Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No 30
Finite Volume Time Domain Method-II

We said that there are three different other options to Godunov scheme and also the MUSCL scheme.

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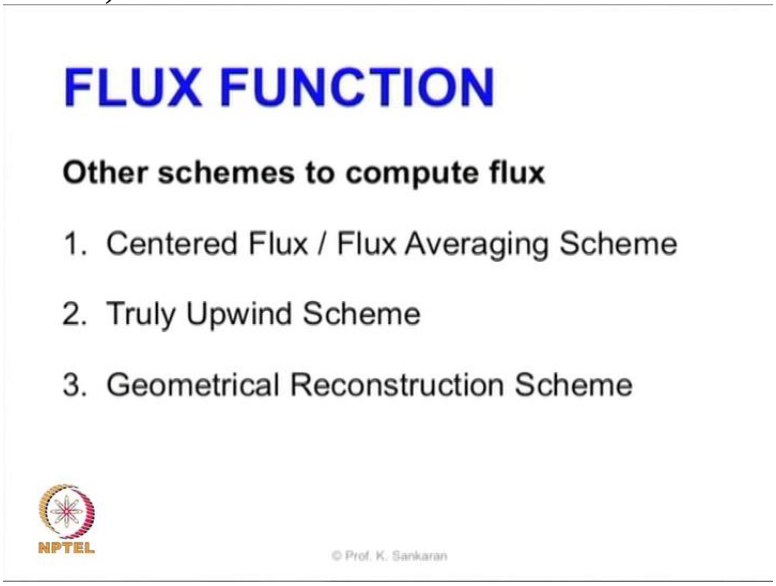

So we are going to discuss now how to compute fluxes using those schemes in this module.

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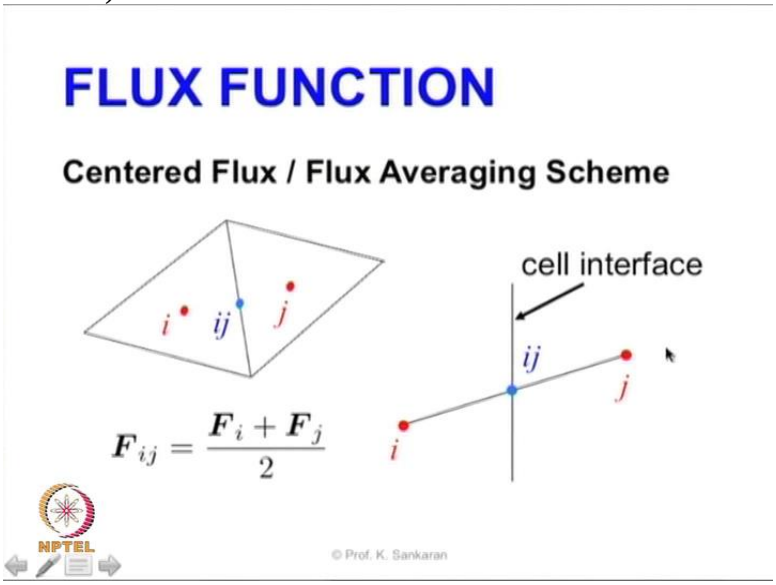

So the first one we were looking to is the centred flux scheme. So centred flux scheme is basically taking into account very simple averaging process so let us say we have two elements which are there to interface, so this interface is called as ij like here. And there are two triangles neighbouring to that.

The field that we are going to compute here at the interface is nothing but average of those two fluxes. So whatever value that we are talking about.

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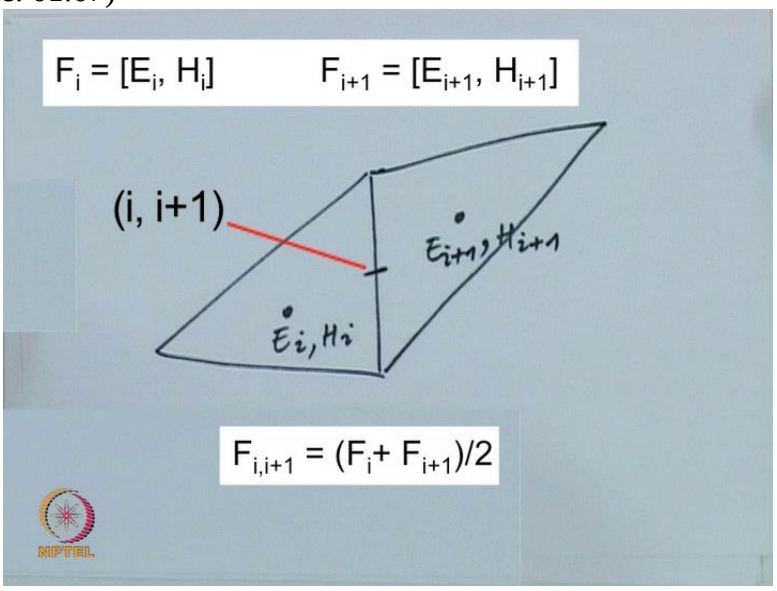

so we might have E field given at the centre. We say E i, H i and here we have E i plus 1, H i plus 1, so we are going to say the field at this point will be the flux that is basically average of these two. So if we say flux on this left side is F i and the flux on the right side is i plus 1, so we are taking the average. This particular schemes should be used with care. I will explain you what is the potential problem by doing this it works in most cases but we have to take it into account how we are going to do the time stepping. When we discuss about the time stepping I will say when we can safely use this scheme for most of the problem. So that is a pretty easy approximation.

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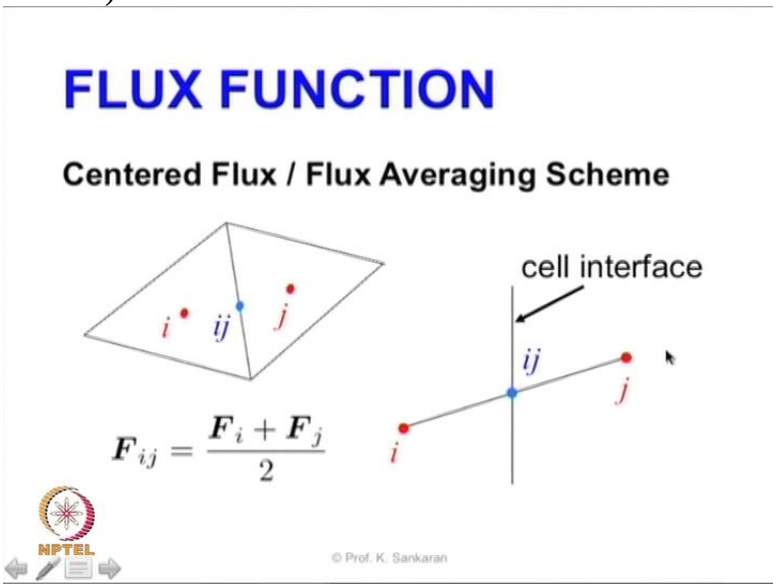

So that is a simple centred flux scheme.

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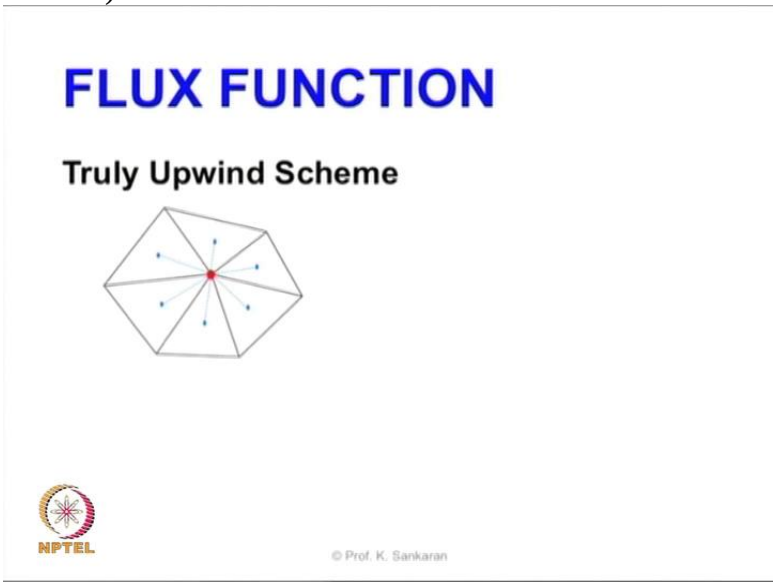

So the second one is truly Upwind scheme let us say we have node that we are interested in. And this node let us say we call the node i. And what is happening now is we are interested in knowing what are the ways in which this node is connected to other triangles.

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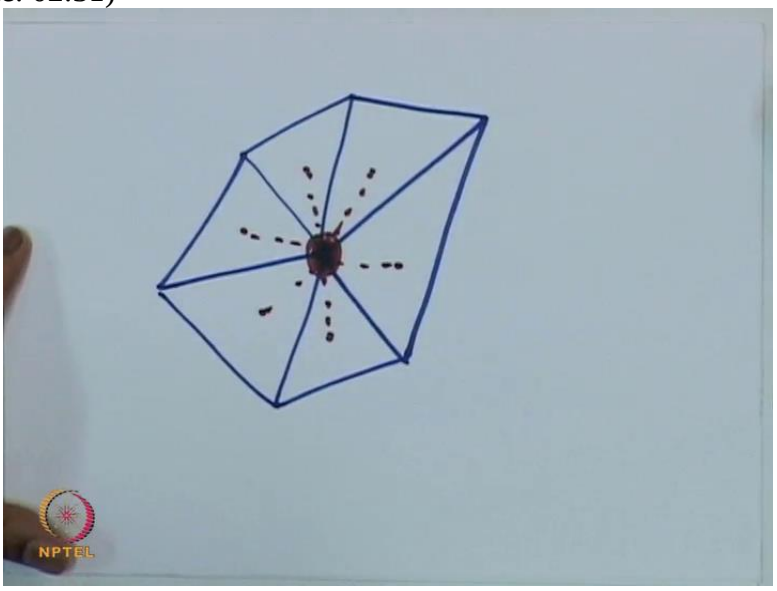

So we might have triangles. So we are interested in the node let us say this node. So what we are going to get is this node is connected to the neighbouring bary centres in some way. So we can be very specific about two triangles and then their dimensions.

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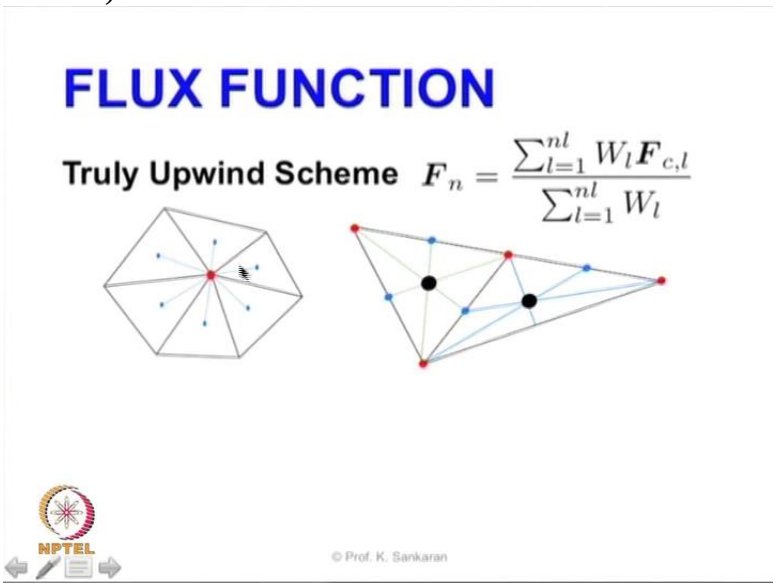

So as you can see in the slide we have two triangles we just take node values which are the nodal values. And we are going to compute the value of the field in a point as the summation of all the neighbouring triangles weightings. So whatever is here is nothing but summation of all the neighbouring triangles and their respective weights.

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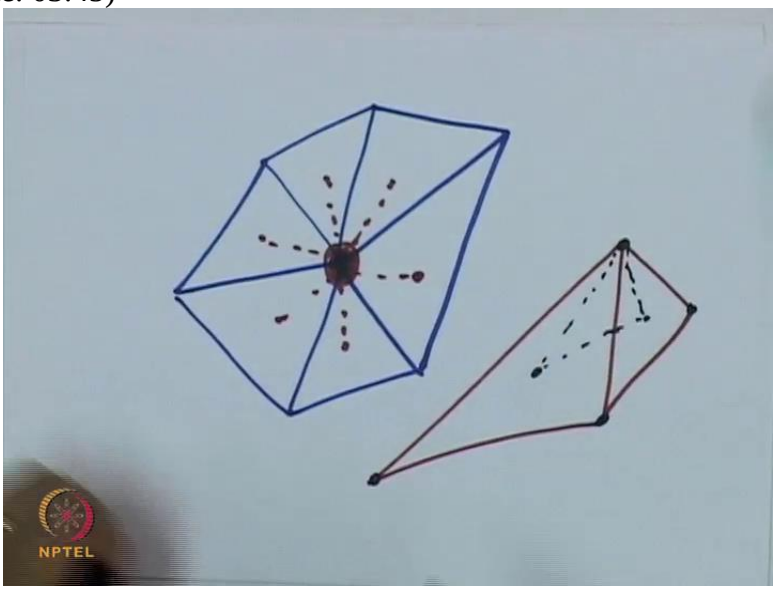

So the weights will be assigned such that if a node is very close to a Bary centre this triangle will have more weight compared to an another one. So let me extrapolate this. Let us say you have two triangles like this and you are computing the value of the flux here and there are two bary centres here and we are computing also the nodal values here. So when you take a node this bary centre will be very far compared to this bary centre. So when you are computing the value of the field. The node the field function will be dependent on the distance of the bary centres away from that particular node. So the weight for this triangle will be more compared to the weight of this triangle.

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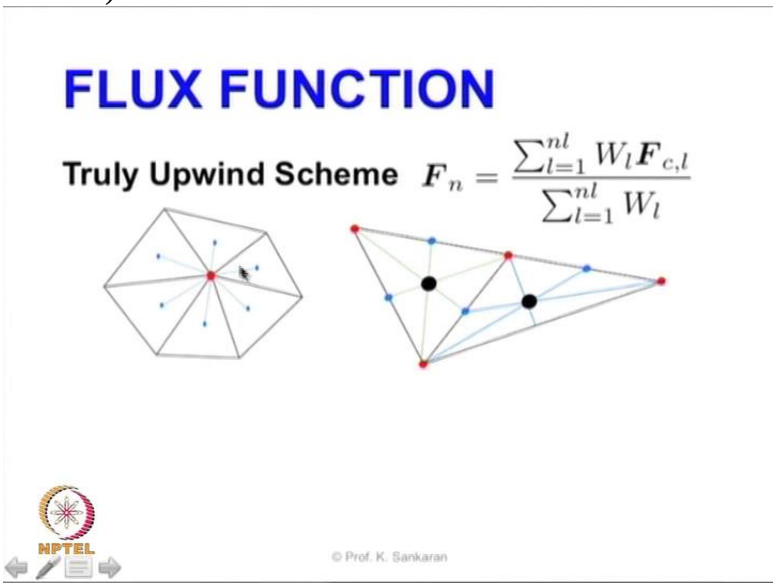

So that is what you see here in this equation. So this equation is saying there is a weighting of the field.

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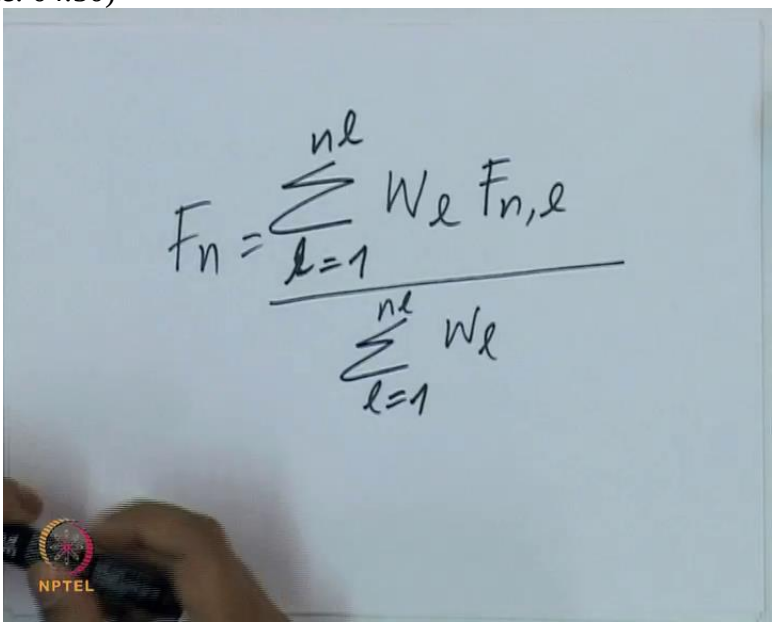

So we can write the equation in a much more simple form. So F n is equal to sigma l goes to 1 to nl and then we are talking about W l F n,l divided by the sum of all weighting. So the top part will be weighted sum as I explained to you, so whatever node is closer will give more wieghtage and then it will get added up. And then finally we are dividing it by the sum of all weights. This thing is a little bit more heavy as you see we are kind of moving away from the bary centric approach. I told you the flux values are computed at the interface the values of the E field and H field are located in the Bary centres.

By going to this Truly Upwind scheme we are actually trying to compute the values at the nodes in a way it is slightly similar to finite element method which has its pros and cons. You have to do more computation, doing more computation you might get better accuracy. And also you can play around with the weighting function itself. I have never used this particular scheme for most of the practical problems because simply the computation is too heavy. I do not want to have too much heavy computation for just the fluxes. So in that sense I just wanted to give you an idea about the Truly upwind scheme but we never use it for most of the problems.

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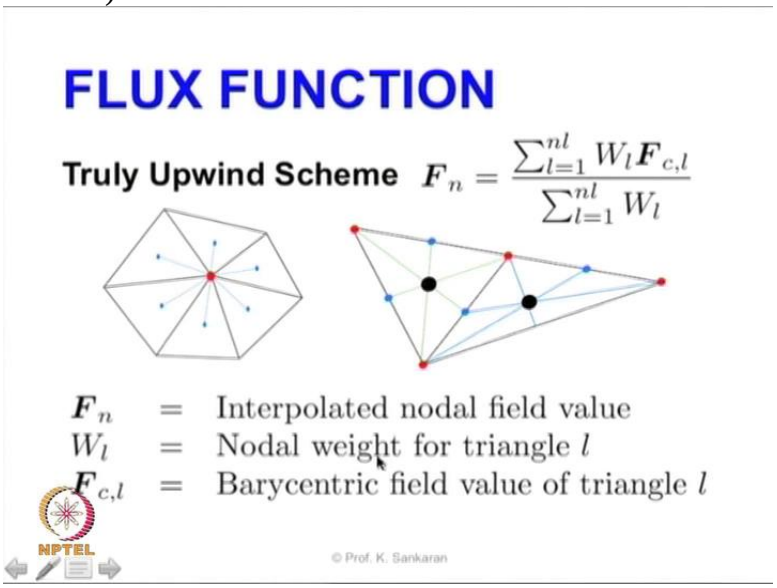

The idea is as I explained F n will be the Interpolated nodal field, the nodal weights of each of the triangles the bary centric field values of those triangles.

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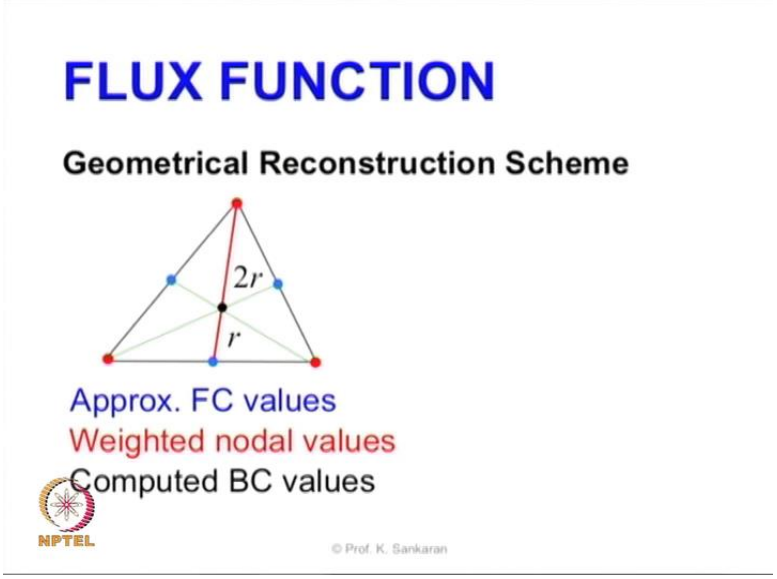

So the next one is called as Geometrical Reconstruction scheme as the name says instead of looking at a particular triangles neighbours I look into a control volume itself. So I can compute the value of the flux based on the geometry of each of the control volumes. So the geometry tells me approximately what will be this r and 2r, they are basically you know they are coming from a vertex to a face centres. So you can compute what this r and 2r are for the individual triangles. And based on that you can approximate the value of the face centres. Because you know this distance, you know this distance and you know this distance. So basically you can extrapolate the values from the centre to here.

So that is a very simple approach but it has also certain computational restrictions. We are increasing our computation test cells. So each of the cells we have to do it. But one way is you can do this already in your pre-processing. So you do not need to re run this every now and then because you already know based on the domain you are having these values already fixed.

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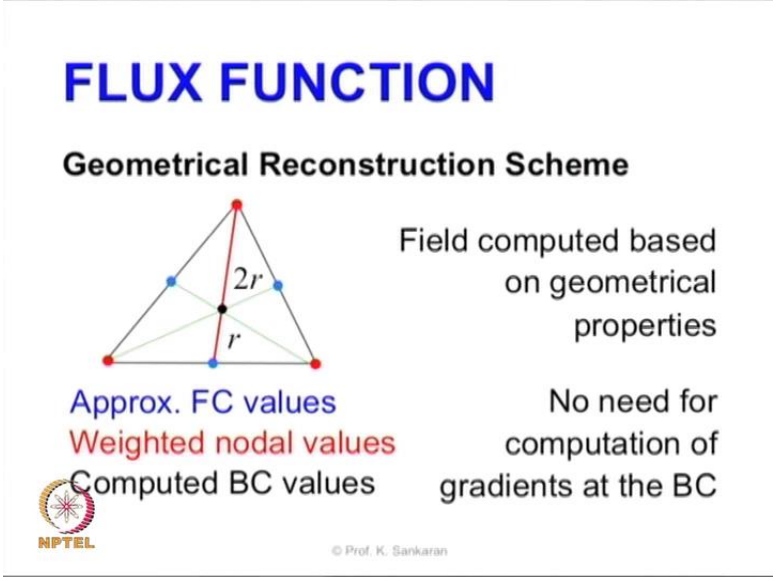

So field computation is based on a geometrical properties no need to compute gradients at the bary centres. Because in the case of the MUSCL scheme as you remember we are going to compute the value of the gradient. The gradient is the value between the face centre and the bary centre. This we do not do it here so in a way there is a improvement but still its not a bigger improvement, but this scheme is much better than the truly upwind scheme where you do not do too much computation.

So having discussed all the various approaches of computing fluxes the most important thing is to look into the time discretisation itself.

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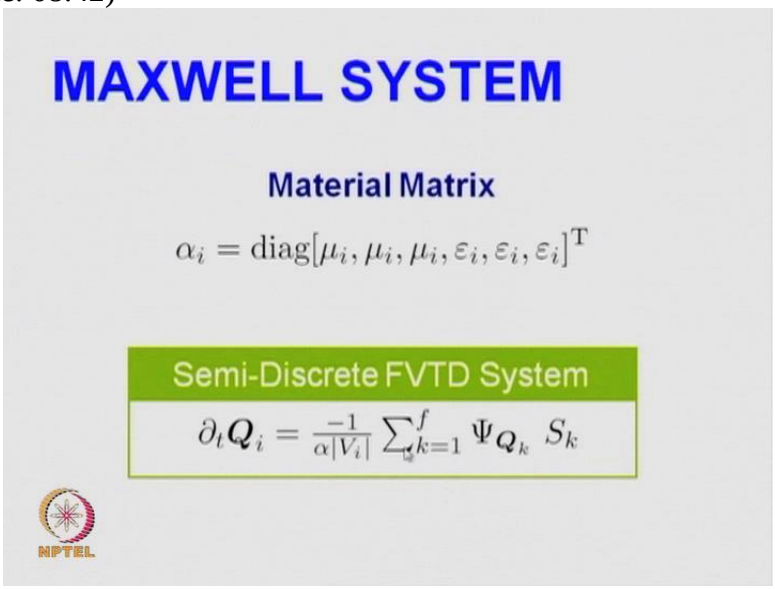

So far we have not talked about time discretisation which is one of the main chunk of the computation itself because only doing spatial discretisation does not help we have to also look into the time discretisation.

And we want a scheme that is simple enough but at the same time also stable. So as you know some of the schemes like Euler method which we discussed in the Finite difference method lectures. It is not that stable becuase it creates unstable solutions and its mostly considered to be unstable. So the kind of scheme what we are going to use now in the case of Finite volume Time domain is a method called as Lax Wendroff scheme. Let us look at it in detail.

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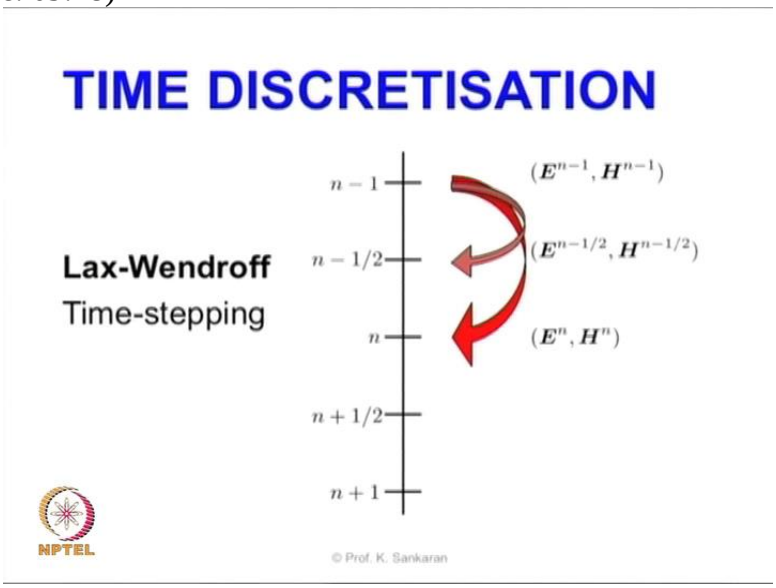

So as you can see in the slide we are interested in time stepping like in the case of leaf frogging method we are interested also in the half steps. But there is a difference here the

difference is our E and H field are located in the same point, which is very different to finite difference method. If you remember in finite difference method in our earlier lectures E field is in one point and the H field is in another point. So if you talk about E field in the n minus 1 and n plus 1 H field are located in the half step. But in the case of Finite volume remember we said it is going to be collocated. When it is collocated we can compute the half time stepping for both E and H at the same time. And we will use this value to improve our guess of what is going to happen in the next step.

And we will move forward; so let me explain this in this slide here.

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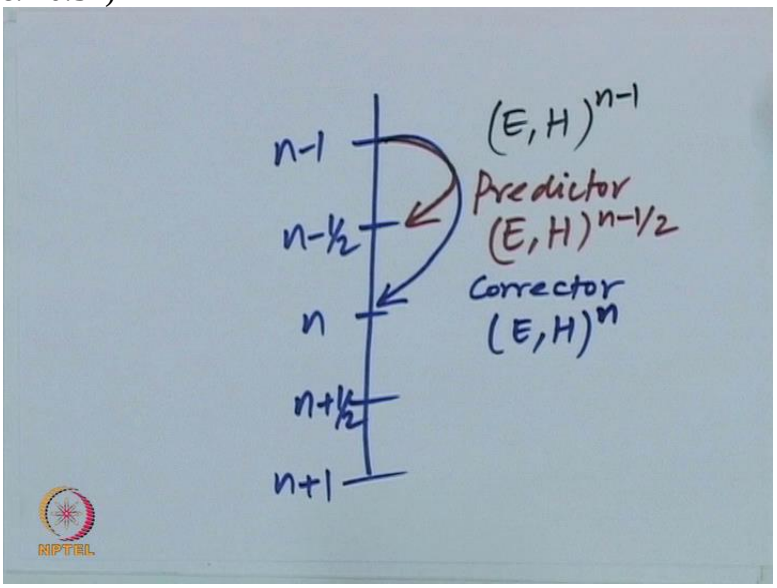

So the first step what we are doing, let us say we are going from so this is n minus 1 and n minus half, n, n plus half, and n plus 1. So the first thing what we are doing is called as the predictor, I am predicting the values of what is going to be my E and H in the next step. So I have the values of E and H here and they are in the time step n minus 1. So I am predicting the value of E and H in the n minus half. And I am using that value to improve my prediction so this is called as the corrective step. So in the corrective step I am using instead of n minus 1 I am using n minus half. So I am correcting my prediction.

Corrector so I get my value at n. So this is basically a kind of a clever way to say I do not just use my value n minus 1 to compute at n then it will be a simple Euler method. I am doing an half time stepping. In a way you can say this method is kind of a Euler method with small time steps. So instead of taking a big time step I take a small time step, I predicted I used this value to compute the value in the front. So this will keep on going after here what I will do is I will use this value to predict the value here and I will improve that value to the next step. So it keeps going

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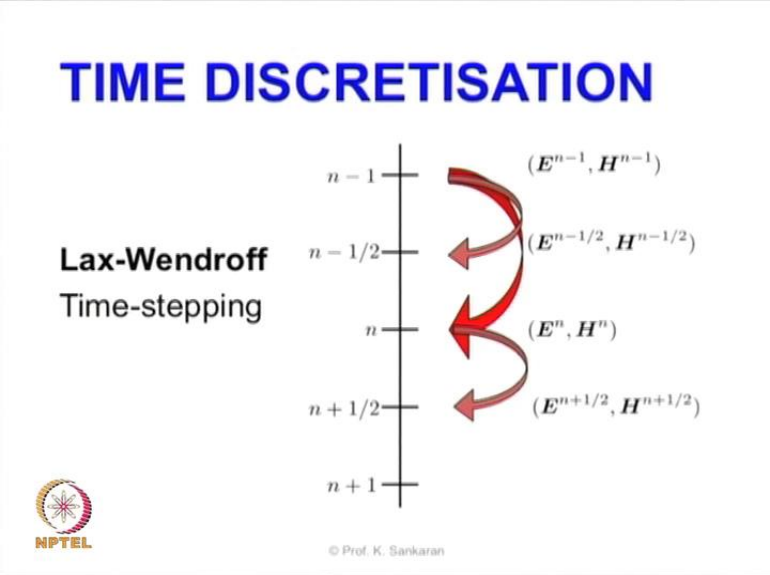

So that is what we have described in the slide.

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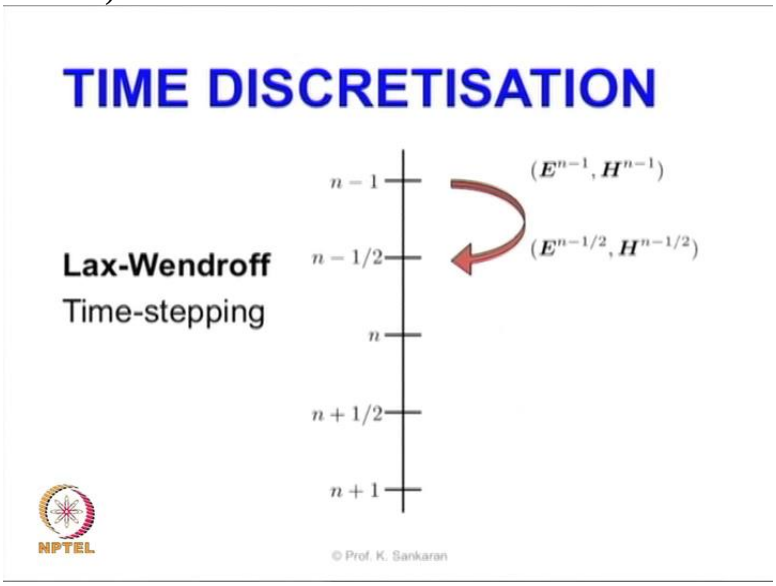

So If I go back

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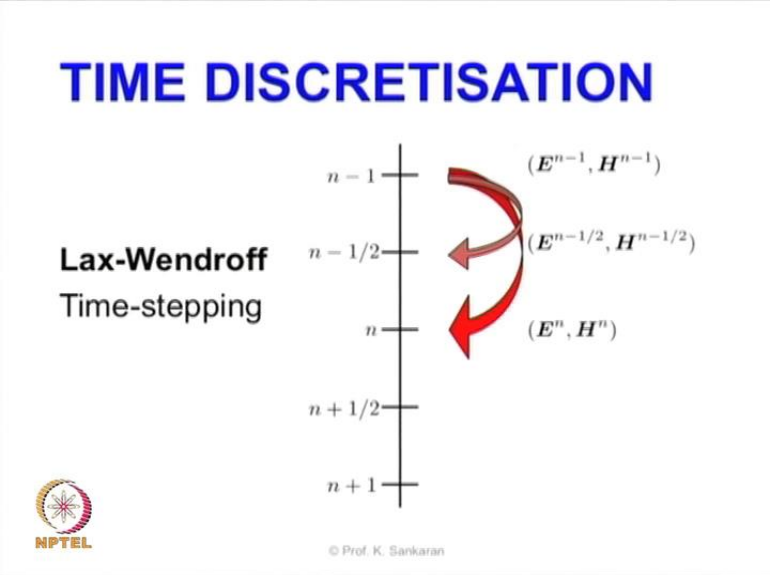

so I used this value to compute it here .

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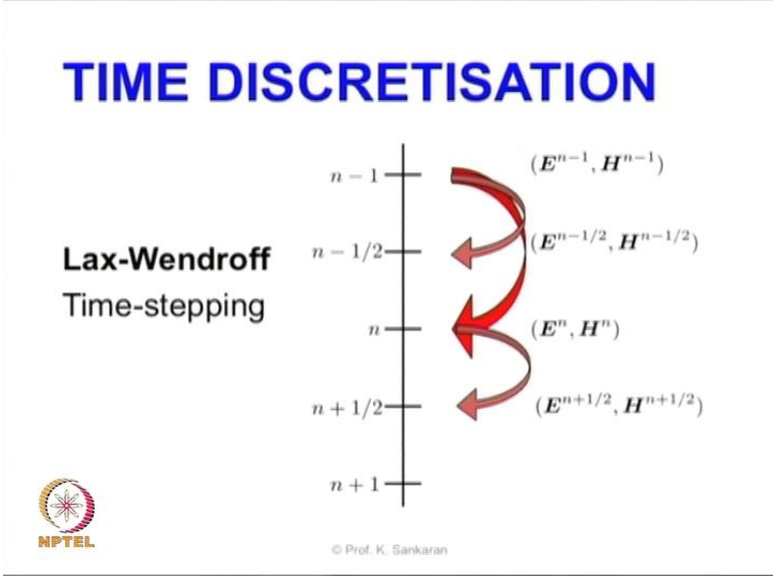

I jumped I compute here

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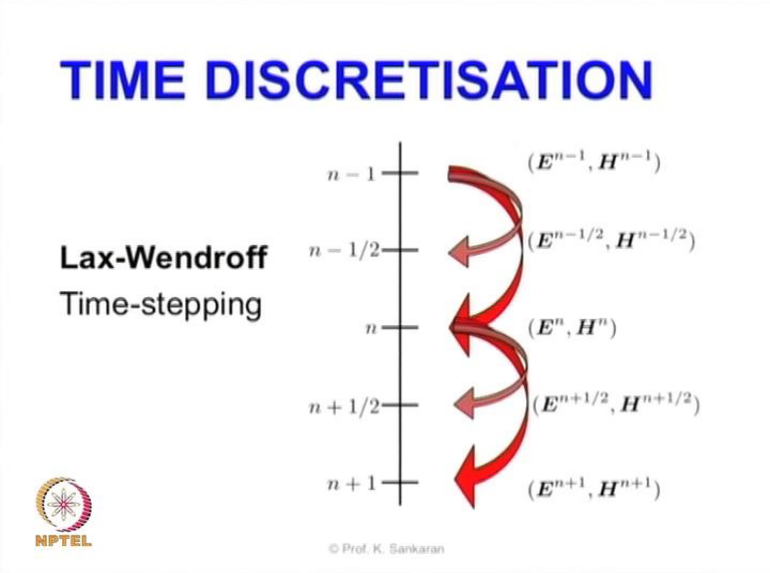

And then I jumped

So depending on which literature you are following people call it Lax- Wendroff scheme or Predictor Corrector method.

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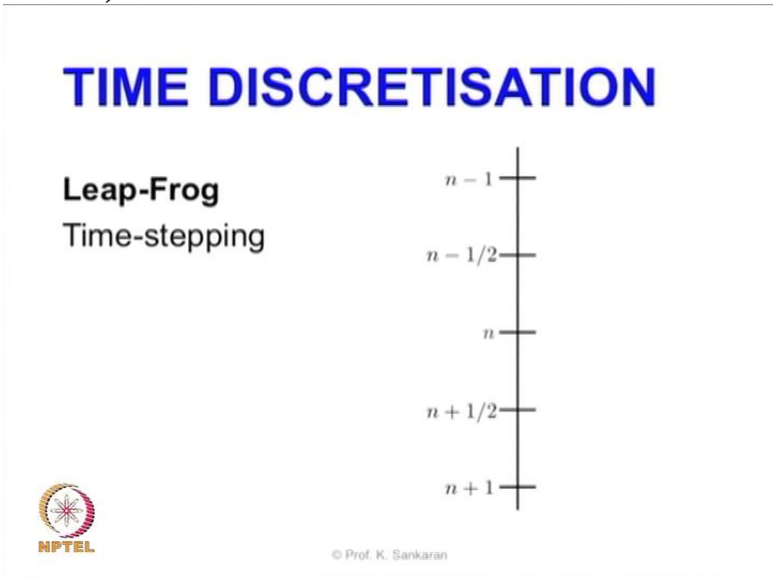

I just wanted to repeat the Leap frogging time stepping which we normally use in the finite difference method for a particular reason. Leap frogging method can also be used here but we can only use it in a flux computation method. Remember I told you while we are discussing theflux averaging scheme you have to pay attention to how you are computing the flux averaging scheme and when to apply that flux averaging scheme.

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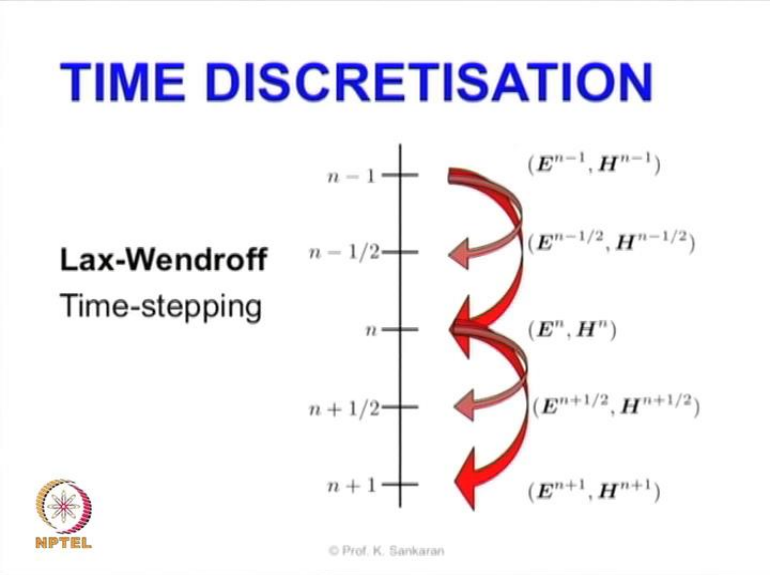

So flux averaging scheme can be used in the case of leap frog method. So where you are basically computing the values of E n minus 1 and H half and then you are jumping E n and then H n plus half and then E n plus 1.

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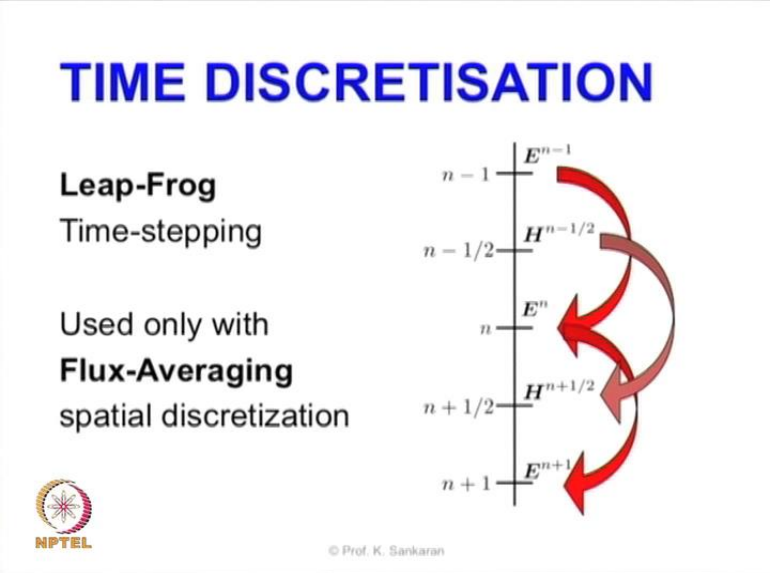

So this is the standard approach of doing the leap frogging method you can use this also in the case of finite Volume time domain method instead of just using only in the finite difference method you ca also use it in the finite volume method but then you have to compute your fluxes differently. So in this case in some of the problems we tried we also used this method for doing the time stepping. Although this is not the main way to go forward. So with this we have covered pretty much the various aspects of time stepping we have looked into various aspects of how we are computing the fluxes and also the basic way of structuring a problem and getting the finite volume formulation.

But that is not everything right? So If you wanted to run a problem you need inevitably certain boundary condtions. Again when we talk about boundary trucations there are different ways to go forward with it. So we will discuss in modules coming next what are the different accurate truncation possibilities that we have for finite volume time domain problems.
So we will look at it in the next module.