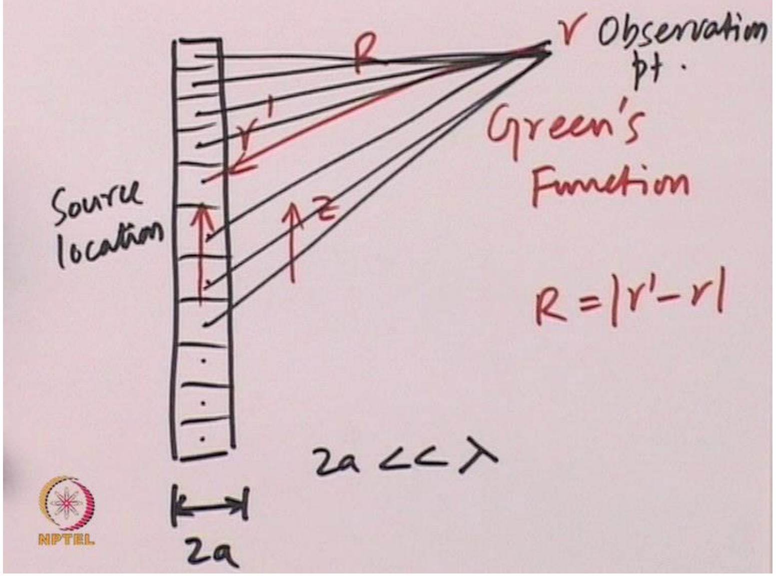
Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No. 24
Method of Moment

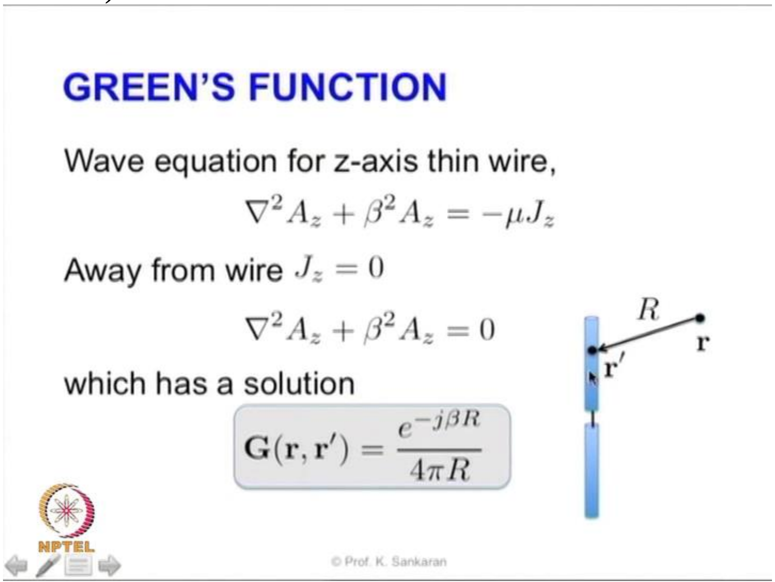
We are in the mother of all methods which is otherwise called as the method of moments which involves quite a bit of mathematics we are almost there so follow up; keep up the spirit, so we will follow into the Green's function discussion which we began in the last module.

(Refer Slide Time: 00:36)



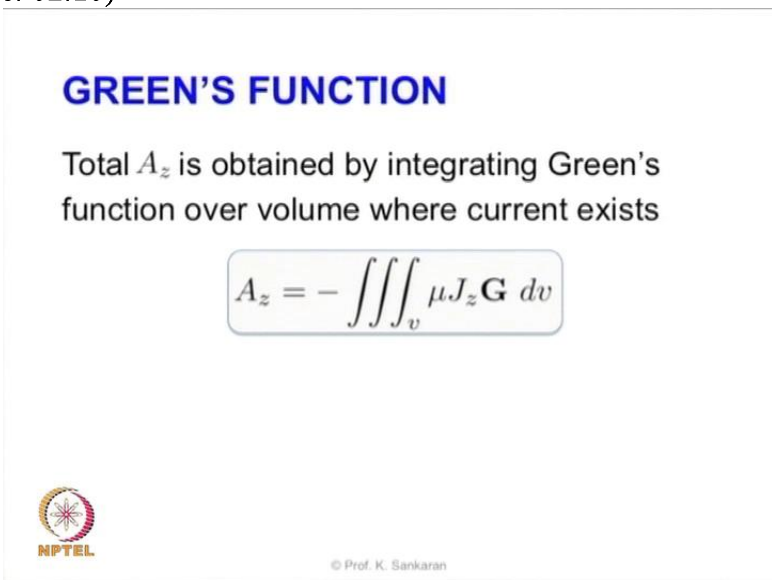
As I said the Green function is a response function to a source that is located at a particular location r prime and we are looking at the response of that source at r which is the observation point. In the case of the thin wire antenna we are going to sum up all the Green's function response and then going to the overall response. So let us start with the wave equation which is the for a z axis oriented thin wire antenna.

(Refer Slide Time: 01:05)



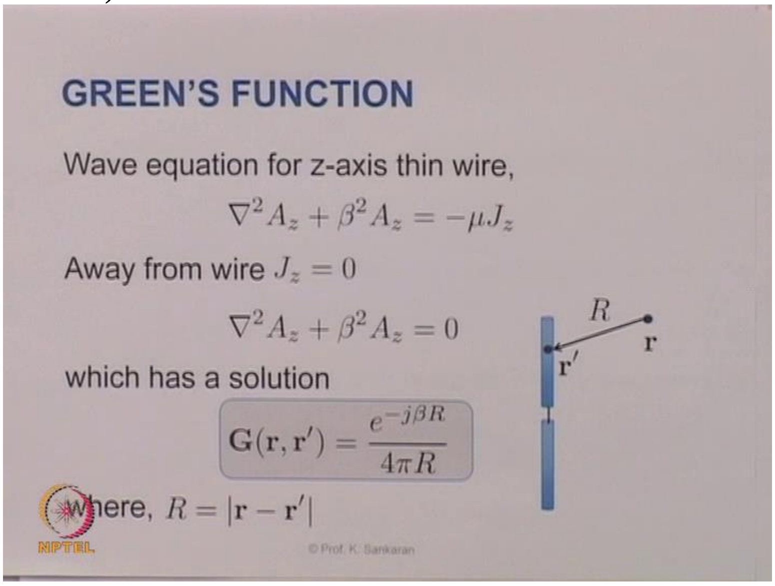
So what we essentially have here is an antenna which is in the z direction. And we have the wave equation which we got from our earlier discussion, we have used the Lorenz gauge to come to this particular point. And now we what we are going to do is going to go away from the antenna, so the Jz is going to become 0. Sowhat we will get is a homogeneous wave equation with 0 on the right hand side, which has a solution of the form which is nothing but the Green's function solution which is going to have an exponential aspect and 1 by 4 PiR where R is the distance from the point of observation to the source location. So now this for one particular source location and when you are going to integrate it along the entire source location what you will get is the total A.

(Refer Slide Time: 02:10)



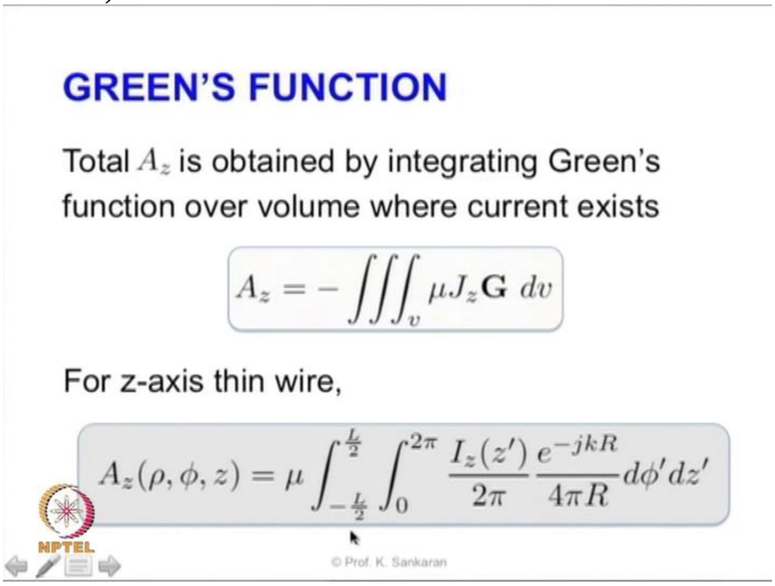
So the total Az is obtained by integrating the Green's fucntion over the volume where the current exists. So what we are doing is we are going to integrate along the entire volume.

(Refer Slide Time: 02:23)



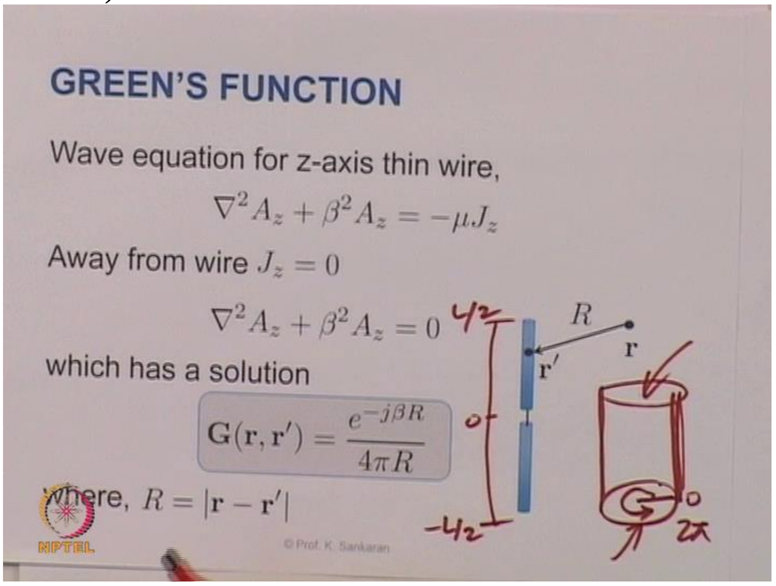
When I say the volume what we are gong to do here is basically this entire volume. And the beauty of the method of moment is as you will see in a bit is to transform this complicated equation which is Az.

(Refer Slide Time: 02:31)



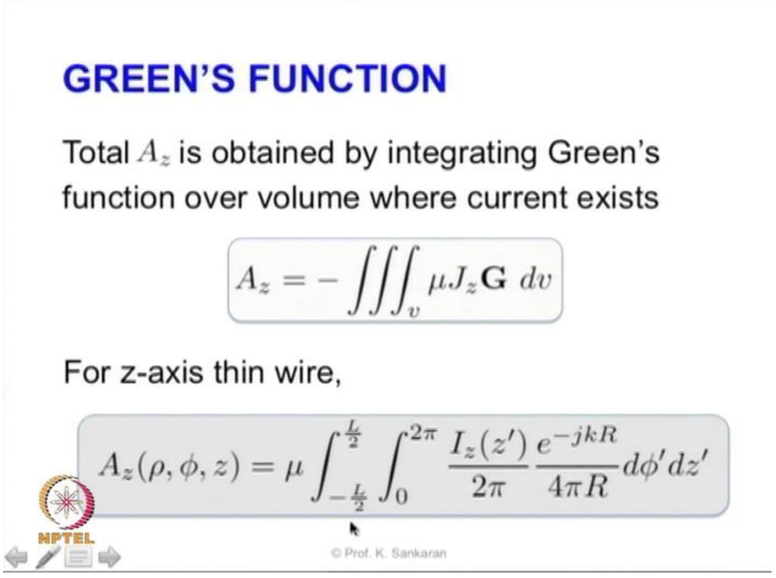
Which is a volume integral into a simple surface integral. So what we are going to do now is we are going to transform this expression using the value of the Green's function and with the understanding of what this value of Jz is going to be we know Iz which is the current that is flowing through the conductor. And we are transforming this volume integral into a surface integral and the surface integral is going to be from a distance which is 0 to 2Pi.

(Refer Slide Time: 03:21)



So what we have here is this entire distance is going to be value that goes from minus l by 2 to l by 2. So the entire distance is going to be l and the centre point is going to be 0 and the integration surface is going to go from 0 to 2 Pi. So we are integrating along the entire surface which is going to be the surface of the wire itself right? Since the value of the thing will going to be opposite to this surface and the bottom surface. The integration will cancel out. So the current if it is flowing in one direction What is going to happen is whatever is coming in is going to go out, so the surface integral on the top this part and the bottom part is going to cancel out. So we are interested only on the surface which is the area that is on the top of the wire.

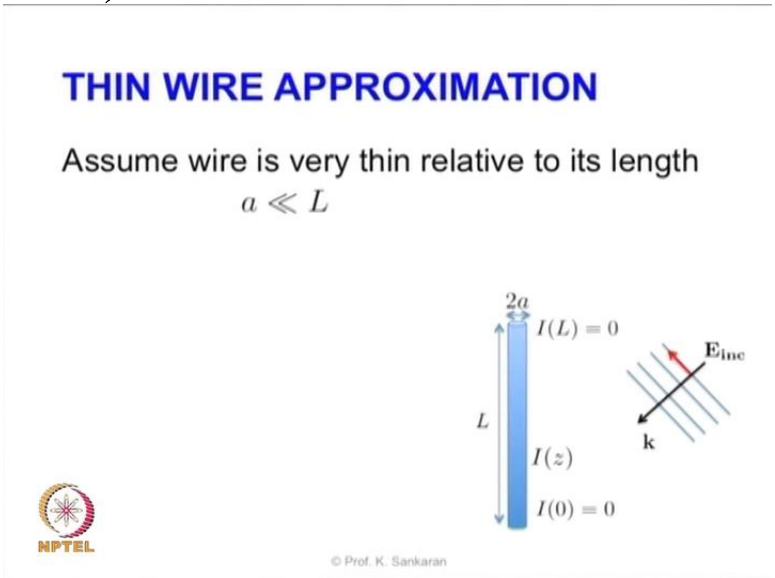
(Refer Slide Time: 04:32)



And this integration as the situation shows is going to be minus l by 2 to plus l by2 and 0 to 2 Pi and we have a variation that is interms of phi and z alone. And there is no variation on

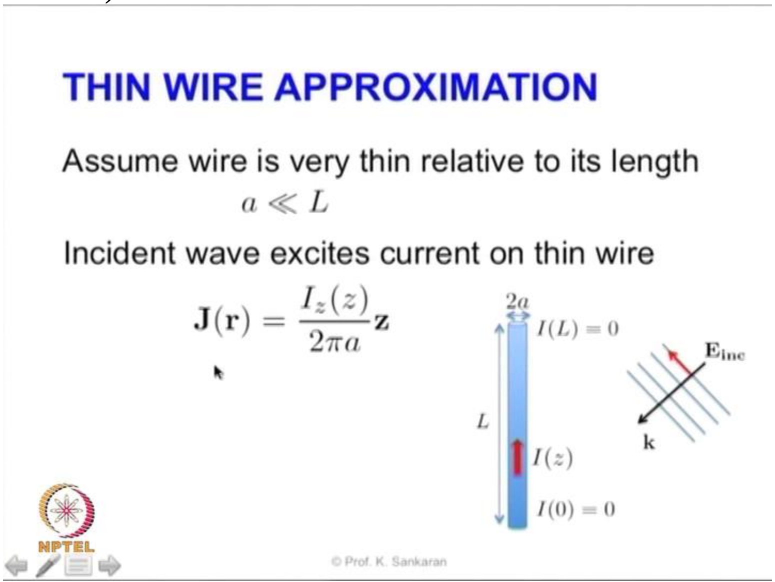
Rho, Rho is here the radius of the wire itself and this we will keep it here as a constant here you donot have the Rho value here we are interested on what is happening on the surface.

(Refer Slide Time: 05:03)



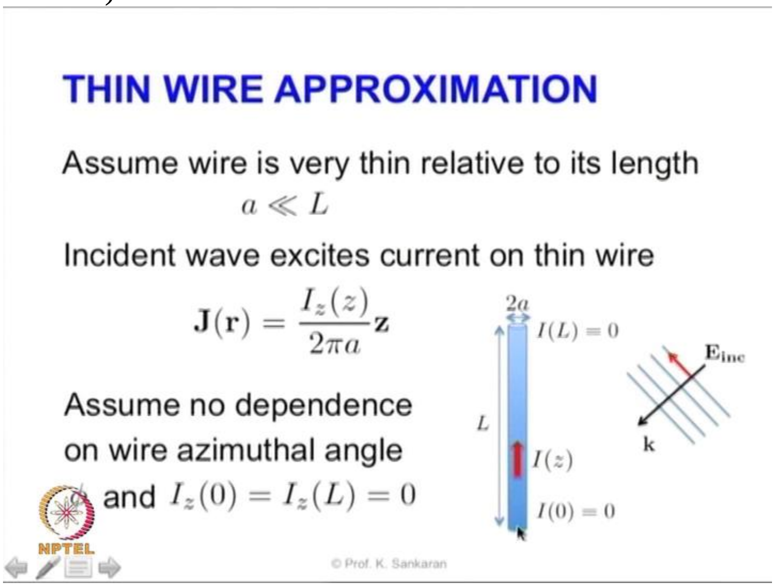
So what we have now is assume that the wire is a very thin with respect to its wavelength or the length itself here when we are talking about a thin wire antenna what normally you will look into is the lateral dimension versus the cross section itself is going to be very important. We do not talk in the terms of wavelength here we talk in terms of the length of the wire versus the cross section of the wire. When the length of the wire is going to be extremely large as compared to the cross section of the wire we can do this approximation. So what we are going to do here is when A is very very small compared to the length itself we can do the approximation as follows.

(Refer Slide Time: 05:45)



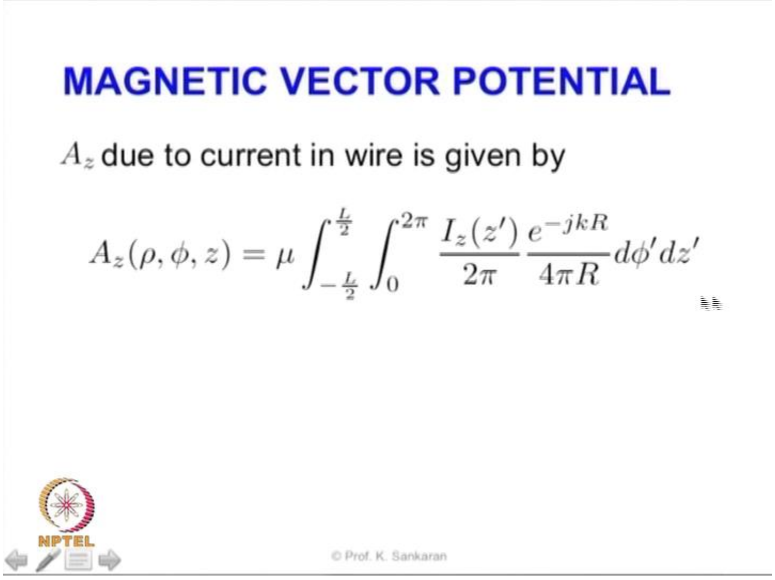
What we have is an incident wave that is coming with direction of propagation given by the vector k and the incident electirc field as certain orientation and it is going to be incident on this particular wire. What we see is the incident wave excites the current on this thin wire. So what happens here is the J(r) is going to be given by the expression Iz which is function of the z axis divided by 2Pir where r is equal to the radius a and z of course is the direction of the J vector.

(Refer Slide Time: 06:30)



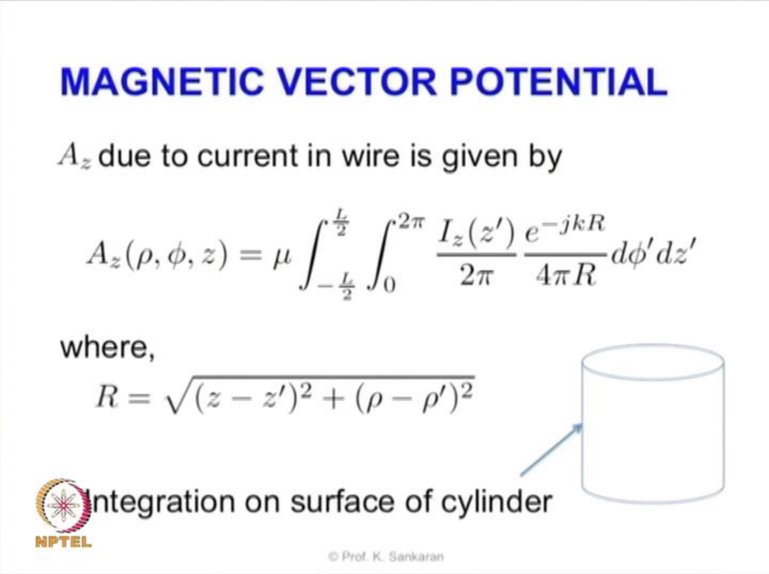
And now assume that there is no dependence on the wires azimuthal angles which is given by the Phi and Iz of 0 where when we start with the point here and at this point the current goes to 0, so the current goes to 0 at this terminal and at the other terminal.

(Refer Slide Time: 06:55)



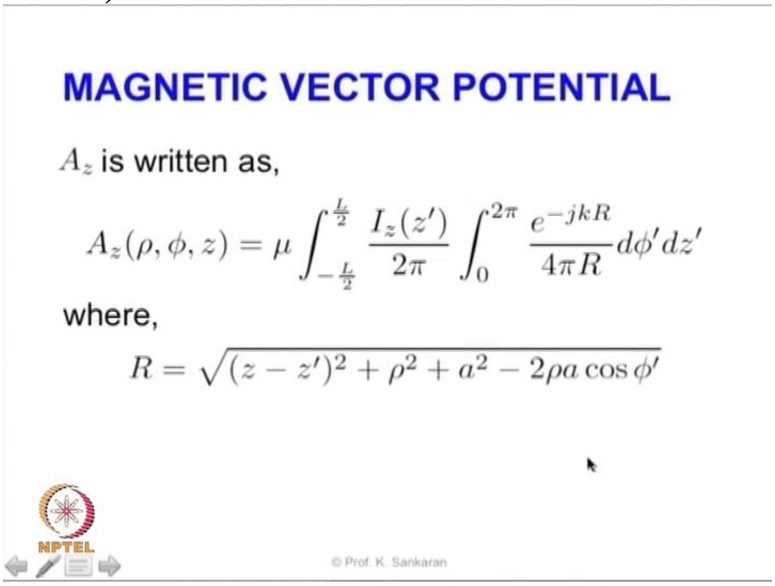
So when we assume these two things what we get is the Az due to the current in wire is given by the expression, it is the function of Rho Phi and z as usual is equal to Mu we have the two integration which is going form minus l by 2 to plus l by 2. And we have the integration on the surface of the angle which is the 0 to 2Pi and we have the current vector the current density vector and the Green's function here and then we are integating on those two integral quantities.

(Refer Slide Time: 07:33)



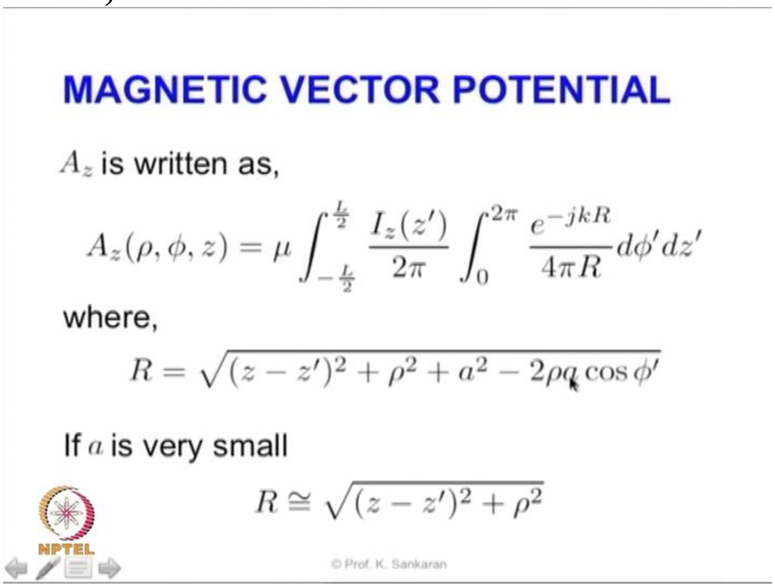
And now when we substitute the value for r which is located at a distance r from the point of the source itself. remember the source locations are given by z prime and rho prime. And the value of r is equal to the square root of z minus z prime square plus Rho minus Rho prime square so the integration is done on the surface of the cylinder as I mentioned.

(Refer Slide Time: 08:00)



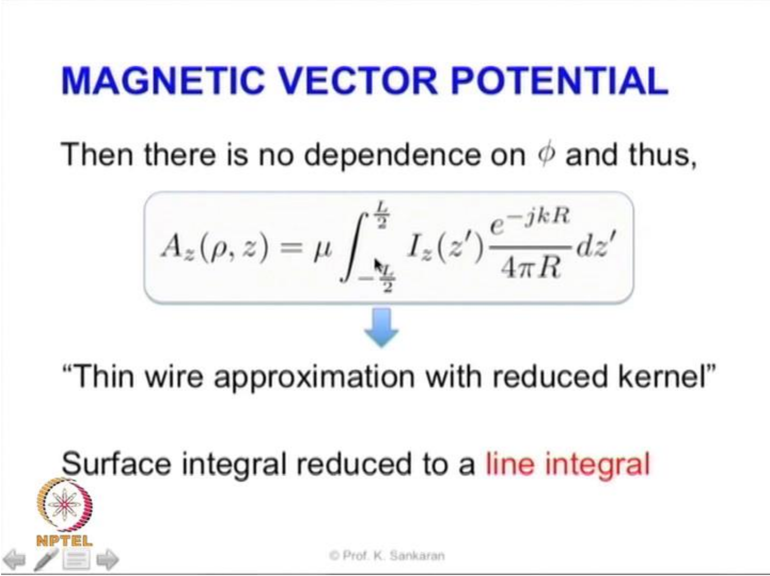
And now since Az is written on the surface of the wire Rho prime is equal to A. What we can write here is Rho minus Rho prime square is equal to we can expand this a minus b square a square plus b square minus 2 ab. So we get the value of the different things. And here of course we have a cross dependence which is basically the value of Rho a and the angle the cos theta is going to be given by the difference of the angle Phi minus Phi prime. Due to the cylindrical symmetry what we have considered we can replace the value of c with just this value. So now what we get is Rho minus rho prime square is equal to Rho square is equal to Rho square plus r square minus 2 Rho r cos of this angle.

(Refer Slide Time: 08:58)



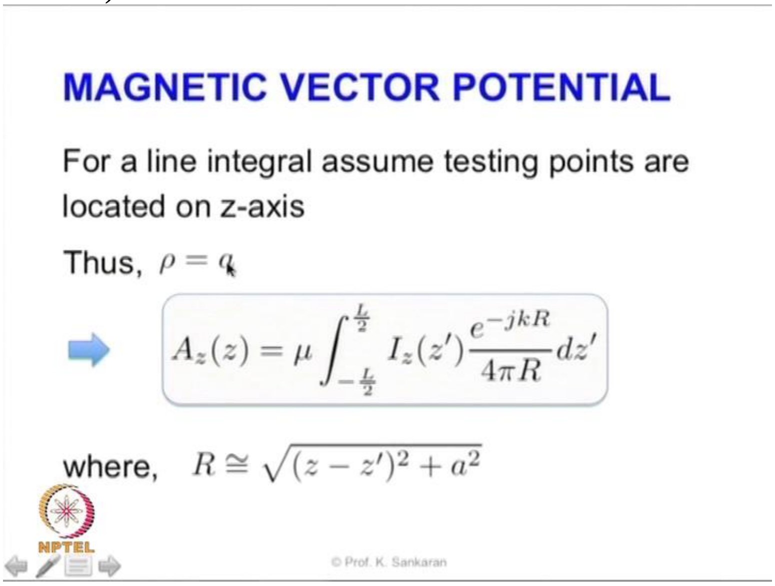
And now Az is written as like before we can bring the Iz outside of the both integral and we have only the integration along the line and this thing is to stay for the second integral inside and as before we have substituted the value of r. And if a is small then what we can do is this term goes to 0 and this term also goes to 0 what you essentially have is a square root of z minus zprime) square plus Rho square.

(Refer Slide Time: 09:39)



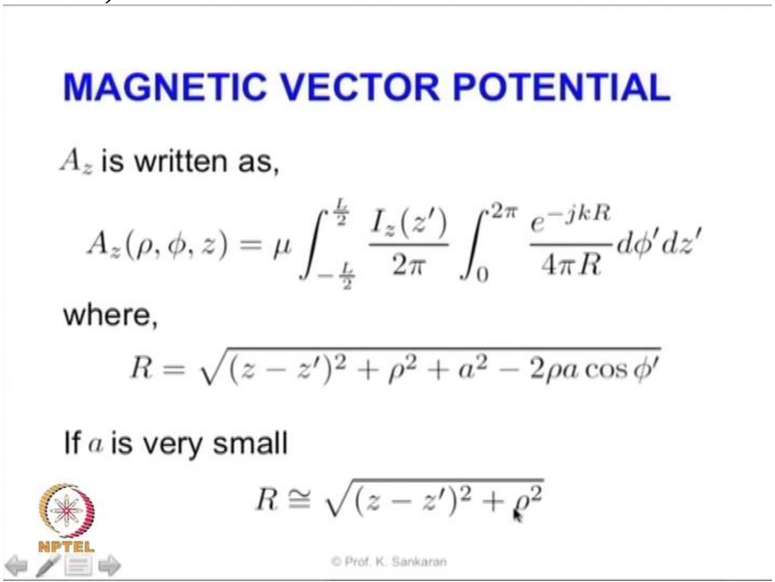
Then there is no dependence on the Phi and thus we can write the value of the value of Az without the angle only using rho and z. And it is given by this expression here which is Mu intergral over the entire line Iz multiplied by the Green's function and we are integrating it over the entire line. So what we are getting now is a thin wire approximation and this is the way where we have introduced when we started remember that we started with the volume integral and then we have transformed the volume integral into a surface integral and now we are going to reduce the surface integral into a line integral that is what we are going to see here. What we have done is basically reduced the value another dependence to from three dependence we gone into only two dependence. This is essentially a line integral here and the surface integral is further reduced into a line integral.

(Refer Slide Time: 10:51)



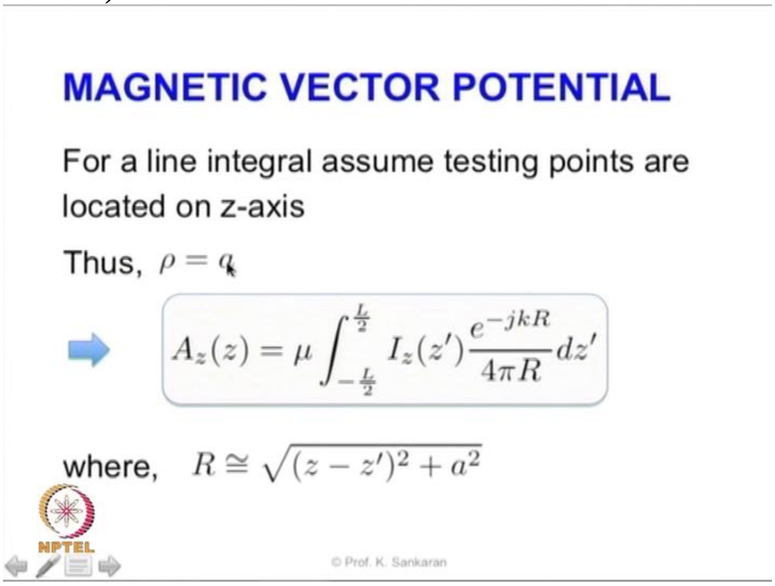
For a line integral assuming that the test point is located at a point let us say at Rho equal to a. What we have got as an expression as before A z is going to only depend on the value of z. So instead of taking Rho as a dependent variable we are only interested on what is happening on the surface of the wire. So we substitute the value of Rho equal to a

(Refer Slide Time: 11:18)



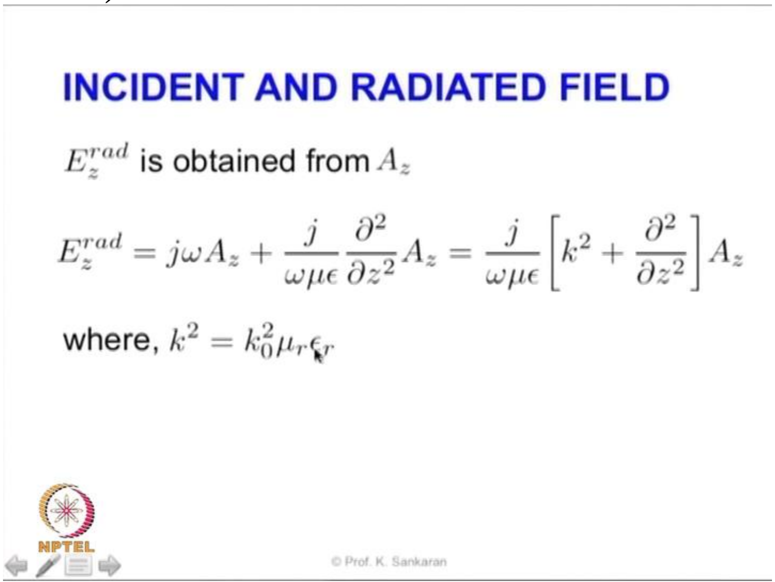
Into the previous expression here and we can substitute the value accordingly. Remember in the previous expression the R value was here Rho and we are now substituting the value for Rho as a.

(Refer Slide Time: 11:37)



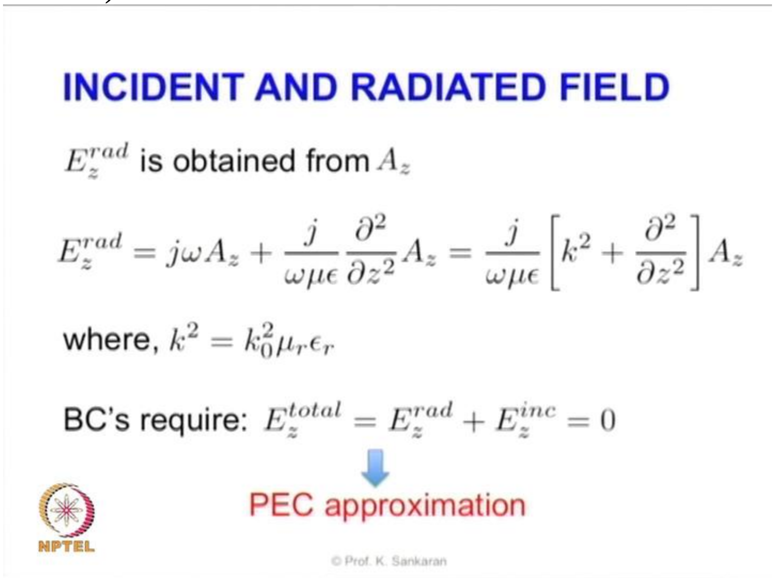
So with that what we have done here is transformed the entire equation from a volume integral to a surface integral to essentially a line integral.

(Refer Slide Time: 11:46)



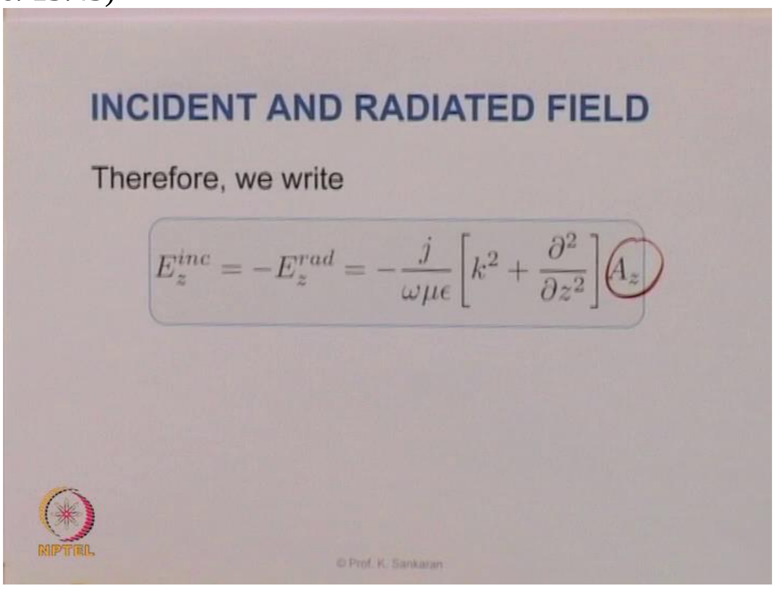
Now let us look at the incident and Radiated field. So the radiated electric field is obtained from A z using this expression, remember this is nothing but the initial equation what we had in the case of computing the electric field only using the value of A z. And there where we had we had the time derivative of A z plus certain coefficinet and then the Laplatian of A, and here the Laplatian of A will only have only one term which is the Doe square by doe z square term and you can take the j divided by omega Mu epsilon outside and essentially you will have an equation which is given by this term which is k square plus Doe square by Doe z square and then you take the A z out. And we can substitute the value k square is equal to k 0 square Mu r Epsilon r which is the value that we are substituting k 0 is the wave number for the free space propagation. And Epsilon r and Mu are the relative permeability and permittivity of those medium what we are considering

(Refer Slide Time: 12:59)



And now we can substitute the approximation that we essentially in the beginning talked about, we are approximating the value of the total field as the summation of the radiated field and the incident field. And then we are saying it is equal to 0. So these are the boundary conditions we are essentially forcing for the problem. On the boundary of the wire the total electric field is gping to go to 0. So this is the PEC approximation the tangentral component of the electric field will go to 0. Here the tangential component will be the Ez component. Hence the total E z component on the boundary of the wire should become equal to 0.

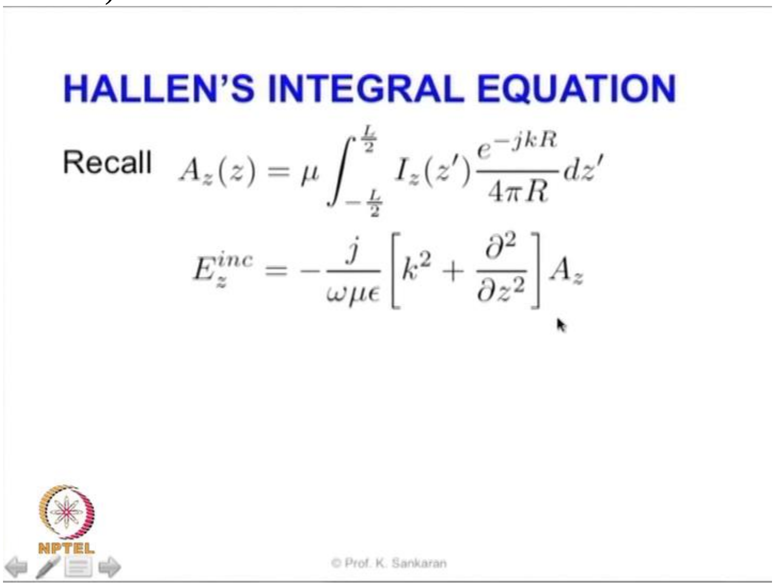
(Refer Slide Time: 13:43)



So what we have essentially got is a term an expression for the incident and the radiated E field E z field purely in terms of the magentic vector potential. And now we have to integrate this along the z to get the value for the total electric field, so that is what we are going to do now. So we have got an expression for this one. And this has to be integrated along the z axis and for doing that there are two ways to go ahead. And those two ways are due to two scientist who have proposed two different approaches one is due Hallens formulation and the other one is due to Poklinton which is called as the Poklinton formulation. Both of them has its own merits and demerits, we will look into it briefly in this next module.

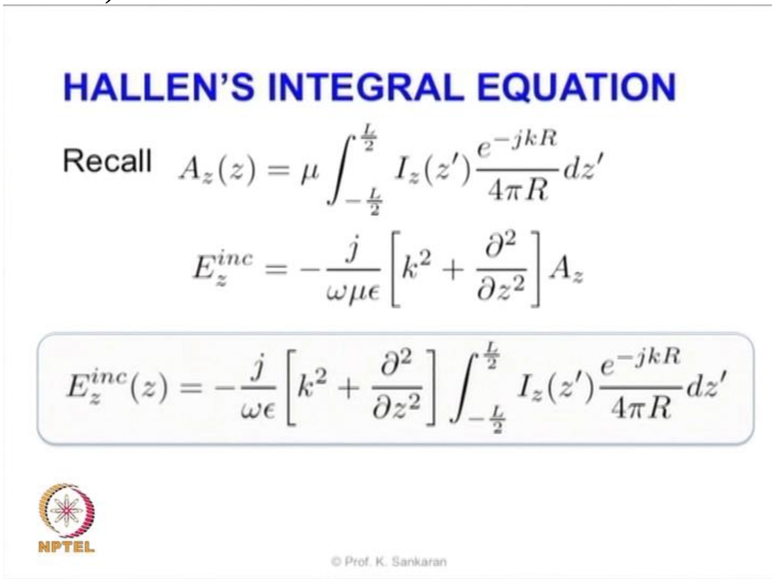
So what we will start in the next module is going from where we are now to the idea of approaching the entire problem using the matrix formulation whatever we discussed in the Galerkin method or so. but right now what we will see very quickly how we can use either the Hallen fromulation or the poklinton formulation to get a close form expression for this problem.

(Refer Slide Time: 15:04)



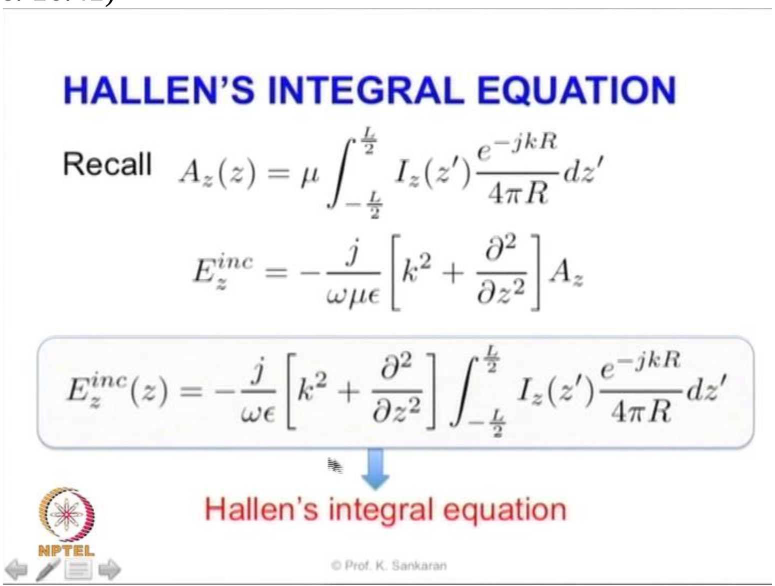
Remember that we used A z as a line integral which goes from minus l by 2 to plus l by2 and the value I z is the current that is flowing in that particular point z prime . And we are going to multiply it using the Green's function here. And then we are integrating it along d z. And once we have that we can substitute the value in the E incident expression for the Az.

(Refer Slide Time: 15:36)



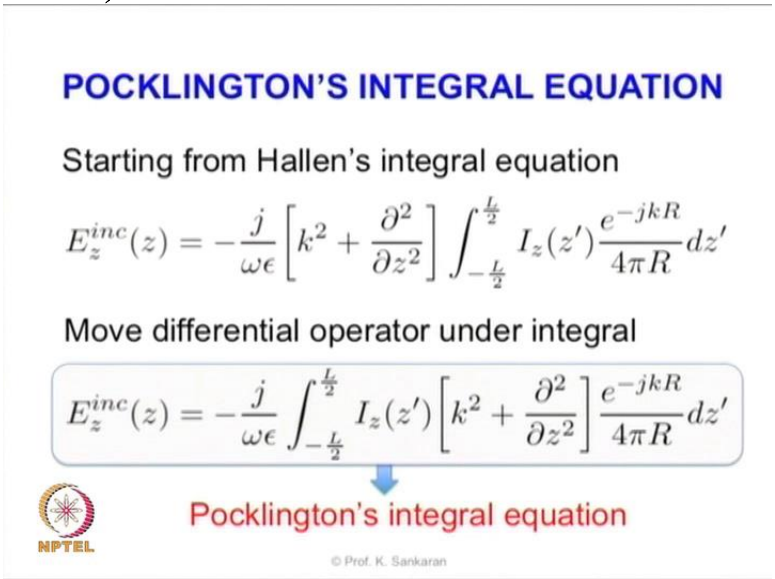
And now what we know is E z is going to be given by this expression and we need to compute the E z by substituting this value for A z here. So what we have is essentially in integration which has certain second order derivative here and the Hallen's formulation is basically going to keep the derivative outside the integration. So it has certain errors that are going to come becuase of this approximation. But there is certain reason for doing this. The reason for doing that is the convergence is going to be fast. The convergence as we see as in number of elements are going to increase along the line. The solution reaches to the final solution quite faster that is what we mean by convergence. While we do this we have to do a little bit more mathematics, however the merit of this is going to be a faster convergence rate.

(Refer Slide Time: 16:41)



So the Hallens formulation is basically going to give us an expression which is basically having the derivative outside of the integration.

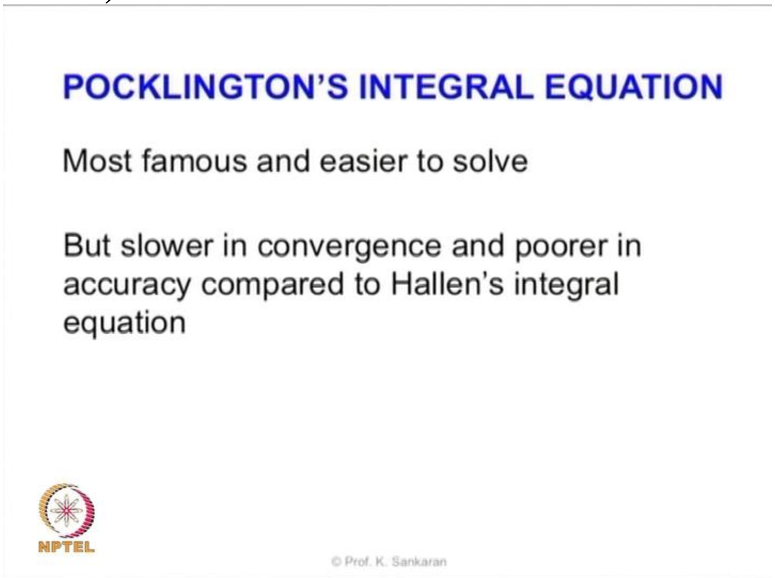
(Refer Slide Time: 16:52)



The next formulation is due to Poklinton as I said which is also called as Pocklingtons integral formulation. It starts with its basic Hallens formualtion. And now it is going to move these differentials into the integral itself. So what we have is basically the ksquare plus Doe square by Doe z square is going to go inside the equation.

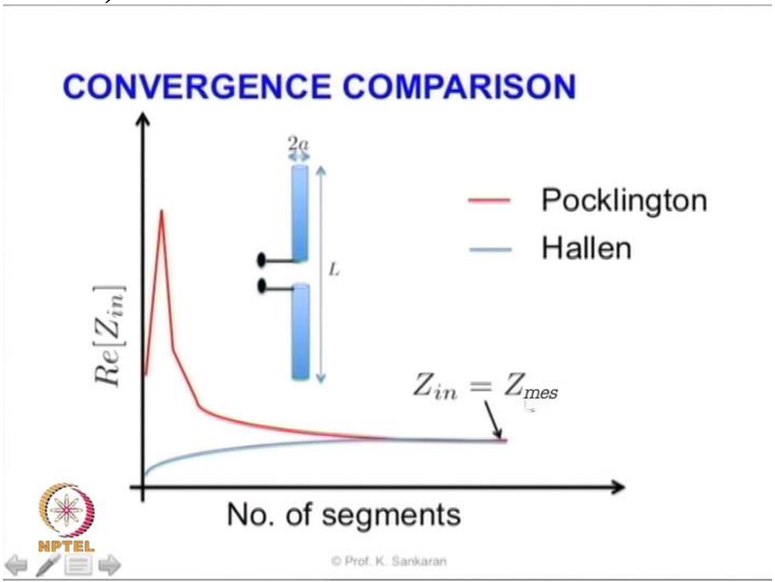
And this thing is going to simplify the entire process of compputing this integral. However its giong to have problems in terms of convergence itself. And also remember that there is going to be a problem also with respect to the singularity and we will see that in the later stages. But what we have essentially done in the Pocklingtons integral equation is moved the differential operator inside the integration itself. And this is the idea of the Pocklingtons integral, and we will most probably use Pocklingtons equation because its easy to compute although the covergence is slow it s the easy and the formulation is more manageable for us to follow through. So we will use it for that prupose. So we will start with the most famous Pocklingtons form which we have discussed now.

(Refer Slide Time: 18:10)



And we will also show how this is going to affect the convergence rate but before that I wanted to repeat that its easy to solve but it has a problem with convergence.

(Refer Slide Time: 18:26)

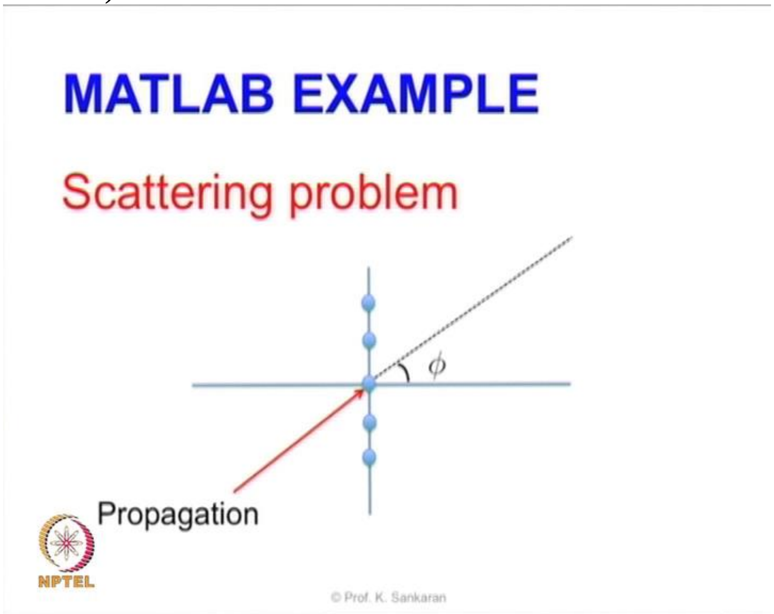


And what is that convergence is going to look like is shown here schematically in this conceptual idea when the number of segments along this line or the wire antenna is going to increase. You see that the Hallen formulation is going to go very close to the value. here what we call as the real part of the z impedence Z in is calcualted and the z calculated here can also be the one which we are measuring in the value here. So the covergence says that as we discussed before the Hallen formulation converges to the measured impedence value quite fast. And also it has a nice curve in terms of convergence whereas in case of the Pocklingtons equation. You see that there is a spike here and this spike is due to the singularity which we will discuss later on but the convergence is quite slow and also as you can see you need to have quite a large number of things to come to the value that is closer to the measured value.

So we will stop at this point before we further into the computation of this problem using Galerkin method, we have now come to the point where we have discussed two approaches to go forward with the Method of Moments for this antenna problem one of them is using the Hallen integral and the other one is the Pocklington equation. And we have set the pros and cons of both of these methods. Both in terms of the convergence and in terms of the ease of computation itself. So for most of the applications we might end up using Pocklingtons integral equation.

With that being said we will stop at this point and we will come back in the next module to compute the entire problem using the pocklingtons equation. We will also show step by step algorithm to compute this entire problem in the Matlab.

(Refer Slide Time: 20:23)



And also we will look into one simple problem of scattering case. The scattering problem is an exact opposite of the antenna problem itself, where the scatterer and the antenna is only on the definition of the source location if the source of the electromagentic field is going to directly sit on the object of interest then the object is going to behave like antenna. But if the source of Electromagnetic radiation is going to be far away from the object of interest then that object is going to behave like a scatterer. So in this case when we talk about scattering problem we are talking about a problem where the source at certain point and we are interested in finding out what is happening when the electromagnetic wave is coming incident on the scatterer. So we will do some example using the scattering problem as in test case using matlab. But we have also discussed about the antenna problem more in elaborate form right now so that will essentially complete the idea behind the method of moments for practical applcations. We will come back to look into the algorithm for computing this problem. Thank You!