Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No 28
Finite Volume Time Domain Method-I

In the earlier module we gave our motivation and also the background about Finite Volume method, we also gave you a little bit on the theoretical side how it really connects to the Maxwell system.

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In this module we are going to look into the discretisation itself. So let us go into it so we are talking about spatial discretisation we will come back to the temporal discretisation in our later lessons.

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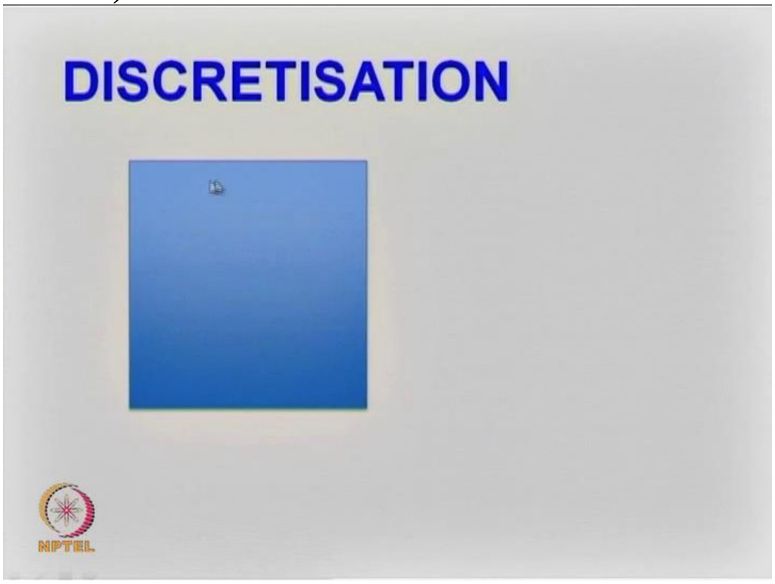

So what we have is let us say a domain which is represented by let us say square. I mean it need not be a square it could be any random shape but just to prove certain ideas I have taken a simple square surface. So you can basically discretise this using the standard squares what you have in finite difference method. And also you have to have a dual mesh in the case of the difference method.

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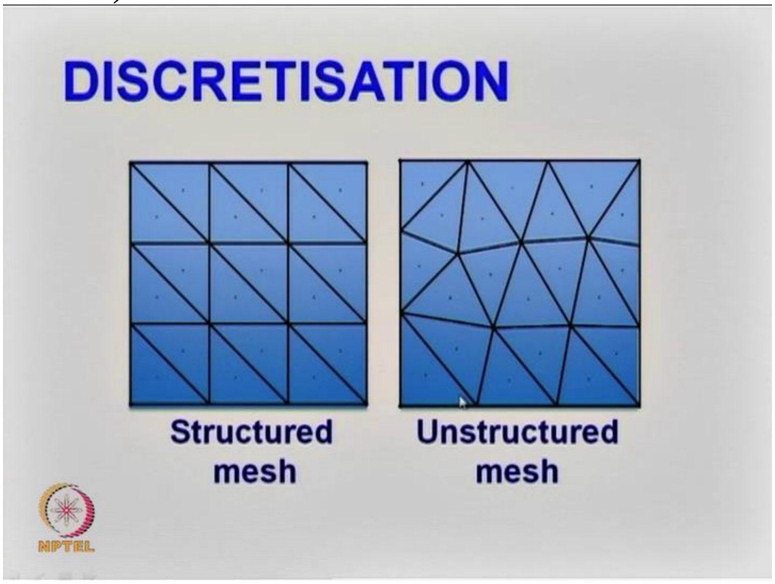

But in the case of finite element you can basically have a structured mesh so what you have here is a mesh which is structured I call it structured because there is certain patterns in which these elements are individually placed. It need not be like this because one of the beauty of the conformal method is to go for unstructured mesh so it can have certain random things it can really have small triangles, big triangles what not. So this is the kind of discretisation we are going to talk about particularly unstructured mesh.

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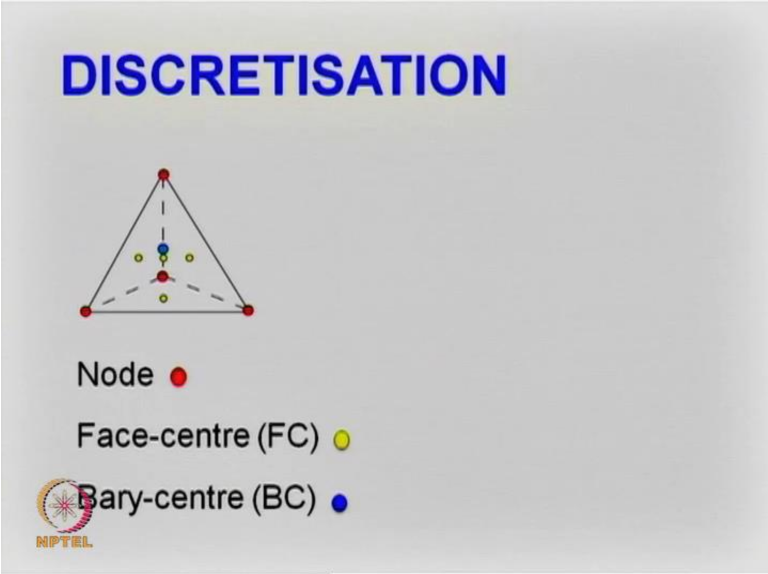

So let us say we take a standard three dimensional case where you have tetrahedral mesh that we are talking about ; e and h field are basically stored in the centre point which is the Bary centre point. As compared to finite element method in the nodal method finite element nodal discretisation where you will have value stored in the nodal points. And of course you can also talk about edge elements where you have the elements edges will have the value stored in the case. Let us say we are comparing it to finite elements where the values are stored in the nodal. So as compared to the finite element we will have only one particular point where the E and H field is stored. Then the question is what are these yellow points. The yellow points are the face centre points these are the centre of these faces. If you are talking about a three dimensional case these are the bary centre of these triangles which are forming the four faces.

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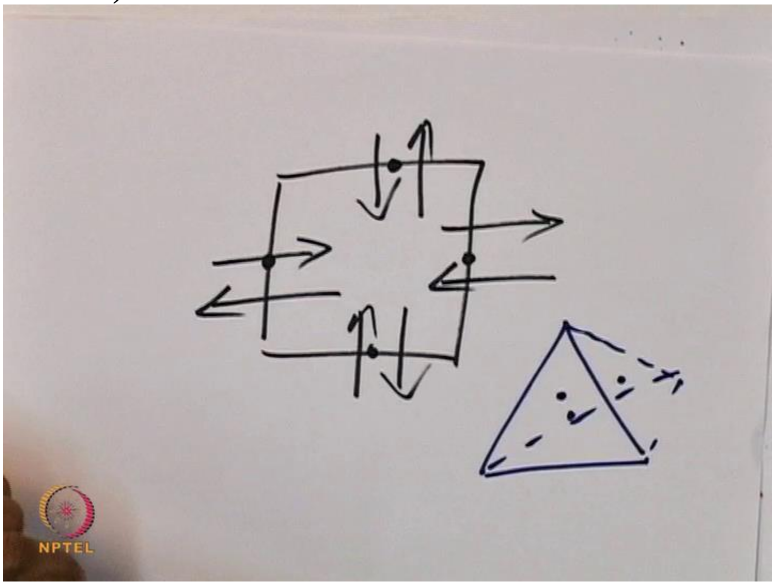

Why is it more important is because we are going to compute the fluxes remember that in the case of the finite volume we talked about a case where we are going to have a volume and we are talking about flux that are going in and out on each of the faces. So we are assigning one particular point which is let us say the centre. So this is in the case of a tetrahedron will be the centre of the face of each of the sides. So that is the point what we are talking about the face centres.

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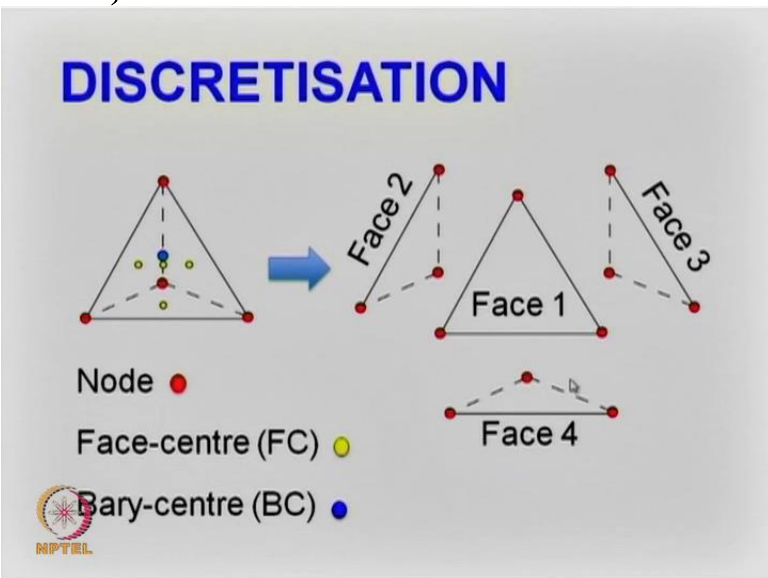

So that being said we can get to know that there are 4 sides call them faces and these faces have individual components each nodes and each of them will have centres

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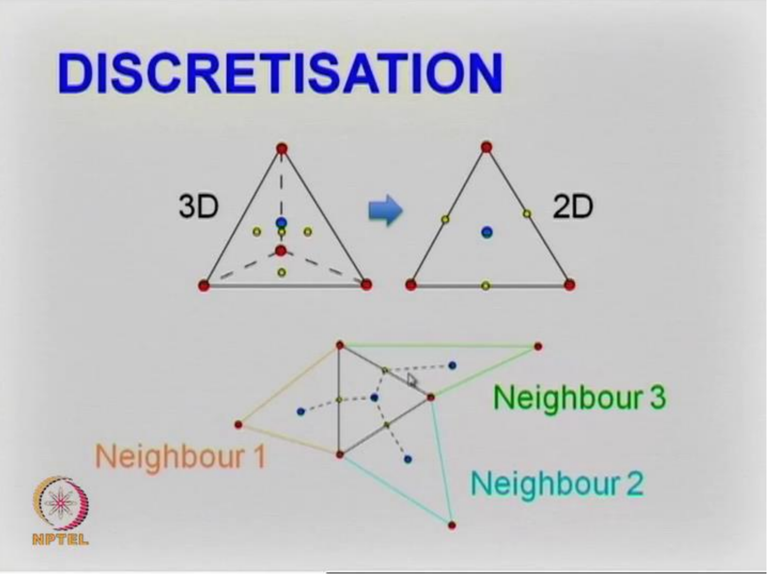

So lets say we are interested in the 2D problem how do we translate this from a tetrahedron we can go to a triangle and the triangle will have also a Bary centre which will be the center of the face of this particular triangle and the centre of each of these sides are nothing but the centre of each of the edges. So this in the case of a 2Dimensional problem can be seen along with the neighbours for example this particular triangle has three neighbours if it is a boundary triangle one of the edge will not have a neighbour whereas the two sides will have neighbours so assume that you are talking about a case where this particular side is the boundary side. What you will have is only these two triangles and this triangle will be not existent. So if it is a main bulk triangle it will have 3 neighbours.

And one more thing is the number of triangles that are attached to a particular node is quite depended on the mesh itself. Sometimes it could be just 3 it could be 4 so depending upon the mesh refinement the number of triangles attached to a node will change. And as you can see we are talking about the projection from the centre of the bary centre of the triangle to the face centre similarly we are talking about projections on each of those sides. These are nothing but the projections. So in other words in a 2 Dimensional case these are the normal projection what we are interested in.

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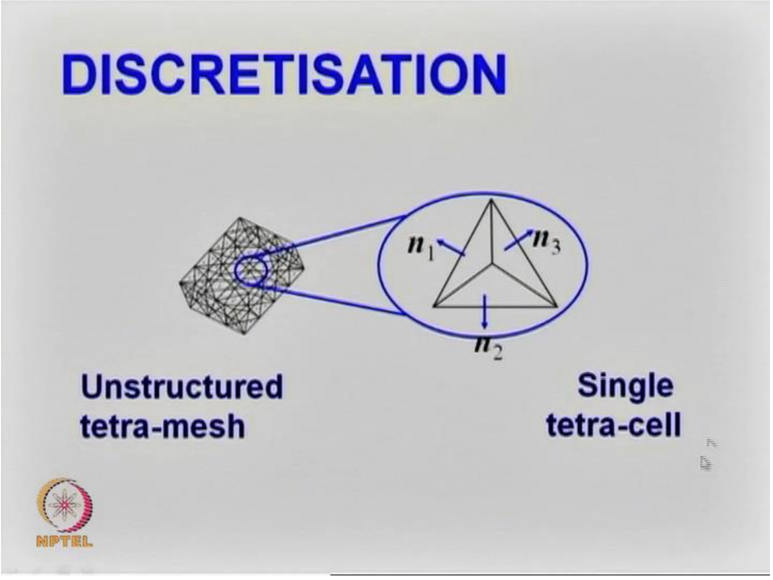

So let us say we can extrapolate the normal projection a little bit further in the three dimensional case what we will have is the normal vectors pointing on all of those sides and this is very important to know because these normal vectors will be used in the flux computation. As we will see in the next slide.

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So we will go now in the Maxwellian system itself.

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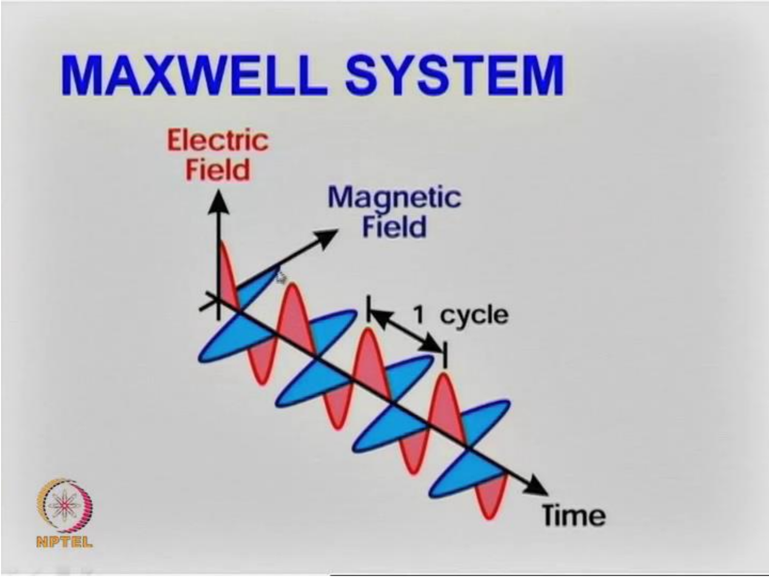

So like in the case of the example problem which we solved in the earlier module we see that the wave magnetic field is varying along one plane and we have the E field varying along one plane.

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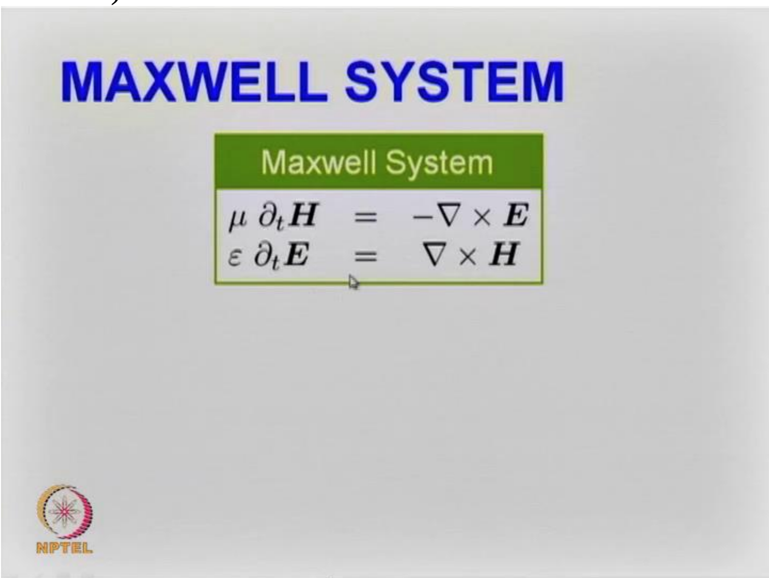

And we can write this form in a more generalize forms as a 3D component the bold letters represent their vectors and we are considering only scalar quantities for epsilon and Mu.

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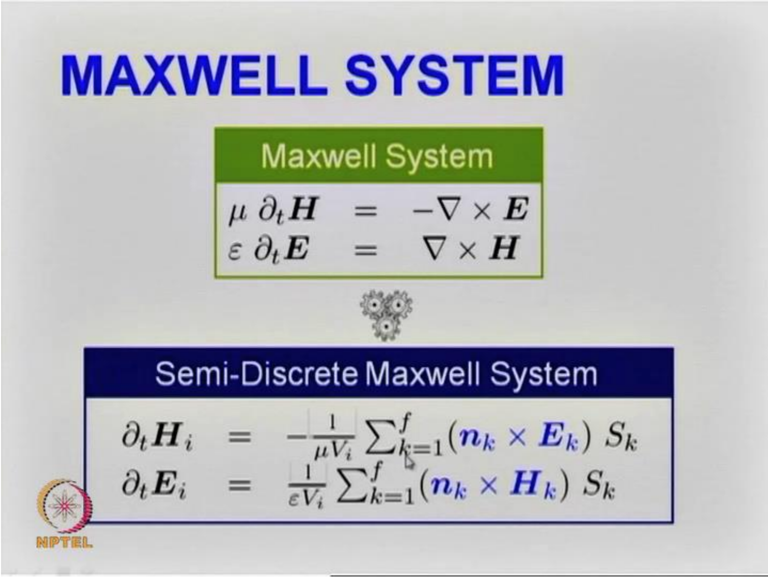

So this can be written in a semi discrete form as follows as you can see the flux components what we talk here are the flux component that we use from the earlier slide. And these flux components are computed along each of the sides whether we are in a one dimensional case or two dimensional case or three dimensional case the flux components will be changed accordingly but still they are defined on the edges or on the faces of each of the control volumes. And S k as I mentioned before its nothing but the surface area of the k th face. And there are k faces for a particular contol volume v i so k goes from 1 to f you will define that in the next slide.

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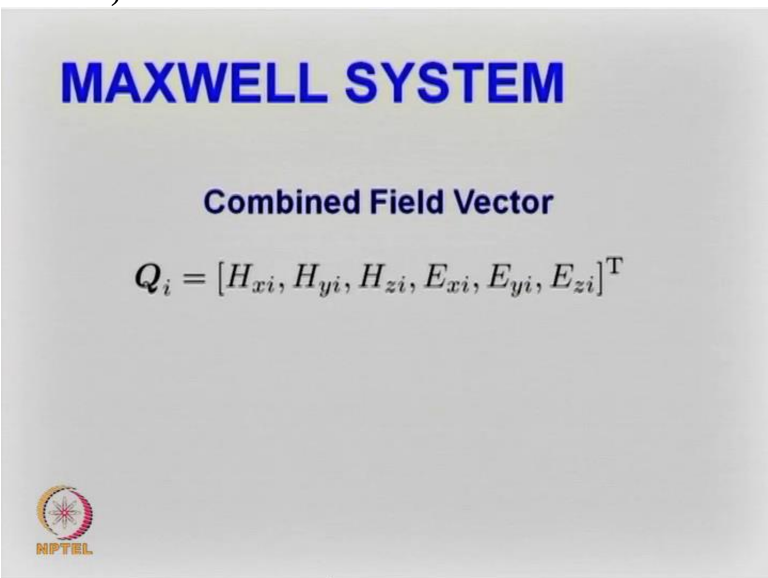

So what we are having now is a combined vector in a case of a three dimensional problem six component vector. And j correspond to the magnetic field and the electric field. As you can see the i represent the i th cell and each of these components are the magnetic components along those sides. And the k represents the transpose of that field component because for mathematically to model them we need to have the matrix form properly represented.

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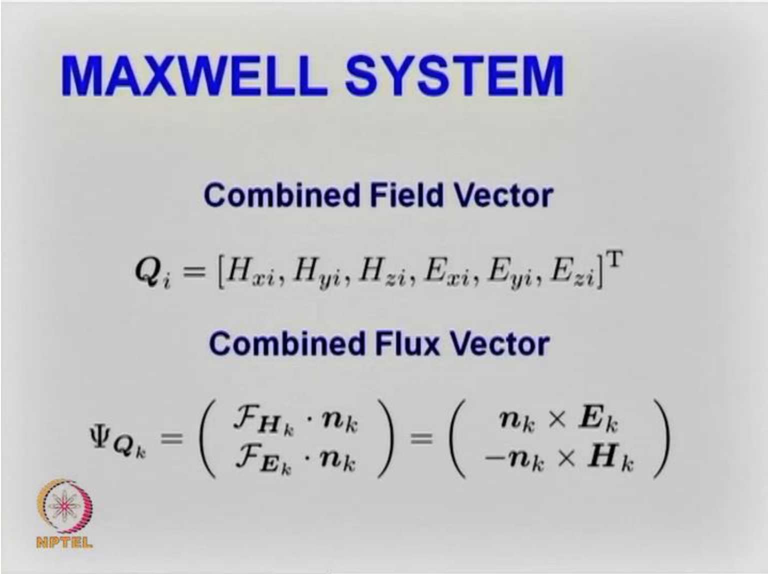

So we have here Phi Q k that is the flux component on the k s face which is the function of Q which is represented as the dot product between the flux function and the normal component. As you can see here we are having the particular case where we have n cross E and n cross H. So if we compare that with our earlier slide where we had a formulation for fluxes we will see these are the components that we have in this slide which is seen here.

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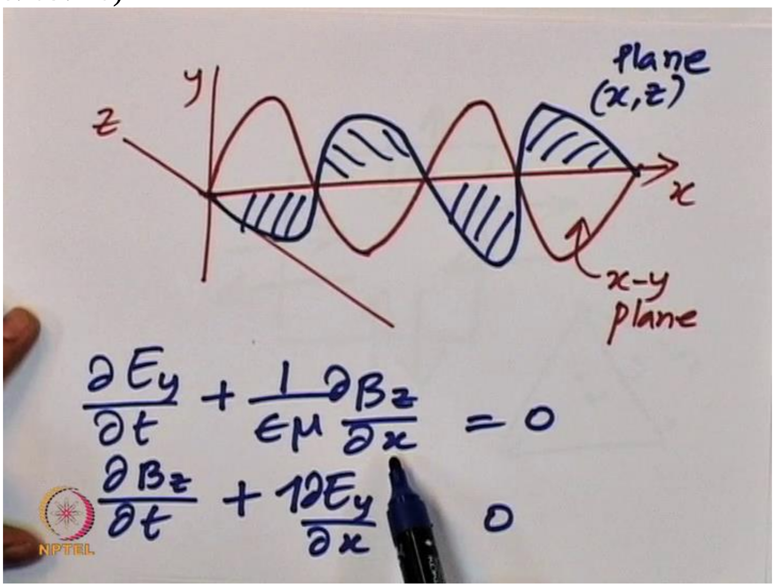

So basically these are the n cross E term or n cross H term.

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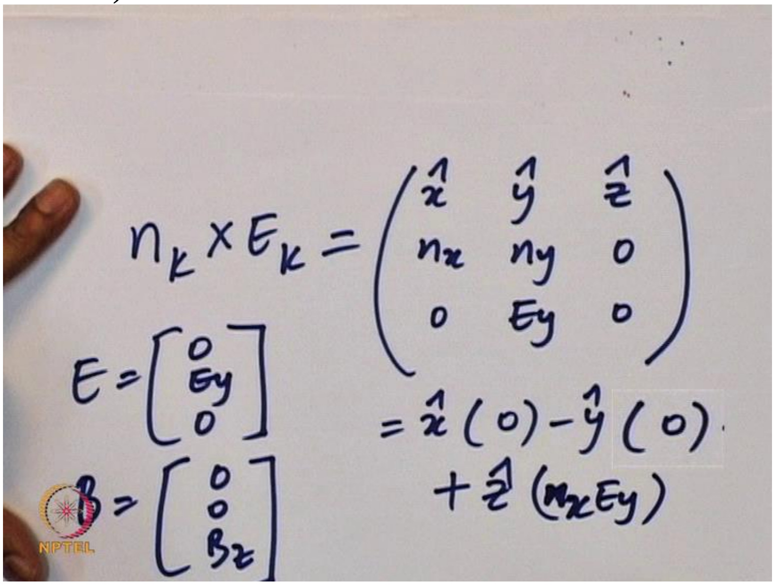

So if we say that we are having only s n cross E in a case where we had only E y component will be given as the x component the y component and the z component and we have nx ny and 0 because we do not have a normal component on the z axis, and we have 0 Ey and we have 0 again) So for the test case where we had E is equal to 0 E y o] and B is equal to [0 0 B z]we will see the n cross E will be given as the x component multiplied by (0 and 0 will be 0) minus the y component multiplied by 0 plus the z component multiplied by (n x E y) minus 0.

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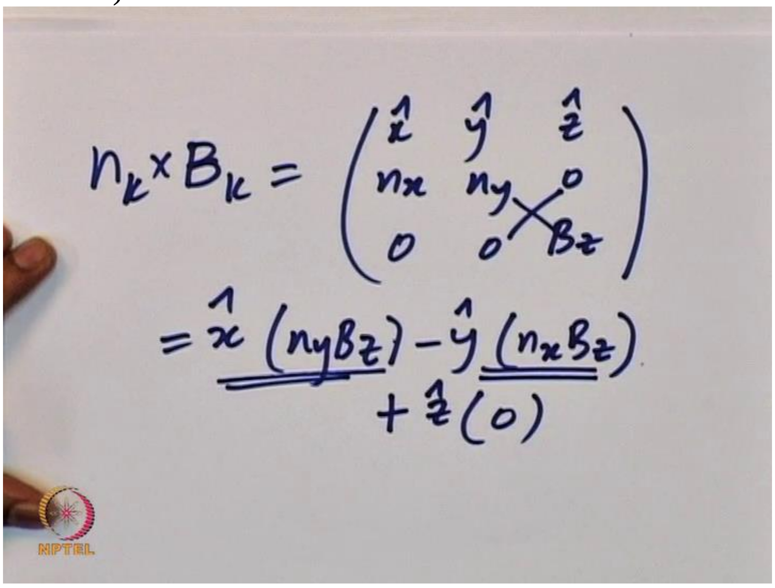

Similarly the n k cross H or B whatever you wanted to compute have the components that are given by (x component y component and z component; n x n y 0; 0 0 B z) this is nothing but x component multiplied by (n y B z multiplied by 0 similarly minus y (n x B z) plus the x component and (0). So we will have components along the x coordinate and y coordinate. So what we will get is the flux function accordingly represented in this particular slide.

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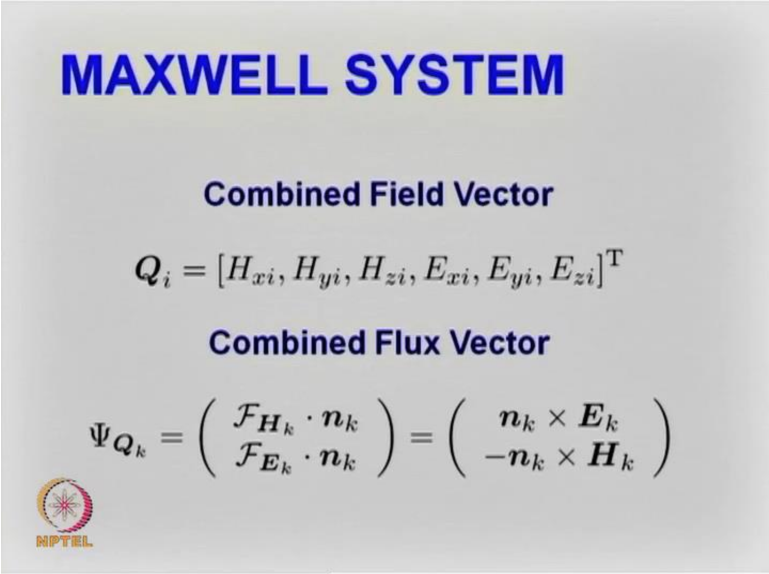

So this is the definition for defining the value of n k and E k n k H k we are using here E and H instead of E and B because we are going to compute E and H directly, we are not going to use B factor but in the example I showed you B because I wanted to use the definition of a which is the a the matrix the diagnosable matrix. So with that being said this is the form that will be very useful for coding the Maxwell equation.

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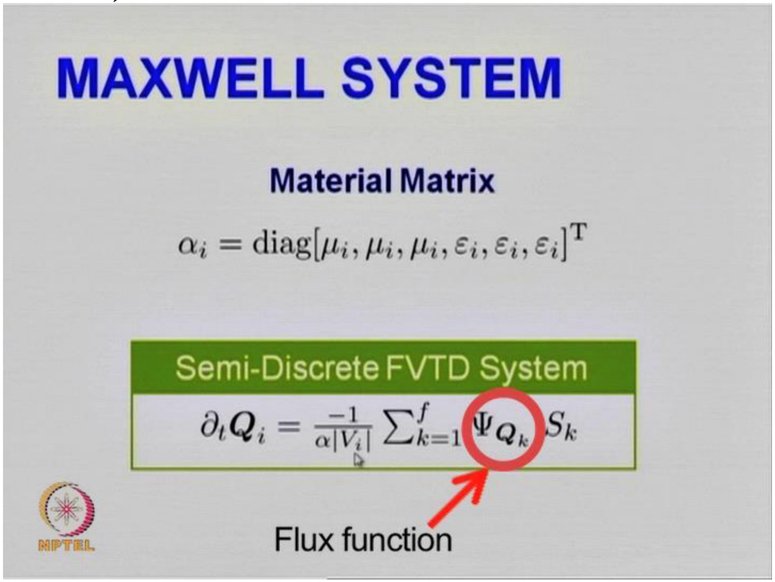

So in this particular form we have to also define the material parameters and the material parameters are defined in the diagonal matrix given by this matrix here so the semi discrete form will have alpha which are defined here and the volume which is the volume of i itself. And the flux component which comes directly from the previous slide here.

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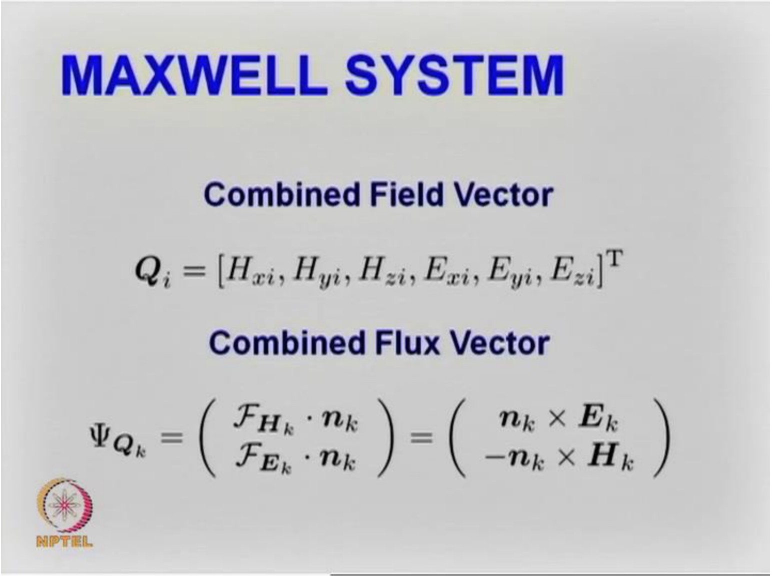

As you can see

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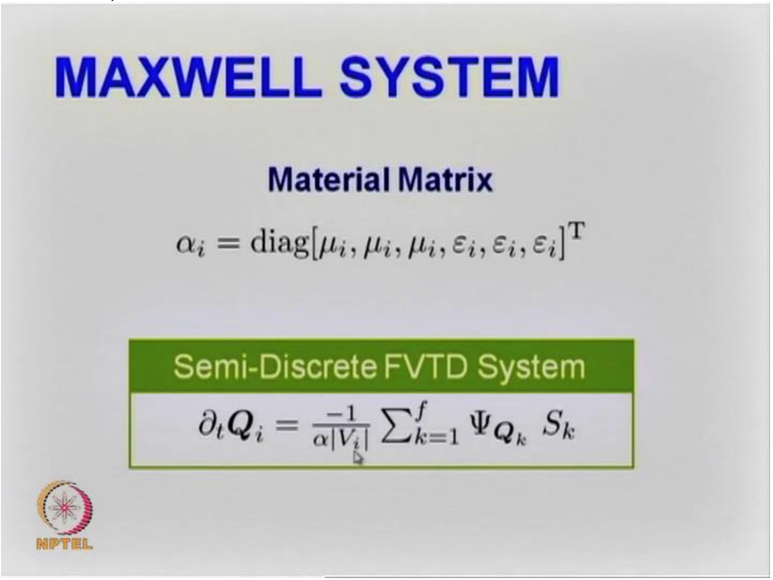

And we are using it to compute the semi discrete form this form is the most useful form for us to go forward because it basically tells whatever we need to know about the finite volume method for us to do is a temporal discretisation which we will do in the following lectures.

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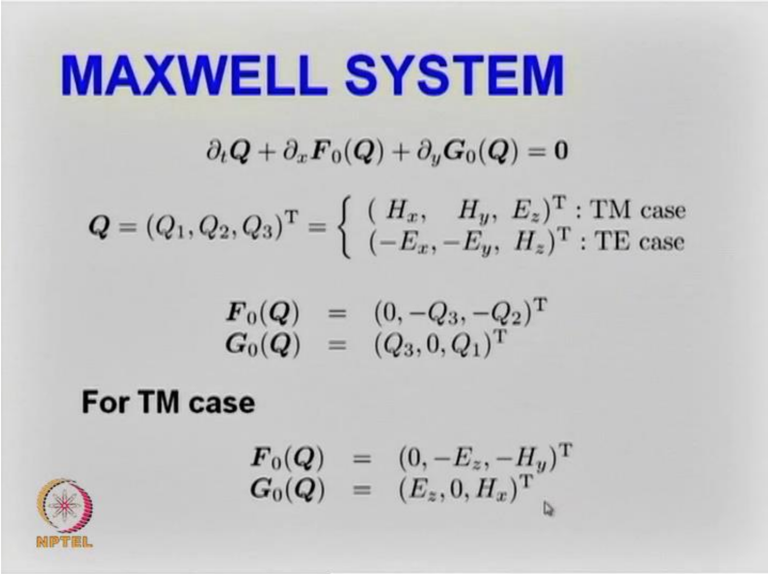

With that I wanted to also give you a simplified form for a 2D case where you have TM or TE case where you have H x H y and E z the magnetic field are only in the xy plane low magnetic field components in the z axis. And similarly the electric field components are only in the x y plane no electric field component in the z axis will be the TE case. So if you have this form we will compute the fluxes accordingly so Q3 Q 2 are nothing but these components. The case are the TM case minus Q 3 means minus E z and minus Q 2 means minus H y and Q 3 is E z and Q 1 is H x and this will give us the definitions of various things so in the case of a TM case we have definition as follows.

So in most of the examples in our next slides what we will do is we will be focusing on a TM case to showcase some of the practical applications of modelling finite volume systems where we will use either a plane wave travelling in a x axis or a wave guide solution and we will also look into some of the advanced techniques namely the perfectly matched layers or absorbers layers how do you define them. What are the constraints so on and so forth.

With this we come to the end of this particular module Thank you!