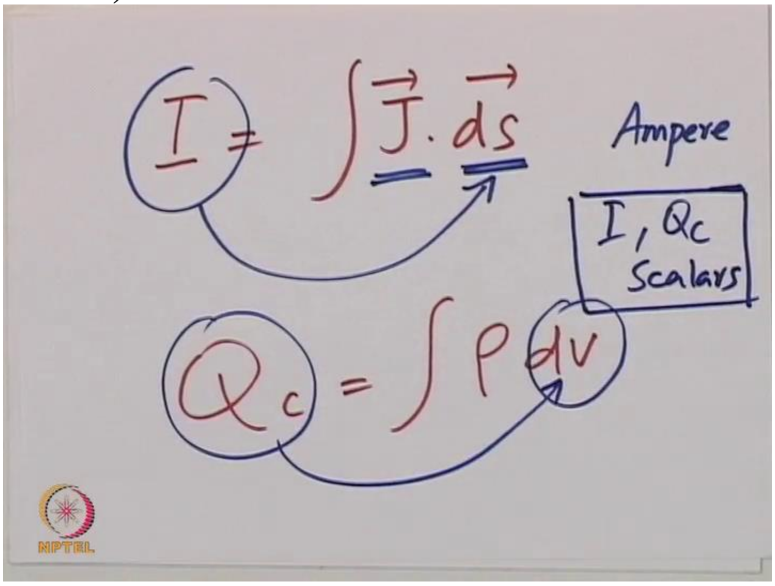

Find the volume association we have quantity which we call it as Q . So Q is the search content search content is equal to integral Rho dv. So Rho is the volume charge density X the volume please pay attention that this is not a dot product they are both values scalars. So here the association is with respect to the volume so again I and QC they are scalars.

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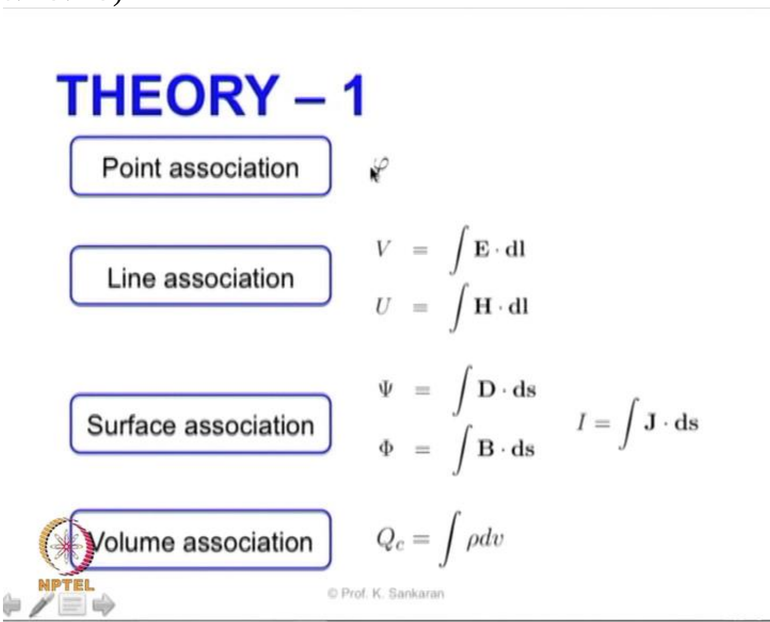

So with this what you have got is the basic definition of electric field magnetic field and electric flux density and magnetic flux density all measured or all represented by the scalar counterpart the scalar counter parts are the most fundamental things we will be using so we have 1, 2, 3, 4, 5, 6 and 7.

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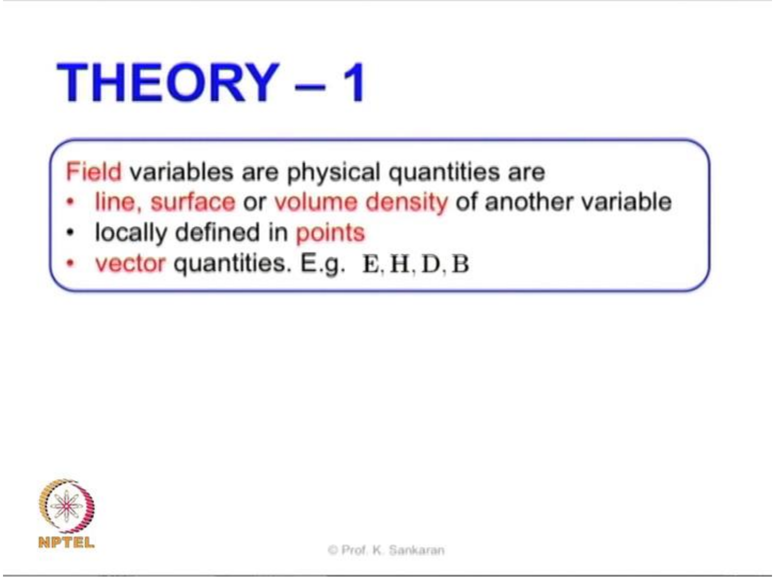

So what we have got here is the field variables are physical quantities and they are associated with line surface or volume density of another variable.

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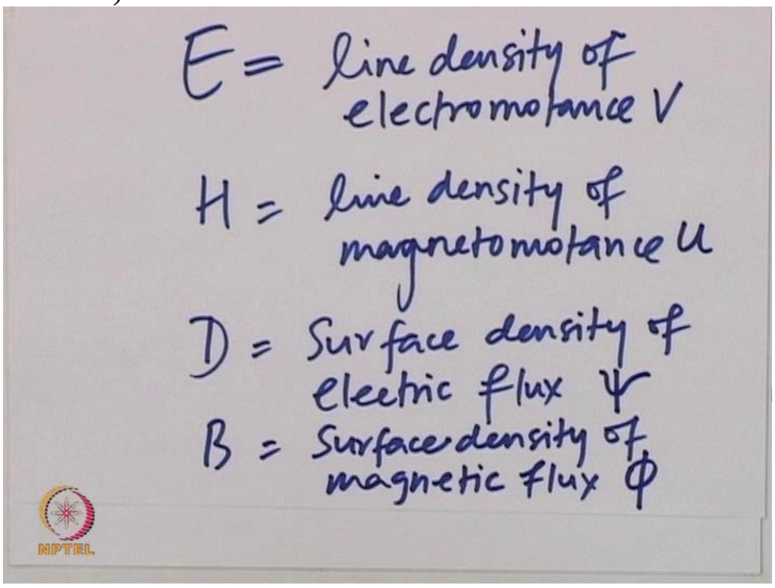

So for example electric field is the line density of Electromotance. So voltage difference actually and magnetic field is the line density Magnetomotance and similarly D the electric displacement is the surface density of electric flux and b is a surface density of magnetic flux this is we this is you this is Phi so on and so forth so in fact the most important thing is we are not used to think of electric field as a line density or you know the electric displacement as a surface density but inside they are what they are so they are the line densities of the surface density and in some case it’s also volume density.

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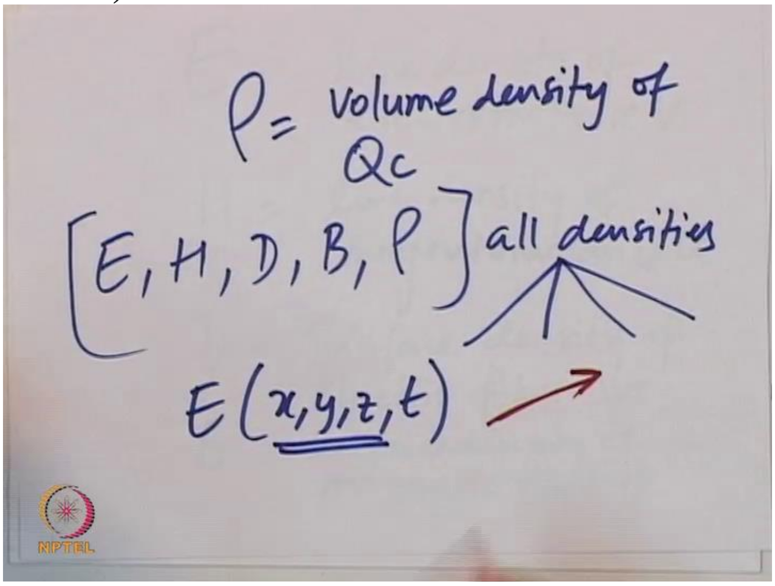

For example when we talk about Rho is a volume density of charge content so if you understood this we are able to understand all the quantity is basically all the field quantities E,H,D,B and Rho they are all densities whether it is a line density or volume density or surface density it doesn't matter they are all densities and they are all define local in a point so when you say e you are defining e at x, y, z, t. So these are the point definitions so they are all locality find in a point which is Defined by x, y, Z and they are all vectors I told you they have particular magnitude and a particular direction.

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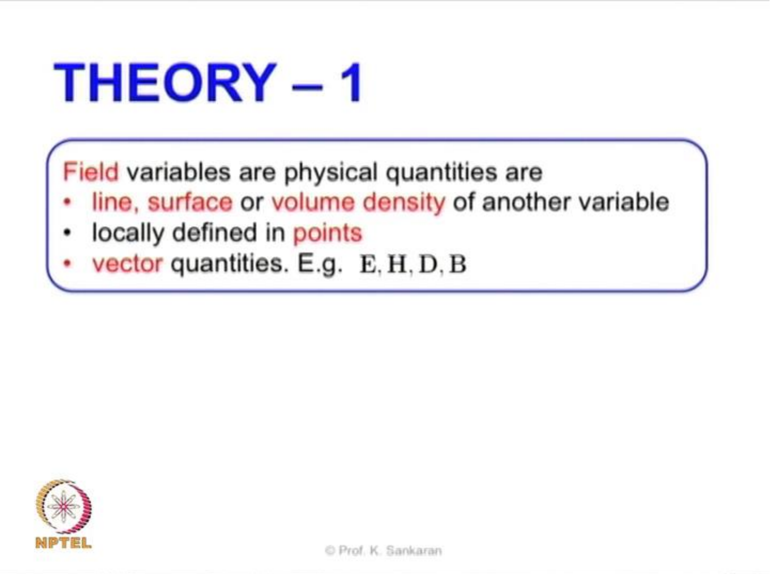

So with this we are able to understand at the field quantities are fundamentally line surface of volume densities I mean the main point here is they are associated two points but not associated to fundamental topological objects they are we are associated all of them irrespective of their topological association to a particular point in space and of course in time and they are all vector quantities.

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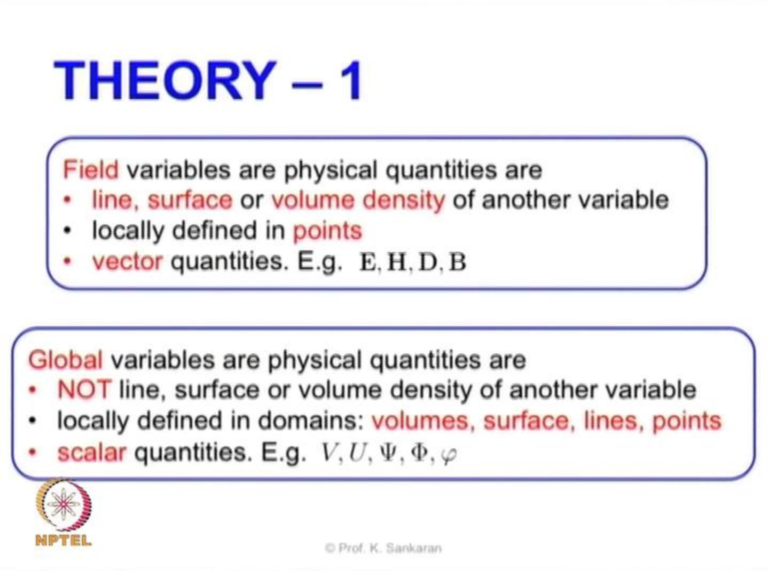

So the counter part is which we have already discussed are the Global variables are used the word Global variable cautiously because in some cases we can also call them as integrable variables I don't use the word integrable variable for the simple reason that when we look into a point Association it is difficult to or in fact it is not possible to talk about integration of a point when you have a line you can integrate it along the line when you have a volume you can integrate it within the volume or if it is a surface you can integrate it along the surface but it's a point it's very difficult to integrate so that's why all integrable quantities are also Global quantities but not all global quantities are integrable quantities.

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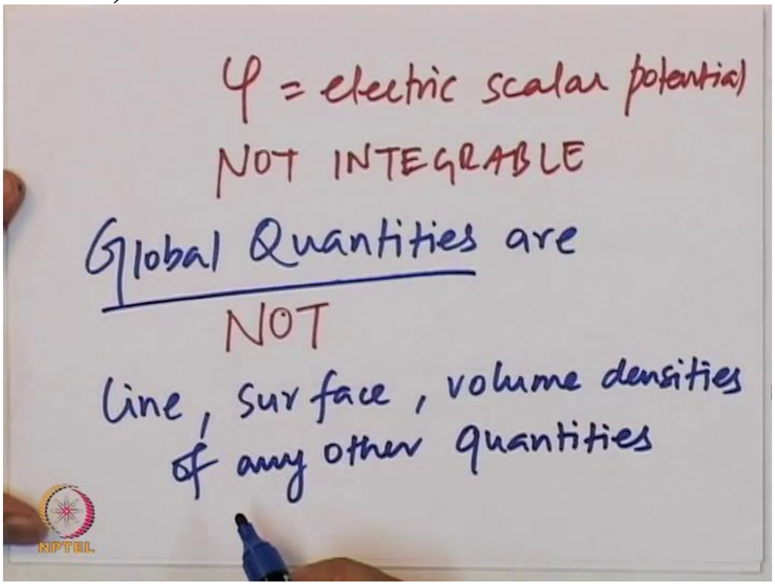

So in the case where we have phi which is the electric scalar potential it's not integrable so that's why we use the term global Global quantities so what are these Global quantities a global quantities they are not line or surface or volume densities of any other quantities so this is the basic definition of what is the Global quantity the Global quantities cannot be defined as line surface or volume densities of any other quantities like the way we have done it for the field quantities they themselves are integrated along the topological space wave whatever they are associated to and they are not defined on the points like in the case of field quantities they are all define in on a particular point but here the Global quantities are associated or defined over the domains if it is associated with the point The point is a domain self like in the case of electric scalar potential similarly electric flux is define on the surface.

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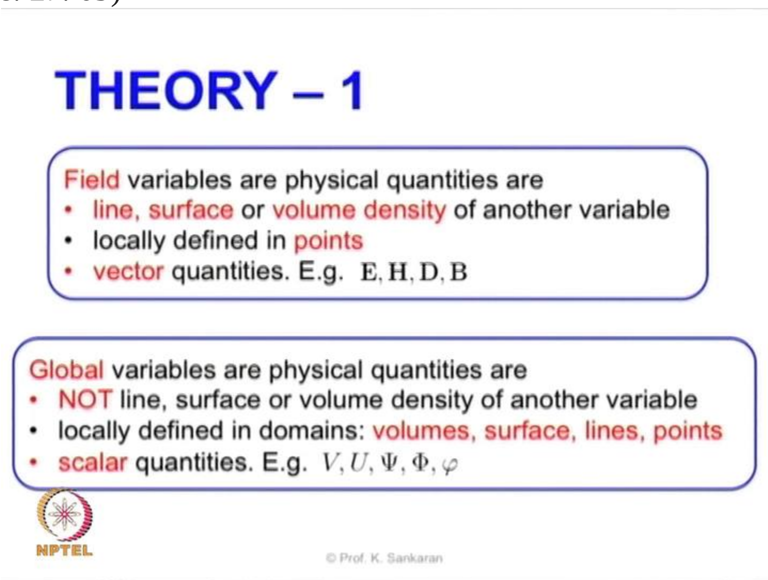

And these are the domains they are not line surface and volume densities of any other variables they are all scalar quantities as compared to the field variables which are all the vector quantities and they are all define on the volumes surfaces are lines and and points which are the domain which are associated to those particular Global variables so this is the basis of the Global variables so with this we have from some basic understanding and also the terminologies that we need in the next step for associating them in algebraic topology kal manner but we have no covered the basic definition of those quantities that we will be using they are all scalar quantities and we are not going to use any of the vector quantities and the scalar quantities are associated to certain domains with this we end is module and when we go into the next module we will look at the algebraic topological Framework thank you.