

Computational Electromagnetics and Applications
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Summary of Week 8

This week we introduced the method of moments which is also rightly called as the mother of all methods.

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MAGNETIC VECTOR POTENTIAL

Substituting constitutive relations

$$\nabla \cdot (\epsilon \mathbf{E}) = 0$$

$$\nabla \cdot (\mu \mathbf{H}) = 0$$

As $\nabla \cdot (\mu \mathbf{H}) = 0$,

$$\mu \mathbf{H} = \nabla \times \mathbf{A}$$

where \mathbf{A} is magnetic vector potential

Not a physical quantity

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We presented the mathematical formulation using magnetic vector

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$$\nabla \times \nabla \times \mathbf{A} = -j\omega \epsilon \mu \mathbf{E} + \mu \mathbf{J}$$

$$\nabla \times \nabla \times \mathbf{A} = \epsilon \mu \partial_t^2 \mathbf{E} + \mu \mathbf{J}$$

$$\mathbf{E} = -\nabla \varphi - \partial_t \mathbf{A}$$

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J} + \epsilon \mu \partial_t^2 (-\nabla \varphi - \partial_t \mathbf{A})$$

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu \mathbf{J} + \epsilon \mu \partial_t^2 (-\nabla \varphi - \partial_t \mathbf{A})$$

$$= \mu \mathbf{J} - \epsilon \mu \nabla \partial_t^2 \varphi - \epsilon \mu \partial_t^2 \mathbf{A}$$

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and electric scalar potential

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LORENZ GAUGE CONDITION

More variables than DoFs, so “to fix the gauge”
let

$$\nabla \cdot \mathbf{A} = j\omega\mu\epsilon\varphi$$

From Lorenz
Gauge
condition

Substitute \mathbf{A} into curl equation,

$$\nabla \times \left(\frac{\nabla \times \mathbf{A}}{\mu} \right) = -j\omega\epsilon\mathbf{E} + \mathbf{J}$$



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And we discussed the significance of Lorenz Gauge while modeling the method of moment problems.

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WAVE EQN USING POTENTIALS

We have $\nabla \cdot \mathbf{A} = j\omega\mu\epsilon\varphi$

$$\Rightarrow \nabla^2 \mathbf{A} + \omega^2 \mu\epsilon \mathbf{A} = -\mu \mathbf{J}$$

Recognizing $\beta^2 = \omega^2 \mu\epsilon$

$$\nabla^2 \mathbf{A} + \beta^2 \mathbf{A} = -\mu \mathbf{J}$$

Vector **wave equation** that relates magnetic vector potential and current



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We described the wave equation using Vector and Scalar potentials;

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Z-AXIS THIN WIRE

In matrix form,

$$\nabla^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \beta^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = -\mu \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}$$

For a z-axis oriented thin wire,

$$\nabla^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \beta^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = -\mu \begin{bmatrix} 0 \\ 0 \\ J_z \end{bmatrix}$$

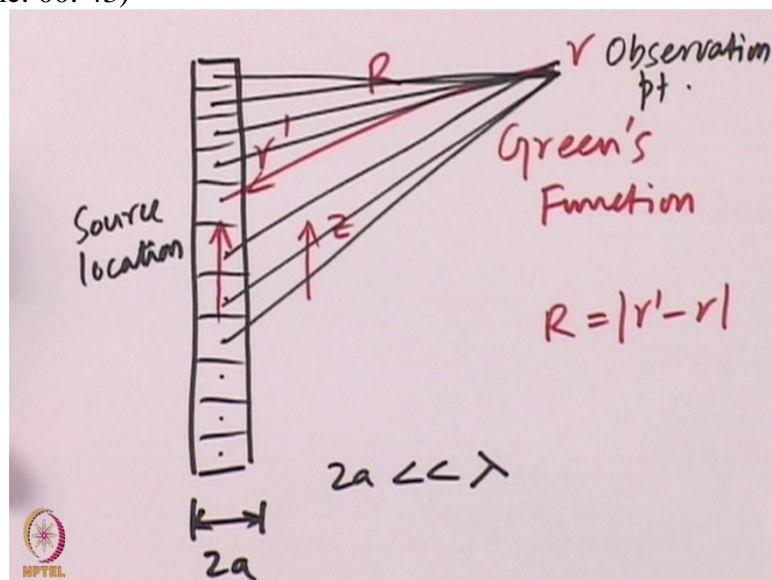
➔ $A_x = A_y = 0$



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And discussed an application of a thin wire antenna

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We explained how to compute the electric field radiated by such thin wire antenna at a far waway point using the help of Green's function

We explained briefly the physical meaning of Green's function while solving such problems,

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GREEN'S FUNCTION

Wave equation for z-axis thin wire,

$$\nabla^2 A_z + \beta^2 A_z = -\mu J_z$$

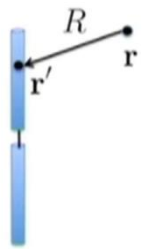
Away from wire $J_z = 0$

$$\nabla^2 A_z + \beta^2 A_z = 0$$

which has a solution

$$\mathbf{G}(\mathbf{r}, \mathbf{r}') = \frac{e^{-j\beta R}}{4\pi R}$$

where, $R = |\mathbf{r} - \mathbf{r}'|$



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we later brought in some practical assumptions related to the geometry of the thin wire antenna to solve them using Matlab environment.

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THIN WIRE APPROXIMATION

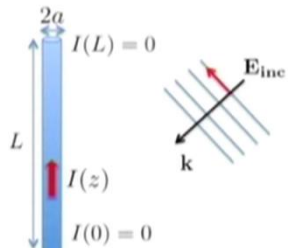
Assume wire is very thin relative to its length

$$a \ll L$$

Incident wave excites current on thin wire

$$\mathbf{J}(\mathbf{r}) = \frac{I_z(z)}{2\pi a} \mathbf{z}$$

Assume no dependence on wire azimuthal angle ϕ and $I_z(0) = I_z(L) = 0$



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We also assumed that the electric field will be independent of the azimuthal angle and the electric current in the thin wire will go to zero at its 2n points.

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INCIDENT AND RADIATED FIELD

E_z^{rad} is obtained from A_z


$$E_z^{rad} = j\omega A_z + \frac{j}{\omega\mu\epsilon} \frac{\partial^2}{\partial z^2} A_z = \frac{j}{\omega\mu\epsilon} \left[k^2 + \frac{\partial^2}{\partial z^2} \right] A_z$$

where, $k^2 = k_0^2 \mu_r \epsilon_r$

BC's require: $E_z^{total} = E_z^{rad} + E_z^{inc} = 0$

↓

PEC approximation



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We later used the perfect electric conductor approximation to solve such problems.

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HALLEN'S INTEGRAL EQUATION


Recall $A_z(z) = \mu \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$

$$E_z^{inc} = -\frac{j}{\omega\mu\epsilon} \left[k^2 + \frac{\partial^2}{\partial z^2} \right] A_z$$

$$E_z^{inc}(z) = -\frac{j}{\omega\epsilon} \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

↓

Hallen's integral equation



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After that we discussed two important integral techniques namely the Hallen and

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POCKLINGTON'S INTEGRAL EQUATION


Starting from Hallen's integral equation

$$E_z^{inc}(z) = -\frac{j}{\omega\epsilon} \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

Move differential operator under integral

$$E_z^{inc}(z) = -\frac{j}{\omega\epsilon} \int_{-\frac{l}{2}}^{\frac{l}{2}} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz'$$

 Pocklington's integral equation

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Pocklington integral equations used in the method of moments problems

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POCKLINGTON'S INTEGRAL EQUATION

Most famous and easier to solve

But slower in convergence and poorer in accuracy compared to Hallen's integral equation

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We also briefly mentioned a couple of pros and cons of these approaches.

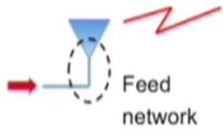
In Hallen's approach resulted in a faster convergence way however it is normally difficult to solve them using computational methods. The Pocklington's approach showed a slower convergence rate and poor accuracy compared to Hallen's approach. However it is easier to solve in terms of numerical computation than in Hallen's approach.

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WHAT IS EXCITATION?

Antenna parameters are easily calculated when it is treated as a transmitting device

Excitation is manner in which energy is fed so as to get radiated



Antenna properties depend on how and where energy is applied



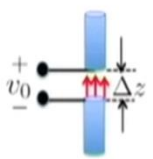
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We highlighted a few techniques to model the source excitations in such problems.


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DELTA-GAP SOURCE

It models the feed as if incident field exist only in small gap at terminals


$$\mathbf{E}_z^{inc} = \begin{cases} \frac{v_0}{\Delta z} \mathbf{z} & \text{at gap} \\ 0 & \text{elsewhere} \end{cases}$$

Simplest source to implement, but less accurate for impedance calculations



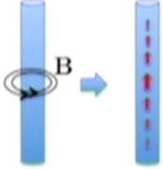
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These techniques include the Delta Gap source techniques;

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MAGNETIC FRILL SOURCE


It models the feed magnetic field circulating around thin wire at feed



$$\mathbf{E}_z^{inc}(z) = \frac{1}{2 \ln(b/a)} \left(\frac{e^{-jkr_a}}{r_a} - \frac{e^{-jkr_b}}{r_b} \right)$$

$$r_a = \sqrt{z^2 + a^2} \quad r_b = \sqrt{z^2 + b^2} \quad b \approx 3a$$

Difficult to implement and more calculations, but more accurate



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Magnetic Frill Source technique and

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IMPEDANCE LOADING

Pocklington's
Integral Equation

$$\frac{j}{\omega \epsilon \Delta z} \int_L I(z') \left(\frac{\partial}{\partial z} + k^2 \right) \frac{e^{-jkr}}{4\pi r} dz' = V(z)$$

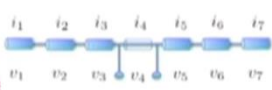
MOM

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Matrix
Equation


$$[Z]\{I\} = \{V\}$$

Perfectly Conducting
Dipole



Impedance Loaded
Dipole

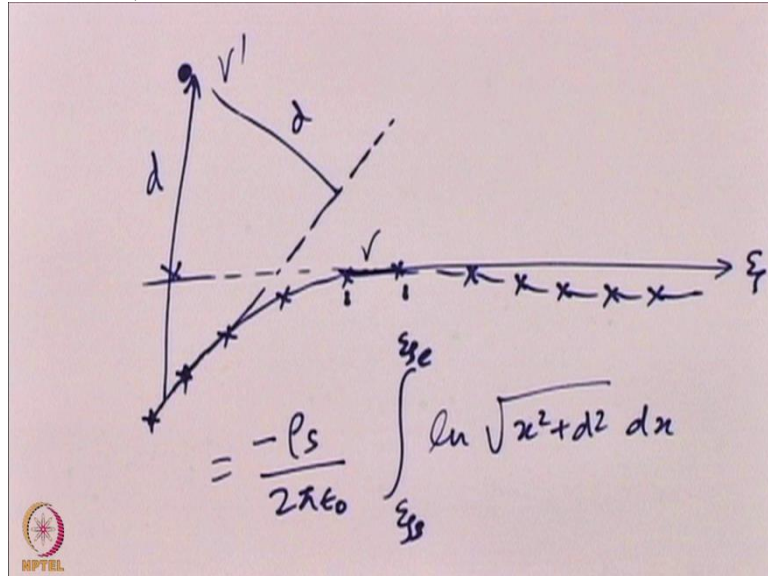




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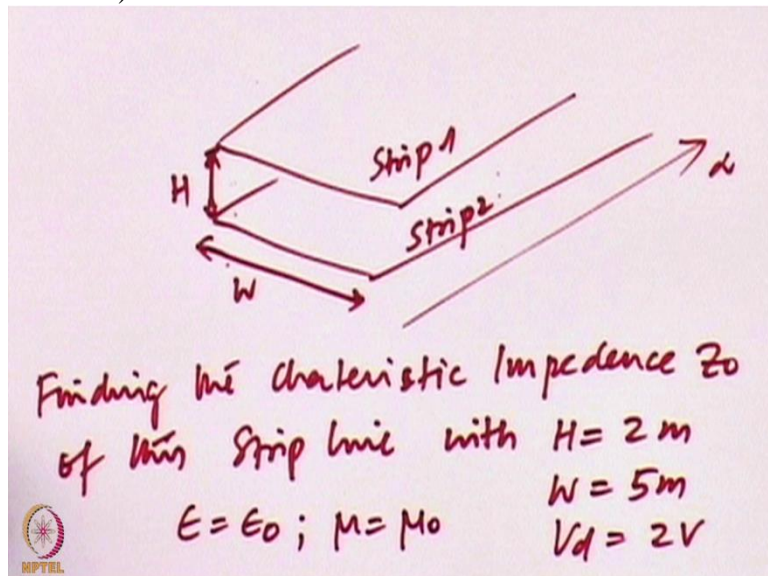
The Impedance loading technique

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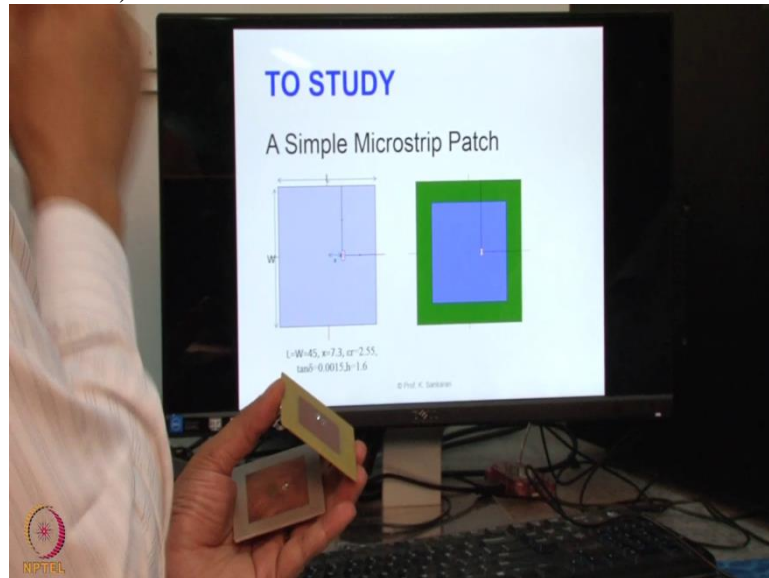
The first problem we modeled using the method of moments in the Matlab environment was that of arbitrary shaped capacitor plates.

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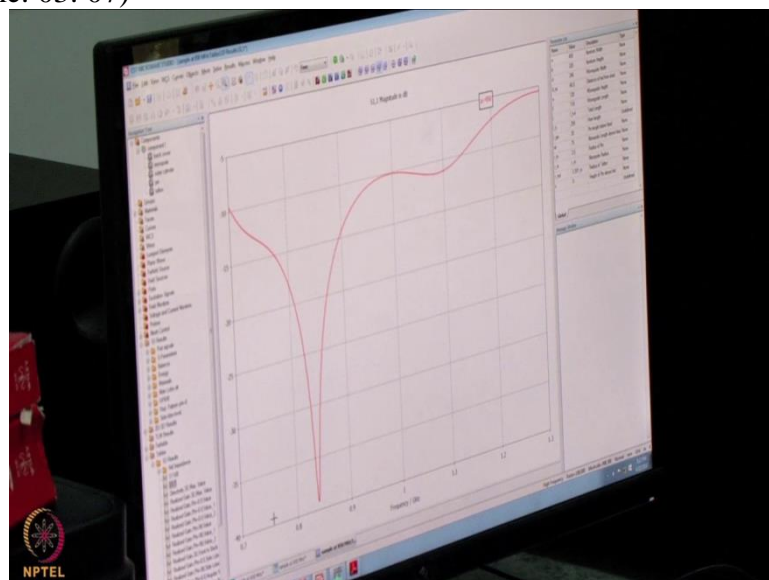
We also demonstrated using Matlab simulation How to compute the characteristic impedance of a metal strip modeled as a transmission line using method of moment.

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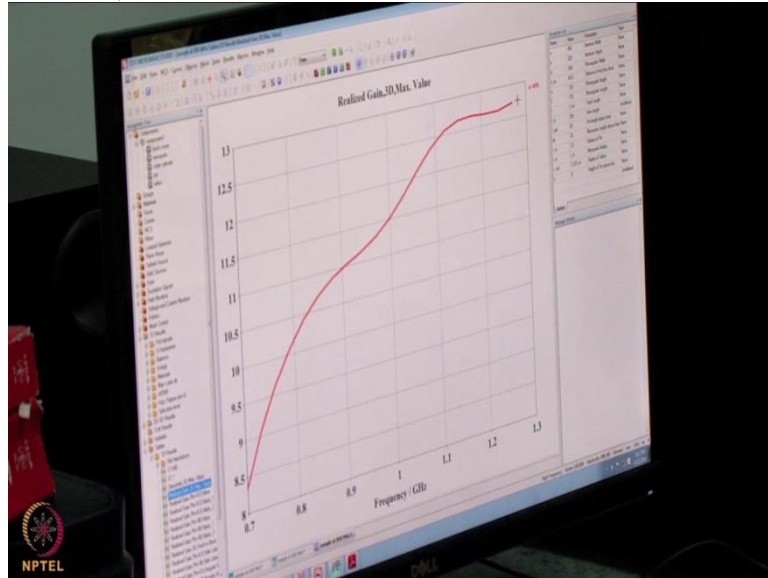
In the lab tour we discussed how to model a practical antenna using commercial solvers.

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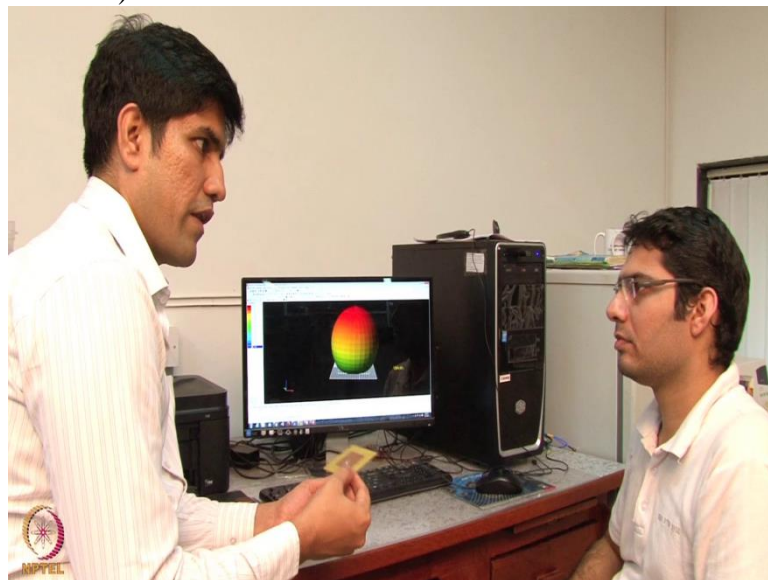
We plotted the S11 parameter.

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Describing the reflection coefficient and computed the antenna gain and radiation pattern.

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We also learned a thing or two about these antennas in terms of their physical aspects and application.

Please go through the concepts and examples that we discussed in this week and try coding simulating such problems yourself or in a group. We are more than happy to support you on the coarse forum to clarify your doubts and answer some of the questions that you might have. Get ready for the next week and until then Good Bye!