Computational Electromagnetics and Applications
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Lecture 06/Exercise 03
Finite Difference Methods –1

The Example which we are going to look into now is umm heat diffusion equation. So we will try to use the finite differencing schemes which we have learnt. Like the forward differencing scheme or the Central Differencing scheme for this particular problem. Before going into the problem itself let us look at the equation what we have at our hand

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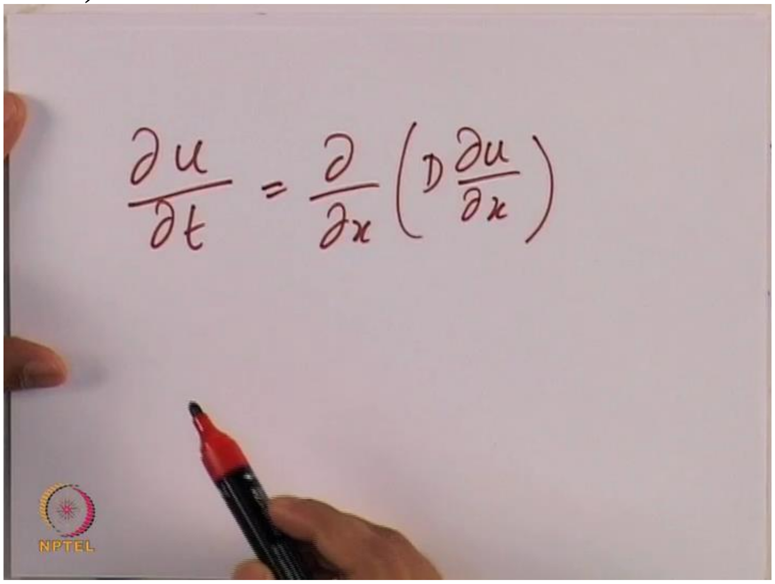

So the heat diffusion equation is going to be of the form (du by dt) the partial differentiation with respect to time is equal to (d by dx) like we had in the case of the advection equation, partial differentiation with respect to x of some quantity which we call (D du by dx). As you can see this is second order differentiation in space, first order differentiation in time.

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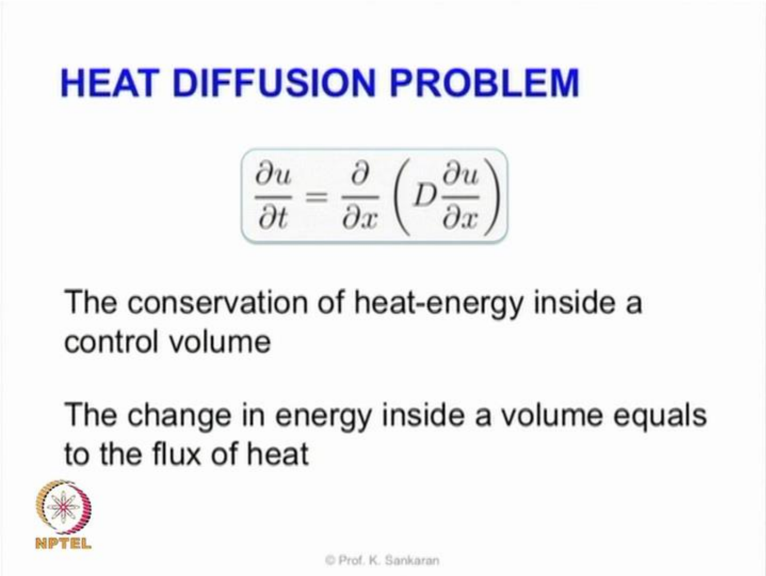

So that is what we have got in this slide. And this is the equation that talks about the conservation of heat energy inside a control volume. And the change in the energy inside the volume is equal to the flux of the heat that is going and coming out of the control volume.

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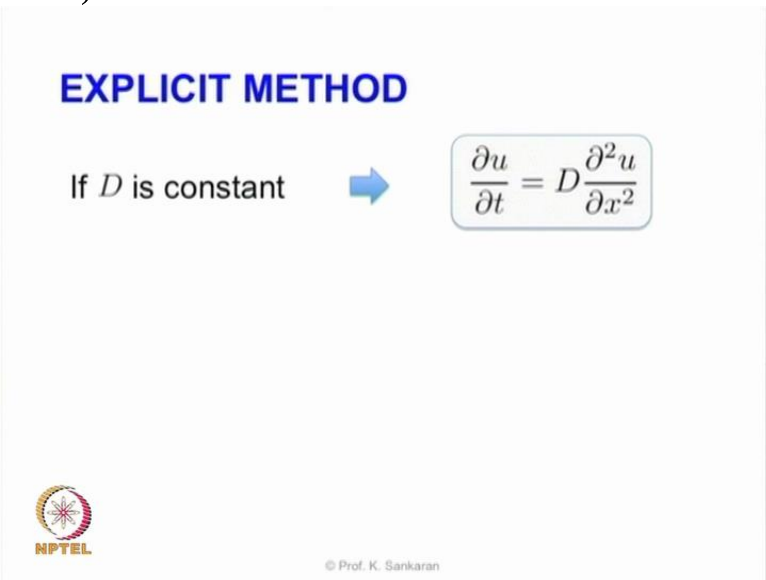

So if we set d equal to constant in this particular equation. We get the second order in space partial differential equation of the form given here.

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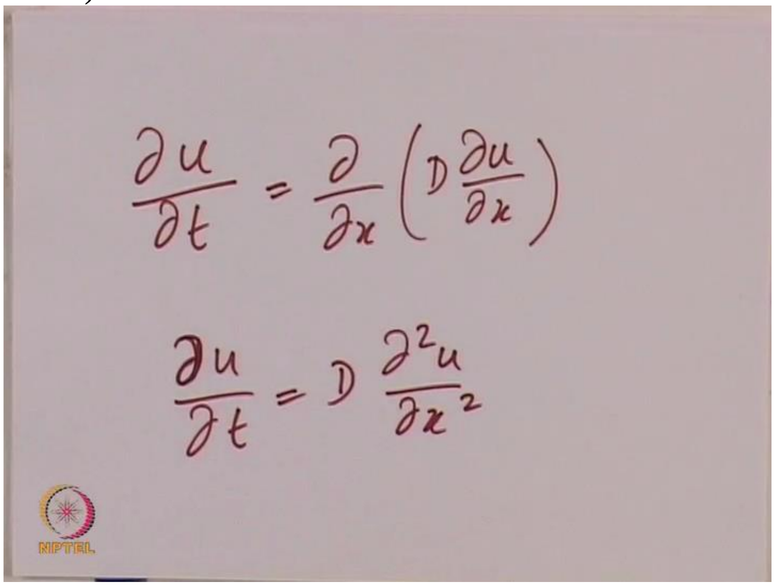

So we will have du by dt is equal to D times d square u by d x square. This is the second order in space and first order in time.

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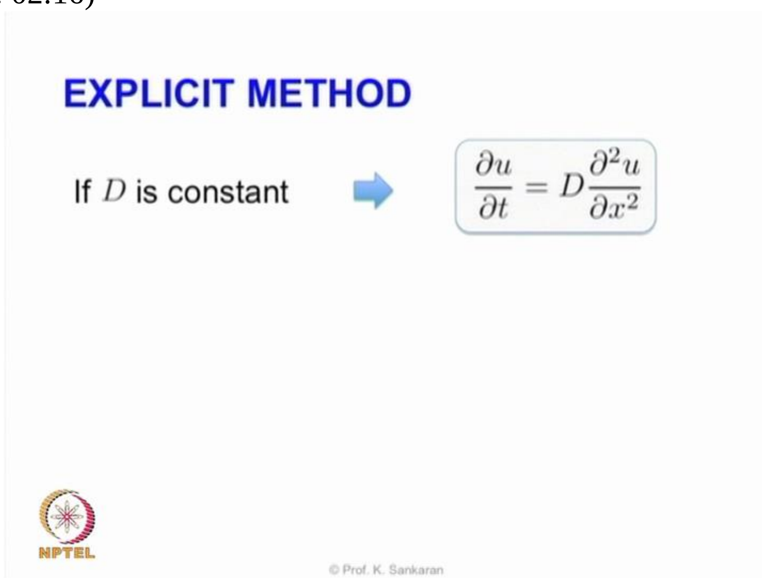

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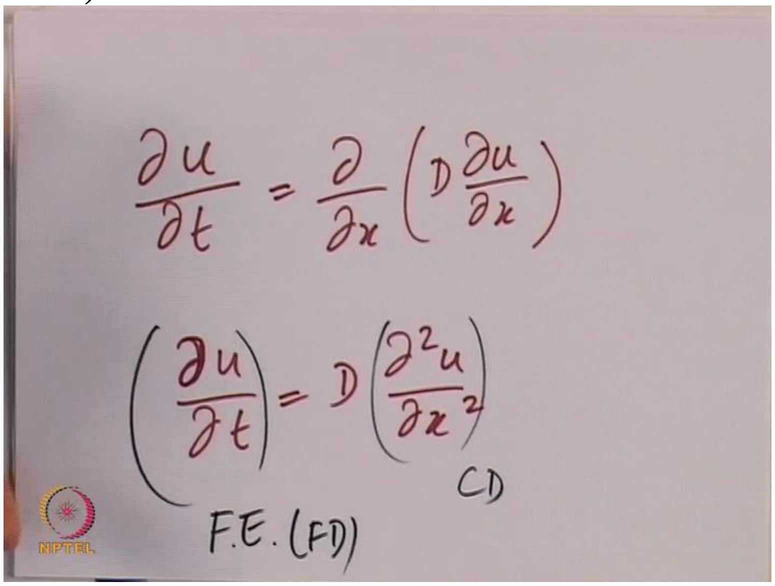

So what we are going to do now is we are going to apply for this particular term forward Euler method that is the Forward Differencing scheme. And for this particular term we are going to use Central Differencing scheme.

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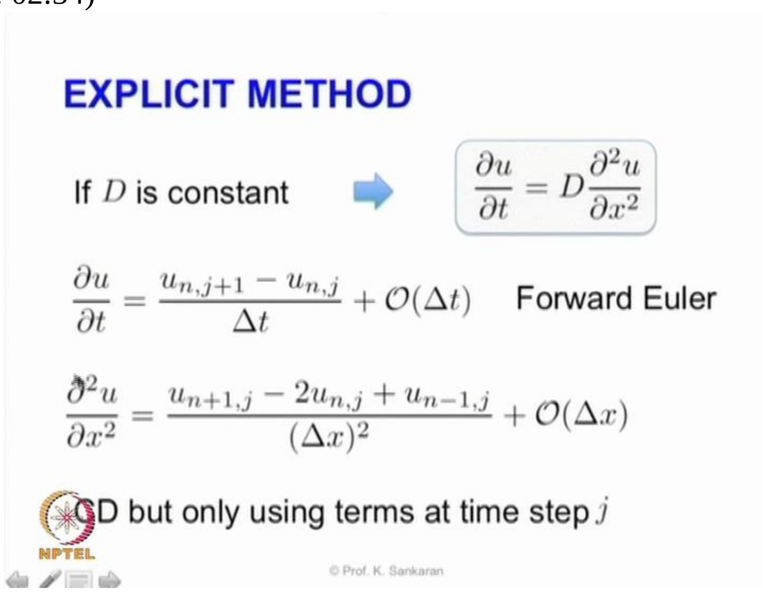

So that is what we are going to see in the next slide. So we have approximated the partial differential do u by do t is equal to (U n, j plus 1) minus (U n, j) divided by delta t plus certain order of truncation which comes from the Taylor series expansion. And we will do the central differencing as i said for the second order partial differential with respect to x as follows:

Again you will have certain order of truncation with respect to x. So we are using CD only using terms at time step j that is more important to know. And if we do that we can approximately equate the partial differential equation to the finite difference equation as follows.

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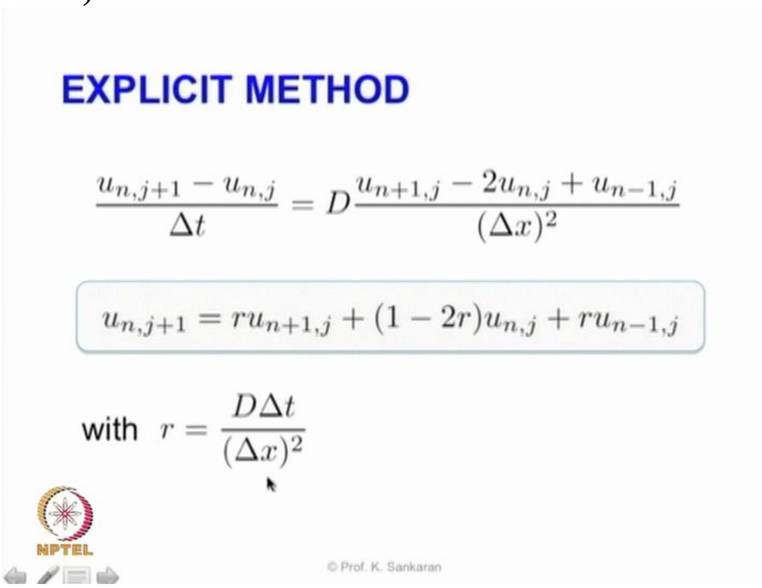

So what we have got now is (U n,j plus 1) minus (U n,j) divided by delta t that is equal to D (U n plus 1,j) minus (2 Un,j) plus (U n minus 1,j) divided by del x square. And as you can see this is the order partial differential with respect to x which we have used the Central Differencing Scheme, and here we have done the forward differencing scheme.

If we rearrange the terms in such a manner that we keep only the (U of n,j plus 1) on the left hand side and bring all other terms on the right hand side and substitute the value r for the constant D delta t divided by del x square, we get this form of an equation. We have seen a similar set of equation while we did the earlier analysis on the finite differencing scheme and this is the same set of equation and r is equal to D delta t divided by delta x square. Again this is a forward in time center in space.

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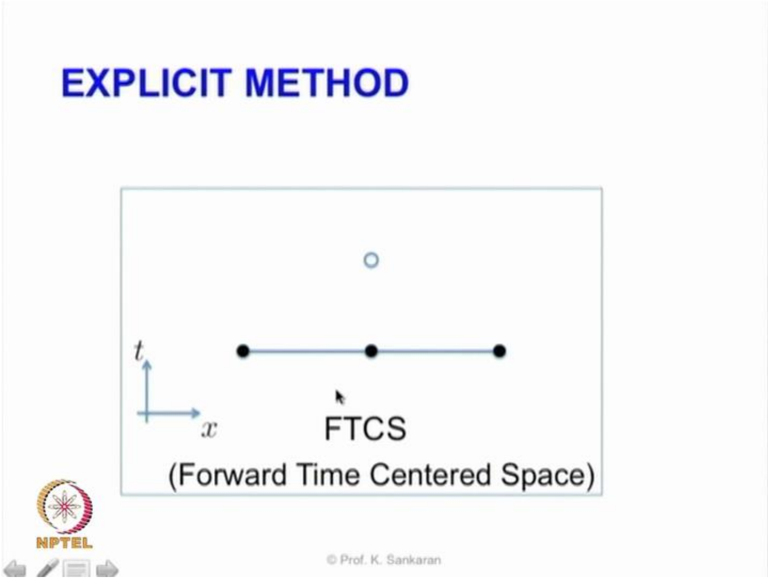

And the stencil for this equation is going to look of this sort. Like in the case of Advection equation, this particular equation is very difficult to model for any real time problem particularly for the reason which we mentioned before, that the delta t is going to be very very very small. So we have to go very very slow in time stepping in order to make the scheme stable. But this is not interesting because you have to wait for a such a long time before anything useful happens.

Assume that you are trying to simulate the interaction of a wave that is going and hitting a scatterer for this propagation you are going to wait for such a long time before it goes and hits the scatterer. So all these things are big bottle neck in simulating any practical problem. So we do not use such a scheme in practical simulations. We will use some other kind of schemes like the staggered scheme or lax scheme predictive corrected method so on and so forth which we will see towards the entire course while discussing other methods as well.

There are different time stepping schemes that we will be using to make any numerical method practically usable.

Yet another method which will be of interest is the implicit method. We have not discussed about it yet, but i will show some examples while we are discussing for this particular heat fusion equation. But before going further let us look into the Matlab simulation. How does the Matlab simulation can be done for this particular heat diffusion equation using the Forward in time Center in space. That is going to be our focus. So let us go into the Matlab simulation now.

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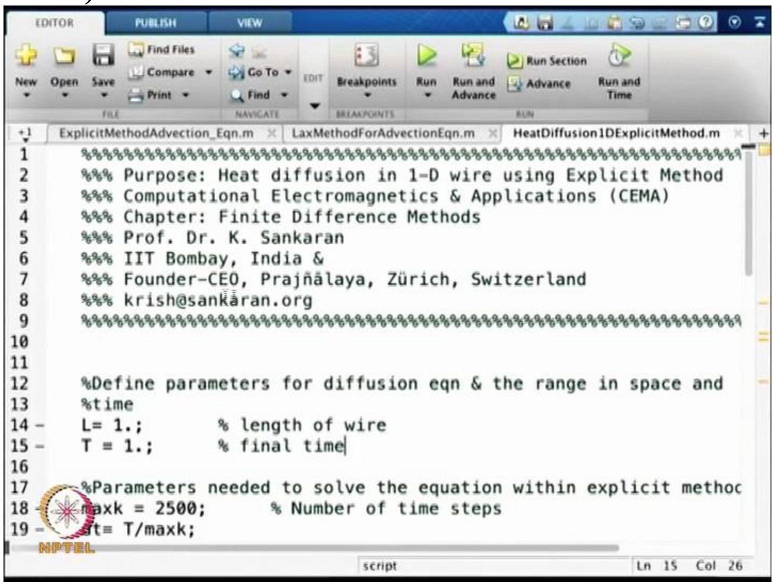

This is the heat diffusion equation which we were talking about. So we are going to model this using Finite differencing method using Forward in time and Centered in space. So before going into the simulation itself let us look into the parameters for the diffusion equation and also the range in space and time.

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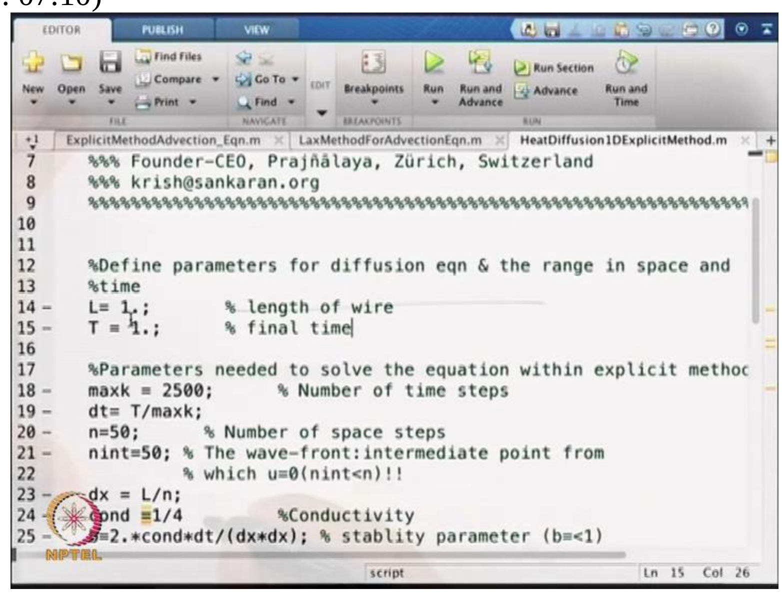

So what we have is we are setting the length of a wire that we are interested in modeling the diffusion of heat on a piece of wire.

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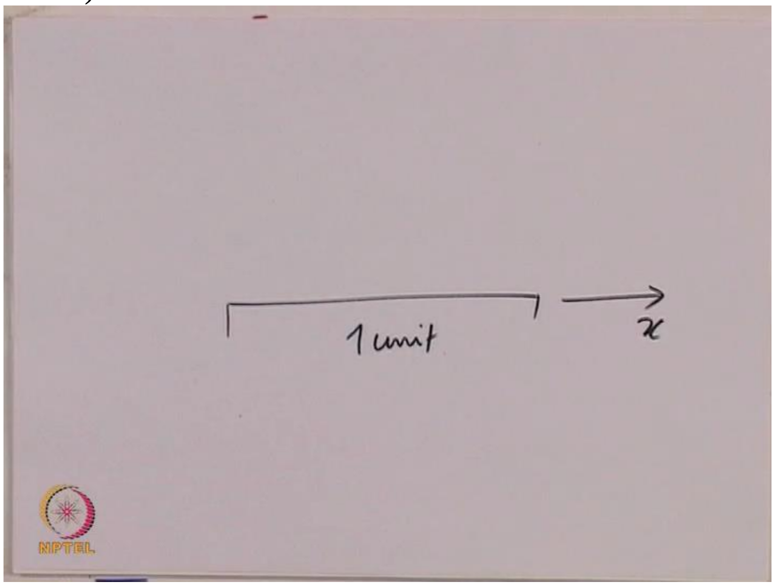

So what we set is this particular length is going to be equal to 1 unit, so this is the one dimensional x axis, so we are interested in what is happening in the heat diffusion in this one dimensional wire.

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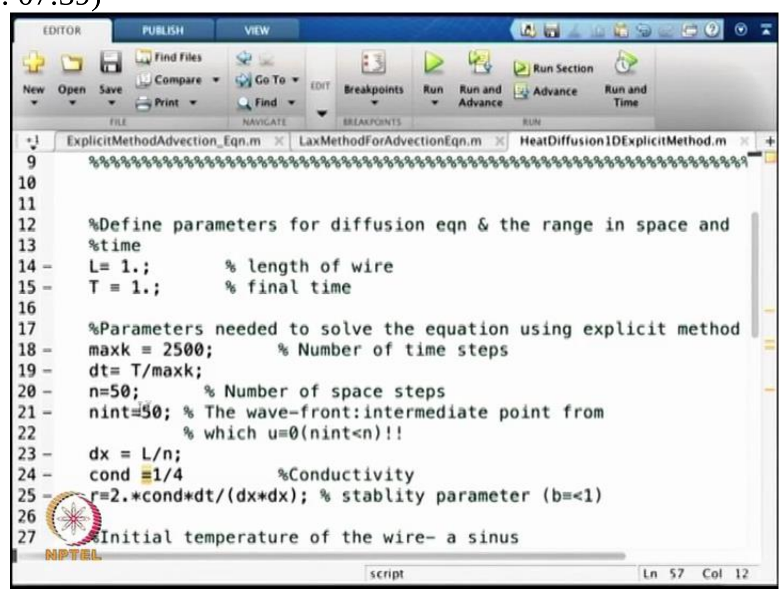

And also we are setting the final time the maximum time that we are going to simulate is also one unit. So the parameters we need to solve is heat diffusion equation using explicit method or discussed in this particular part of the code. So we are setting the maximum time step, as 2500 time step. We are setting the value of delta t as i said this is going to be very very small if you are going to use forward in time centered in space. So that is given by the value t.

The maximum time divided by the maximum time step. So we are going to divide 1 divided by 2500 as you can see this is very small. And also the number of space steps is going to be 50.

So again the space step is not the (()) (08:32) is going to create the issue here it is the time step. But again we have to set the space step which we have done here. And based on that we are going to calculate various parameters

So the Parameters that we need to solve the equation using the explicit method are discussed here. So the maximum time step is going to be given by maxk which is going to be 2500. And the delta t is the time stepping as I said this is going to be very very small. So it is going to be 1 divided by 2500. And the maximum space step is going to be 50. This is not going to be an issue here; it is the time step that is going to cause the issue. We need to set the maximum time step and maximum special steps which we have done here.

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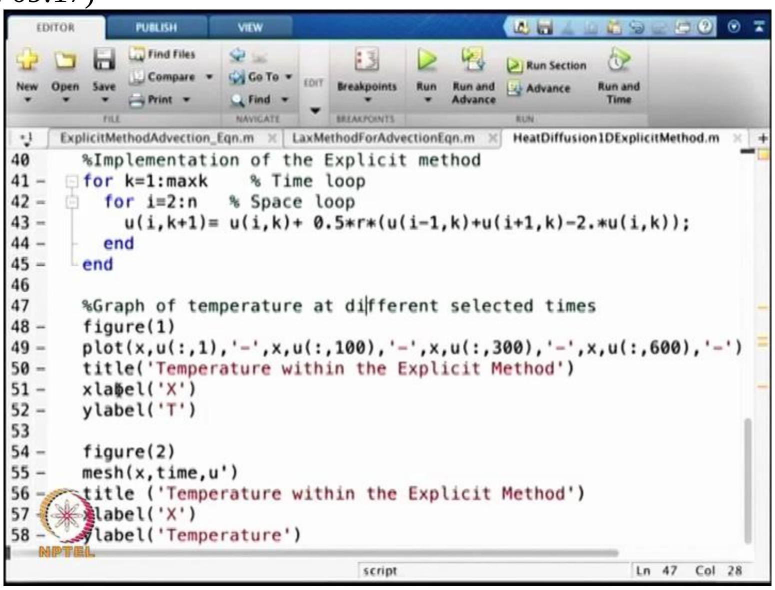

And we see the value of r which is the stability parameter, which is going to be set as r. The stability parameter which we have discussed in the slide before is going to be 2 times the constant d multiplied by delta t multiplied by dx square. So this is what we have here.

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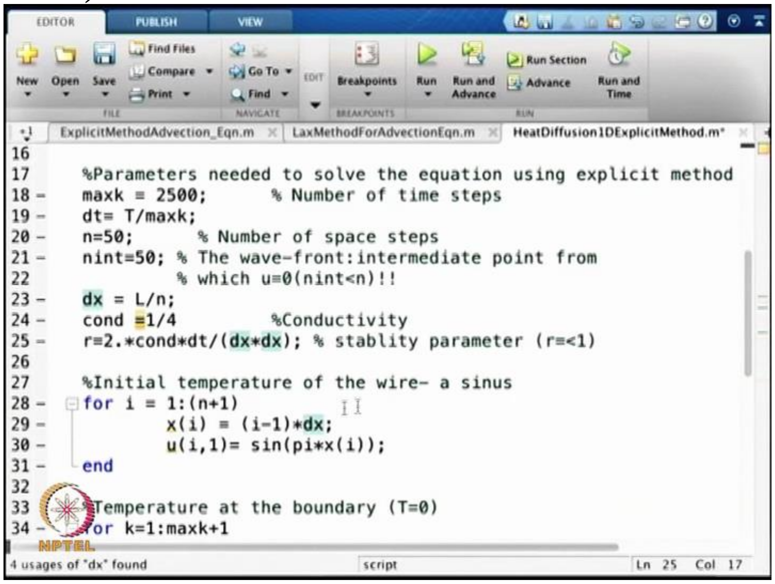

And we are initializing the temperature in the wire. And we are putting the boundary conditions; we are going to implement it using the explicit method. Remember this is nothing but the same equation which we had in the case of the explicit method.

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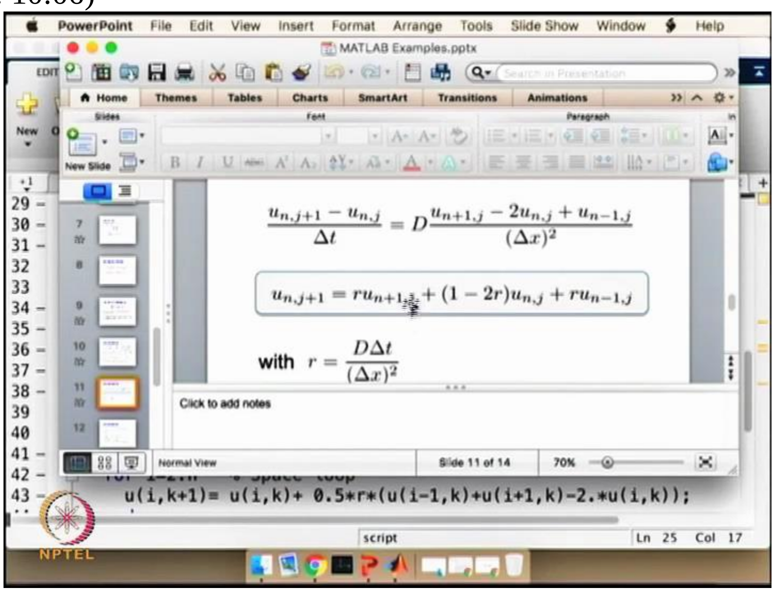

So we will go back in the slides and look at this r condition one more time and also the explicit method algorithm that we have got.So what we see is, so the r value is D multiplied by delta t divided by delta x square.

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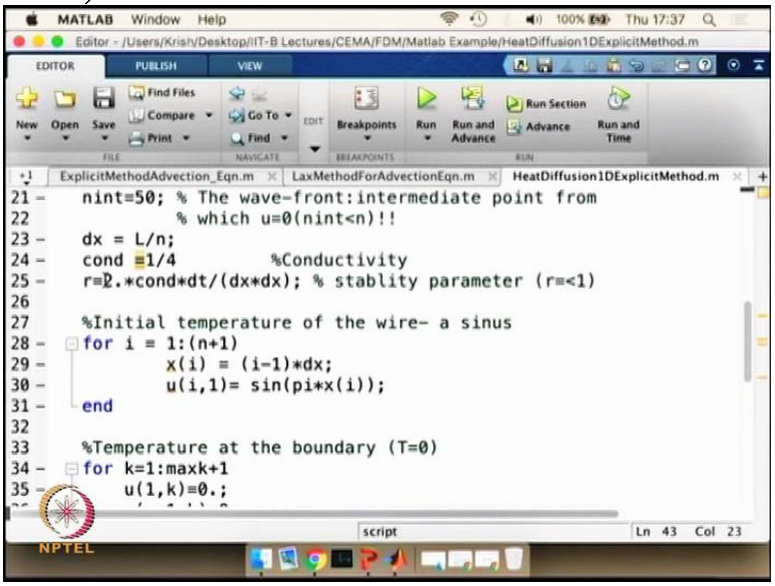

That is what we are having here. And we are using the condition for two times that value for the reason that there will be the term of two that will be on the right hand side.

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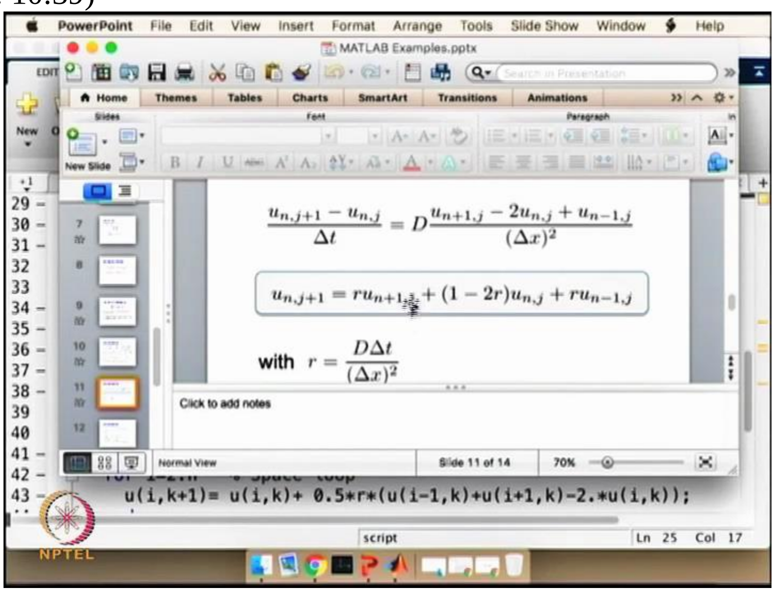

As you can see in this equation. So r multiplied by 2. So this is the value that is going to be the most critical value that is why we are making the value that is 2r not r itself. So let me explain using the stencil one more time.

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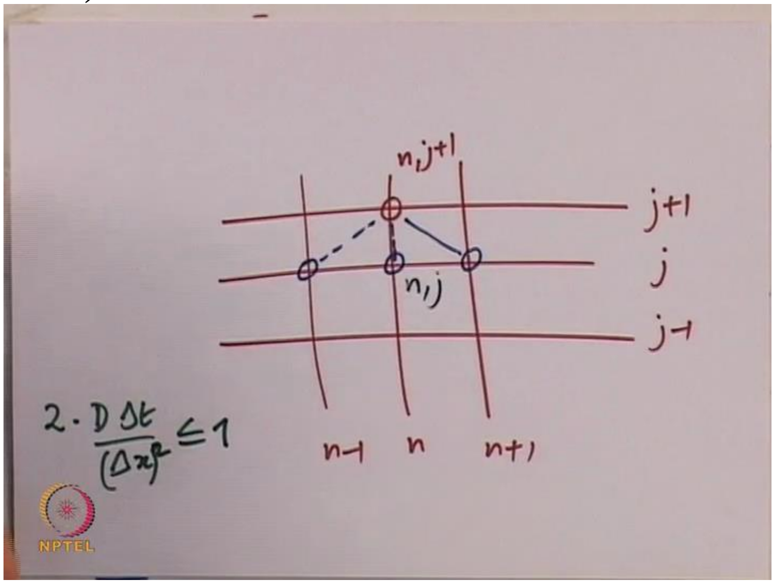

So what we have now is stencils so if you say this is j this is j minus 1 this is j plus 1 and this is n, nplus 1, n minus 1, so we are interested in computing the value at this point which is going to be n,j plus 1. And we are going to use the values that are in n plus 1, j. So n plus 1, j this value, we are going to use the value at n, j and we are going to use the value n minus 1, j. So these are all going to contribute to the value of the solution that we are going to compute here.

Interestingly the time needed for the solution to propagate from this point to this point is going to be shorter compared to the point here here. The time that one solution needs from here to here. So the solution dependence on this point is going to be more critical for the stability rather than the solution propagation from this point or this point. That is why we are going to take the coefficient that we have in the equation for u of n, j. So this n, j.

So we are going to take the influence of the point n, j more significantly than the other point. So that is why we are setting the condition that twice D delta t divided by delta x square should be less than or equal to 1.

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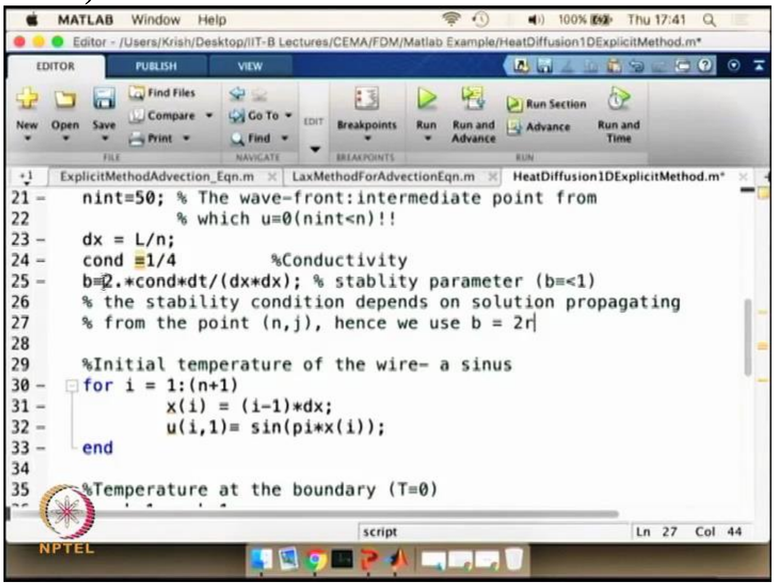

And that is what we have set here for this particular equation. So the reason why we have set it for r as 2 times D multiplied by delta t dx square is for that reason. So may be it might be helpful if we change this one into b and explain this in the comment so that you do not get confused. So we will keep this one as b and explain this in the comment, the stability condition depends on solution propagating from the point (n, j) hence we use b equal to 2 times r. So that is why the reason we have 2 here.

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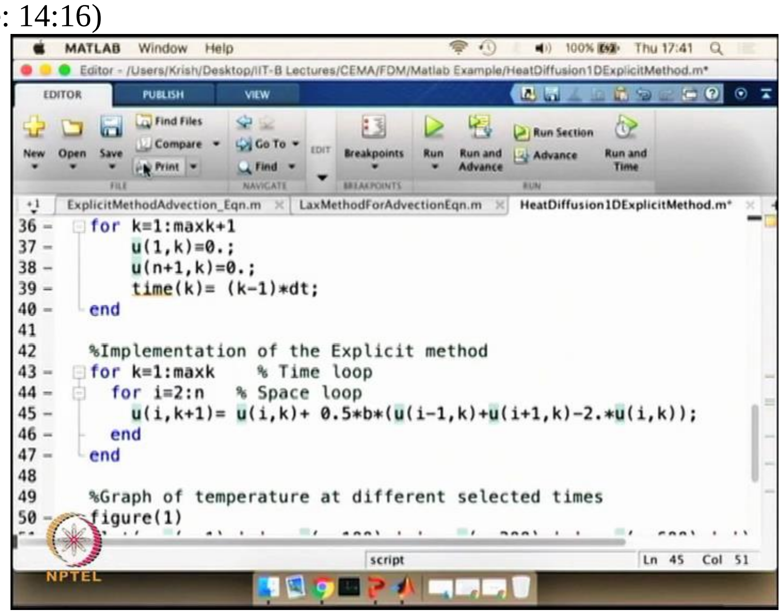

Other than that we can change this back to b here because we have changed on the top, the simulation parameter as b. So this is a stability parameter, so it should be clear now that we are using twice because the stability condition is going to depend, may be we can write it as stability condition depends more on solution propagating from the point (n,j) hence we use b equal to 2rfor the stability parameter.So the code should be self explanatory for you to run it and test it later on.

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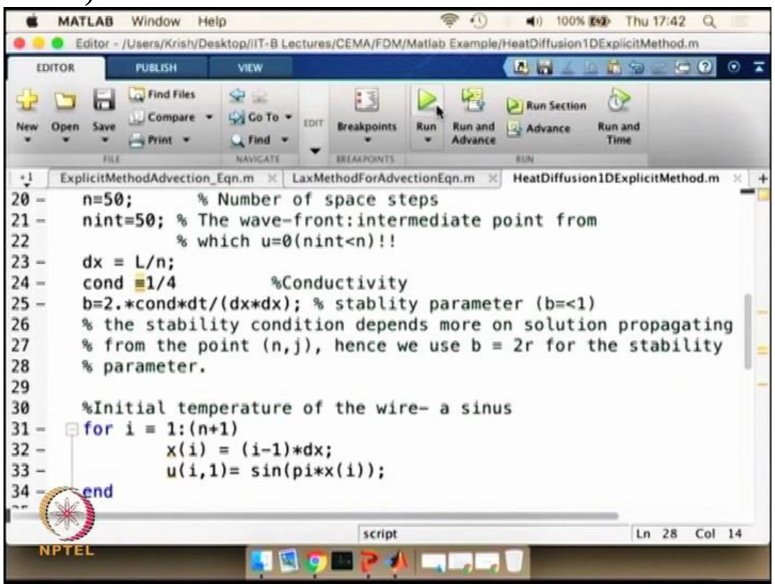

So with this background, so we are going to simulate this particular problem and see what is going to happen. Later on we can manually set the value of b at different limit and see what is happening. For now let us run the code, so we are running the code now and we are going to see what is going to happen.

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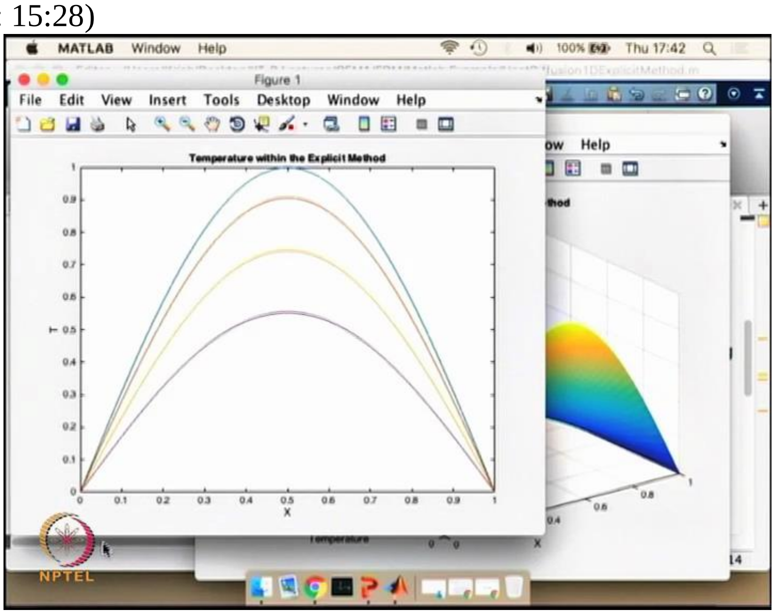

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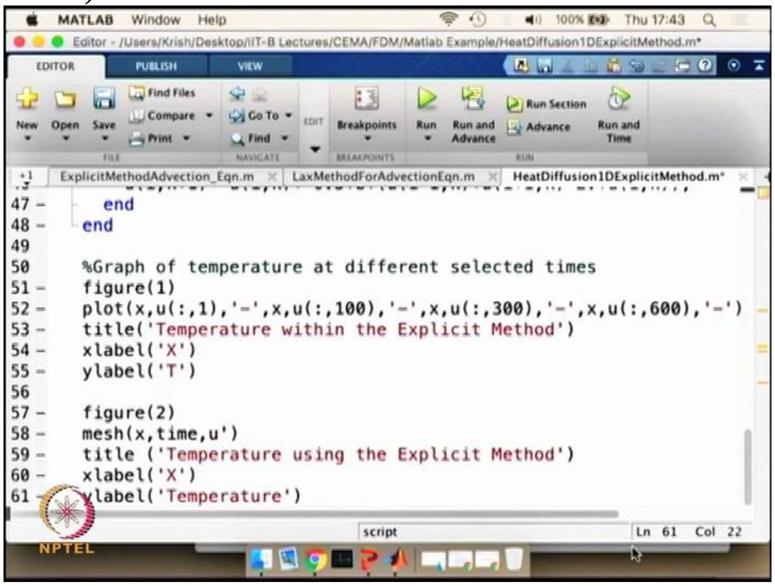

So what we are plotting here is we are plotting the value of the temperature at different selected times. We are plotting at time step 1, time step 100, time step 300, time step 600 so on and so forth using different colors. So what we see is exactly that and then we are also plotting in the 2D mesh temperature within you can write temperature using explicit method and we are seeing the time step and also we are plotting the temperature.

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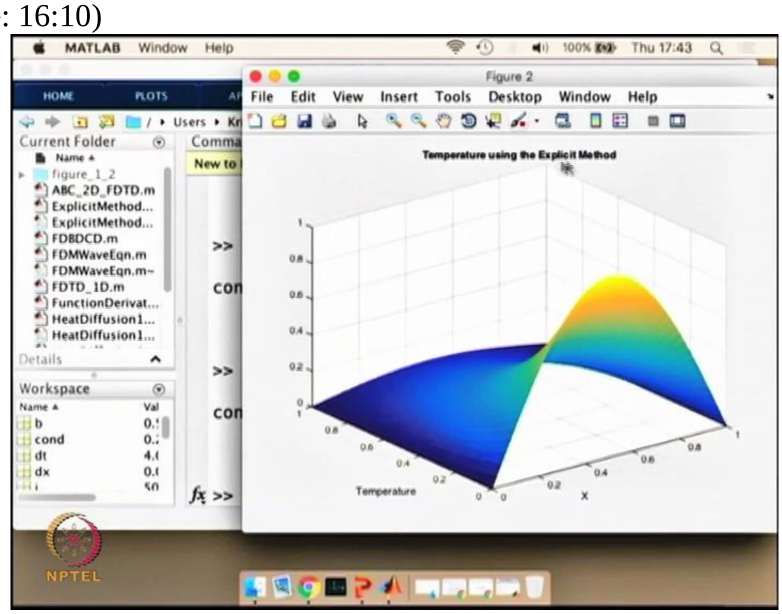

So with that you can interpret the result now. So what we see as result. Let us run it one more time heat diffusion equation so temperature using the explicit method. So what you are seeing are the two graphs, so the first curve is the initial temperature at different times step and then you see that it is changing over a period of time. And also we have to see what is the value of b we are using.

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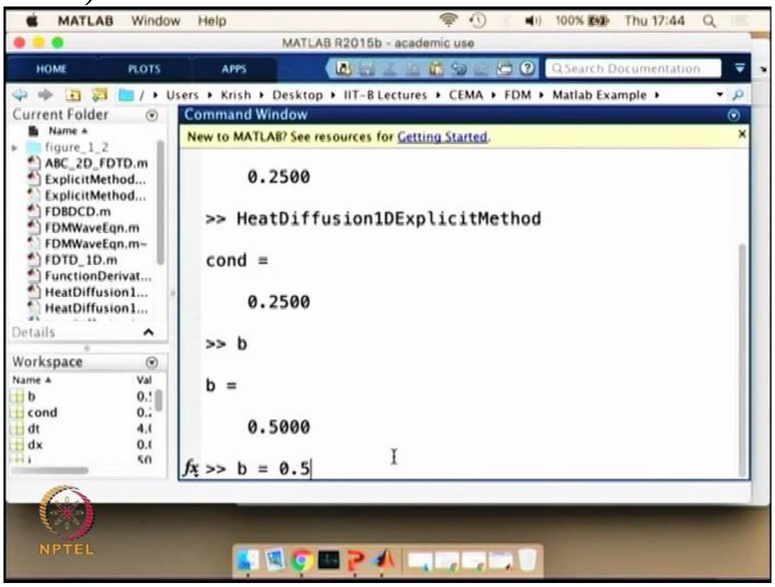

So see we are using b equal to 0.5. But if try to change the value of b Let us say we are going closer and closer to 1, let us see what is going to happen. So for doing that what i am going to do is i am going to comment this particular part of the b and manually substitute the value of b. So i will save this one. So i am going to set the value b equal to 0.5 once again and i am going to simulate this equation.

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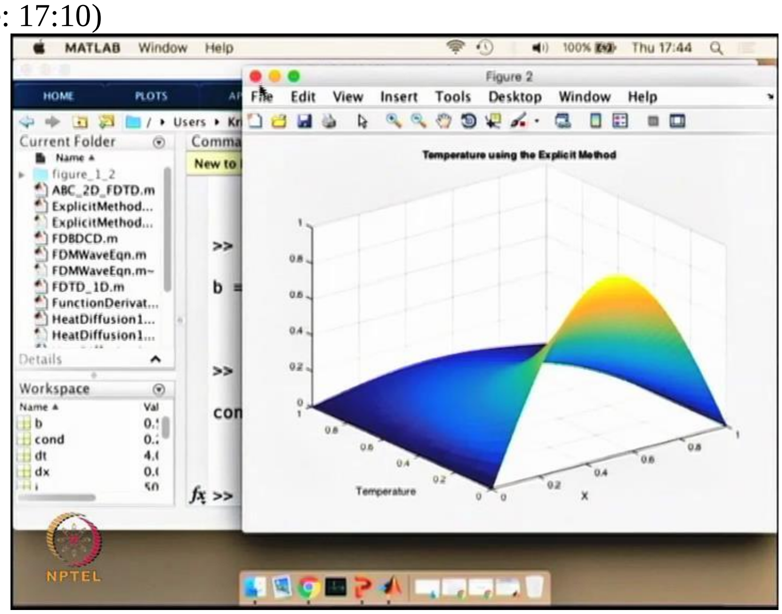

And i am getting the same result which is good.

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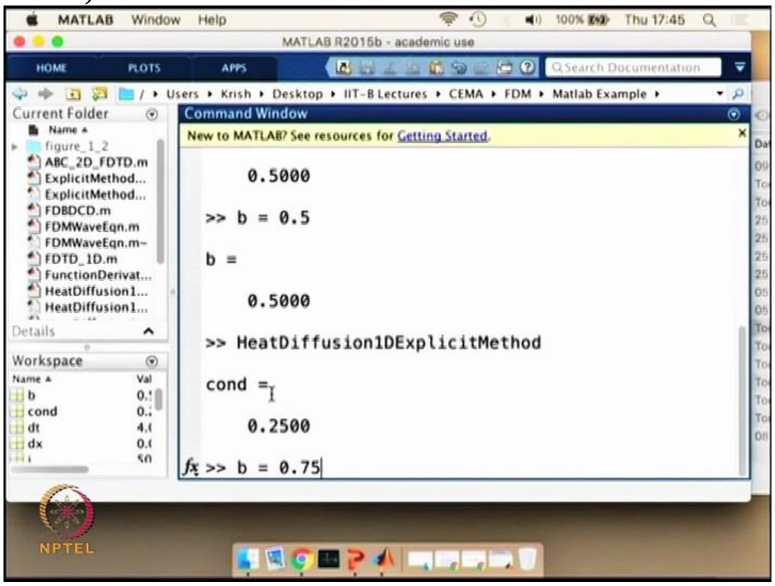

Now I am going to change the value of b to let us say 0.75 and now i am going to simulate the same equation.

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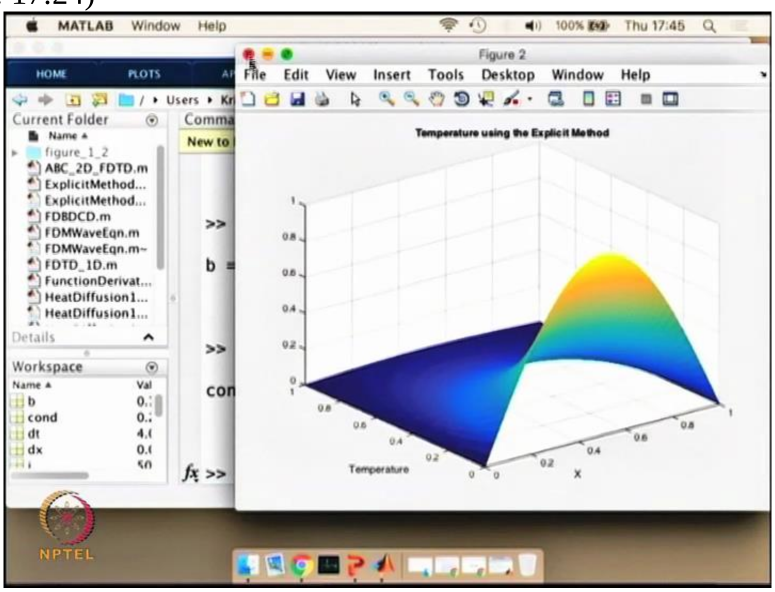

And i am seeing pretty much the same result.

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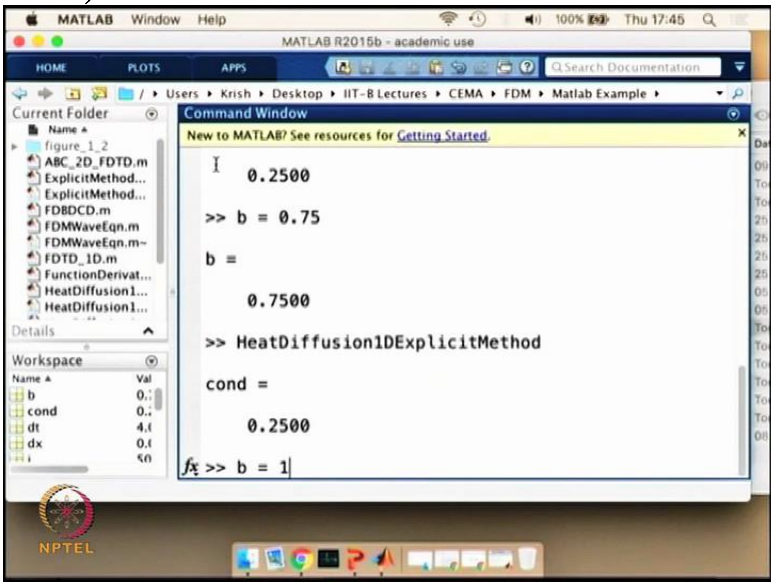

So, now i am going to change the b value to 1and now i am going to simulate the same result.

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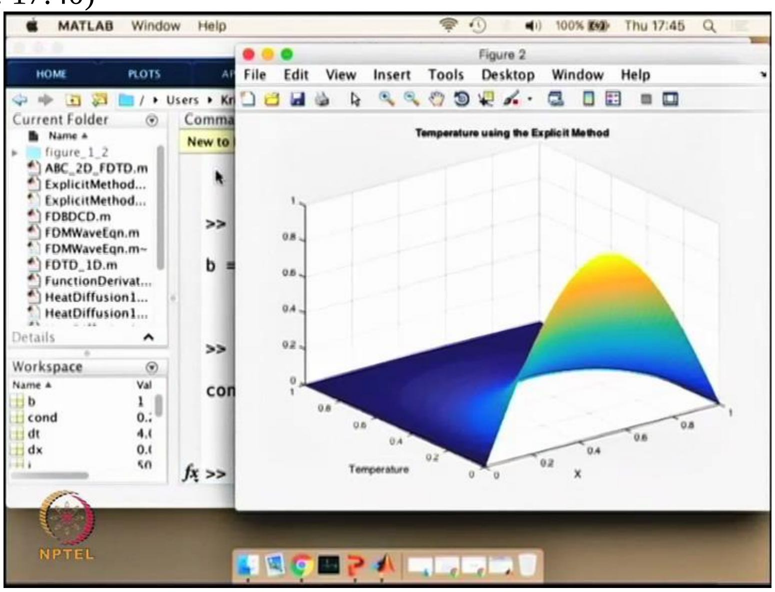

And i will start to see the value is still ok.

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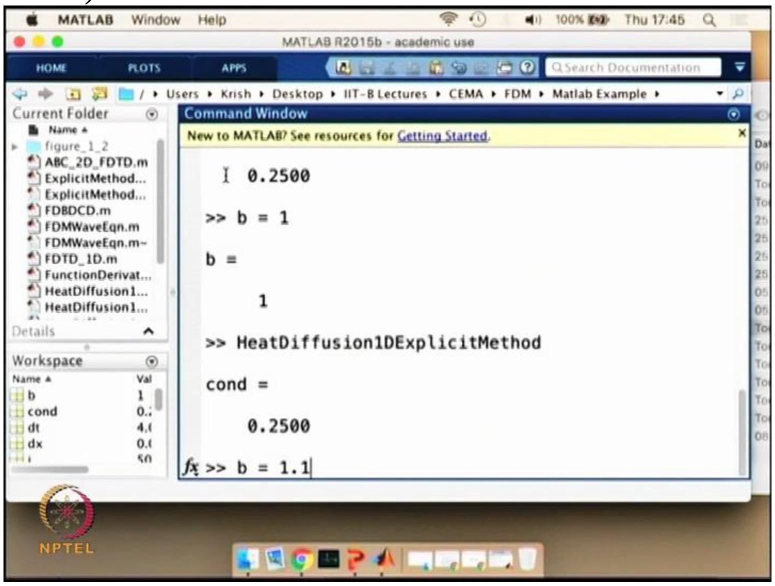

But the moment i go more than b equal to 1, let us see what happens. So i am doing b equal to 1.1.

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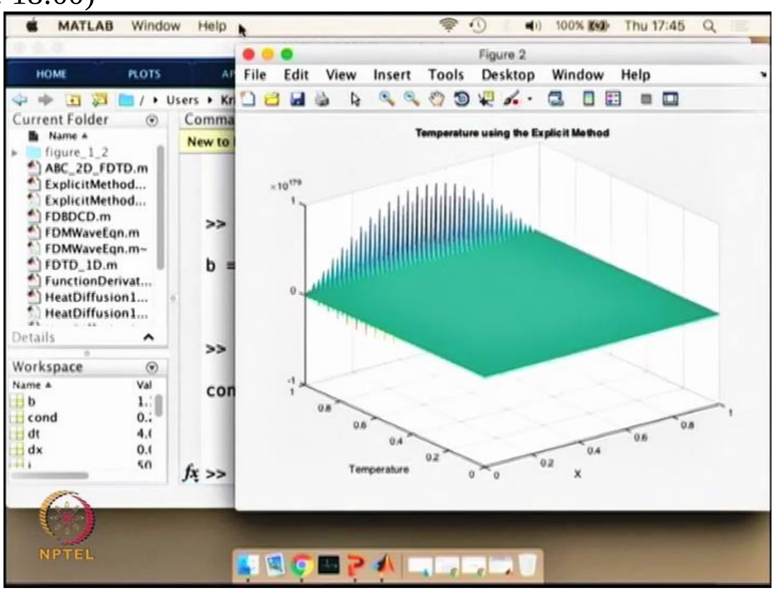

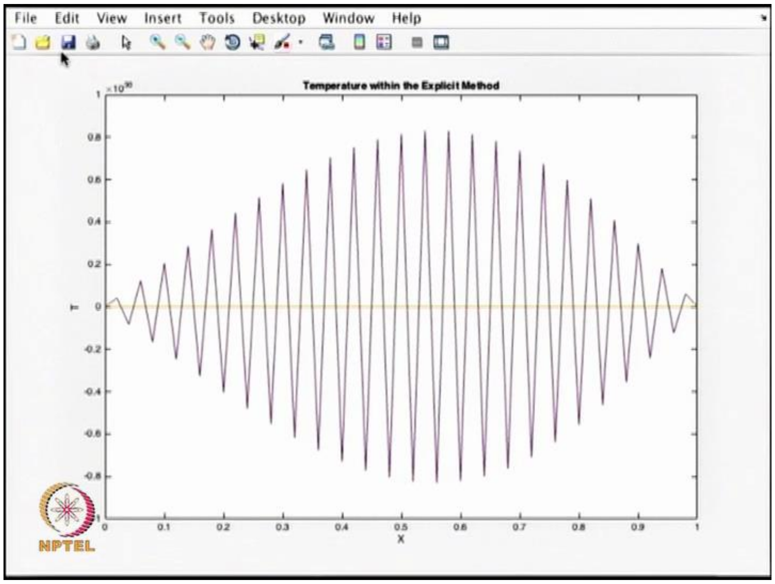

You start to see some kind of error which is you see that it is going to 1 multiplied by 10 to the power 30 which is a clear case for instability.

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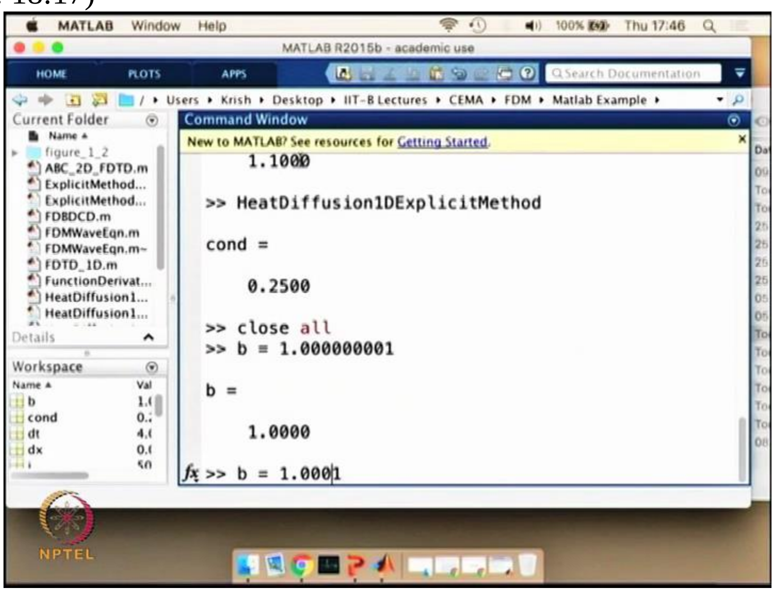

So may be we can try to go b very very low, so i am going to change the value. Now i am going to run the same equation

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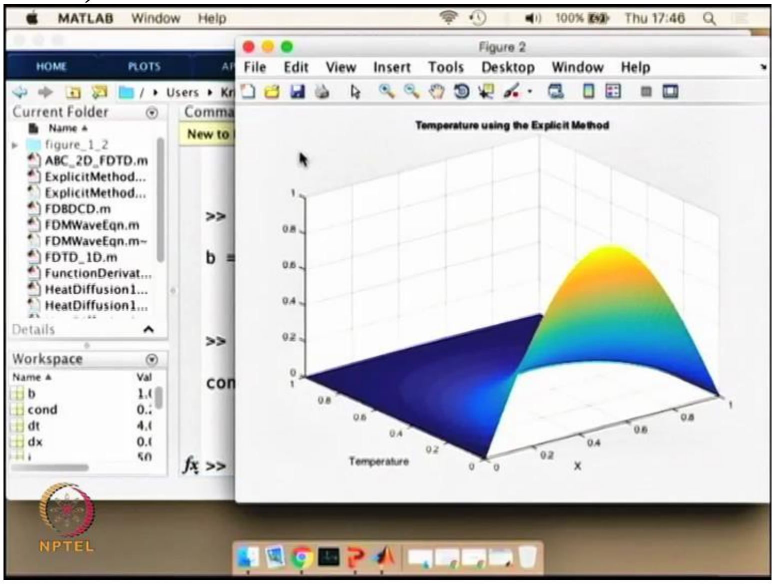

It is still ok, but if I go 100th the point more than the expected value it is still ok.

So 1 10th more than the expected value, now i start to see the instability coming into play. The solution still looks ok but the instability is already starting.

So what we have done now is to show that we cannot go more than the maximum limit by an order of more than 10. So if i start going at point 1 this will not be useful at all. You will see that the instabilitlity is clearly there and the value is 1 multiplied by 10 to the power 30.

So even by going 10 percent more than the allowed value of the maximum simulation time stepping it is already not working. So as I said this is a classical example to know how to program the finite differencing scheme using forwarding time and center in space for heat diffusion equation.

We will provide you this code for you to try and practice a little bit the simulation as well. And this is also a good way to learn certain criteria like the initial condition, boundary condition so on and so forth which will enable you to do advance coding later on. So i encourage you to take this code and practice and get the sense of it.

With that being said we have covered the example for Explicit Method or Heat diffusion equation. And we will come back and discuss further details on the next things in the next module.

Thank you!