Computational Electromagnetics and Applications
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Lecture No 31
Finite Volume Time Domain Method-II

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We will now discuss some of the accurate domain truncation techniques. So the title as it says its Part 1 of the domain truncation. Because there are further advancements to the techniques that we will discuss now. So as I start what I wanted to say is a very simple rudimentary approach is to truncate your domain is to put absorbing boundaries. So if you have a circle you make sure that you have absorbing boundaries on the surfaces of the boundary but in case of finite volume what has been happening in the last few years is basically kind of they make the domain go really far away order to avoid any of the reflections that are coming from the edges so what I mean by that is most of them happen is the absorbing boundary conditions are dependent on the angle of inclination for angle of incidence of the incoming waves so what happens is.

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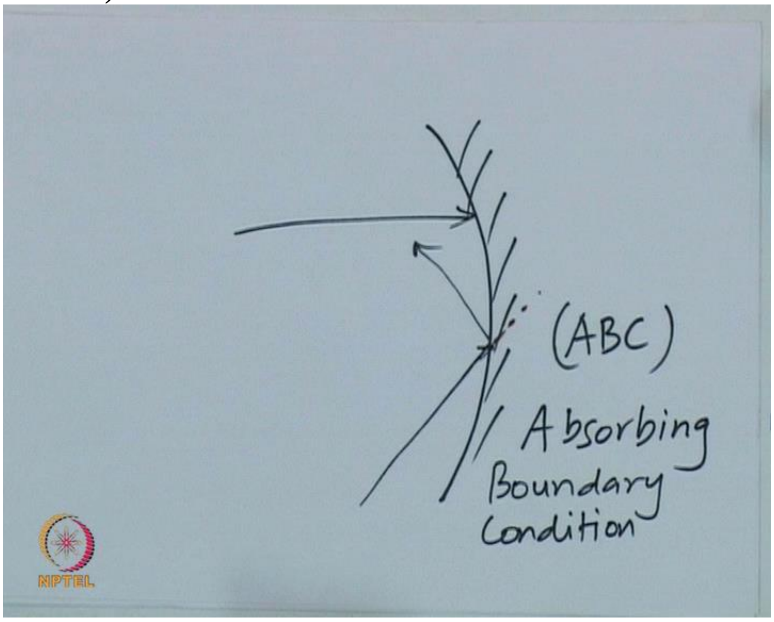

So let’s say you have a surface and I say all this point. They are having absorbing boundary condition. So let’s say a wave is coming in this direction it gets reflected but if the wave is coming in this direction it gets reflected like this and obviously we have to reduce this reflection as much as possible because whatever comes in is getting absorbed but its not getting attenuated but there is still some amount of reflection so this reflection is not zero but when it is having a normal incident as in when a wave comes normal to the surface the reflection is less. So the absorbing boundary conditions which have been used in finite volume method are not really that accurate. So what happens if people with try to put boundaries at very very father distance so as to minimise whatever is getting reflected let’s say your computing something here you put the boundary really far.

So that whatever reflection comes back is taking long time so that way they were able to avoid most of the practical problems but again this means if I have to put a boundary here I have so many cells that are in between which are going to also increase the computational cost so until 2004 2005 the absorbing boundary conditions of silver Muller silver and Muller Thevar to people there boundary condition is the most common one so in 2004 2005 2006 onwards there was a series of paper that were published on improving the boundary condition using perfectly matched layer we discussed perfectly match layer in our earlier module on finite difference method and also finite element method so we will be discussing about the perfectly matched layer so in other words we call it as pml

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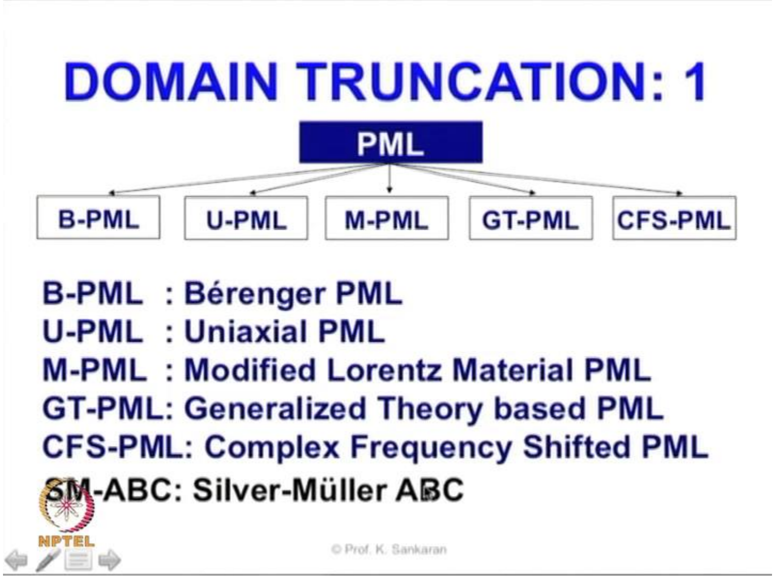

There is a group of PMS so this might be shocking for you that there are so many pml all we know is Berenger is a french engineer first published his work is formulation is a quiet known one and of course there were other modulation other changes to that Berenger PML which is called universal pml modified Lorentz material pml or generalized theory based bml or Complex frequency shifted pml and of course the standard boundary condition of silver and Muller so in this lecture we will be focusing mostly on the first two types namely the Berenger PML and the PML of the uniaxial layer so these are the two things which we will be focusing on for people who are interested in those other pml areas we could give some references and they can look into it but these are the first 2 pm else or the most commonly used pml and that is what we would be focusing on so let us go into the theory of those 2 Pml

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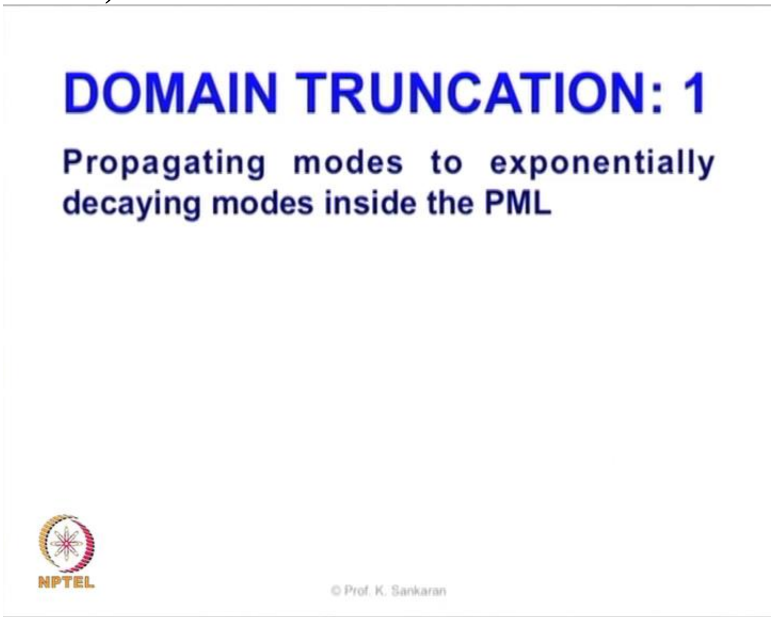

So what is happening is you have a wave that is coming in and it's wave it's not enough for the wave to be let's say absorb but they have to also attenuated so what you are talking about is there is a propagating mode and we have to covert that propagating mode Into exponentially Decaying mode so that is what is Basic Physics behind the DML theory so what you are talking about is propagating mode to Decaying Mode inside the pml.

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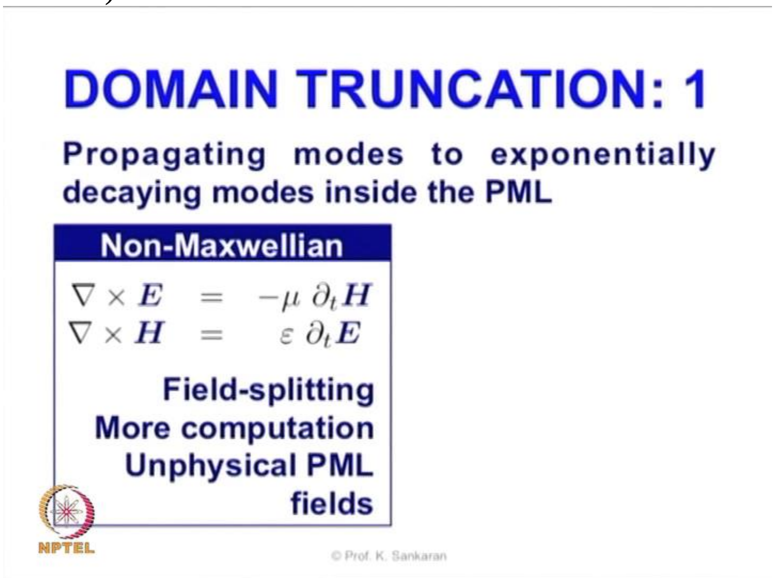

So I said the word physics behind the bml theory so award of a caution has to be made here so when I say physics behind it people expect that they are physically realizable so that's not the case here so the case what is in the case of non maxwellian or Berenger PML there is no physical counterpart of those field that we are computing they are truly mathematical this we have already covered in the case of finite difference method I am repeating for people who have not followed that lecture so this field what we are computing in the case of Berenger pml they are purely mathematical manipulation of the field equations so with that being said we will look at it in more in detail in the following slides so I used the word non Max alien just to make sure that there is no physical counterpart of those fields so in other words they are not physical and they are doing that by splitting of the field we have to do more computation and they are Un physical pml fields this I have mentioned it.

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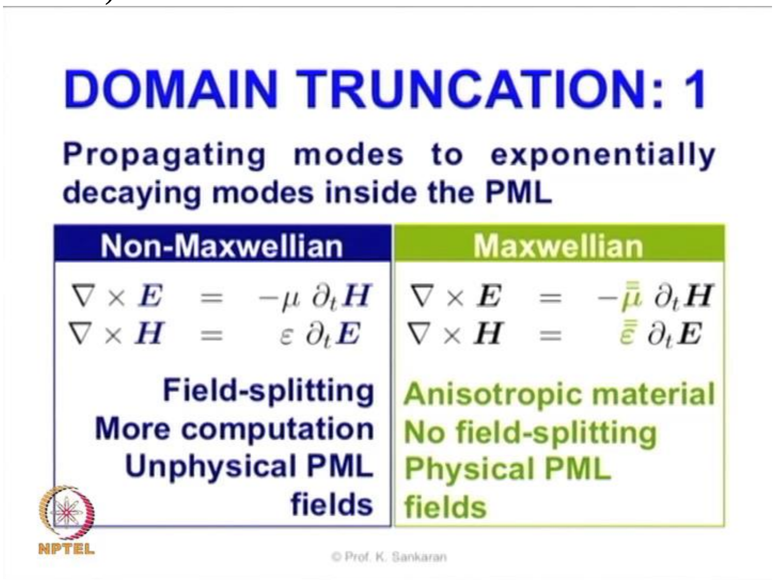

The counter part of that is the maxwellian ML which is also called as the uniaxial pml as you can see there is something different in the case of the maxwellian compared to the non maxwellian so what we are calling them is anisotropic material and these materials are an isotropic because the Mu and Epsilon permittivity and permeability there not scalar quantities but they are tensor quantities and we are not doing any field splitting and there is a physical pml field that we can compute they are physically computable and also there is a physical counterpart of those fields so again the word physically computable or physical counterpart is bit of a stretch of a word the question is field itself something is that you cannot measure electric field or magnetic field is only something that you can deduce based on the value of the potential or the voltage difference.

Or in the case of magnetic field its current field itself is a mathematically reduce quantity but even then in the case of splitting of the field there is no counterpart there is no real relevance of that is purely mathematical but in the case of an isotropic material you can say you can somehow kind of mimic the behaviour using certain anisotropic materials so that’s the reason I said they are physically meaningful or physical pml fields again take this particular thing with A Pinch of salt so we will see now what are the corresponding equations so we will see now how this maxwellian and non maxwellian model r differing for a simple two dimensional problems so we will take a transverse magnetic problem

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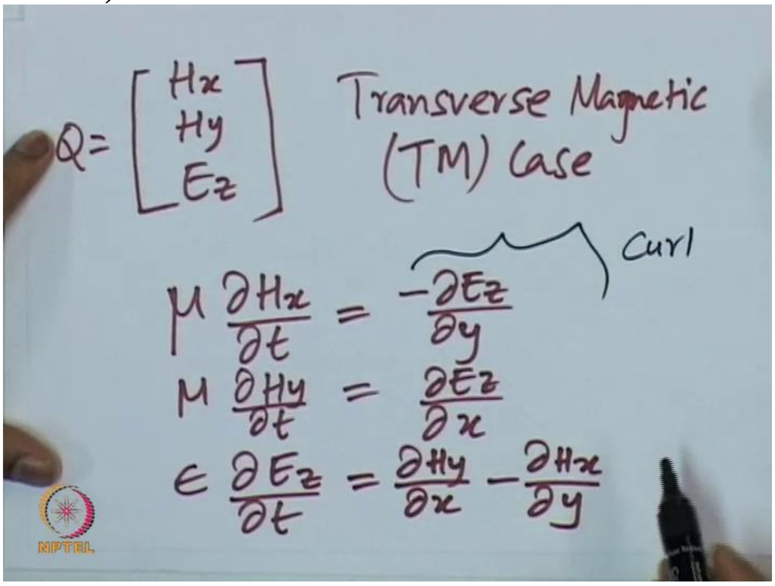

In other words what we have is the field quantities q will have components for the magnetic field in the X and Y direction and the electric field is only in the Z direction so this is the transverse magnetic case so you can also do the same thing for electric field in the X and Y direction and magnetic field in the Z direction in that case it will be a transverse electric case so if you are writing down the Maxwell equation for transverse magnetic case what we will have is Mu dH x by dt so these are the partial derivatives is equal to minus dE z by dy; and similarly we will have Mu dH y by dt is equal to dE z by dx and we will have the third equation which is respect to E z which is equal to we will have two components dH y by dx minus dH x by dy. So what we are having here is we are having components which are basically the curl terms. These are the curl terms.

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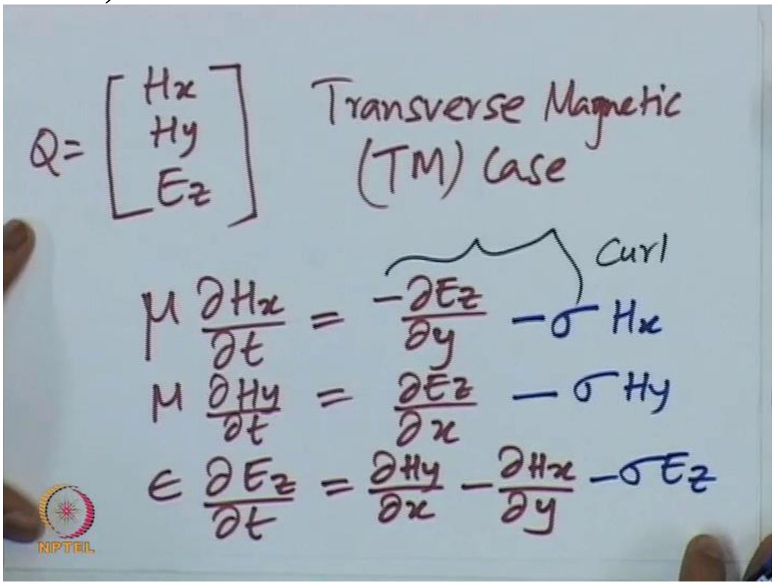

So what we are going to do now is we are going to say this is the standard Maxwell equation in a computational domain.So what we are going to do now is add certain losses to it. So the losses are basically H x similarly there is a loss in y direction and then there is loss in the z direction

So once we do that we are basically making the wave to decay in a particular direction what we want so if we say that we wanted to decay it along the x axis so this is the way we have to do it. And these terms are can be split into losses into x direction and losses in y direction. so let us explain this in the next slide.

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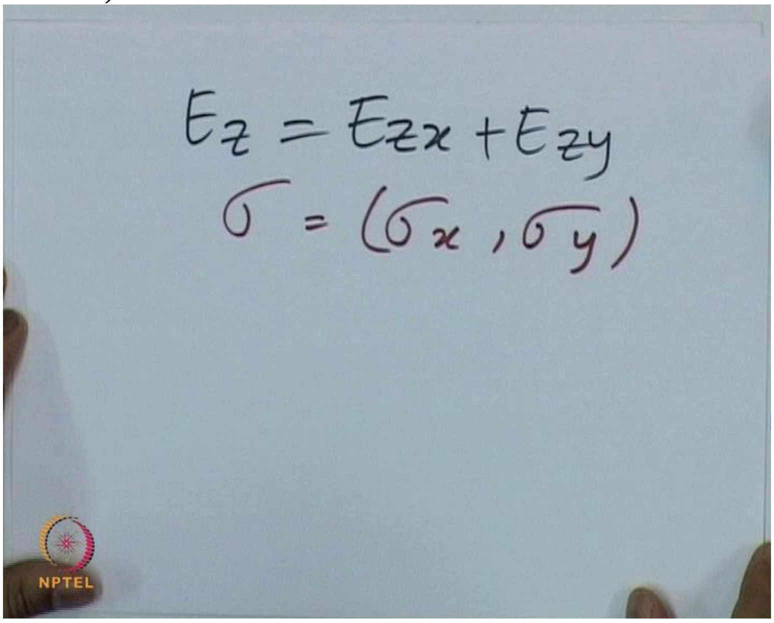

So what is happening now is we have components of E is split into E zx plus E zy and again i am repeating this is purely a mathematical trick. And I am saying sigma will have components which are sigma x and sigma y. So these are the losses of damping x and y direction travelling waves.

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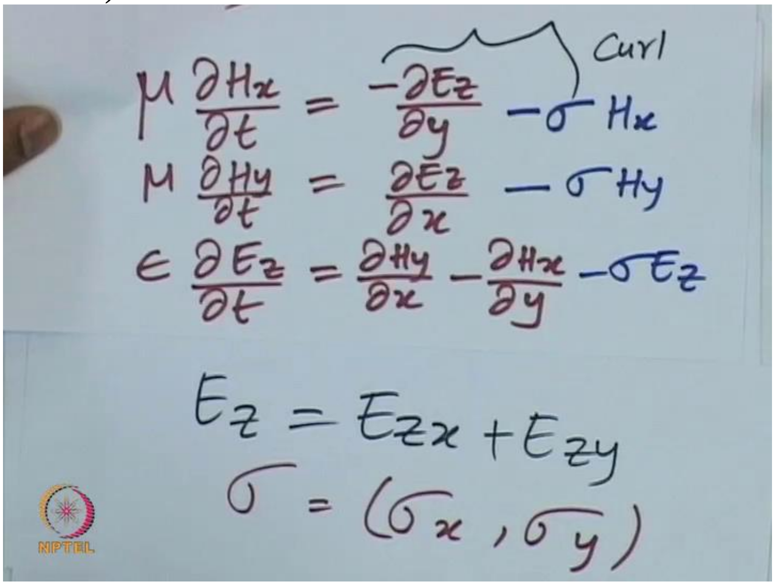

So this will give us a kind of a equation for if we put this value E z in the case of this equation here we will see that we will have 2 equation instead of 1 equation in the case of E z component.

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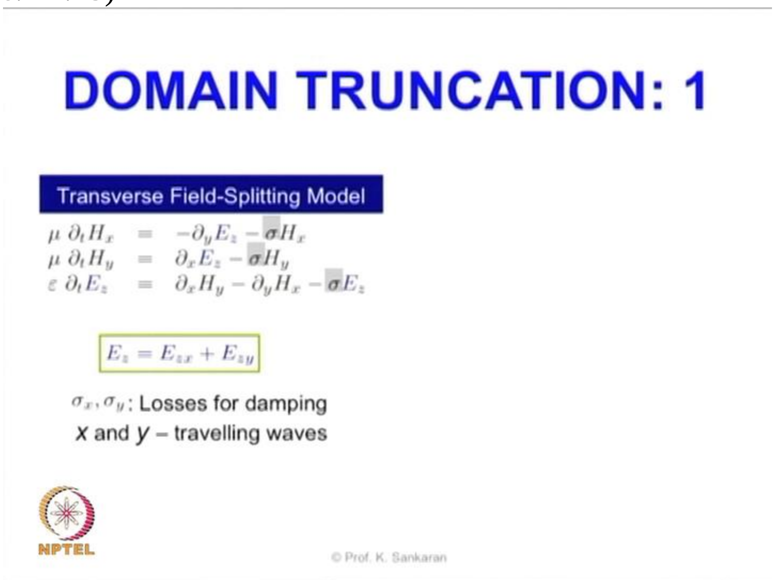

So let us look at how we are doing the same thing in the case of the Maxwellian model so once more we will see we have described in the case of the Non Maxwellian model so these are things what I have set. So let us look at how we can do the same thing in the case of in the Maxwellian model where we are having the values of material components the material parameters. The permittivity and permeability is no longer scalar its going to be a tensor.

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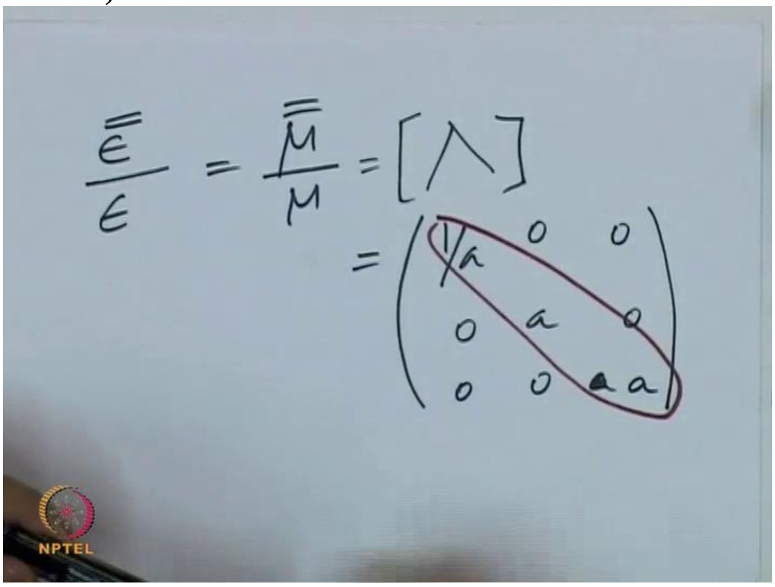

So what we are having is the value of Epsilon by Epsilon , similarly Mu by Mu is equal to a [tensor] let us say I make the tensor look like lambda capital lambda. Which is equal to 1 by a a and a and I will have value 0 0 0 0 0 0). So basically it will be a equation which is diagonal whose components are given by the values here. And if I want the material to be anisotropic along x axis I say it is 1 by a on the first component. If I want to be anisotropic along the Y axis I put a here and 1 by a here. If I want it to be anisotorpic along Z axis I will do the same thing along the last component. So these are the components along which we are going to do.

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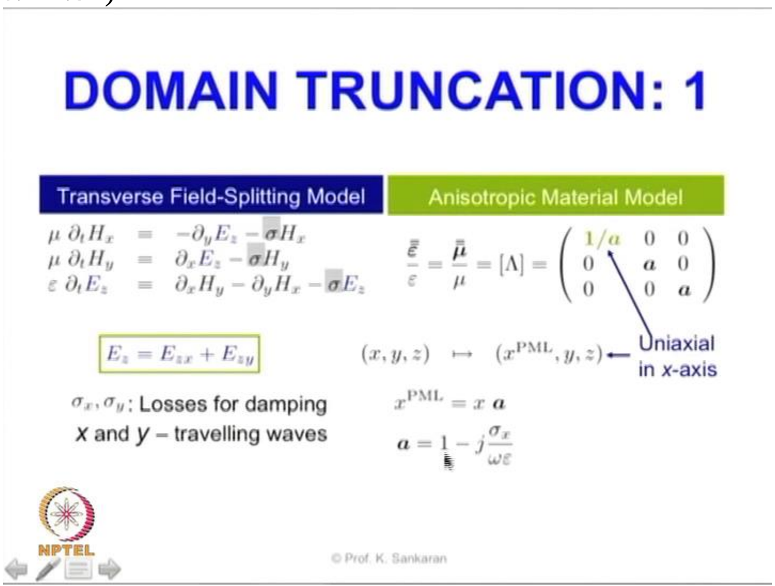

So that is what we will see here in the slide the relative permittivity so when you talk about permittivity of that material divided by the value. So we are talking about the components of the relative permittivity accordingly and the physical meaning of this is I am basically stretching the material in a particular direction where I want the wave to attenuate in the case

I want the wave to attenuate in the X direction I am stretching the X coordinate from X to X PML which is the stretching aspect and the stretching basically happens like a way that each of the component of X in the PML will see the value as x multiplied by a which is where a is given by this expression. So this expression is as you can see has a frequency component.

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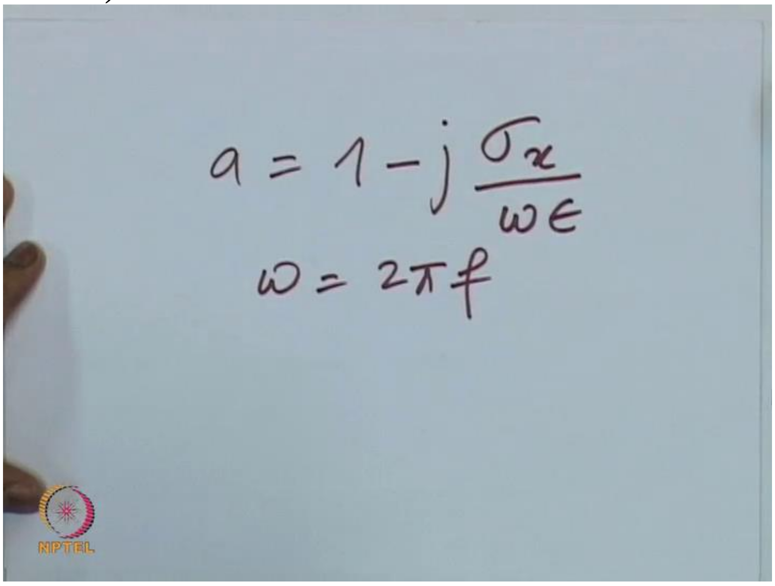

So what we are talking about is a is equal to 1 minus j sigma x by omega permittivity so this value is 2 pi f , so this will depend on the frequency of the wave that is coming. So that being said we can basically compute the values of a for each of the modes. And we can club in the form what we need so as to get a nice formulation.

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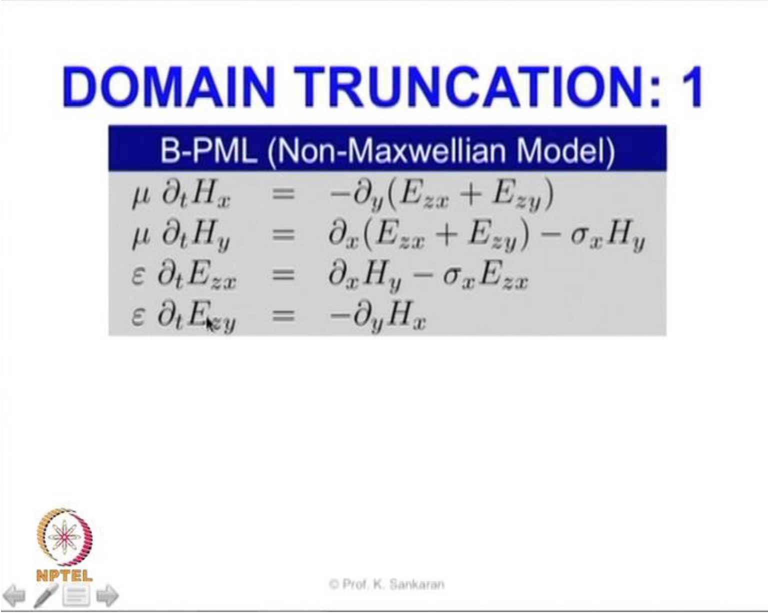

So with that we are able to get a kind of an attenuation what we need. So what we see now is in the case of the domain truncation using Berenger PML instead of three equations where we got now four equations and this is based on the equation what I have said we said E z is equal to E z x plus E z y, so based on that you get two equations here.

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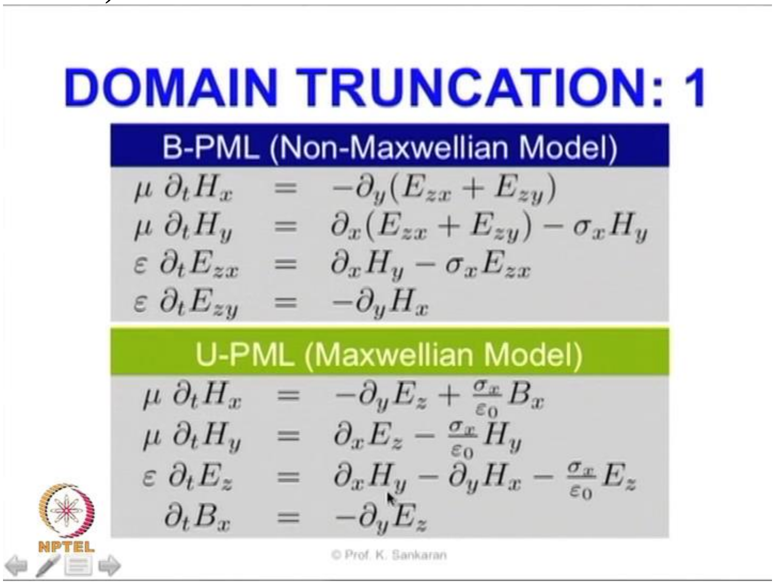

And in the case of the Maxwellian PML you are getting also 4 equation , but the most important thing is this. The value for the fourth field what you are computing here is something that you have not computed in any of the other cases. If you see we have dy partial differentiation with respect to y axis for the Hx field is not computed in others here you have compute the values with respect to electric field here also you computed with respect to electric field.

But here you computed with magnetic field but this is a very different flux it is a differentiation with respect to x not with respect to y. So in the case of the Berenger PML you have to compute four fluxes which is computationally costly. But when you look at the uniaxial PML which we call it as UPML you are also having four equations but the last flux is basically the flux that you have already computed in the first case. So you are not doing any additional work. That is what we will see in the next slide.

So in the case of Finite volume methods we will mostly use uniaxial PML because it is computationally heavy compared to other Berenger PML. So to put things into context so we said we have two different ways of going ahead with the PML regardless of what problem we are solving we can use them as Berenger PML for the truncation or the Uniaxial PML and we said why we are going to use Uniaxial PML but not Berenger PML for the simple reason of the computational cost.

So with that being said let us see the final set of equations what we will get for the TM case for doing any calculation.

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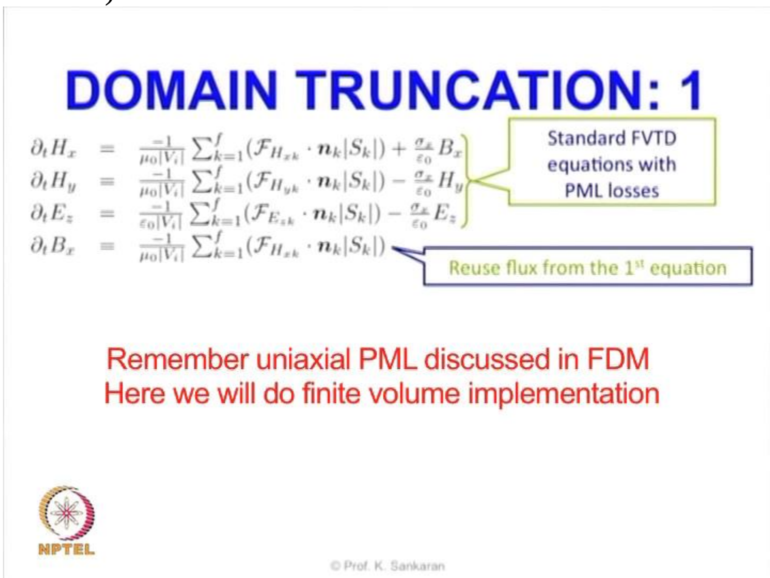

So what we have here is we have problem of TM case and then we have this H x, H y, B z and B x so this B x is the additional component whose value we are computing and this is the attenuation that is also there in the H x side using the B x value and this B x value we are computing using this equation. And this particular expression what we said so these are the standard Finite volume equation with PML losses. The last one we will reuse the flux from the first equation. This we have discussed before.

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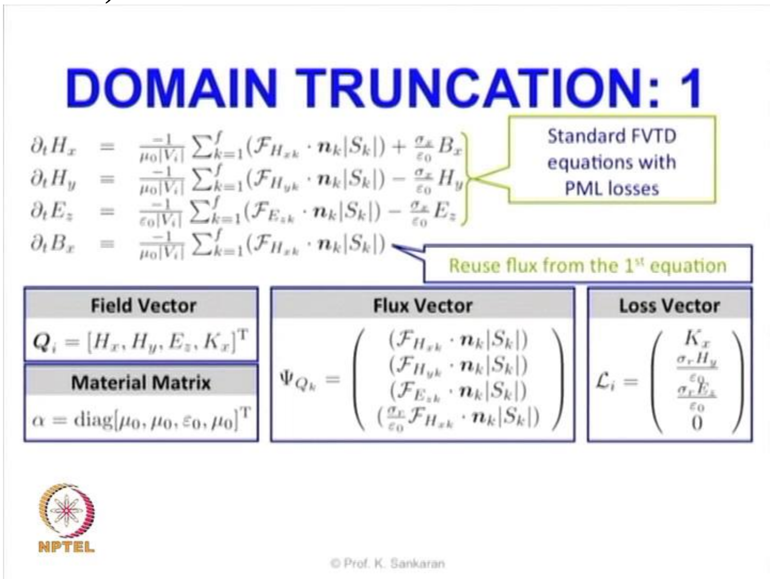

And in the case of the basic TM mode we will have the components given by these values and we can compute them accordingly. So with that we are having a close form solution so as you can see if you need to compute problem you can basically use at this formulation. In this case I have used B x but maybe I have (())(19:17) it as K x. So this should be K x or B x they should be consistent. So let us see it here.

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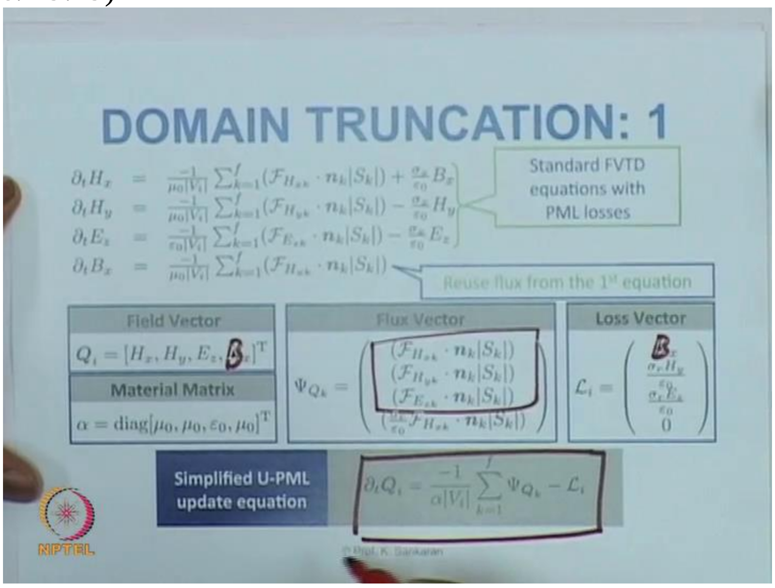

So this particular expression is the same one so we will also call this one as B x. So these are all the B x terms. So we have the first three equations which are the standard equations. And the fourth equation which is basically the equation computed based on the flux that we compute here and we have the additional term B x that is being updated using these form. And finally what we will have is this equation.

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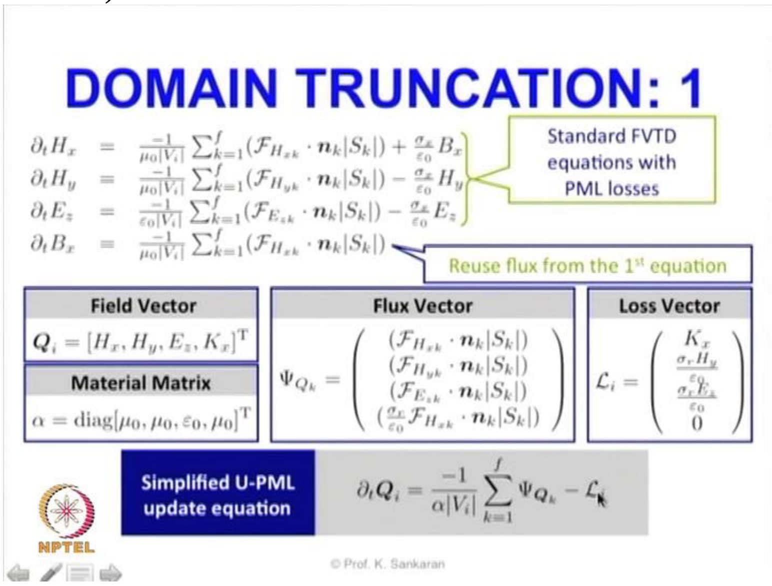

This equation says as you can see in the slide what you will have is the value of the thing what we are updating the vector is computed like this to and inside the domain where there is no PML you can put this loss term equal to 0. Remember this is B x not K x so this last term will be 0. And then you will have a natural update equation inside the boundary. And on the PML side you will compute the boundary losses or the domain losses based on this expression.

With this we will come to the end of this module we have covered quite a bit in this module we have looked into the finite volume formulation itself with certain applications in mind. And we have also introduced domain truncation part 1 where we have looked into the perfectly matched layer for planar applications. So we will see in advanced method which we will cover in the next module as domain truncation technique part II. Some of the extension of this perfectly matched layers for other applications. So with this we will stop here and we will see you in the next module. Thank you!