Computational Electromagnetics and Applications
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Lecture No 27
Finite Volume Time Domain Method-I

Welcome back! In the last lecture we discussed about the motivation for using Finite Volume Time domain method. And we also gave you the background on the mathematical formulation which comes from the computational fluid dynamics. Obviously we do not understand the physical meaning of those formulations. And this is what we will be discussing in this lecture and at the end we will discuss a little bit more about the discretisation itself. So let us begin with the form of the equation which I gave you before.

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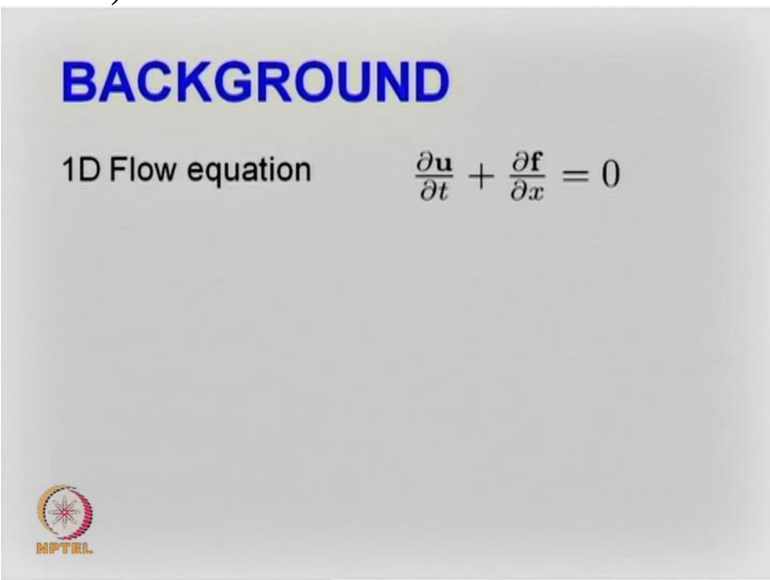

So this is basically a one dimensional Flow equation which I said. So let us take simplify this equation even further;

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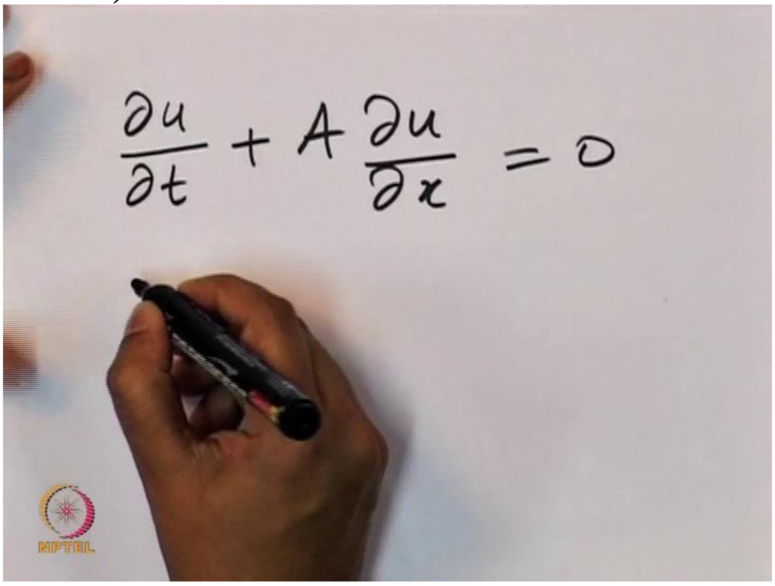

let us say that we are having a form where we are using let us say du of t and then we say that there is A which will be discussed in this lecture and then say du of dx. So this is a very simple form if you compare this with the equation that we have here this is nothing but A equal to 1 in this case and df by dx is nothing but du by dx. So this particular form is the more general form and we are going to use this general form to understand.

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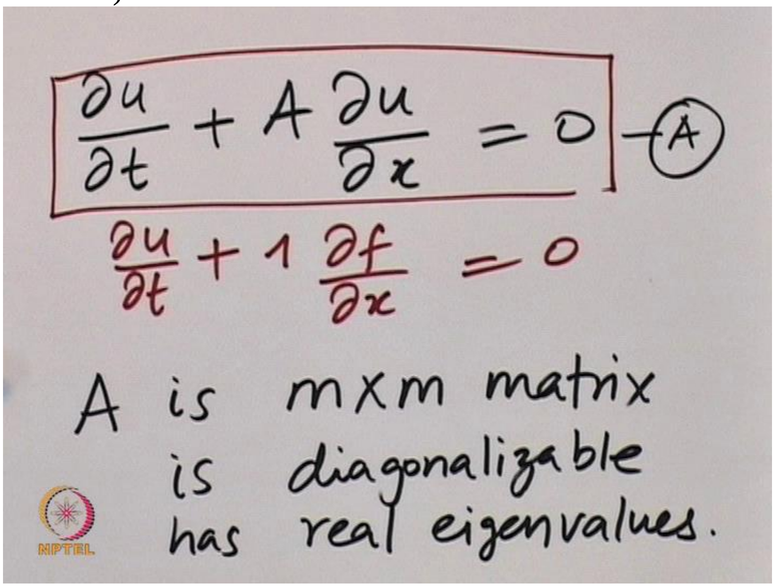

So like I said in our earlier case we had du by dt plus A we said its 1 and its df by dx. So you can see the similarity between these two equations. So we will be starting to work with this more general equation. So as to get you a more better understanding of what do I mean by this A? What is the significance of A? What is the significance of the entire equation? So we say that this equation is of the form hyperbolic if A is a mby m matrix. And A is diagonalisable, and A has real eigen values. So we know that from the basic linear algebra the matrix the square matrix that is diagonalizable will have certain Eigen values. And in the case of this form we call this equation as hyperbolic if A is this m by m matrix, and it is diagonalizable and it has real Eigen values.

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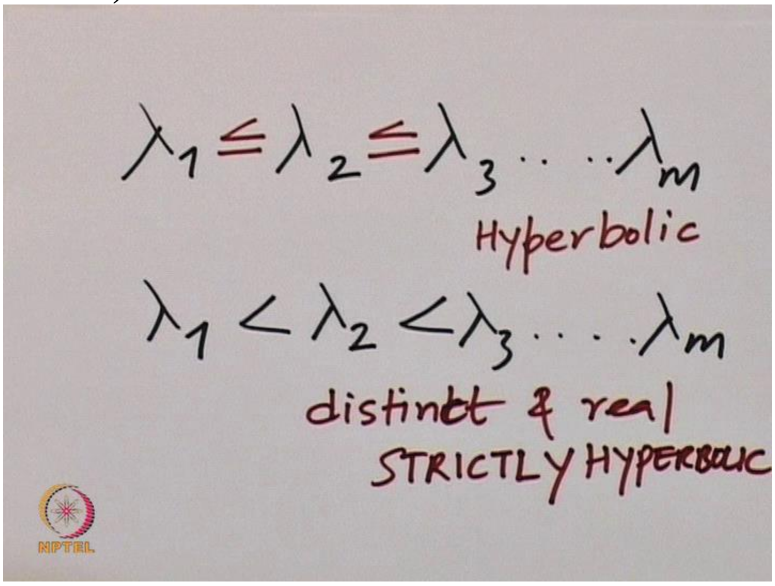

Let us say the Eigen values are represented by so Lambda 1 Lambda 2 Lambda 3 so on until Lambda m. Since its a m by m matrix we will have m Eigen values. So if this particular Eigen values are all having certain properties. The property is if they are all real and they are all in a way that they could be same or you know they could have certain properties. They can go in a you know increasing order or they could be all same. But if the are all real we call them as hyperbolic. But if these eigen values are real but distinct; in other words Lambda 1< Lambda 2< Lambda 3 until Lambda m. So they are all different so they are all there is no equality sign here, and they are all distinct. So if they are distinct and real they are called as strictly hyperbolic.

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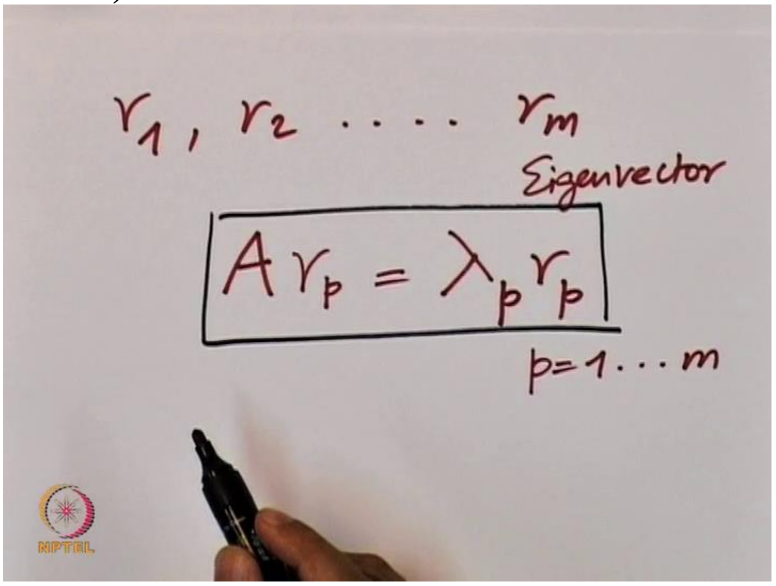

So we know from the linear algebra that once we are talking about let us say Eigen values there is also corresponding Eigen vectors. And these Eigen vectors are let us say are represented by r the letter r. And they go from r 1, to r 2 .... until r m. And these are all the Eigen vectors. The word Eigen comes from German basically means characteristic. so when you say Eigen vector they are nothing but the characteristic vector and Eigen values are characteristic values. So when we say characteristic the question comes what do they characterize? Right? So the relation between the Eigen value and Eigen vector is given by the relation A the same matrix A which we saw before, r p which is the Eigen vectors is equal to the Lambda which is the Eigen value of p times r p. And p goes from 1 to m. So this is the basic equation that defines the relation between an Eigen value and Eigen vector for a given diagonalizable m by m matrix.

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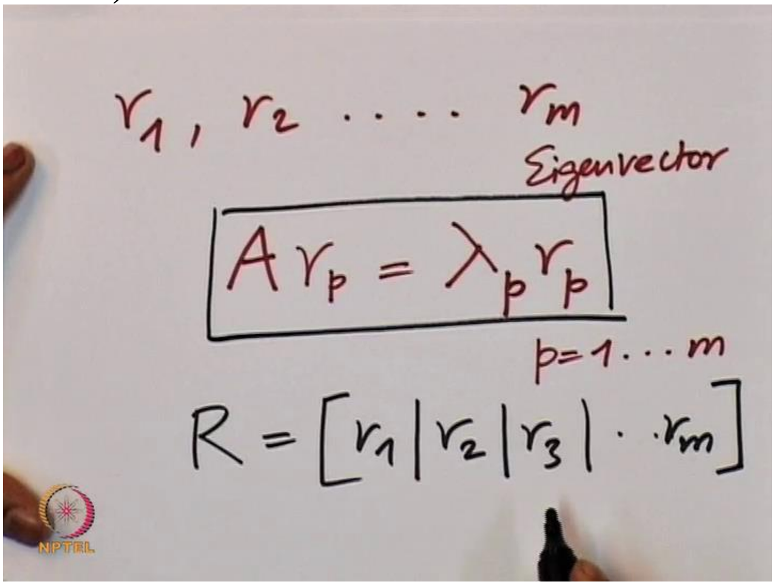

So that being said we wanted to know what is the structure let us say we talk about a particular matrix which is consisting of all the Eigen vectors R is equal to [r 1|r 2|r 3|.......until r m]. So this is nothing but a kind of a column vector which has all the individual columns represented by r different Eigen vectors.

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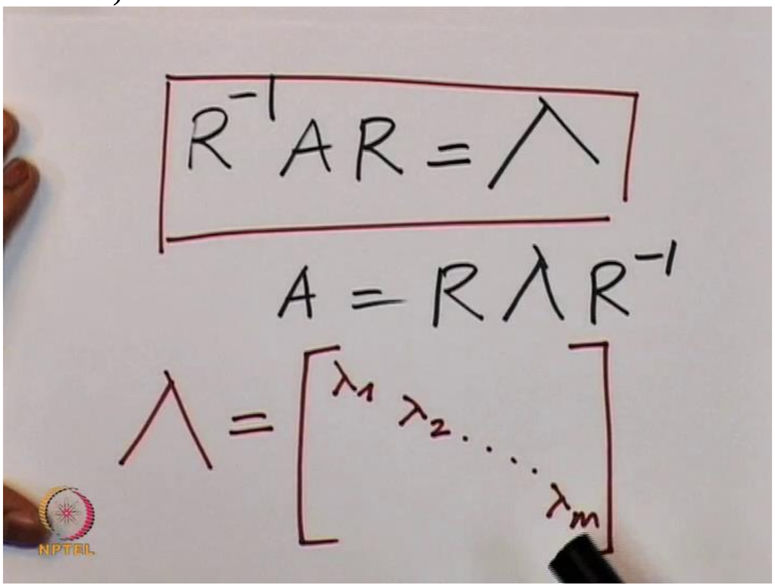

So if we say that r is a column vector and depending on whether we use them as a column vector or column row vector we have to adapt the equation accordingly but the idea is once you have the definition of the column vector the Eigen values and Eigen vectors we can have them written of the form which is basically R inverse of A R is equal to capital Lambda. So the equation here is the definition of the relationship between the Eigen vector and the big Lambda so the big lambda is nothing but a diagonal matrix consisting of all the Lambdas. So you can also see we can write the same expression as A is equal to R capital Lambda R inverse. So I am pre multiplying or post multiplying with accordingly the matrices so I can get from here to here. So and I told you that the Lambda capital Lambda is equal to its a big matrix, diagonal matrix and it has various Eigen values along the diagonal [Lambda 1 Lambda 2 Lambda 3 so on until Lambda m]

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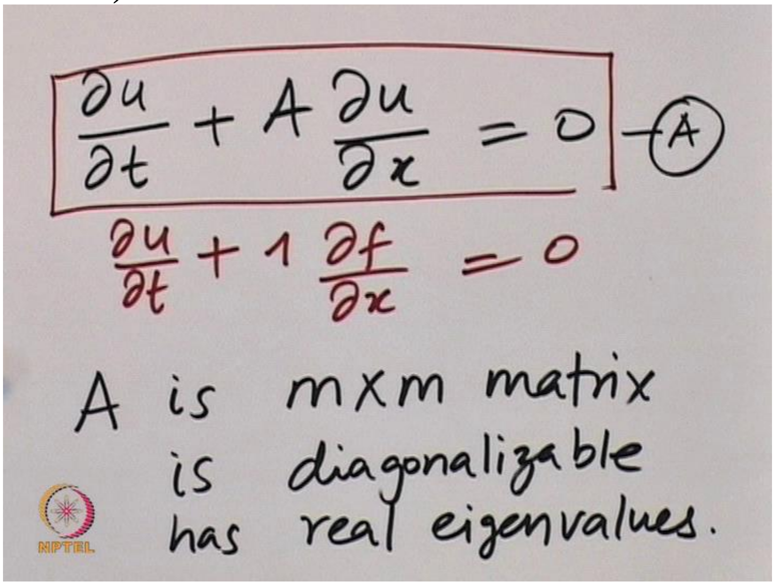

So if this is clear what we are going to do is we are going to find how we can use this particular form in the first equation what we had basically;

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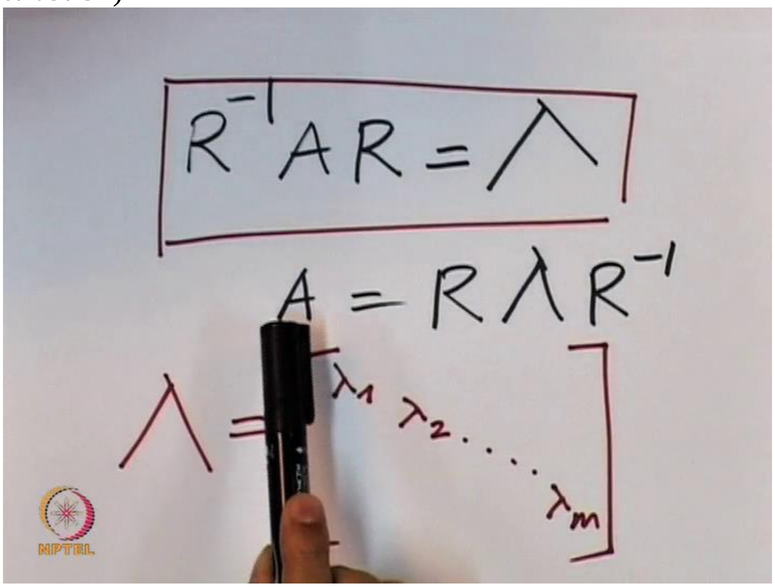

We are going to use the definition of A.

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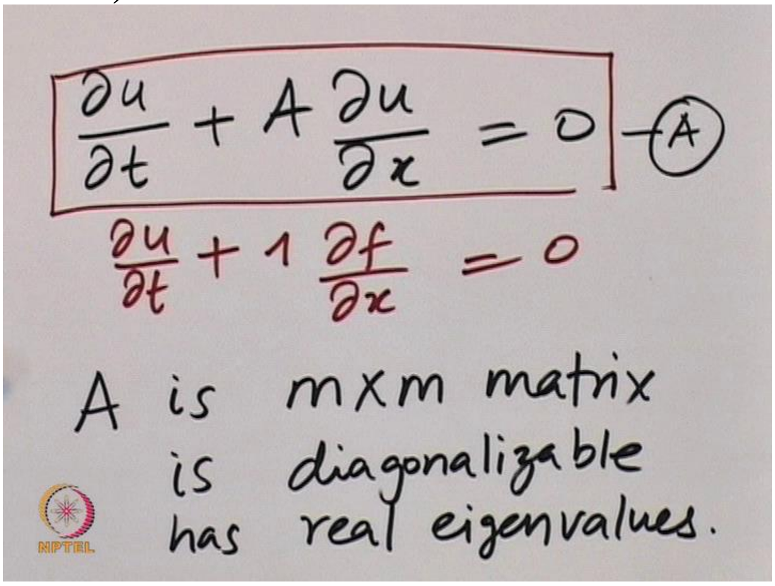

In this particular equation and then we are going to find a way to decouple the equation into various terms.

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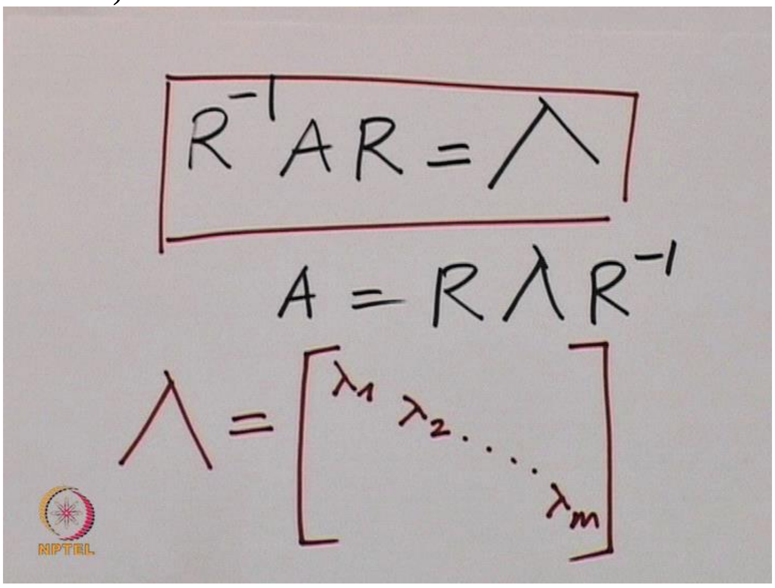

So what we are doing here is we are going to multiply the equation which we had before.

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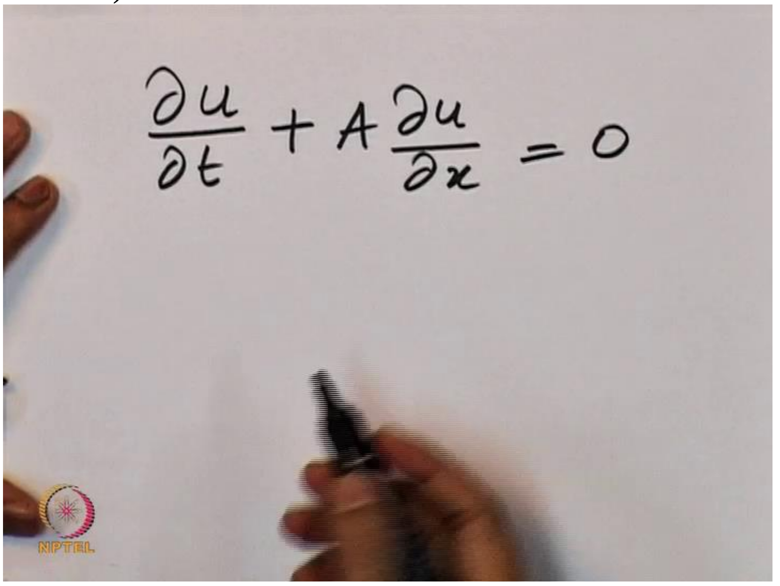

I will write down the equation one more time, so which is du by dt we have plus A du by dx is equal to 0.

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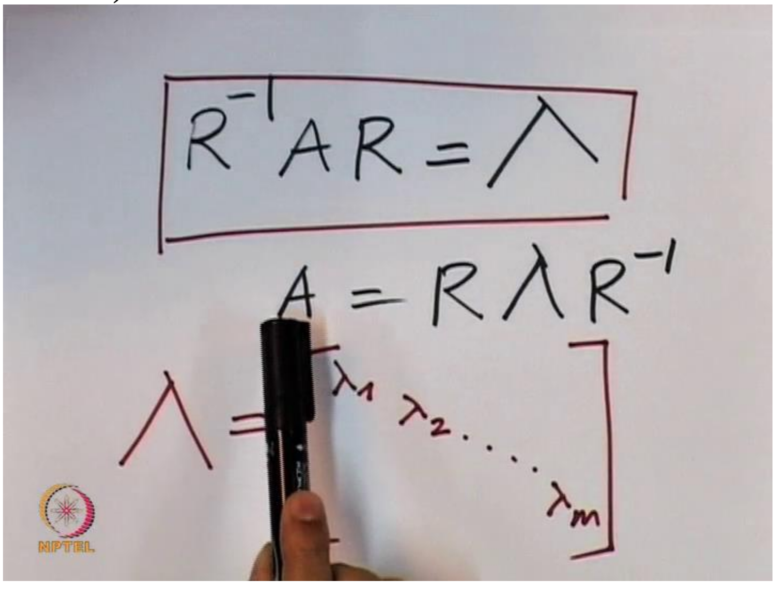

First of all we have substitute for A using the value that we got in the earlier stage.So A is R bi lambda r minus 1.

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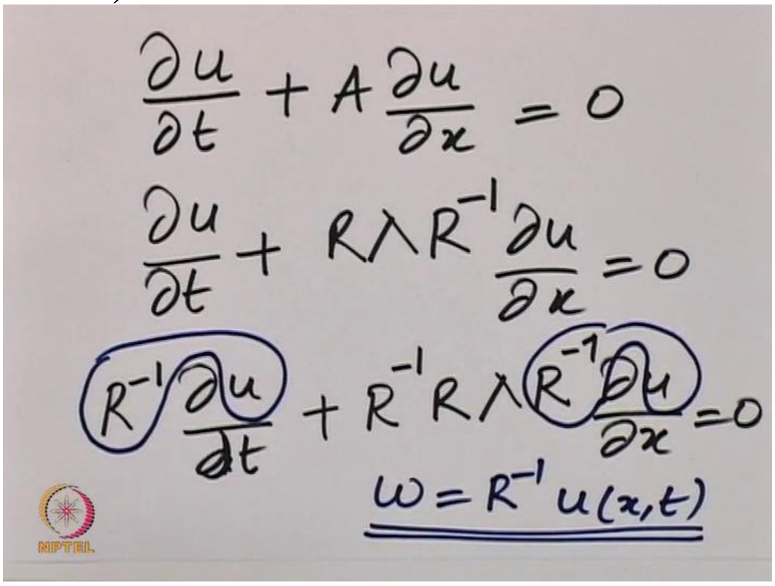

So this I can put it already in this equation so du by dt plus R capital Lambda R minus 1 du by dx is equal to 0. And I can pre multiply this equation using R minus 1 So I will get R minus 1 du by dt plus R minus 1 R Lambda R minus 1 du by dx is equal to 0. So what I can say is? I can say I will define a new quantity which is nothing but a definition for simplifying this equation let us say I call it w is equal to R inverse u(x,t) So what I am saying is if I use this particular definition I can basically simplify this part by putting w and I can simplify this part putting w, this one will become 1 because its R inverse times R and then we will have only the term that is with capital Lambda.

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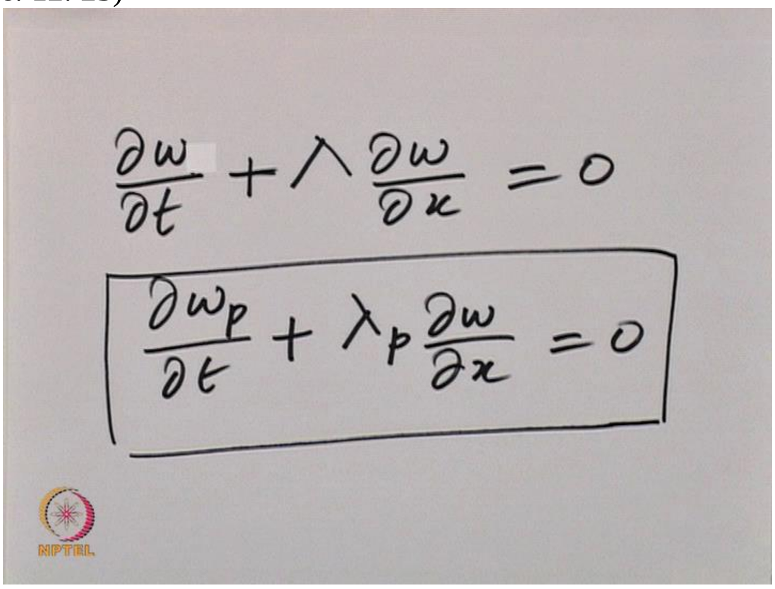

So this equation will be looking like this so what we will have is dw by dt plus the individual Lambdas so we will have the bigger Lambda times dw by dx is equal to 0. So why I have done this is nothing but I have adjusted for I have used this definition. So maybe I will use only w here but then when I am doing the equation you will see dw p by dt plus Lambda p dw by dx is equal to 0. So basically these are the individual components because this is a diagonal element. So when you multiply the diagonal element what you will get is only the p component and then you will get individual equations. With this understanding we will describe how this particular form will be helpful for us to use in the Maxwell equation while we go into the next step.

So as I said right now what we have done is we have got an understanding of what is the meaning of advection equation hyperbolic form. We said that this form will have eigen values and if they are distinct they are called as strictly hyperbolic and we have also shown that this form can have certain meaning in terms of the propagation. So this we will see how it is reflecting in the case of Maxwell equation.

So what we are going to do is we are going to take a simple Maxwell equation and we are going to discuss how this whole thing is going to come into effect.

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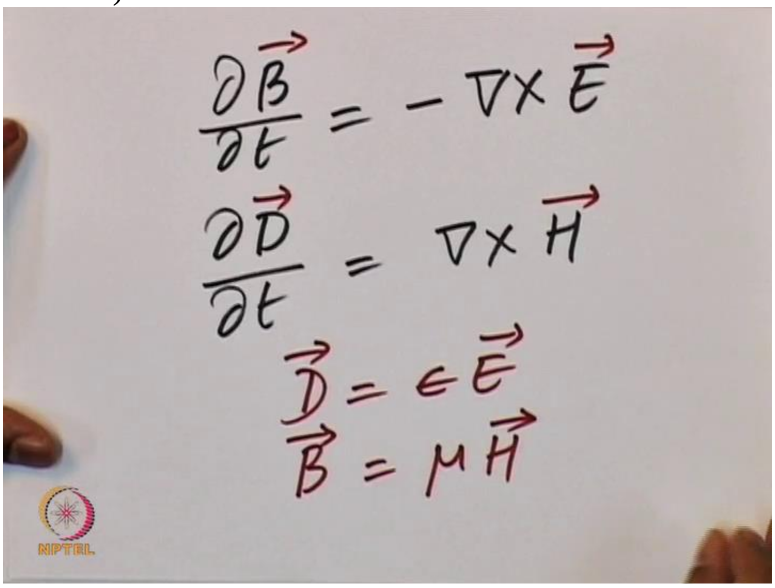

So let us take a simple curl equation which is basically given by the Maxwell laws which are db by dt is equal to minus curl of E. And we will have dD by dt which is equal to curl of H. So this is a form we are going to use obviously we dont take any source terms so we assume the source terms of 0. So this is the form that we will start using. So this form for a simple case for a material parameters are scalars. So that means D is equal to Epsilon E and B is equal to Mu H. Obviously these are all vector quantities. Epsilon is the permittivity and Mu is the permeability and we choose them to be constant.

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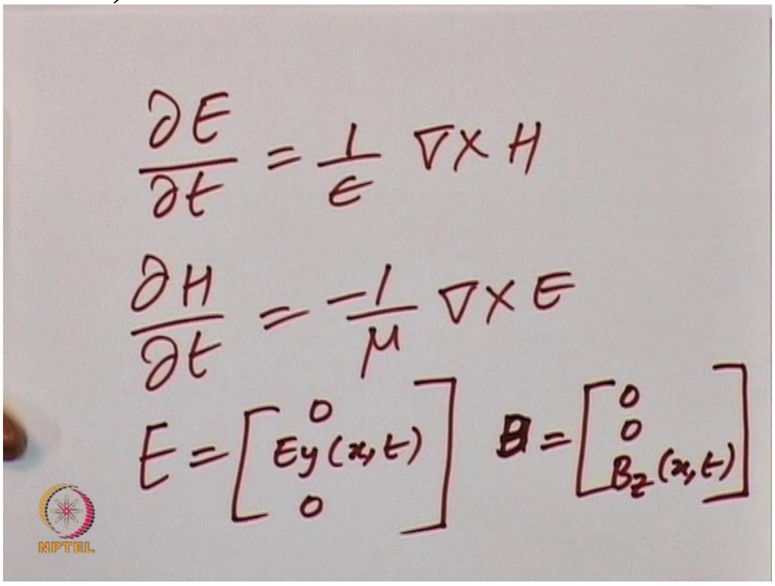

So accordingly this expression will be simplified for the case of simple Maxwell equation as dE by dt is equal to 1 by Epsilon the curl of H dH by dt is equal to minus 1 by Mu the curl of E. This is a three dimensional equation it has totally 6 components. But let us take a simple case where we are talking about E has only component along the y direction and H or let us take B has only component along the z direction [0 0 B z(x,t)]. The reason I am using B here is to make a connection between this particular form and the eigen values and the A matrix that we learnt before. So with this we know that we can model the equation accordingly

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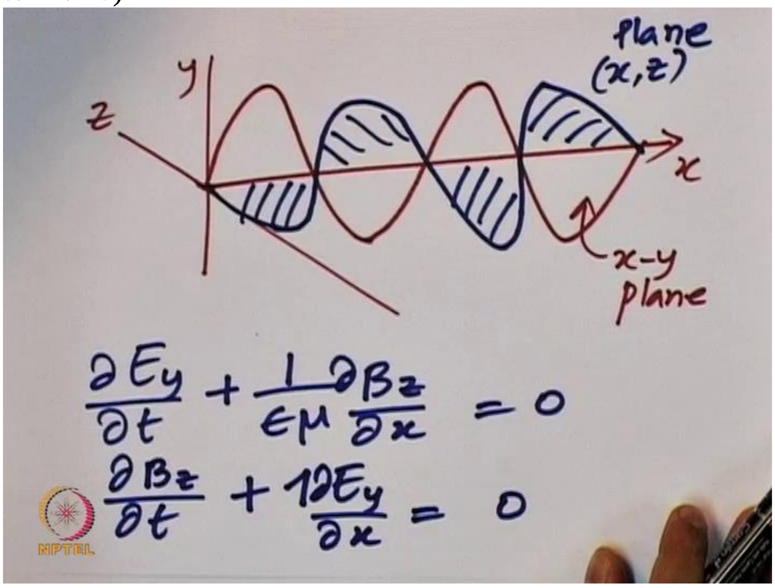

So what we are going to talk about is we have the case where the values are changing this is a z axis and this is the x axis this is the y axis and z axis and what we are tallking about is E field is only along the Y direction and its going along x and we have the b field which has only component along the z direction and its also travelling along the so this is along the palne of x z. So this is in a plane of (x, z) plane , and this one is in the plane of x -y. So for this simple equation what we get is we get a form where can write E y d by dt plus 1 by Epsilon Mu dB z by dx is equal to 0. The B having only component along z direction and its partial derivative with respect to x .

And the B component will have a similar partailly differential equation which is written as dB z by dt plus E so I will put 1 partial derivative of E y is equal to 0. So we have a coefficient equal to 1 but here we have a coefficient 1 by Epsilon Mu. I have specifically taken this form so as to get the matrix formulation where we will see that this is very similar to the equation what we had before;

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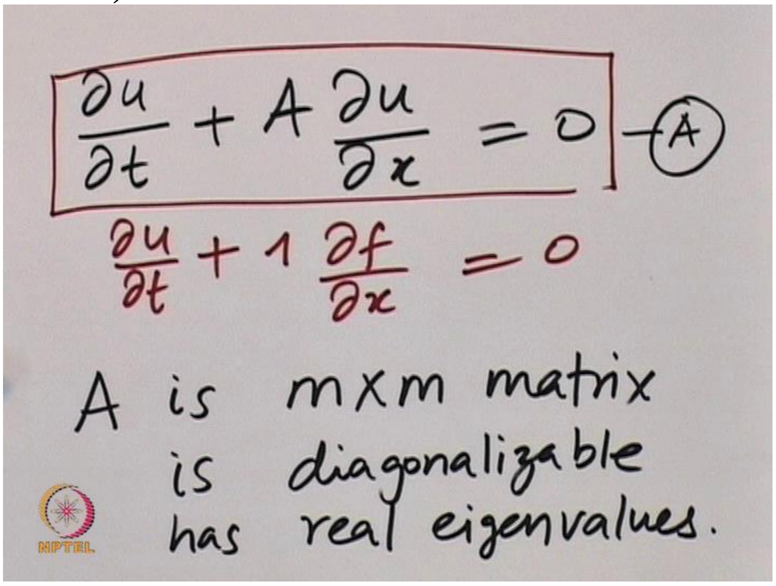

Which is basically du by dt A du by dx equal to 0. So what we have is a matrix form which is A

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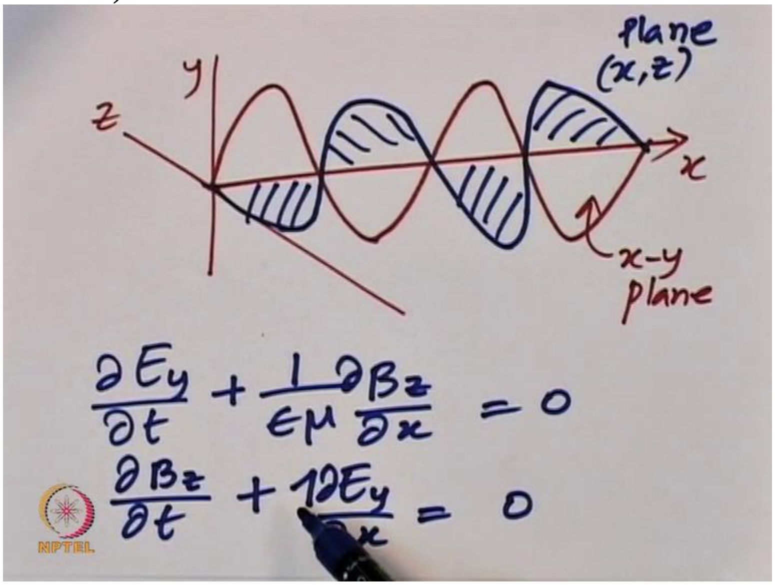

And the components of the A are given by these components here

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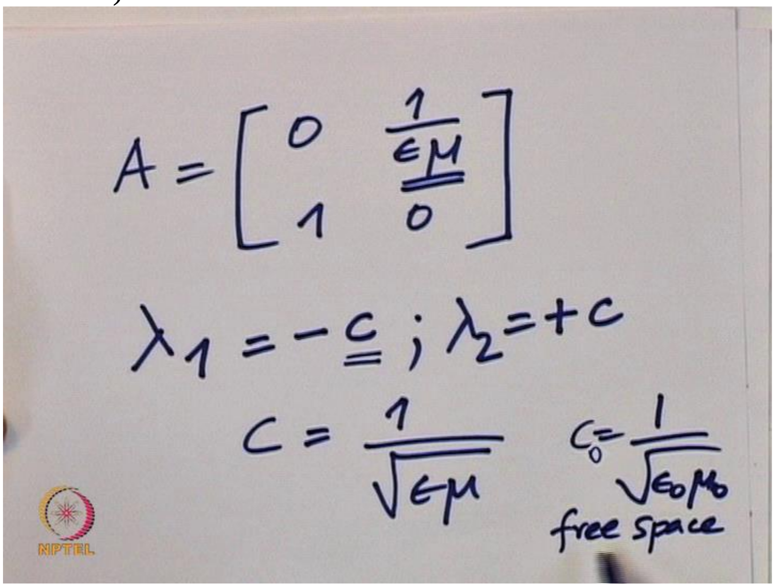

and for this particular case what we will have is A is defined by a m by m matrix. Since its a two dimensional form we will have a 2 by 2 matrix whose values are written as follows: [0 1 by Mu 1 0] And you can verify that this A can be diagonalised and it will have eigen values which are given by Lambda p in this case Lambda 1 is equal to minus c and Lambda is equal to plus c since Lambda 1 and Lambda 2 they are distinct these are a representation for a strictly hyperbolic system. And obviously there is a connection between the c and the components here. So c is nothing but 1 over square root of Epsilon Mu.

So this is the characterisitc velocity in which the wave propagates. So as I said Eigen values are nothing but characteristic values and these values are defining the certain properties of the particular hyperbolic form in this case we see that these are two directions in which the wave propagates. As I said eigen values are nothing but characteristic values and these values are defining the certain properties of the particular hyperbolic form in this case we see that these are the two direction in which the wave propagates. One is the left propagating the other one is the right propagating wave. And obviously when we talk about the free space we will have c is equal to 1 over square root of Epsilon 0 Mu 0. This is the case for a free space or vacuum. We have seen now how the hyperbolic form is useful.

Because you might wonder why that we start talking about hyperbolic form or eigen values or eigen vectors when we can directly go and model the entire equation. Its always better to get a physical feeling for the kind of equation we are talking about. So in the case of Maxwell equation its a hyperbolic equation and these equation will have real and distinct eigen values. And these values are nothing but the propagation and velocity in the free space it is the free space speed of light.

And so with this understanding we will look in the next module about the discretisation itself how are we going to discretise it? What are the benefits of doing that? What are the values that we are storing? And we will also pay attention to the actual Maxwell equation for numerical solution. Thank You!