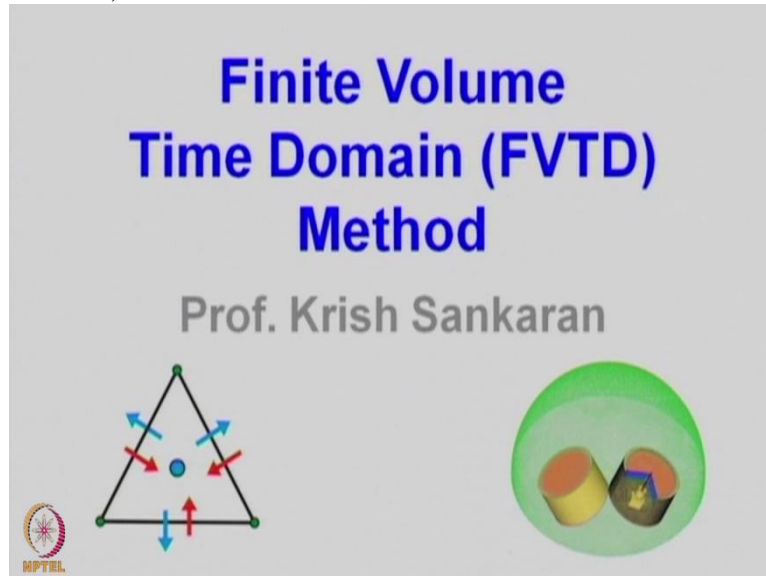


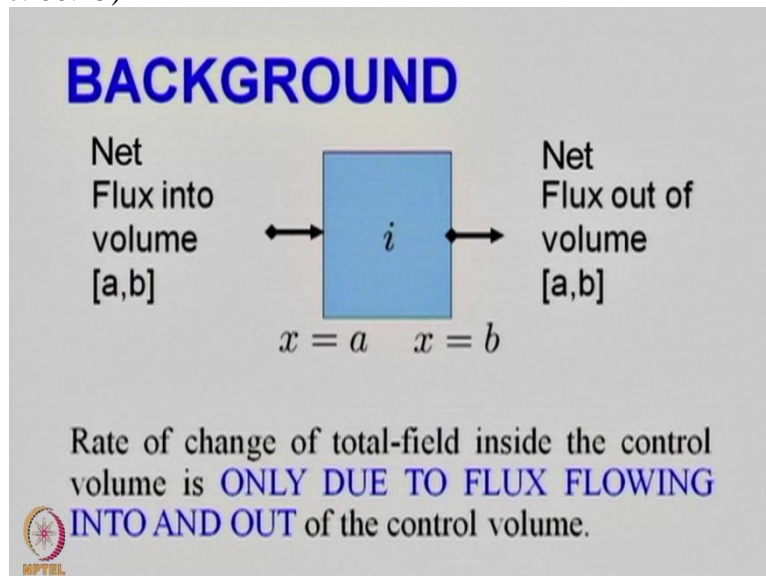
Computational Electromagnetics and Applications  
Professor Krish Sankaran  
Indian Institute of Technology Bombay  
Summary of Week 09

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This week we introduced one of the alternative methods for modelling electromagnetic problems namely the finite volume time domain methods

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We started with the technical background of this method setting the bases for Finite Volume formulation

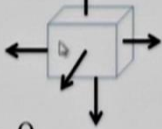
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## BACKGROUND

1D Flow equation  $\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{f} = 0$

Integrating over a finite volume  $v_i$  

$$\int_{v_i} \frac{\partial \mathbf{u}}{\partial t} dv + \int_{v_i} \nabla \cdot \mathbf{f} dv = 0$$

Using divergence theorem 

$$\int_{v_i} \frac{\partial \mathbf{u}}{\partial t} dv + \oint_{s_i} \mathbf{f}(\mathbf{u}) \cdot \mathbf{n} ds = 0$$


To solve a simple advection equation.



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## BACKGROUND

$$\int_{v_i} \frac{\partial \mathbf{u}}{\partial t} dv + \oint_{s_i} \mathbf{f}(\mathbf{u}) \cdot \mathbf{n} ds = 0$$

Semi-discrete formulation

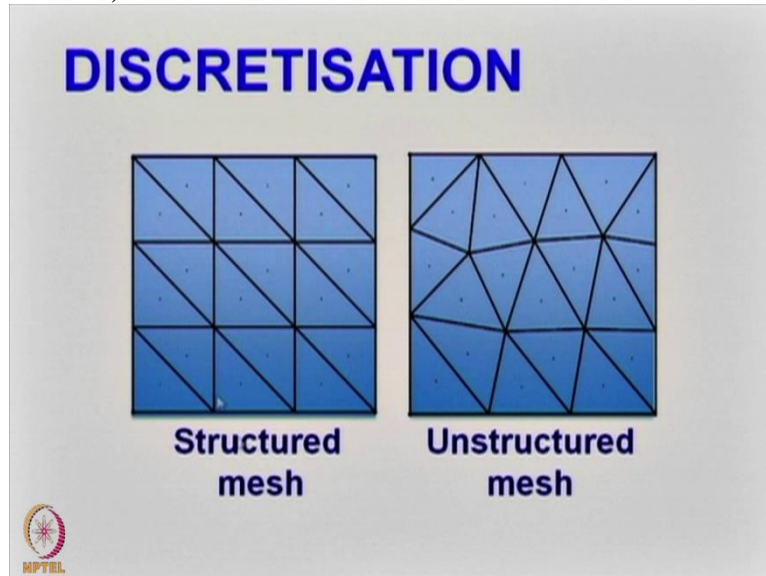
$$\frac{\partial \mathbf{u}}{\partial t} = - \sum_{i=1}^k \mathbf{f}(\mathbf{u}) \cdot \mathbf{n} ds$$

$k = 6$    $k = 4$  



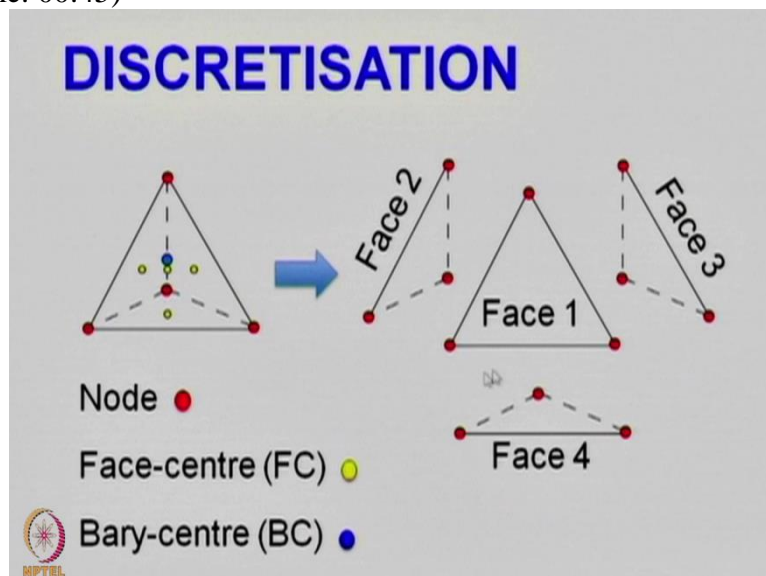
Later on we modelled the one dimensional flow equation using finite volume method.

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We discussed how the spatial discretisation is carried out

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In the finite volume frame work

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**MAXWELL SYSTEM**

Maxwell System

$$\begin{aligned}\mu \partial_t \mathbf{H} &= -\nabla \times \mathbf{E} \\ \varepsilon \partial_t \mathbf{E} &= \nabla \times \mathbf{H}\end{aligned}$$

Semi-Discrete Maxwell System

$$\begin{aligned}\partial_t \mathbf{H}_i &= -\frac{1}{\mu V_i} \sum_{k=1}^f (\mathbf{n}_k \times \mathbf{E}_k) S_k \\ \partial_t \mathbf{E}_i &= \frac{1}{\varepsilon V_i} \sum_{k=1}^f (\mathbf{n}_k \times \mathbf{H}_k) S_k\end{aligned}$$

MPTEL

Particularly for the case of Maxwell equations

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**MAXWELL SYSTEM**

Material Matrix

$$\alpha_i = \text{diag}[\mu_i, \mu_i, \mu_i, \varepsilon_i, \varepsilon_i, \varepsilon_i]^T$$

Semi-Discrete FVTD System

$$\partial_t \mathbf{Q}_i = \frac{-1}{\alpha |V_i|} \sum_{k=1}^f \Psi_{Q_k} S_k$$

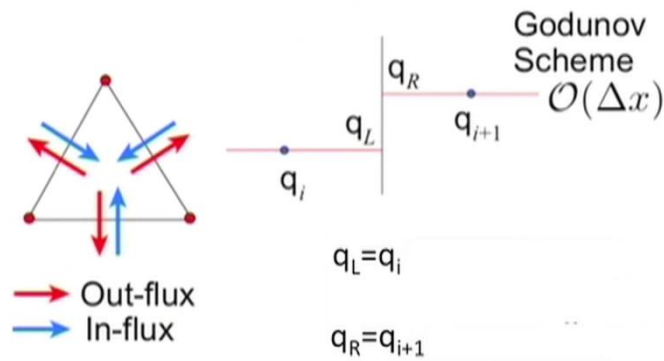
Flux function

MPTEL

We emphasized the role of flux function in the finite volume frame work and discussed various approaches to compute the flux function.

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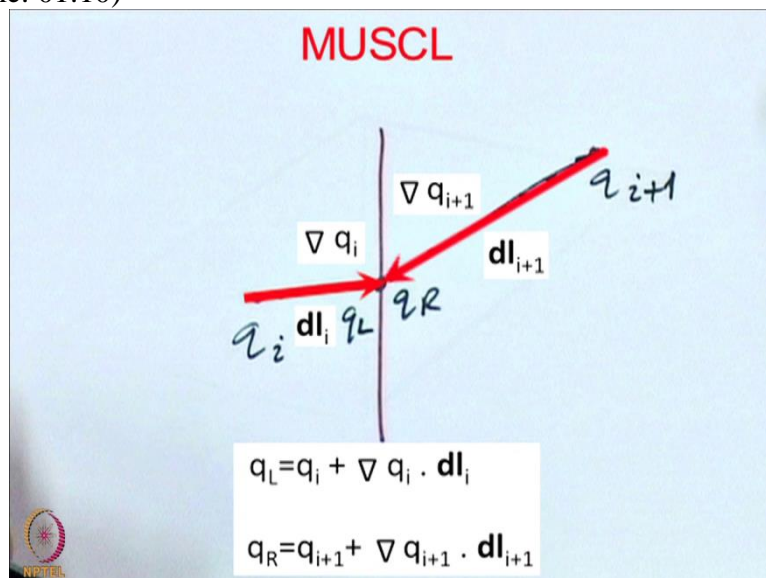
## FLUX FUNCTION



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This includes the famous Godunov approach

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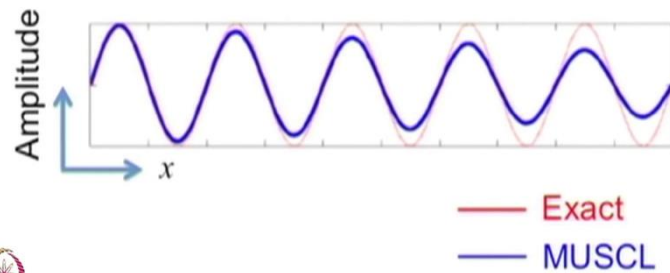
And the monotone upwind scheme for conservation laws shortlu abbreviated as MUSCL algorithm for the Finite volume method

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## FLUX FUNCTION

Godunov scheme is highly dissipative for CEM

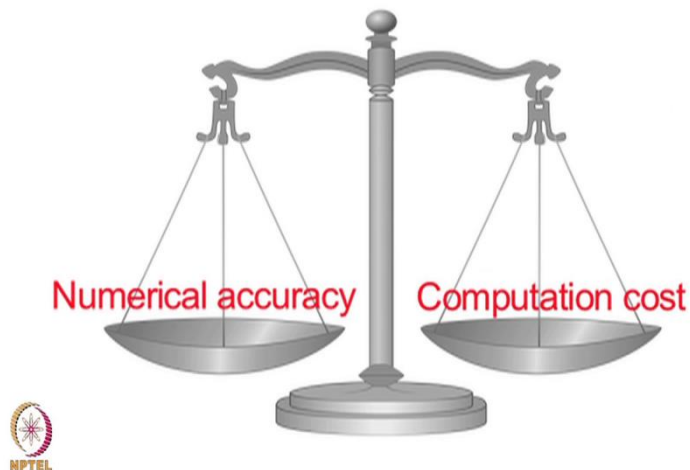
MUSCL is an improvement but still dissipative!



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We remark the pros and cons for these approaches

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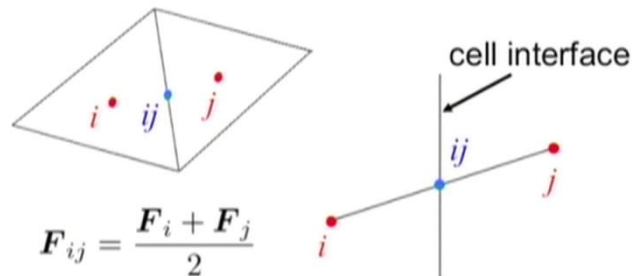


Particularly emphasizing their computational cost and accuracy

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## FLUX FUNCTION

### Centered Flux / Flux Averaging Scheme



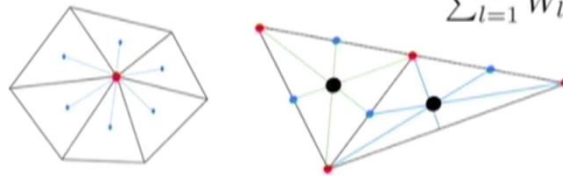
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We later introduced the centred flux averaging scheme

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## FLUX FUNCTION

### Truly Upwind Scheme $F_n = \frac{\sum_{l=1}^{nl} W_l F_{c,l}}{\sum_{l=1}^{nl} W_l}$



$F_n$  = Interpolated nodal field value

$W_l$  = Nodal weight for triangle  $l$

$F_{c,l}$  = Barycentric field value of triangle  $l$



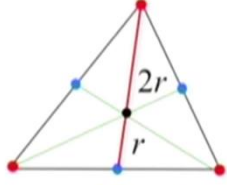
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Truly upwind scheme and

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## FLUX FUNCTION

### Geometrical Reconstruction Scheme



Approx. FC values  
Weighted nodal values  
Computed BC values

Field computed based on geometrical properties

No need for computation of gradients at the BC

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The Geometrical reconstruction scheme as alternatives to the more popular Godunov and MUSCL approaches

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## MAXWELL SYSTEM

### Material Matrix

$$\alpha_i = \text{diag}[\mu_i, \mu_i, \mu_i, \varepsilon_i, \varepsilon_i, \varepsilon_i]^T$$

#### Semi-Discrete FVTD System

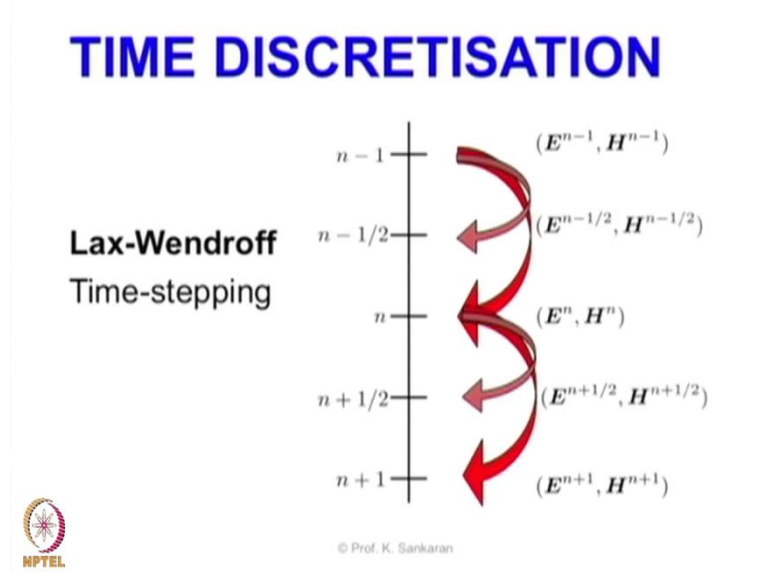
$$\partial_t Q_i = \frac{-1}{\alpha |V_i|} \sum_{k=1}^f \Psi_{Q_k} S_k$$

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Later we explained the time discretisation for the finite volume method

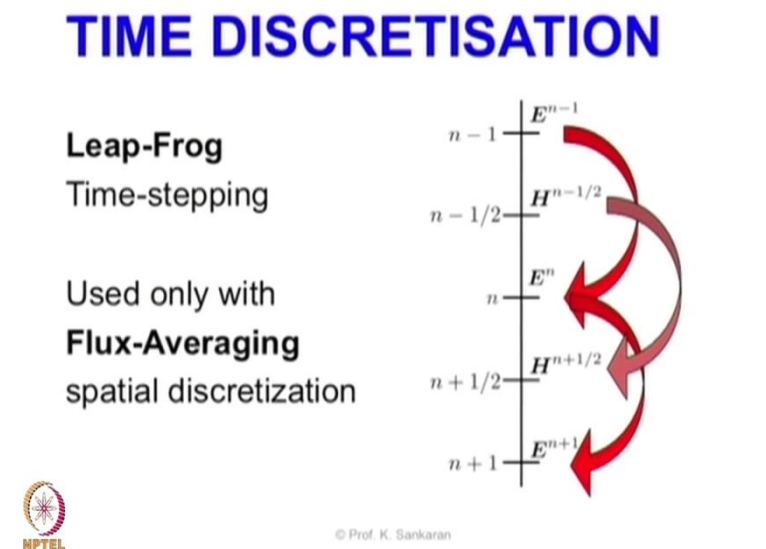


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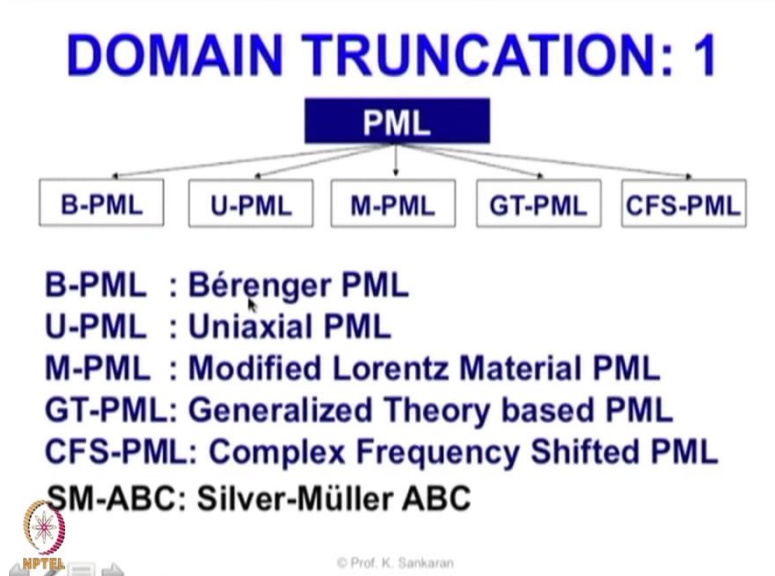
We elaborated the most widely used Lax Wendroff

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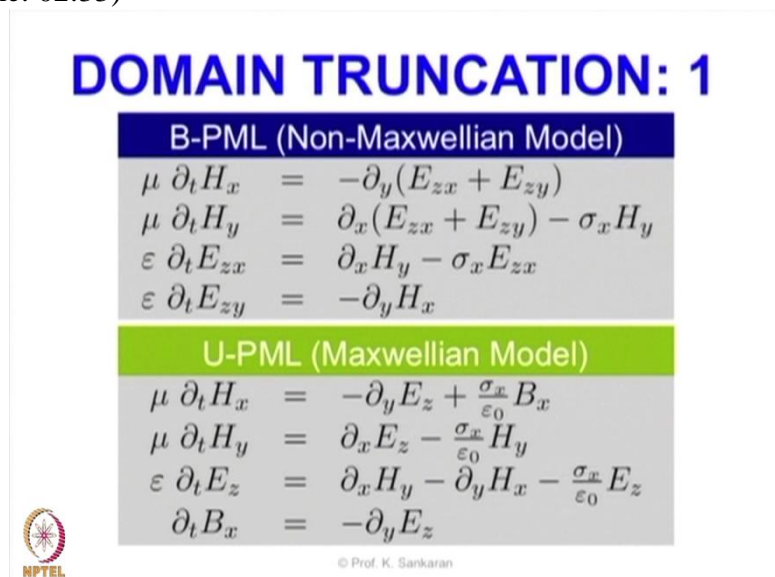
And Leap Frog time stepping schemes for Finite volume method

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Finally we introduced certain accurate domain truncation techniques in the Finite volume framework. We first discussed the simple Silver Muller absorbing boundary condition and then introduced the more accurate perfectly matched layer approach for Finite volume method

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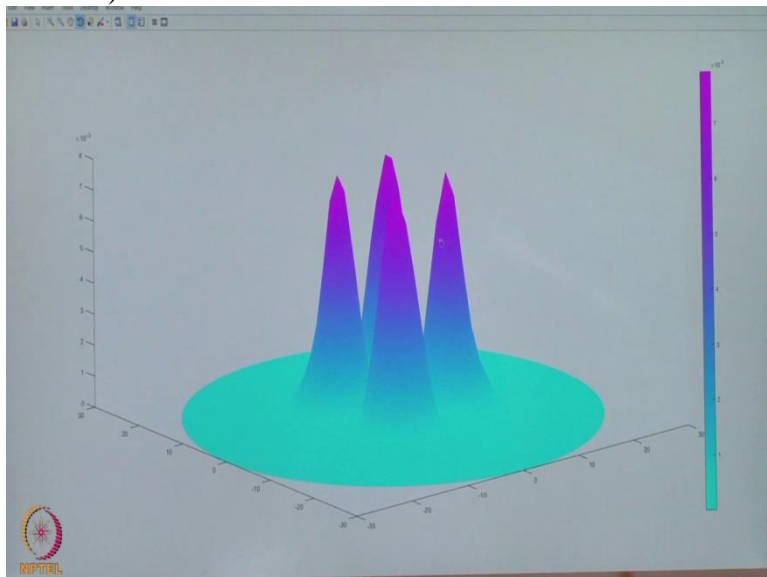
We discussed two broad classes of perfectly matched layers namely the non Maxwellian Berenger PML and the Maxwellian Uniaxial PML.

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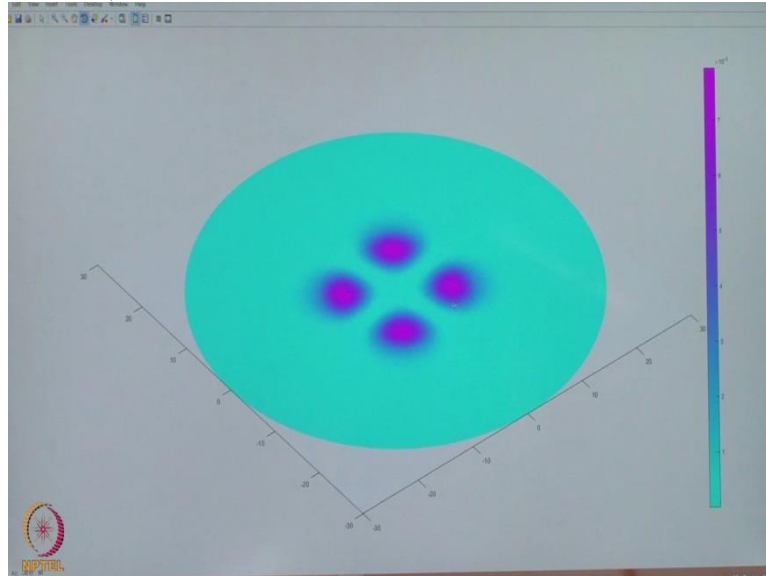


In the lab tour we discussed a modelling exercise involving a multi mode optical fibre

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And we simulated various mode profiles for this multi mode fibre.

Please go through the concepts and examples that we discussed in this week. We will be dealing on these basic ideas in the next weeks lecture. Post your questions in the forum clarify your doubts and get ready for the next week until then Good Bye!