Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No 26
Finite Volume Time Domain Method-I

Welcome back in the precious lectures we have looked into finite difference finite element methods which are one of the most widely used methods in computational electromagnetics or computational mechanics to a larger extent. But there is yet another scheme which is quite popular in computational fluid dynamics which is gaining attraction is the method of Finite volumes. This method particularly gained attention in the last 30 40 years particularly owing to some of the recent developments in computers because we can use excellent modeling techniques that can be applied for various electromagnetic problems. So we will look into method of finite volume looking at time domain applications.

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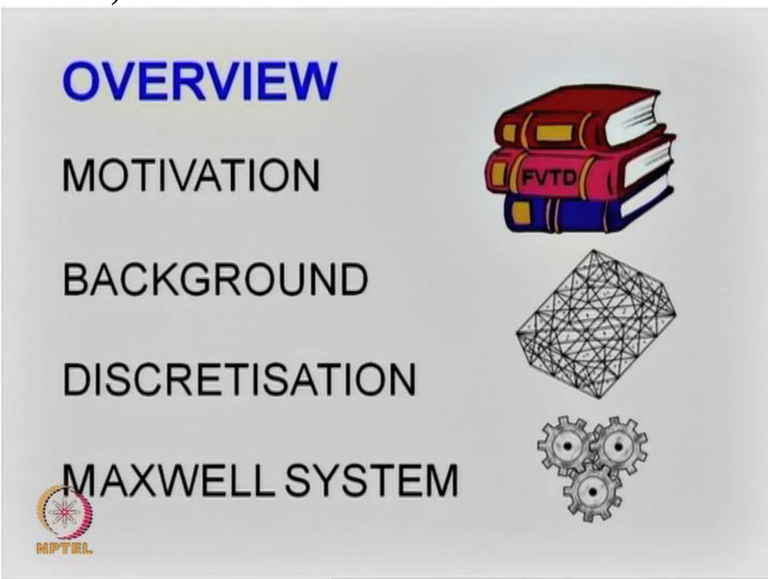

The motivation will be the starting point for our lecture today why do we have to consider finite volume as compared to finite element or finite difference method.

I will discuss on the formulation where it comes from what are the motivation for such a formulation and things of that sort.

And I will describe little bit more on the spatial discretisation. We will of course also look at time discretisation at the later stage. Here we will be mostly interested in spatial discretisation.

And of course we will end this module with a nice way of modelling Maxwellian system basically the curl formulation in Finite volume time domain frame work.

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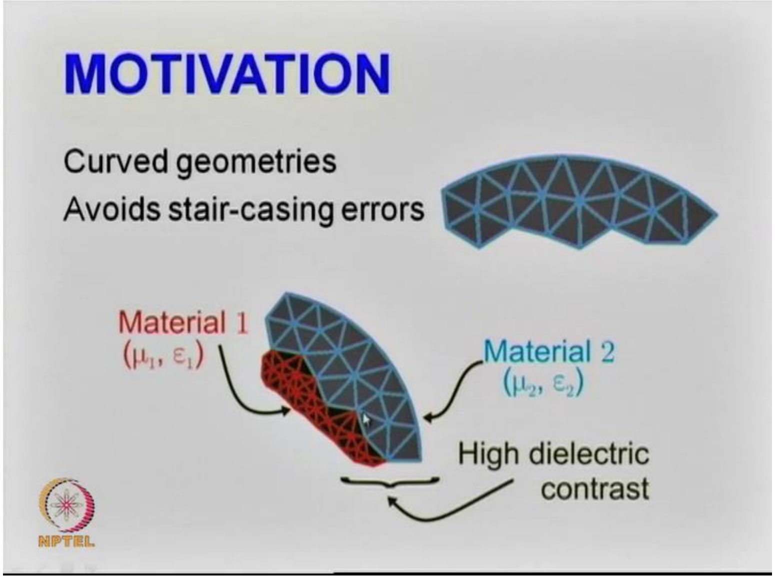

So let us start with the motivation one of the biggest challenges what the finite difference method is facing is the stair casing error. Especially when you have geometries which are oblique or curve which has very fine details and you know complex geometrical and also physical parameters. Finite difference time domain has a lot of difficulty because there is a lot of stair casing error.

The next advantage is basically looking at the material contrast so what I mean by that is you can have materials which have very different permittivity and permeability but they are very close to each other so they are in a way discontinuous or you know high contrast can be mapped.

This is also an additional advantage over Finite difference time domain method of course you can do the same thing also in finite element but the finite volume time domain method has certain advantage over finite element method which I will describe in a bit.

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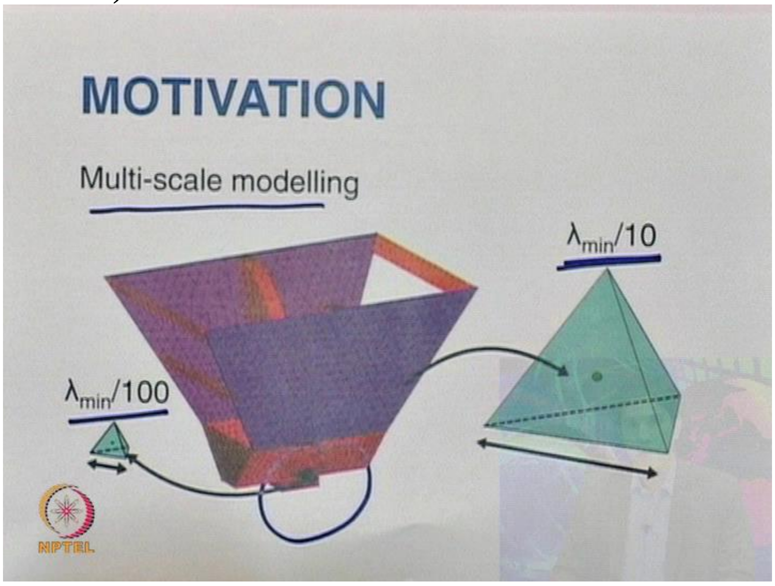

The other aspect is multi scale modelling so what I mean by Multi scale modelling is you can basically think about modelling a double ridged horn antenna. So in this case what you will have is a kind of a structure which is having quite a bit of variation in the spatial discretization. As you can see in the case of the balloon these points we are going from Lambda minimum by 100. What I mean by Lambda minimum by 100 is the minimum wavelength in other words the maximum frequency component will see this discretization as if there are 100 cells wavelength.

Likewise on the outer edges of the double ridged antenna you will have lambda minimum by 10 which is still not as coarse but still coarse enough compared to this is 10 times. So we can basically go from very very small scale to very large scale. And we can even increase this but there are certain pros and cons of doing that but this is something what we call as Multi scale modelling. Finite difference time domain actually allows also the same kind of multi scale modelling but it does not give that elegance. The elegance of the finite volume or finite element is it naturally allows you to really go from a very very small scale to a very large scale.

With that being said this is one of the main advantage of any conformal time domain method. Not only just finite volume but finite volume of course allow this possibility.

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The other one is Broad band capabilities. So what I mean by broadband capabilities is in the case of finite element method what we will see is we will be able to simulate a particular frequency and mostly finite element method is used in the frequency domain formulation. Of course you can also think about the time domain formulation it has its pros and cons you might get an explicit form or a implicit form but any way for a finite element method you are mostly in the frequency domain approach where you have a single frequency that you are exciting and then you are see in the response. Whereas for a time domain method like in any other time domain method you have a broadband capability you can simulate it with a broadband pulse and then you can get the entire spectrum of frequencies that you are interested in. So that is one of the biggest advantages of any time domain method.

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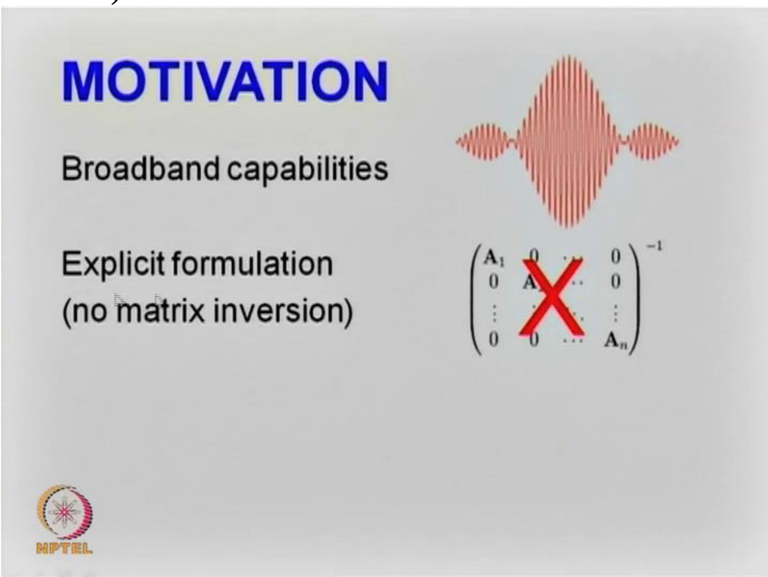

And as I slightly hinted out before in the case of the time domain formulation using finite volumes you are able to get an explicit form. You do not need to do any matrix inversion.like in the case of Finite element method. So the beauty of this is it allows you to do very large scale problems without worrying about the conditioning of the matrices things of that sort. And you do not need to use any specific matrix solvers for inversion and things like that. So it is pretty straight forward approach.

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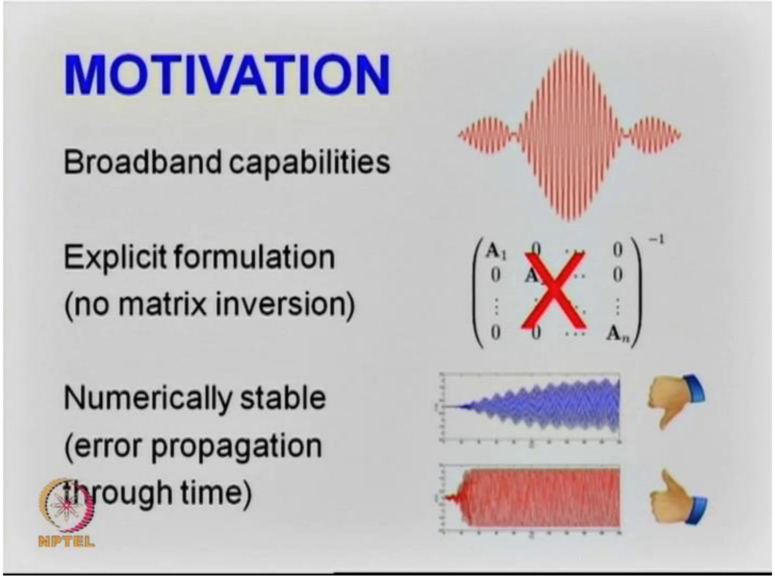

The other advantage is Numerical stability. Let me explain this point a little further. I think we have looked into it in finite difference and also in finite element approach. Any numerical method you want that method to be stable over a long period of simulation. So for example you start with time equal to 0 and you are running it for 1000 cycles you donot want the simulations to crash. Some simulations crash because the error accumulates over a period of time and what happens is you get a kind of a infinite solution which is non physical. So as you can see in this slide this is the energy that is computed over a period of time.

There are methods which are you know over a period of time the energy value increases and then over a period of time it explodes. Whereas schemes like finite volume can give you schemes like finite volume can give you a kind of a stable formulation. THis is very important for running long term simulation I will come back to this particular aspect of finite volume a little later because this is very interesting on one hand it is the advantage of the finite volume but on the other hand it also creates certain doubts about the method usage for various applications .

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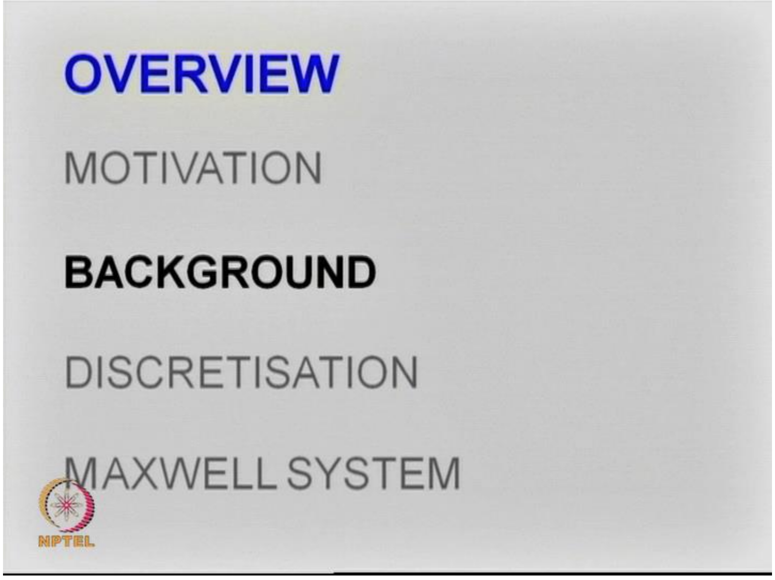

So having said that now we will focus on the backround that we need for this particular method.

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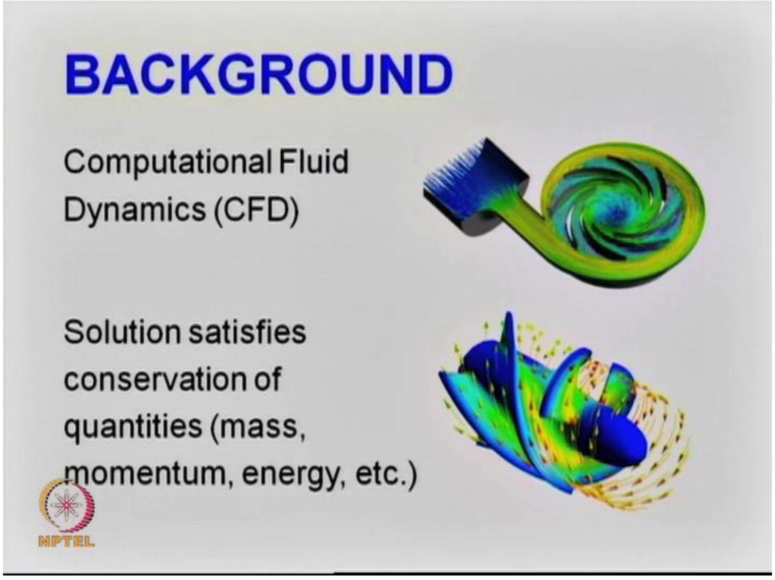

I describe that the method itself comes from the computational fluid dynamics. So it has been used a lot on computing various solutions whether it is mass flow or momentum or energy in various applications and as you can see the various problems in computational fluid dynamics sometimes might be looking very different but in a way they are very similar to the kind of problems what we are dealing in computational electromagnetics.

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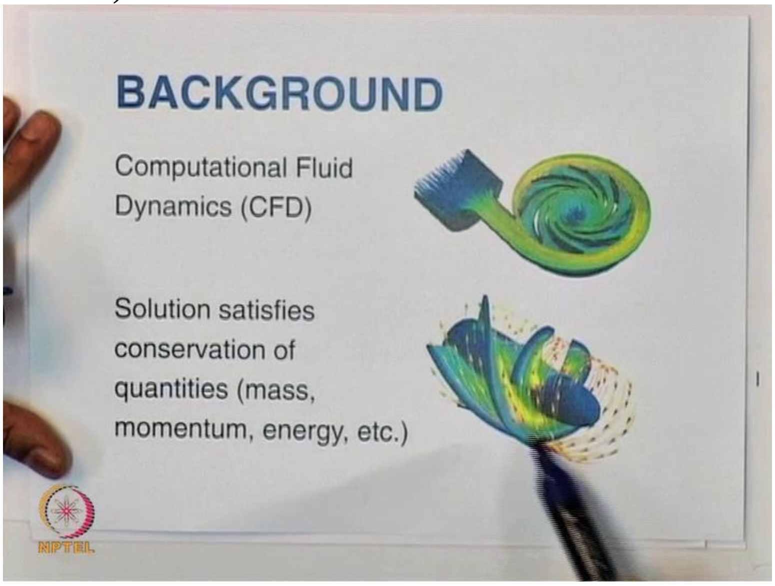

So in this case what you are talking about is the flow of the fluid which is a conserved mass it is conserved not only in individual volumes but along the entire domain and you are able to compute the various field quantities whether its vector or scalar for this particular domain. so having said that what is the basis of finite volume time domain.

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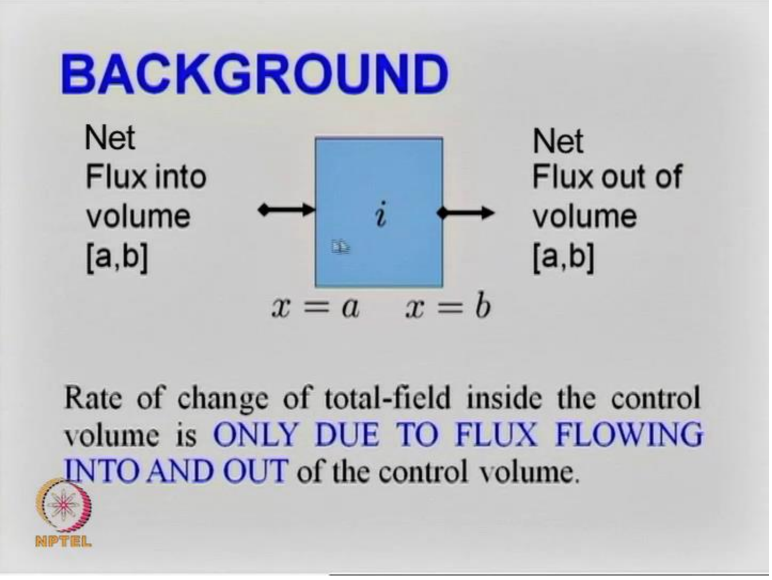

So let me explain this with a simple control volume which I have described here as i. This is a one dimensional problem we are going to look at so we have chosen x equal to a to x equal to b is the domain we are interested in. Having said that we have some flux that is going inside the volume and some flux that is going out of the volume, the only requirement for this particular formulation is the rate of change of the total field inside this control volume is only due to the flux that is flowing into an out of the control volume.

So let me explain this a bit further

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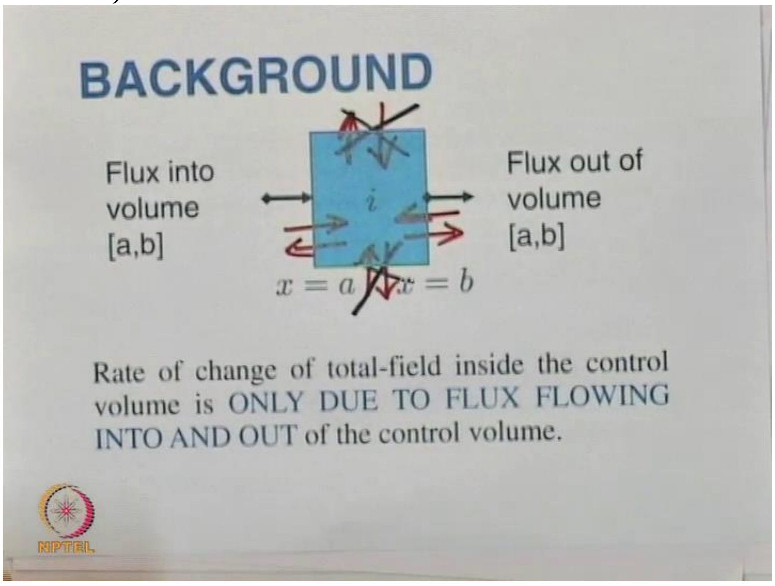

So what is happening here is you are basically having a kind of a control volume and in this contrl volume there are going to be fluxes that are going inside and outside on each of these sides. Since we said that we areo nly going to deal with one dimensional thing we are saying we are not interested about flux changes in the y direction. So this is not the flux that we are talking about, this is not the flux we are talking about. So we are only interested in the flux that is going in the X direction. So what this particular formulation says is the rate f change of the total field inside this volume i is going to be because of the flux that is going into and out. So obviously we are interested in knowing what is the net flux that is going in this side and the net flux that is going in this side.

To know that we have to see how we can know the net flux.

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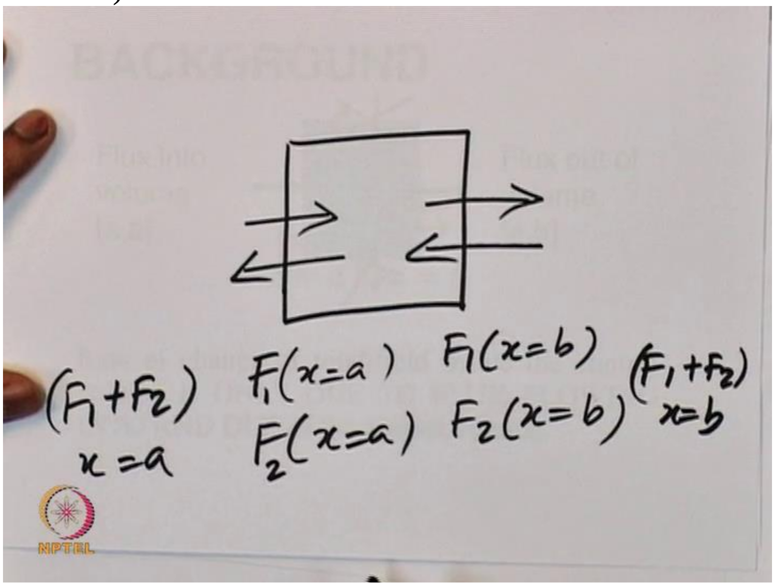

So basically what we are having here is a kind of a volume. And this volume we said that there is a net flux. Let say that the flux going in at x equal to a is F and the flux that is going out is x equal to a let us say this is F 1 and this is F 2 on x a. So this is the flux that is going in x direction calculated at this point and this is the F 1 that is going in x equal to b at this point and F 2 (x equal to b) at this point so what we are interested is the net flux. So the net flux at point a is (F 1plus F 2), this is x equal to a and similarly we will have (F 1 plus F 2) at x equal to b

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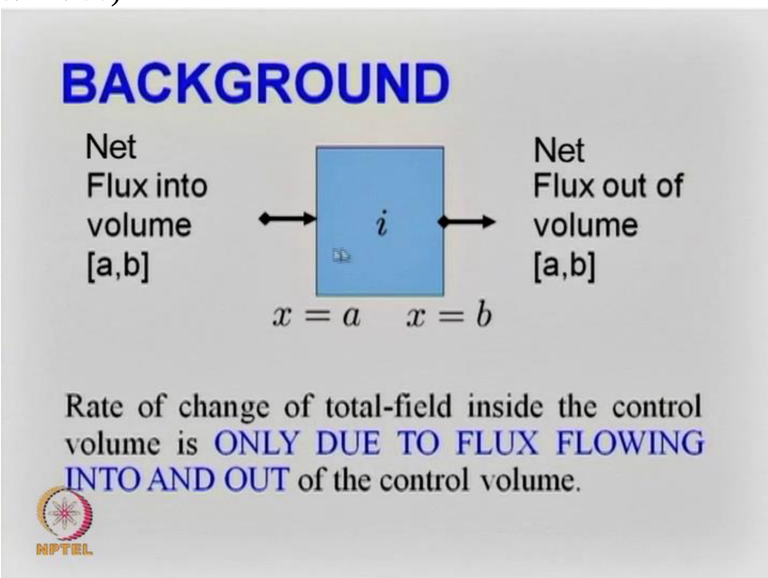

So once we know that we can say that the rate of change of the flux is only due to the flux that is going in and the flux that is going out.

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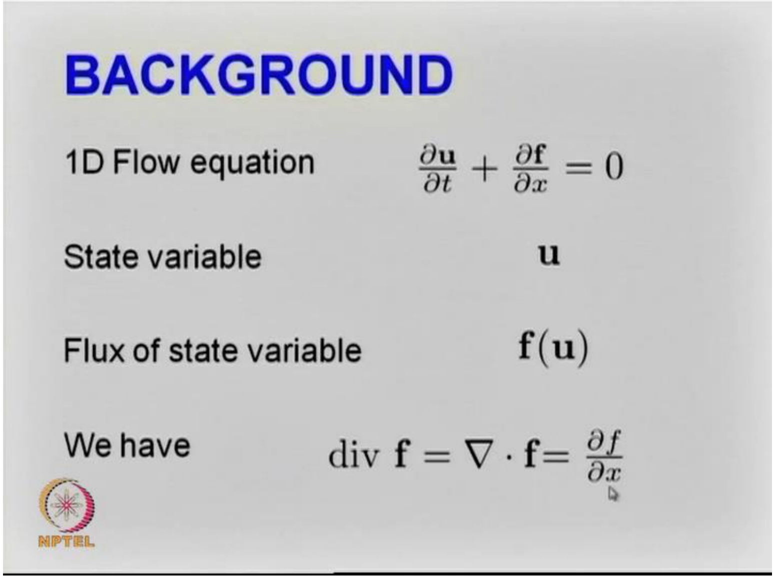

So let us take a simple example of one dimensional flow described by this advection equation. So this advection equation is basically a one dimension equation and we say that the flux is going to change only in the x direction so we have not taken into account any partial derivatives with respect to y or z. So the state variables are those variables that are going to vary as a function of space and time which are the variables of the system which is here u. And the flux of the state variables are nothing but f (u) which is the flux here. And using the diversions we can say that the diversions of f is for this particular one dimensional case is equal to diversions the partial derivative with respect to x. And this is a scalar quantity as you can see it is and it has only one component.

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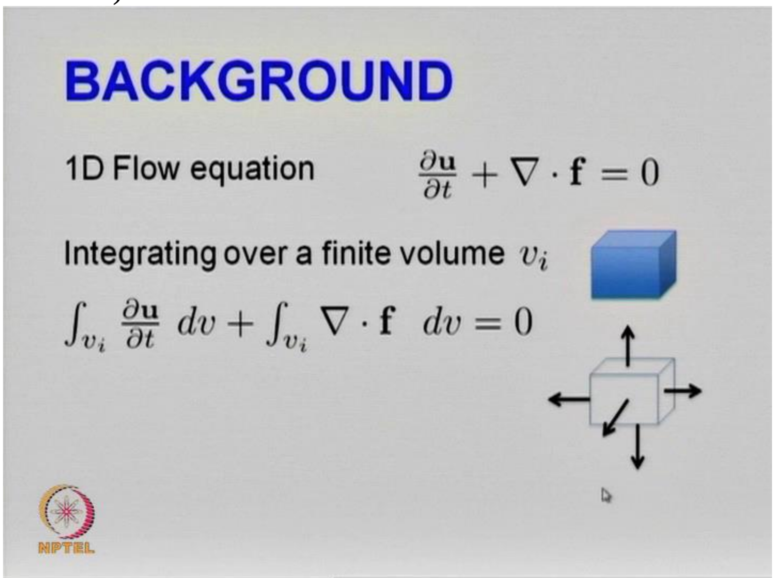

So let us update this left hand side a little bit using our knowledge of what will be the term here.So we have used the diversions of f and we are now integrating this over the control volume let us say v i. So this is the control volume and whose volume is given by v i. So we are integrating this equation over the volume and this is equal to 0. So let us say that the surface of the control volume is represented here which is the s. And this s has individual components if we take it as a cube it has 6 components. So 2 on the x direction 2 on the y direction and 2 on the z direction.

So the normal components of these individual sides are also defined. So a note on the normal component. So if you take let us say a surface and then say the normal is pointing out of the surface you have a made a convention. You can do the same thing using the normal that is pointing inside the surface. So as you might have learned in the basics of the diversions you are talking about diversing from the volume. If you would have read the work of Maxwell he instead of using diversions he might have used conversions basically he is talking about the fluxes that are going inside the volume. Either way it will be a plus or a minus but if you stick to a direction please follow it through.

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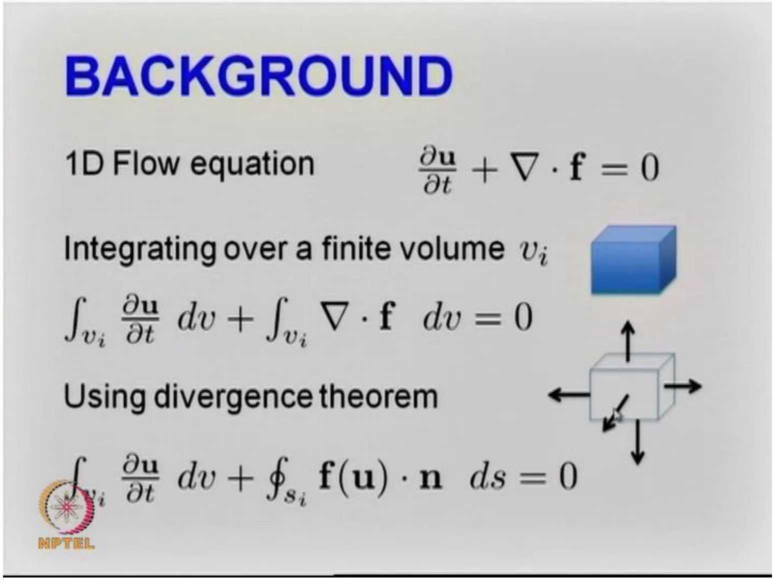

So if we say that we have defined the normal components accordingly what we have now is using the diversions theorem we can change the second term on the left hand side from a volume integral to a surface integral this is a closed surface that is why you have a circle integration and these are the normal projection of the fluxes. So the f.n is nothing but the normal component of the flux that are projected on the individual sides.

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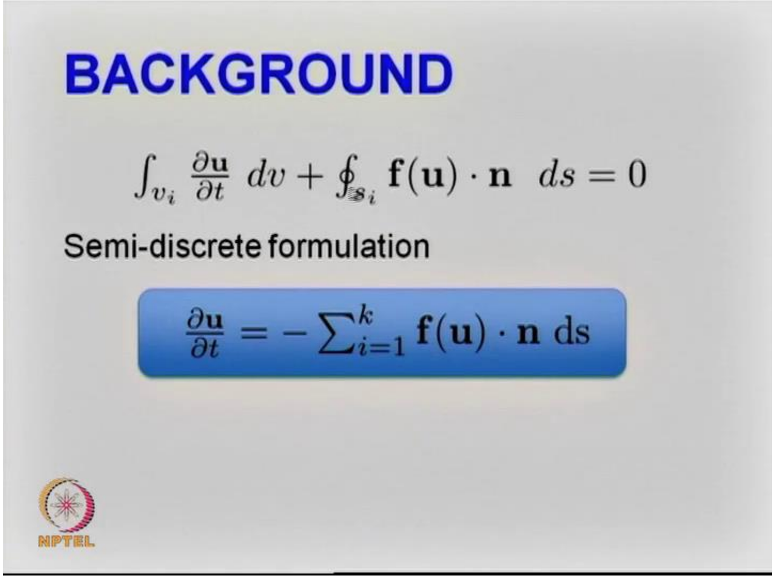

So if this is clear we can go and update this form into a form that we can use in a computational model. So instead of having a continuous second term I can take it on to the right hand side this minus is coming from the right hand side and I will change the continuous integral into discrete summation.

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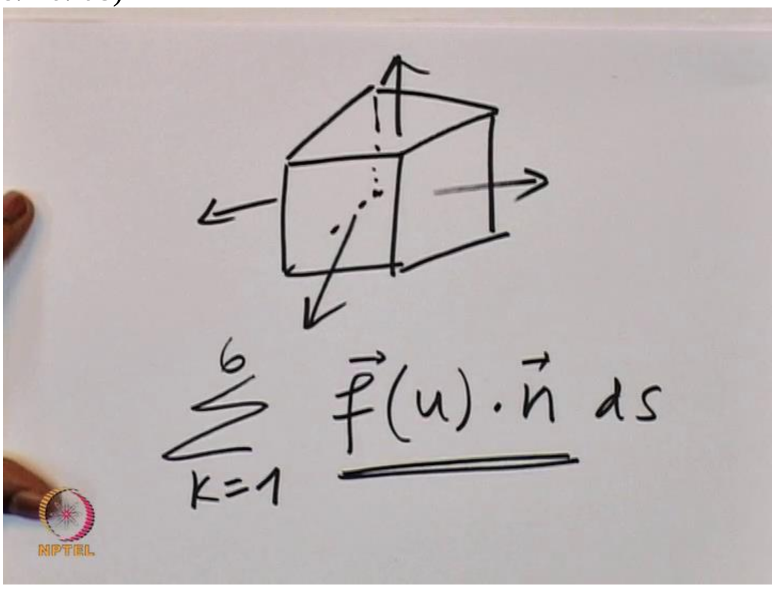

Let me explain this point using a simple cube. So what we are seeing here is basically different sides of cube. And the normal components are marked accordingly right? So instead of saying this surface is a continuous surface I am splitting this surface into sum of all the sides. So k goes from 1 to 6 in the case of the cube. So this continuous equation will become a discrete equation and we will have a flux component . the normal ccomponent times ds this is a dot product because both of them are vector quantities. So what you will get is for each of these sides you will get one unique number that you will use it into this particular equation.

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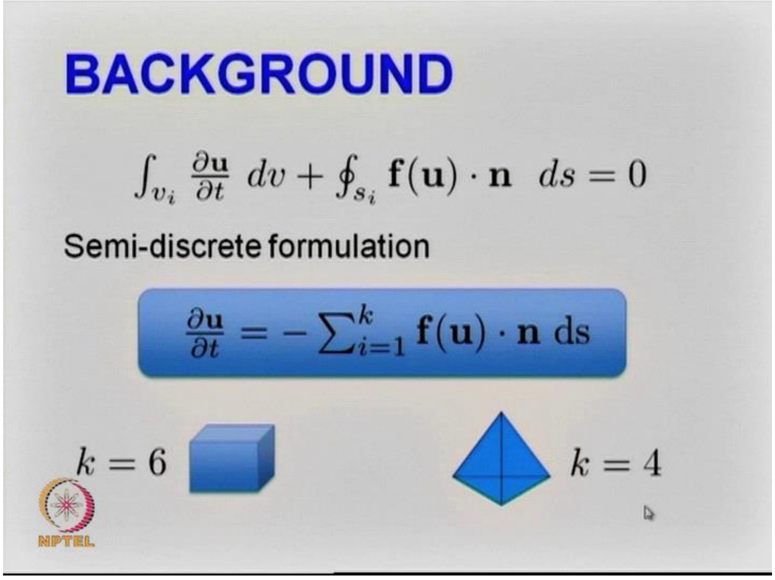

So this being said this form is a form that we will use for most of our application and obviously we need to discretize this particular term as well but we will see this in the following lectures. As I said k equal to 6 if we are talking about a cube. But using a finite volume time domain method on a cubical discretisation is riding a Ferrari and competing for slow car race. So you donot want to do that what you want to use is you wanted to use unstructured grid so you wanted to have the flexibility of the tetrahetron which will give you unstructured grid. And obviously when you use tetrahedron k goes from 1 to 4. So this being said we will now look into the problem of Finite volume using a simple formualtion which I have given you and we will see how this one dimensional equation gives a better understanding of advection problem.

So we will look into example where we are looking at various meaning behind this formulation what the mathematician call as hyperbolic formulation. We will ask the question what is the condition for the hyperbolic formualtion what defines a particular partial differential equation to be a hyperbolic equation and what are the physical meaning behind each of the eigen values for a particular hyperbolic equation.

So with this I conclude this module and let us see in the next module Thank You!