Computational Electromagentics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No 32
Finite Volume Time Domain Method-II

So we are here in the third module of Finite volume time domain lecture. So we have seen so far the various dimensions of formulating the finite volume equations for Maxwell system. And also we discussed quite a lot about the discretisation and the time discretisation to be specific also on the spatial discretisation, we discussed a lot about also advanced techniques. So we are today going to follow up on the advanced techniques for domain truncation. And we are going to expand that to a radial case and also spherical case.

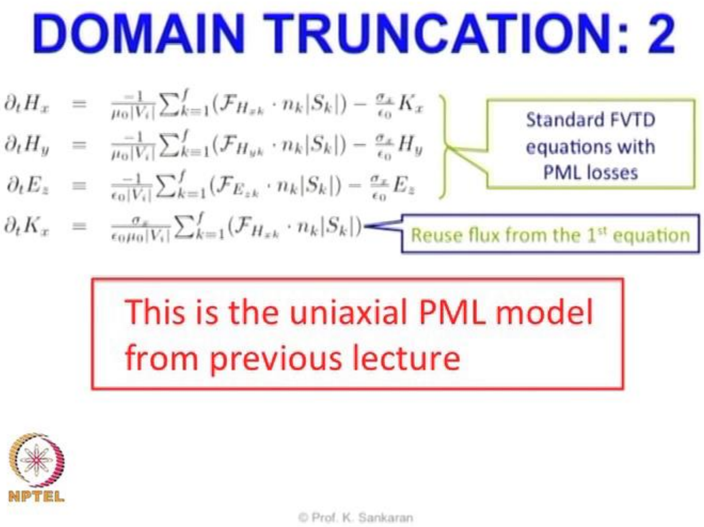
Because so far we have been truncating domains on a rectangular manner so which is quite good for plane wave and also good for let us say wave guide applications. But its more convenient to go for radial or spherical domain truncation techniques. So that is what we will be focusing today and in the later stage we also show some applications of Finite volume time domain for practical problem solving either in antennas or in waveguides. And also conclude this module on Finite volume time domain. I am openly discussing about some of the challenges that this method is facing and what are next further research that is needed. And also giving a kind of a direction for future research.

(Refer Slide Time: 01: 59)

With that let us go into the todays lecture, So we will start with the domain truncation technique 2, which I call 2 because we started with domain truncation technique in the earlier modules.

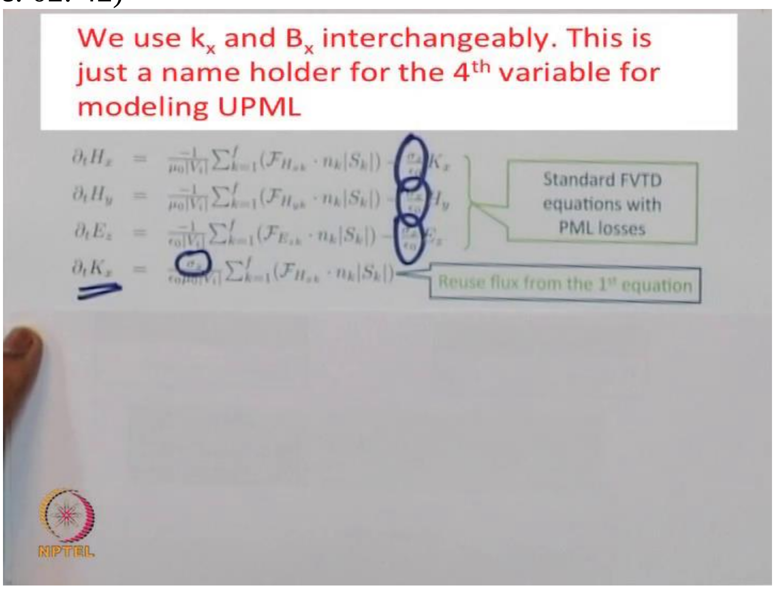
We will show some applications and some of the challenges. With that let us begin with domain truncation part 2.

(Refer Slide Time: 02: 19)



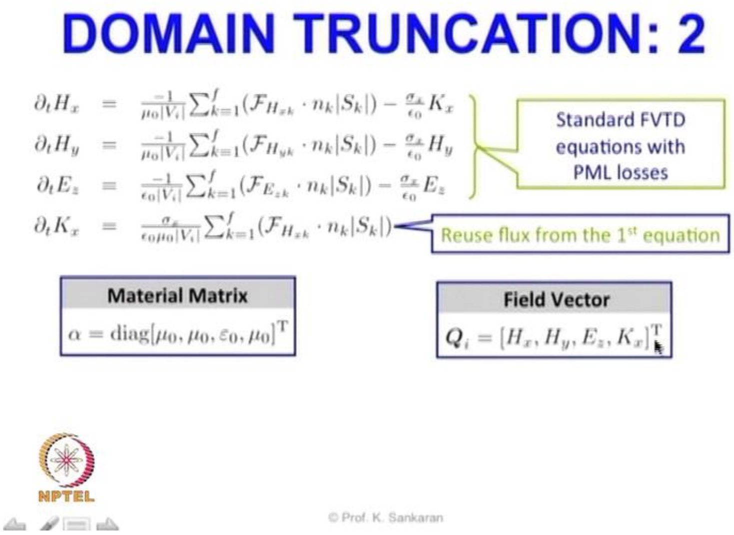
If you remember we had this equation that is coming from our system of equation of Finite volume, let us see this in this slide. We said that we can reuse the first flux directly computed from the first equation

(Refer Slide Time: 02: 42)



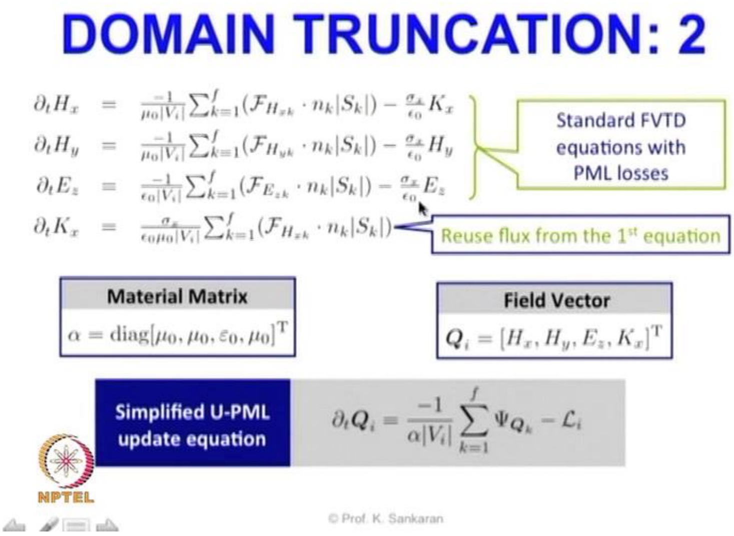
So irrespective of the fact that we have 4 equations, the last equation's the flux value is recomputed. And the first three is nothing but the standard Finite volume time domain equation with PML losses. And the losses are given using the terms here. And these terms are going to be the losses in the x direction. Because we were interested in truncating the domain main wave application or wave guide application along x axis. You can also notice that this fourth equation will have a loss term already sitting here. So it is going to be coupled as per the equation So that being said let us go back to a slide.

(Refer Slide Time: 03: 29)



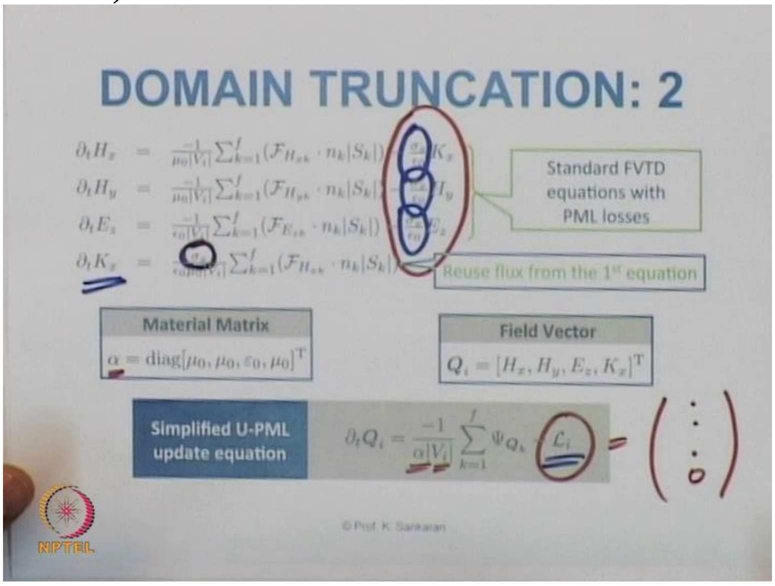
We said we will have the matrix material parameter given by alpha which is here as you can see in the slides. And these values are Mu 0 and Epsilon 0 accordingly and the field vector the combine field is given by Q i which is H x H y E z and K x. We are talking about transverse magnetic case the counterpart will transverse electric case where you will have E x E y and H z and accordingly the fourth value will also change.

(Refer Slide Time: 04: 11)



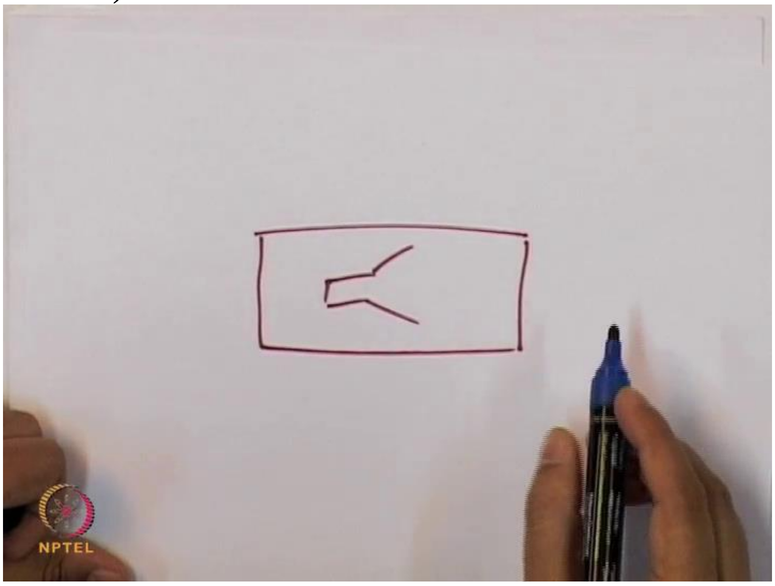
So we have the final form of the simplified equation which is given by the final equation here. As you can see this is semi discrete this is continuous still in time. But we have already covered how to discretise the time variable. So it is enough that we focus now on this one. So this L i are these loss terms. Let me explain further in the slide here.

(Refer Slide Time: 04: 38)



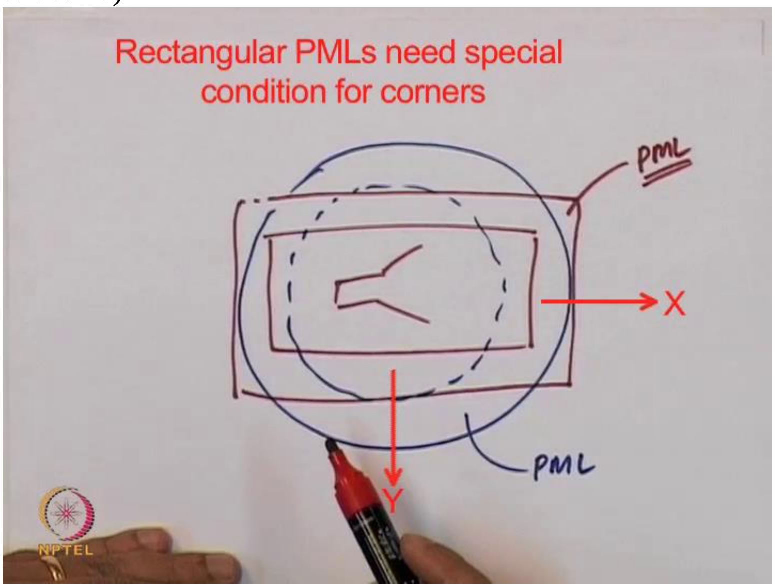
So if you can see here these L i terms are those loss terms. So these terms are sitting here. So it will be of vector whose first value, second value, third value are given by these equations. And the last value will be 0 because there is no losses here the loss is already sitting here. And we discussed about alpha which is given here. And v i as we already know in the finite volume formulation will be the value of the volume of the cell. In the case of 2 dimension this will be the area and then we will have the surface or the counter part of the 2D case with the length of the edges sitting here. So with this equation we are able to understand that this is good enough for us to model plane wave or also in the case of wave guide truncation. So let us look at in the case how we can expand this for a radial case? Because what we are interested is we wanted to have a kind of a truncation.

(Refer Slide Time: 05: 40)



As we can see here for example we might have an antenna and we donot want to truncate the antenna like this. Because this way we are kind of in one sense we are increasing the surface that is needed to be meshed and uncomputed. And secondly in some cases the angle of incidence is quite close to normal in some cases its gazing angles. As we already saw the performance of the PML and also absorbing boundary conditions are good in the normal incidence case but already its over when we go away from the normal incidence case. So this will be something we wanted to see.

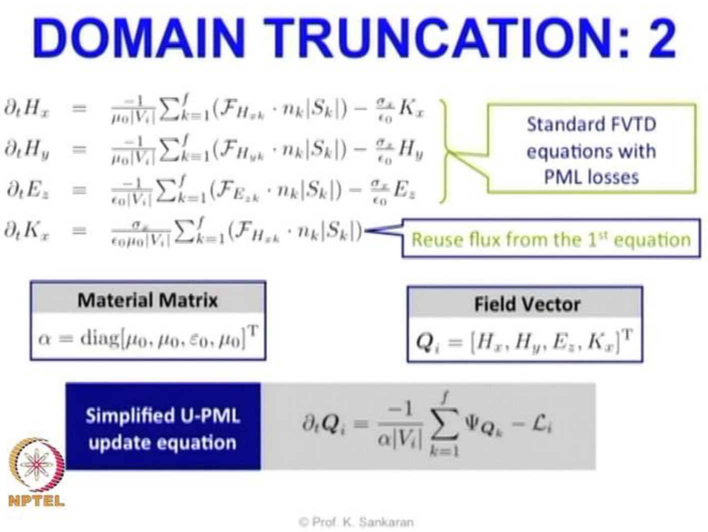
(Refer Slide Time: 06: 28)



So in this kind of case what we want is we want to truncate the antenna like this. So in the case of a rectangular truncation we have to put the boundary conditions like this. But in case of a radial truncation our PML will be in this area. So this will be the PML area in the case of radial truncation in the case of rectangular domain this will be the PML. And also we already saw that we need to have special conditions for the PML that are facing in the x direction and y direction and so on and so forth. So this is something we need to avoid in the sense what I wanted to say is we need to have a general condition which is good enough for various angles of incidence and also it is enabling us to model geometry in a compact way and also accurate way. So in that sense going for you know rectangular truncation you need to specify the way in which the PML will behave for X axis and Y axis. This is something we can avoid if we go for radial truncation.

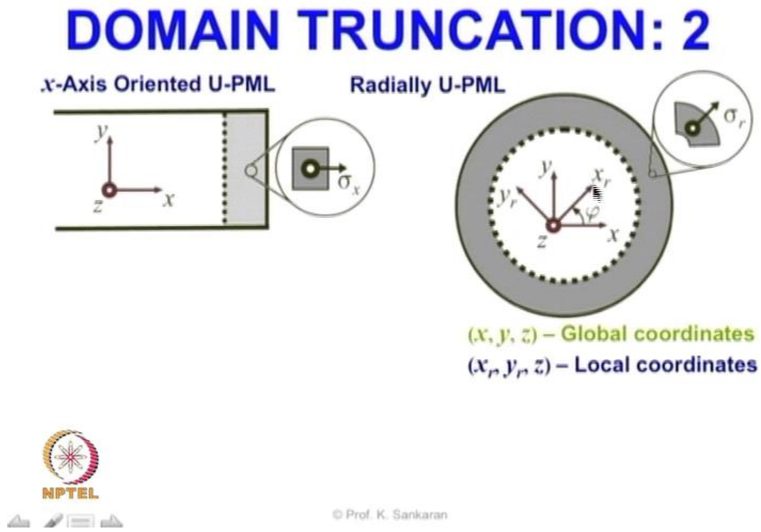
So this is what I am going to explain you in this class. This has certain advantages as I already mentioned in terms of formulation but also it is good for numerical accuracy point of view also. Because more or less we can come close to normal incidence if we are able to keep the domain in a way that we will see in the examples.

(Refer Slide Time: 08: 14)



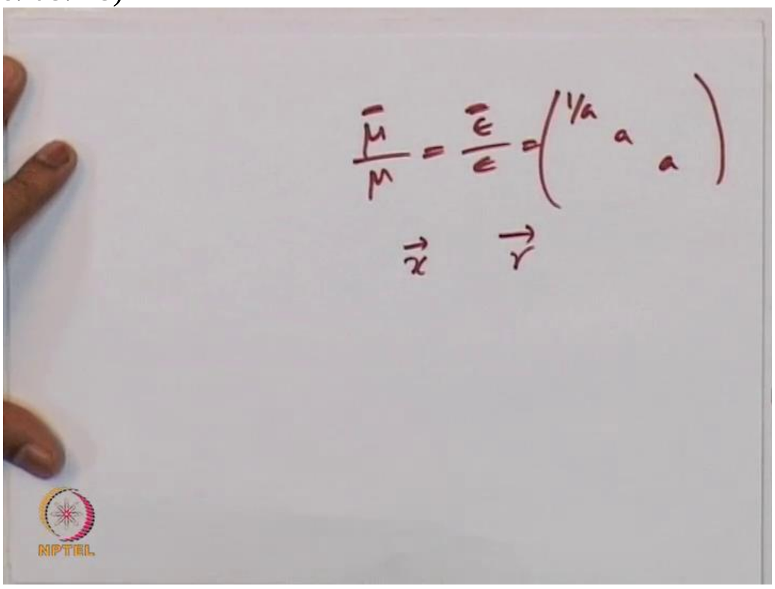
So let us go into the theory of radial truncation.

(Refer Slide Time: 08: 16)



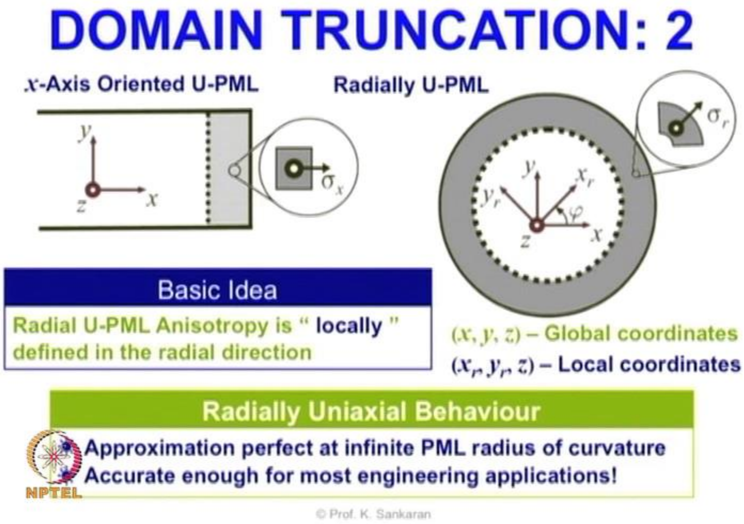
So right now we have got a simple condition where PML is sitting on the X axis. So the PML is actually an X oriented PML. So X axis oriented uniaxial PML. So what we want is we wanted to have a PML that is radially oriented. So in that sense regardless of which direction I am coming from the anisotropy which we called.

(Refer Slide Time: 08: 48)



So let us go back anisotropy is in the Epsilon by Epsilon is equal to Mu by Mu we said if it is a x oriented PML we will have the anisotropy in this direction. And the rest it's a diagonal matrix and we will have 1 by an if it's an x oriented PML. So what we are trying to do is we are trying to make sure that this 1 will be the anisotropic direction will be instead of x it will be the radial direction. So instead of x we are going into the radial direction.

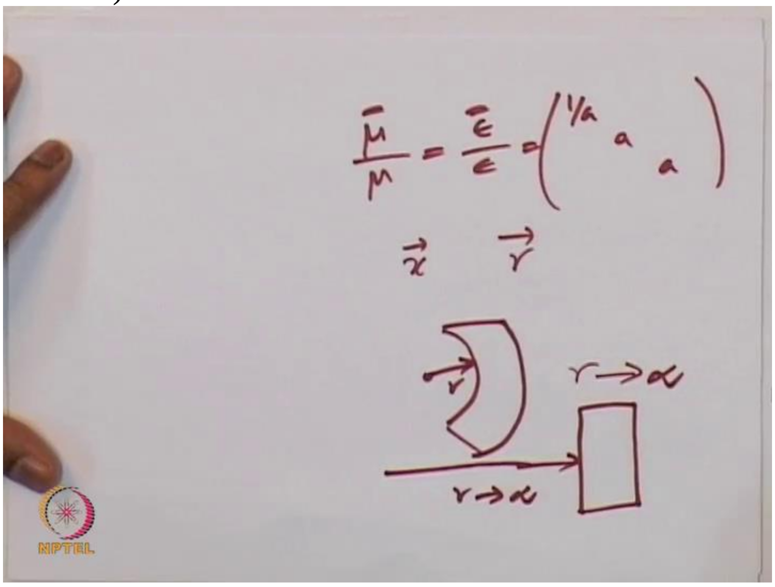
(Refer Slide Time: 09: 31)



So that is what we are seeing here in the slides if you see the x r what we have in the rotated symmetry here is the direction in which the PML is going to be anisotropic. And we are moving from locally we are talking about a radial axis which are x r, y r, and z. We have kept z here because we assume that along the z axis the PML is going to be symmetric so we are not going to worry about the z axis. What we are interested is only the x and y axis. So globally we are sitting in x,y,z coordinate. Locally we are in the radial x r, y r; where x r and y r are actually defined using this particular transformation. So what we are going to get is a radial PML , radial uni axial PML anisotropy is locally defined in the radial direction as I mentioned here in this case.

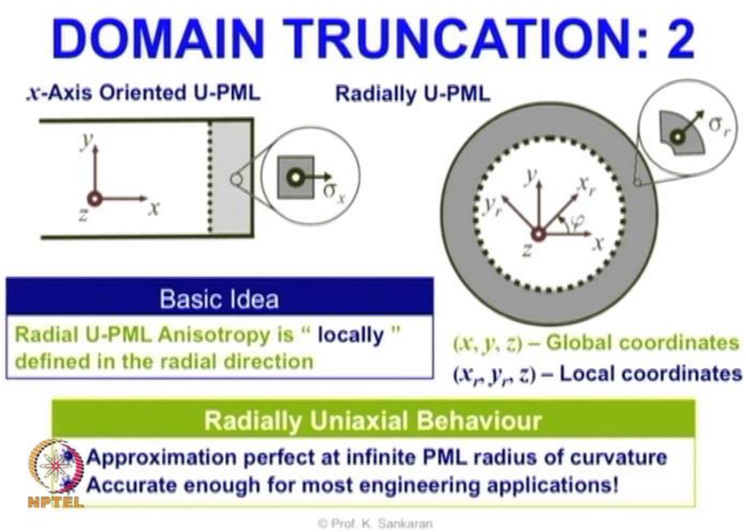
And the beauty is approximation is perfect at infinite PML radius of curvature because when we are talking about infinite radius of curvature we are basically talking about a clean PML because radius of curvature we will see in the next slides.

(Refer Slide Time: 10: 49)



So if you can see here so this is the way we define the radius of curvature. So if we have a PML like this so this is the radius of curvature. So when r is tending to infinity so this will become almost flat. So here r is going to infinity so this is the kind of transformation or approximation we are tend into.

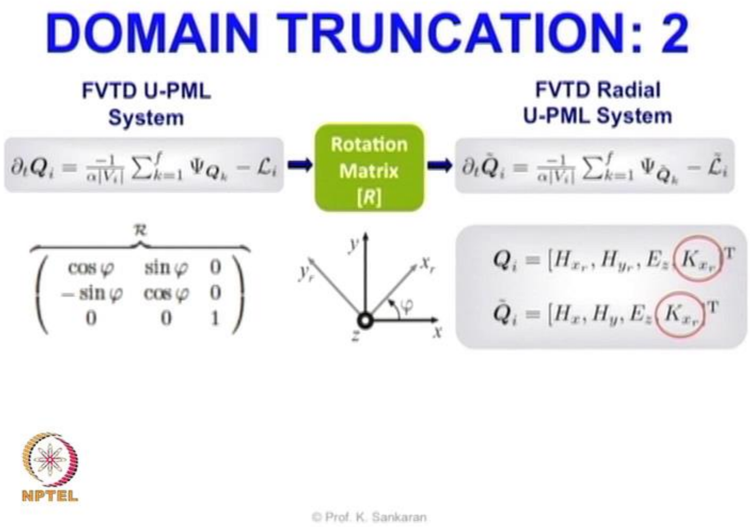
(Refer Slide Time: 11: 26)



For most of the practical application this approximation is good so for most engineering application which we will show in the next slides this formulation has been good enough with respect to absorbing boundary conditions that we have already discussed like Silver Muller or any of the other absorbing boundary conditions. And also this approximation is enabling us to come closer to measurement also. In the case of antenna chamber measurements and things of that sort 40 db minus 40db or 50db is good enough and for most application this is the range which we wanted to be in.

We can even go lower if we can reduce that spatial discretisation that means we are going finer and finer per wavelength and also you can improve the accuracy by making the radius of curvature larger and larger. So we wanted to have a compromise; in the compromise we wanted to make sure that the radius of curvature is more enough at the same time large enough for most of the application.

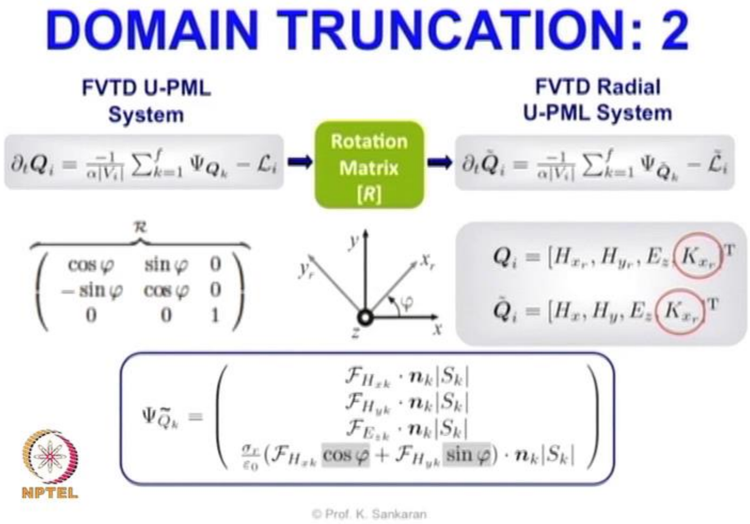
(Refer Slide Time: 12: 38)



So we will have now the formulation for the uniaxial radial PML we will begin with the equation which we are already familiar with. And we are going to take it forward to the radial case. So let me explain this how we are doing it so we are aware of this equation from our earlier slide so we have here the material parameters we have the Los terms and the flux terms so what you are doing here is we are doing the rotation as I explained you in the earlier slides so this rotation is going to impact only the X and Y line components as you can see here only the H x and H y and k x are going to be affected.

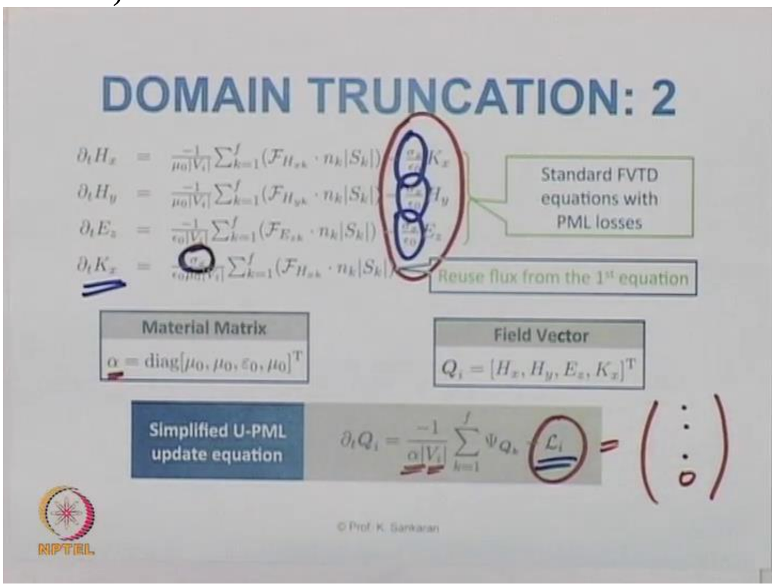
E z value will not get affected because E z is oriented in the Z direction and symmetry in the Z direction we are not doing any transformation so you will not have any impact on the is E z field what you will have is only the variation in the H x and H y and k x fields so what we have here as rotation is simple transformation that is going to take each and every component in the H x and H y area and going to transform it using the coefficient of rotation Which is cos phi sin phi minus sin phi and cos phi this is simple rotational thing and you will see that there is no influence on the EZ because this will be the same its 1 here because if it is not going to be affected the cakes are the quantities we are already computing.

(Refer Slide Time: 14: 17)



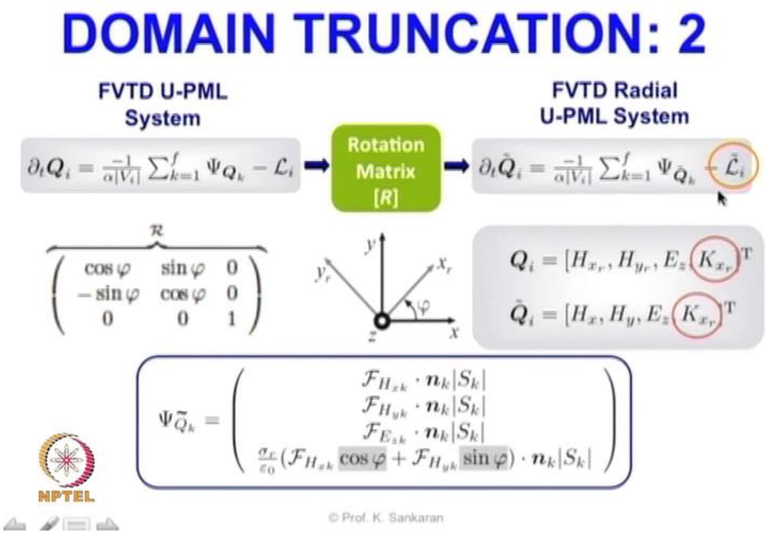
And we will have the values accordingly given by the last equation as you can see these are the initial terms the main Term that is going to impact in the radial PML are this k x terms these are the times that you have here so these are already the original terms that we have got from the Maxwell equation for finite volume but these terms are going to impact and as I already told you we are not going to do any additional computation because this value is already coming from here and these values already coming from here one important thing to notices in the case of planer PML we didn't have two terms we have only 1 terms so maybe we can see this once more when we go back into the slides

(Refer Slide Time: 15: 12)



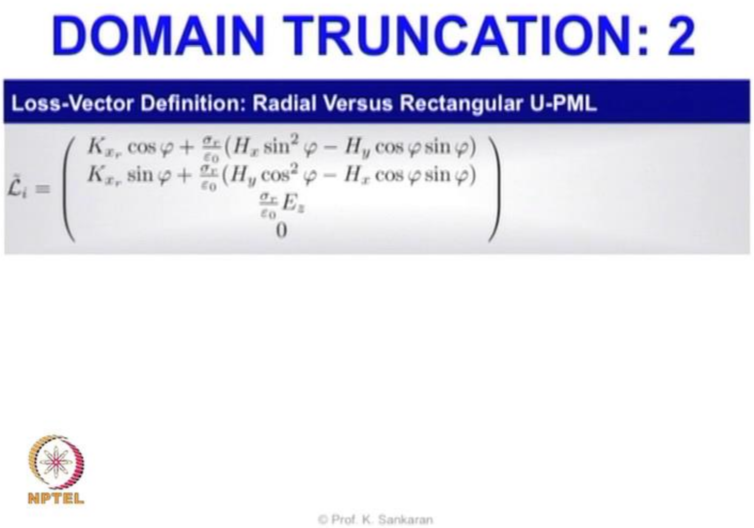
and I can explain you this in this case we don't have to terms but we had only one term we had only X component for K x but in the case of radial PML we not only have the access component but also the flux of H y component which is nautical but it has to be seen also in terms of the value of Phi which is the rotational angles

(Refer Slide Time: 15: 35)



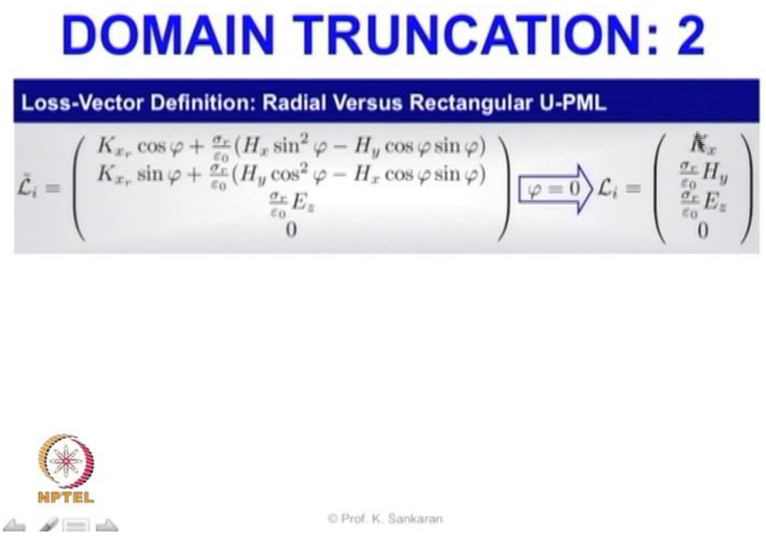
as you can see here in this slide there are two terms but they are sitting in the case of HX will be cos Phi and it is H y it will be Sin phi and you will see in the next slide why this makes sense and we will see that this is the loss term which is also given by the earlier slides

(Refer Slide Time: 15: 59)



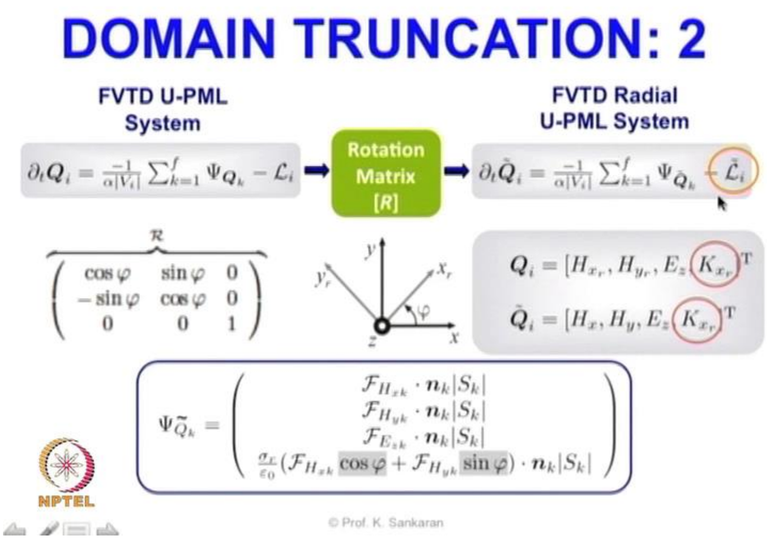
So I will see here the loss term is given by K X values and H y values and he said value is already the same and we don’t have the last term like in the case of earlier uniaxial PML in X direction we did not have any time on the IV vector component so that is the same way any Z component is the same we see that we are only impacting the terms on the H x and H y are the x I mean the two components

(Refer Slide Time: 16: 31)



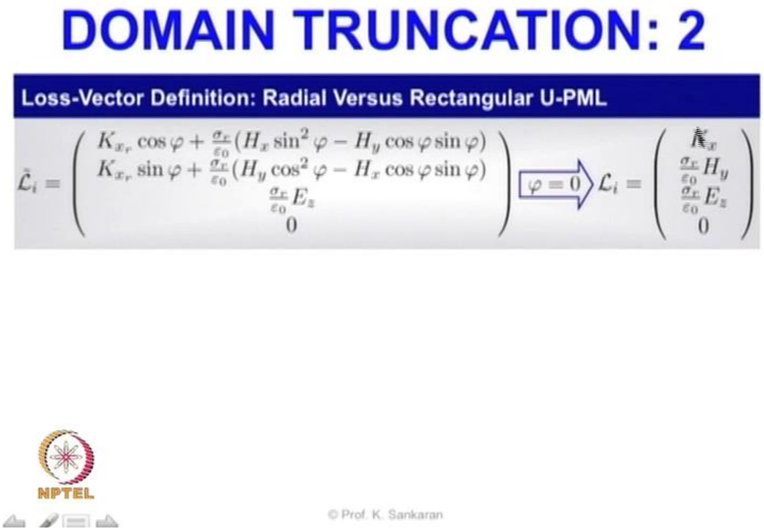
We will see here why this make sense for example if you put p h i equal to zero in this equation so what we will see is all the components of sin will become zero except for the component of Cos phi because Cos phi will be 1 so we will have kx and this is exactly the equation what we had for a uniaxial axe oriented PML so it gives us confidence that this formulation is correct for all the different combinations for the rotation so if this is not correct when you put Phi we have made some mistakes in the formulations but once we put Phi we are getting back to this equation and also once phi is zero

(Refer Slide Time: 17: 23)



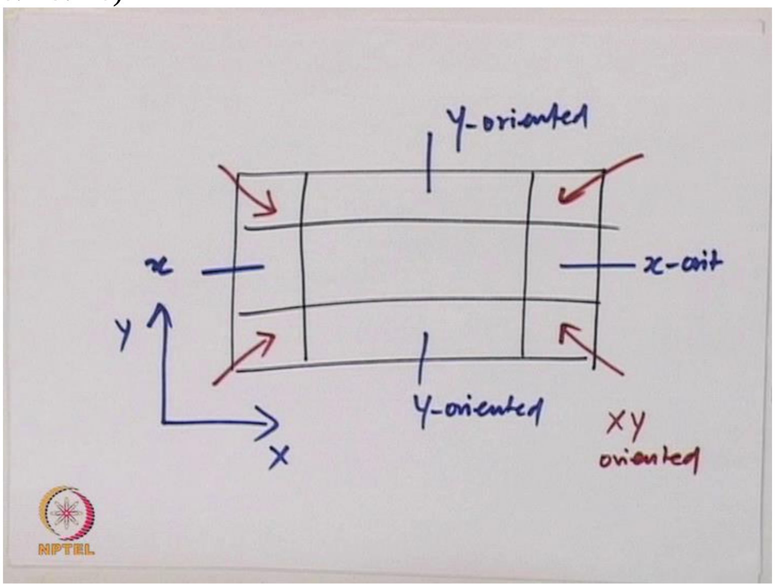
as you can see here in the earlier slide when Phi is 0 this component the Y component will become zero but this component will be there which is exactly the same as in the case of uniaxial oriented PML.

(Refer Slide Time: 17: 33)



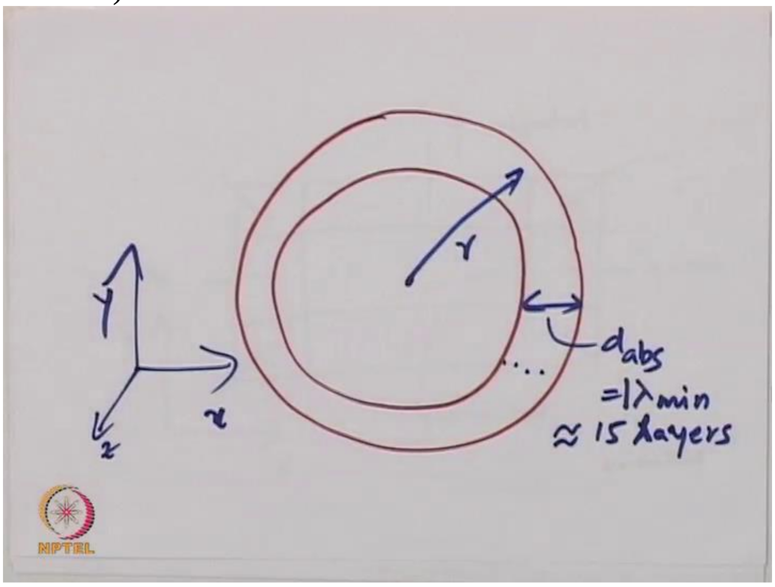
So putting things together this makes sense so now we are coming too unique properties of this PML we said that we are able to extend the uniaxial PML oriented the X direction to radial direction for a nice formulation for it and we wanted to see how much of this formulation can be practically used for problem solving that being said the advantage of this one is it gives us only one generalize formulation so instead of taking a uniaxial in X direction or uniaxial in Y direction or uniaxial in Z direction I can just live with one particular equation

(Refer Slide Time: 18: 16)



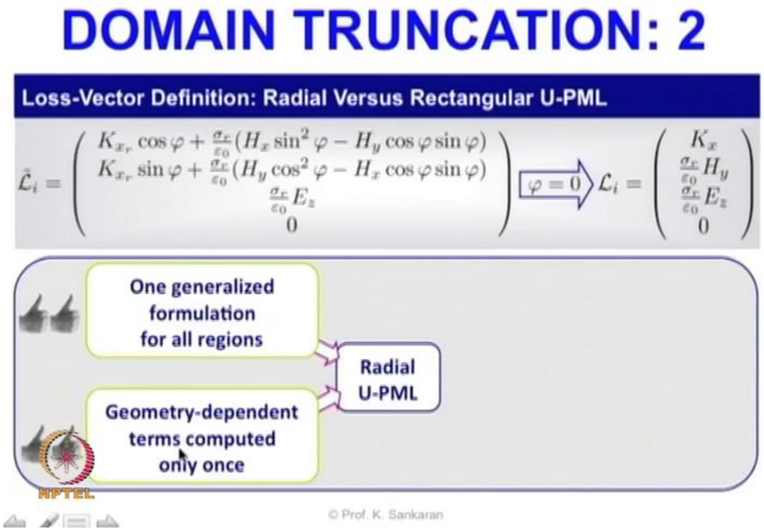
Let me explain this with the slide here so what I am talking about is so in the case of Benergen PML we said which is the split case we saw this also in our earlier lectures in finite difference method and finite element method so you have this is let us say this is X and Y so this will be the X oriented unit so this will be also X oriented this will be Y oriented this will be also Y oriented but these corners are special cases I think we saw this also in our earlier lectures in finite difference methods so this one this one this one and this one hour special cases so they have both X and Y oriented so we have to handle them specially we have to have computer different fluxes so that we can avoid when we go from split formulation 2 the uniaxial formulation

(Refer Slide Time: 19: 39)



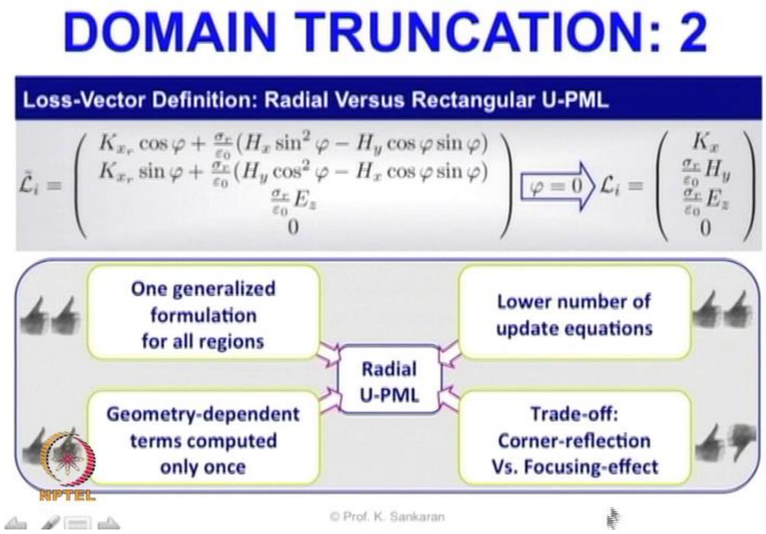
so what I am talking about is regardless of what is my direction if I have to truncate it so I will call this as my thickness; thickness of my absorber I use the word de absorber so normally what I will do is I will go for a thickness that is good enough for most practical applications so I will say I will go for 1 Lambda minimum so whatever is my minimum wavelength I will choose that will be the thickness of my absorber and I can say this is roughly in the case of the practical discretisation level we are talking about something around 15 layers of PML so when I say 15 layers what I am talking about is 15 cells will be sitting so the number of triangles in each of the directions what you are talking about will be 350 so this is clear what we can use as a rule of thumb and we can live with it because most of the applications we are talking about 10-15 cells so that is fine and the beauty of this formulation is we don't need to have any special conditions because we said this is our radial direction and it is uniaxial radial direction so regardless of which direction you go whether it is x axis or y axis and I will also see in the case when you are talking about Z axis we can do the same thing also in the three dimension where the radial PML will be still valid

(Refer Slide Time: 21: 30)



So that being the case we are able to formulate it in a much more easy manner and what we get as advantages we get one generalize formulation this is something I already discussed right now for all regions and we are able to say that geometry dependent terms are computed only once so most of the geometry dependent terms what you are talking about are values of this Phi and also the values of some of the material parameters and also the volume and the edge length and all these kind of things we are computing only once so we don’t need to go to them we don’t need to compute each of the time loop this is one good thing as well and this is not particular to finite volume this is generally the case if you want to optimise your code you have to make sure that you are computing the geometry dependent terms only once the most important thing is we get 1 generalized formulation.

(Refer Slide Time: 22: 33)



And the lower number of update equations as we also discussed we don't need to compute separately for the loss and also for the additional variable this is already integrated in the earlier computations that we do and there is a trade off everything doesn't come for free there is a trade off so we will discuss what this trade-off is in the next module so what we will see is how we can optimise this what is a trade of you are talking about what are the ways in which it will affect our numerical measurements and how we can make sure that this is good enough for most of the applications so until then we leave here and then you will come back and see applications in the next module thank you.