Computational Electromagnetics and Applications
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Exercise No. 14
Finite Element Method-II

We looked into the problem of coaxial cable and capacitance calculation and voltage calculation in the earlier exercise.

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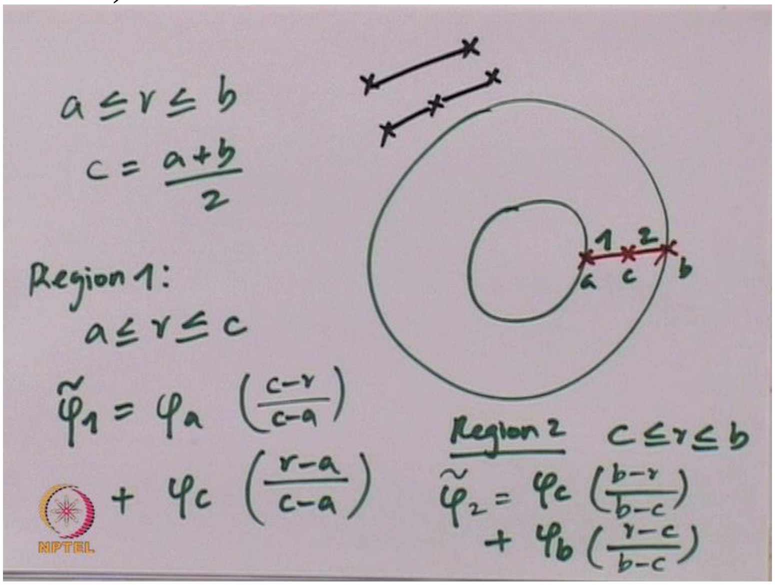

And I said we are going to improvise on the method that we used before. And this time we are going to go for higher order approximation. But before going to do that I would like to give you one or two important remarks on the problem what we did before. So let us look into the problem geometry one more time.

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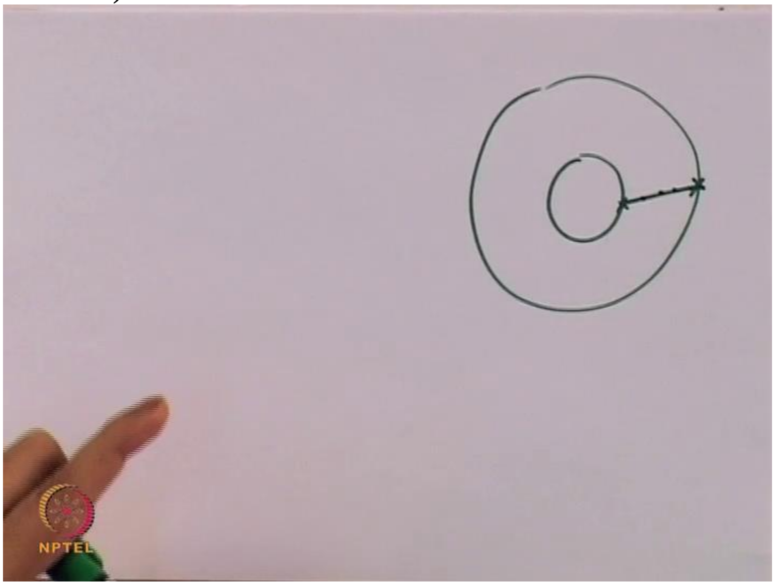

We have an inner conductor and an outer conductor. And we had points that are sitting in the radial direction. We had several points several nodes. So what happens is when you have a single line as the value and you are able to see the result of those values both in the theoretical calculation and in the numerical calculation.

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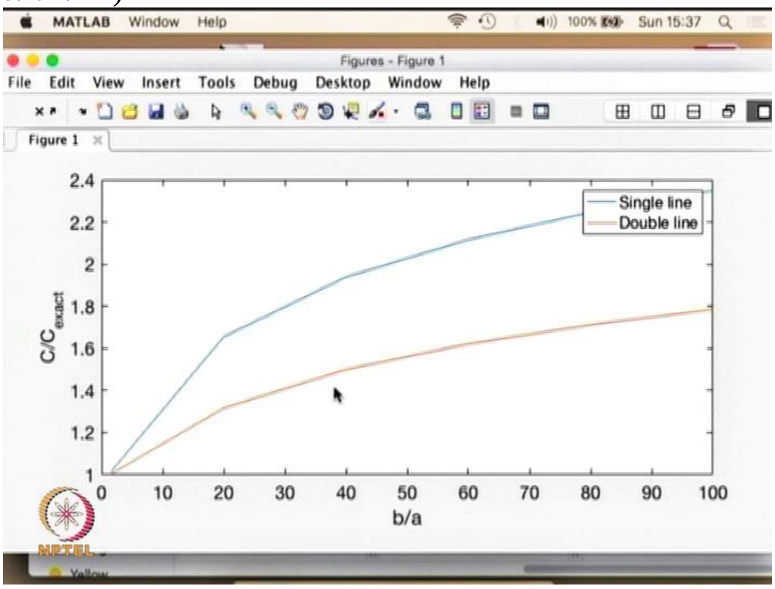

What is happening is when you have single lines what you see is the error value what you are computing here is going to be quite high whereas when you have double line.

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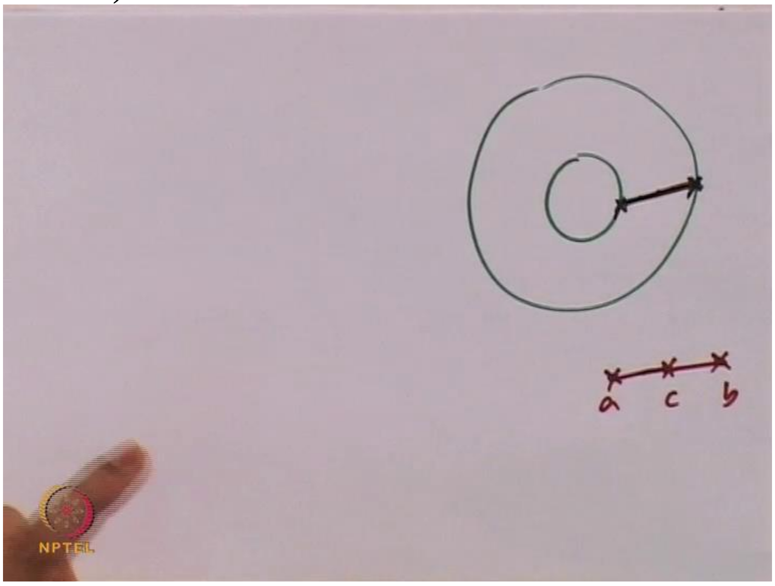

That means for example instead of having one line segment joining them. If I have 2 line segments so this is a, this is c, this is b, my value of computed capacitance is going to be much better compared to only one line.

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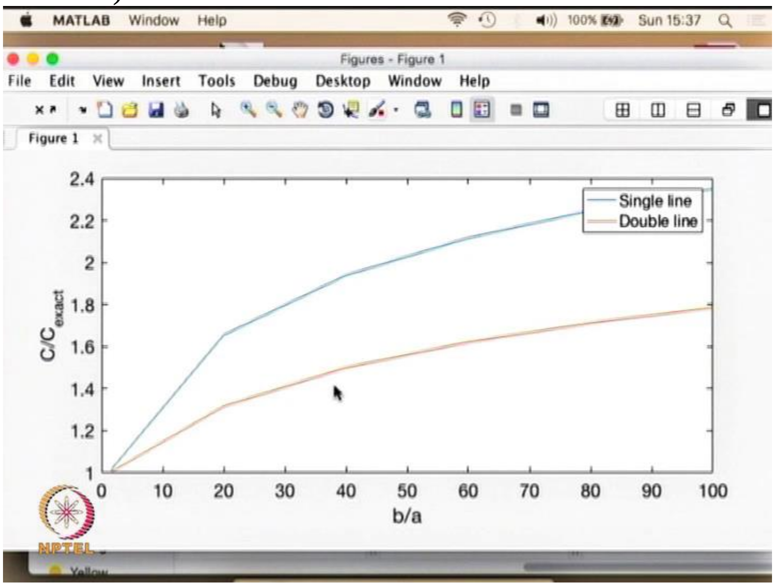

This we saw using the error value that we computed in the code. But what this graph is going to show us is a very different and important information. So this graph is a ratio this plots the ratio on the x axis for b by a. And it computes the error value, so if c computed value is exactly equal to c theoretical value then you will have 1. If not you are going to overestimate the value of capacitance using finite element method than what it is actually in the theoretical value.

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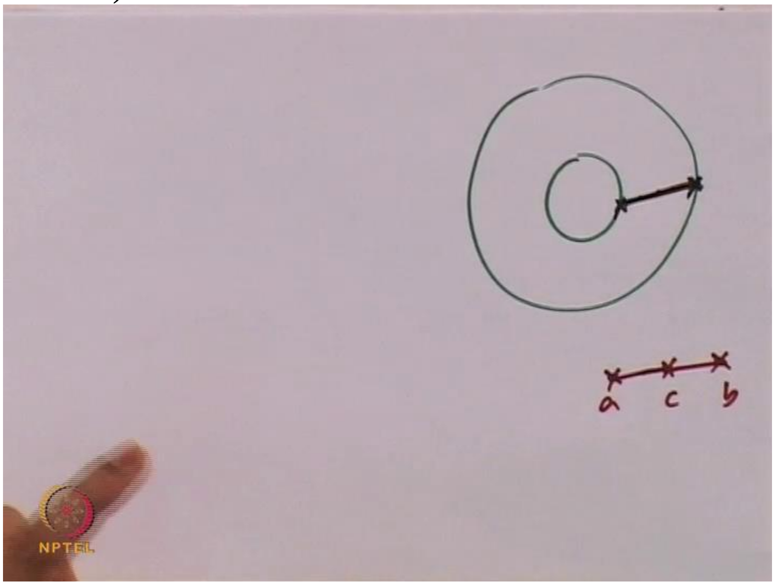

So for example when we have the b by a ratio that is quite small. That is b divided by a is equal to let us say it cannot be 1 because then the radius of this conductor fall exactly on the radius of the internal conductor. So what is important is when we are going from different radius of the outer conductor.

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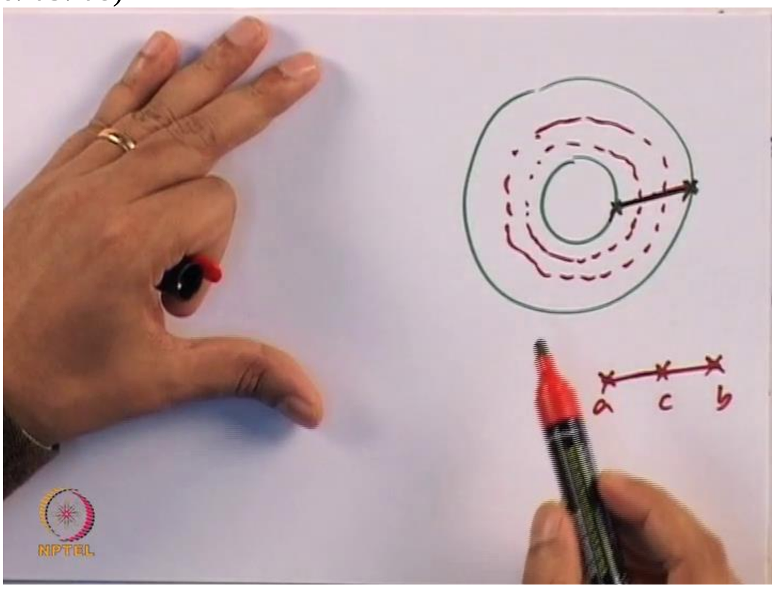

So we are keeping the inner conductor same. We are just increasing the outer conductor radius. So let us say when the radius of the outer conductor is almost as same as the inner conductor, then a single line is enough to approximate it as good as the double line. Because it does not make any sense to have a two points that are so close to each other. And then we have a third point in between them. So in that sense the single line approximation and double line approximation converge to the same result when the radius of the outer conductor becomes as comparable to the radius of the inner conductor.

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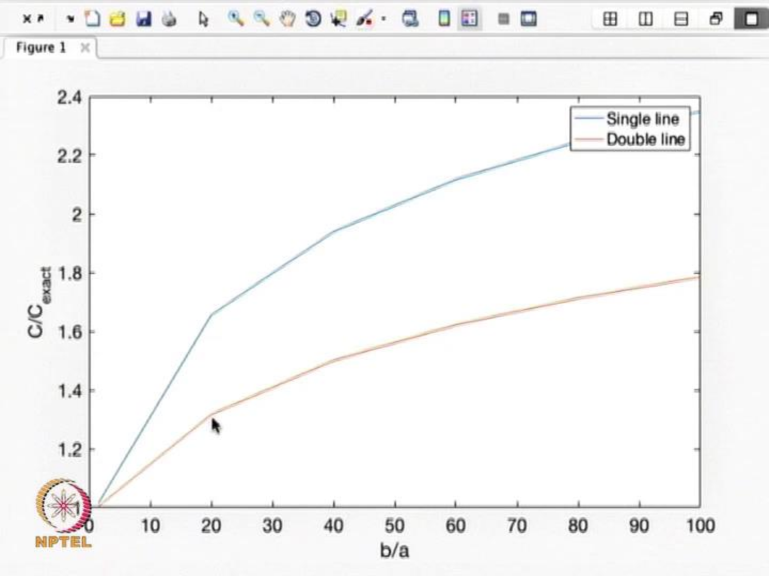

Whereas when the outer conductor is increasing you see that the double line as in we have two points, here in this case we have two points, in this case three points, the error that we get from the three point approximation is much lower than the error we get from two point approximation. And the error of the two point approximation keeps growing at a higher rate, whereas the error of the three point approximation grows at a slower rate.

So this is an important analysis that is given in terms of the radius of curvatures of the two conductors and the computed and the exact value. So this is way you can understand how the convergent is going to happen and also you can appreciate that when the outer conductor becomes comparable to the inner conductor radius. But still larger than the inner conductor. Then the number of approximations points that we have can be smaller and smaller. Still with two points we are able to capture the capacitance as close to the exact value. That is what we are seeing here in this particular graph.

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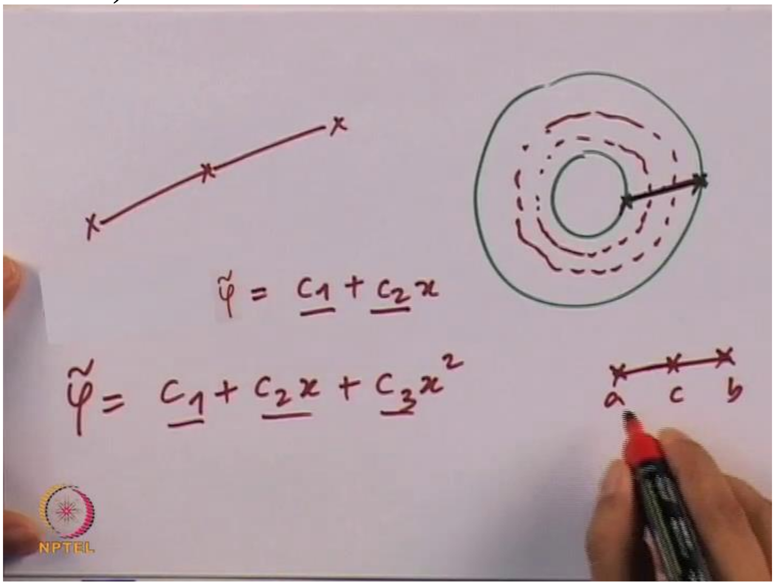

So now I said we had in the previous problem linear elements. And these linear elements have the form, but now instead of doing that we are going to say that the potential is going to have a variation that is going to be higher order. So in this case we are going to for second order. So we will write it as Phi tilde equal to c 1 plus c 2x plus c 3 x square. So this is the same equation if I write this in the same notation for the first problem we had c 1 plus c 2x. What I have done is I have added one more term for the higher order function. So now we have instead of two parameters we have three parameters for higher order. Once again we would like to express the voltage that we are computing in terms of the nodal voltages and the basis functions. So what we are going to do is we are going to call this as a basis function and we are going to write them for different elements.

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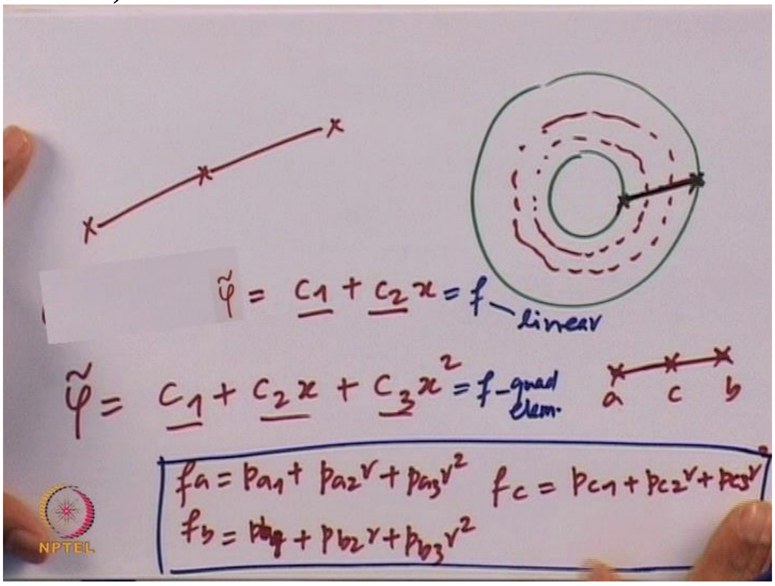

Let us say we are having three points a, c, and b. And the basis function for those a, c, and b are going to be on the form f a, f b, and f c. So f a is going to be given by p a 1 plus p a 2 times r plus p a 3r square. So this is nothing but the same one. I have changed x to r because we are talking in the radial axis still it holds because it is a one dimensional problem. You can use x or r, but it is more convenient to write r because we are talking about radial aspects in this geometry.

For the second one it is going to be given by p b 1plus p b 2 r plus p b 3 r square. For the third one it is going to be given as p c 1plus p c 2 r plus p c 3 r square.

What you are seeing is a kind of a basis function that I have introduced here initially we had a basis function which is nothing but a linear element. And in the second case we had a basis function which was, so maybe it is easier to write it. So this was the first basis function. So this is the second basis function so linear element. This is quadratic. So for these problems we are going to use these three basis functions. And these basis functions are going to be on a, c and b. So let me write it down on a separate sheet.

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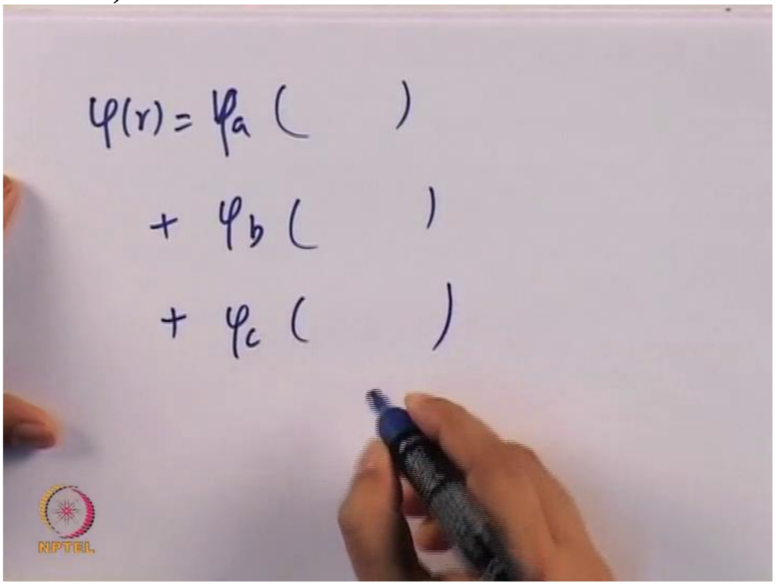

So our potential value that we are going to compute Phi(r) is going to be the value that I am going to assign on the inner conductor and the outer conductor the potentials, and also the central point. So Phi a multiplied by something plus Phi b multiplied by something plus Phi c multiplied by something. Remember in the earlier case for the linear element we used Geometrical data and they are based on the linear approximation. And here we are going to do quadratic approximation.

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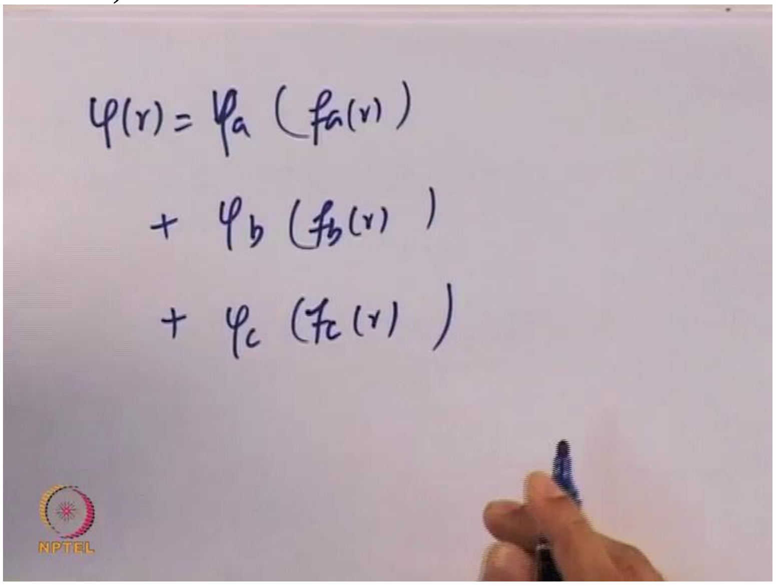

So we will have f a, f b, and f c. And they are all going to be functions of r.

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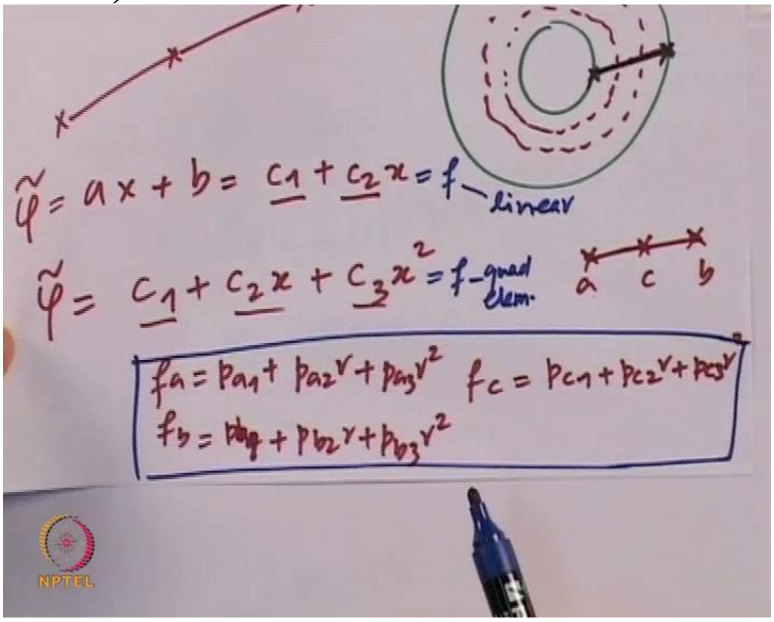

As I have described here. So f a is going to be a function of r, f b is going to be a function of r, and f c is going to be a function of r. And P a 1, P a 2, P a 3, these are coefficients like c 1, c 2, and c 3.

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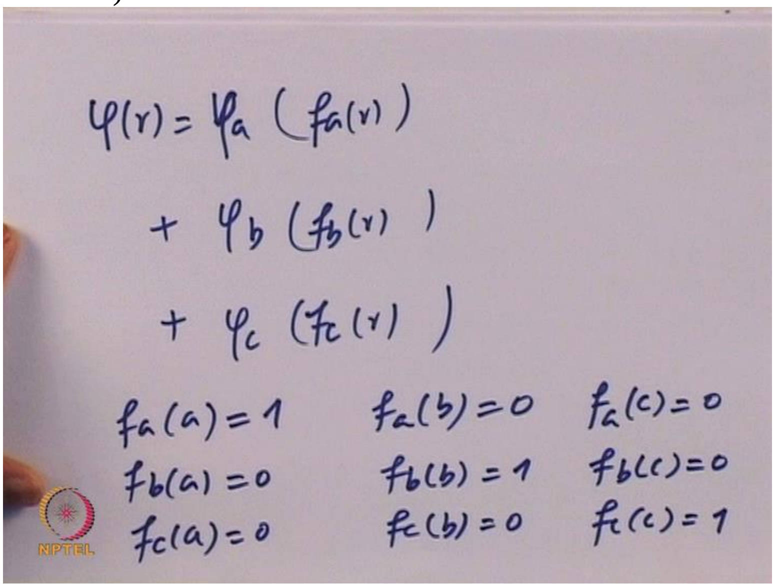

So this being the case we can write the equation subjected to certain conditions. The conditions are f a (a) is equal to 1, so this is when r equal to a for f a value will be equal to 1. f a (b) r equal to b is going to be equal to 0, and f a (c) r equal to c is equal to 0. That means the first basis function what we are using will have a value 1 for the node point a whereas it will be 0 for the other ones. Similarly the f b value will be 0 at a, it will be 1 at b and it will be 0 at c. And the third one will be f c(a) will be equal to 0, f c(b) will also be equal to 0, f c (c) will be equal to 1. So this is the assumption. This means the basis functions are going to behave in certain manner and that is what we are going to see. And when we plot the basis function you will see and you will appreciate what we are talking about. So inserting these values and talking about the boundary conditions into the equations what we had.

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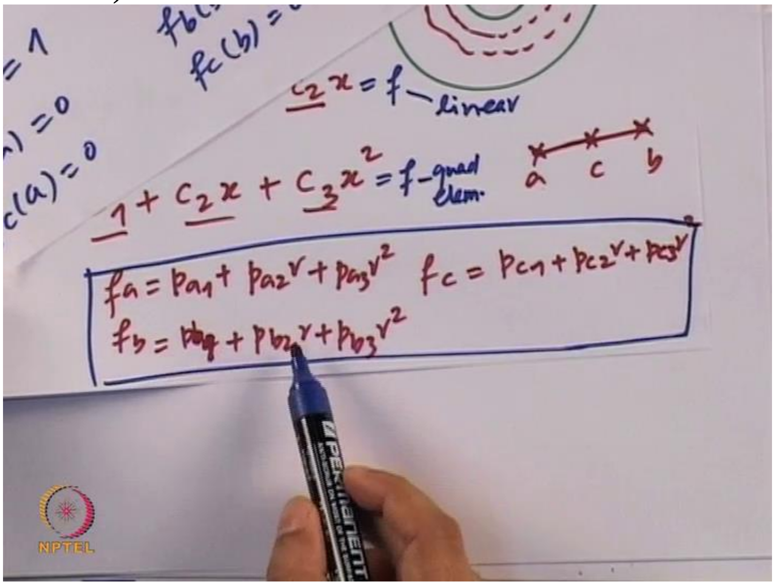

What we get essentially is a kind of a matrix equation. The matrix equation will be of the form, So I know the value for f which are given by these values and I know that the boundary conditions of those basis functions are going to behave like this.

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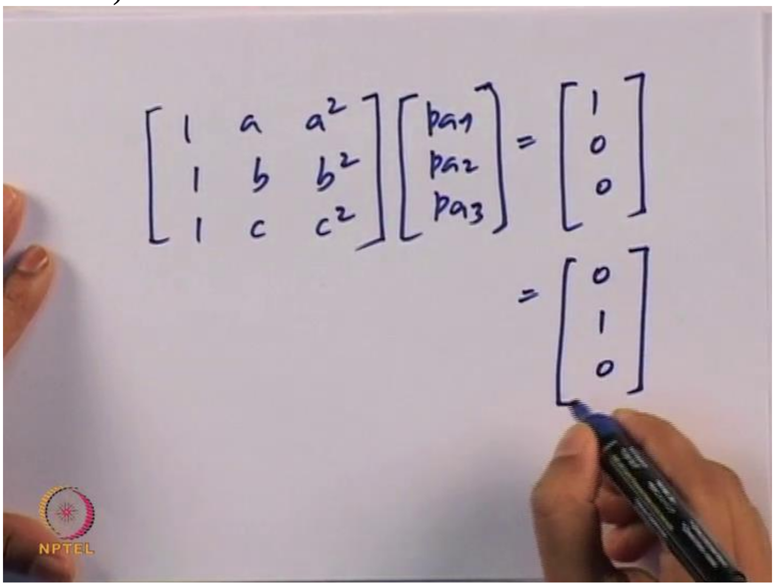

And I know the equation itself. The equation itself is going to be of the form as we discussed before. And so we will have the following conditions which is [1 a a square, 1 b b square, 1 c c square] and we are multiplying it with [p a 1 p a 2 p a 3,] . This is going to be for the first node, so this is going to be [1 0 0]. And for the second one this is going to be [0 1 0].

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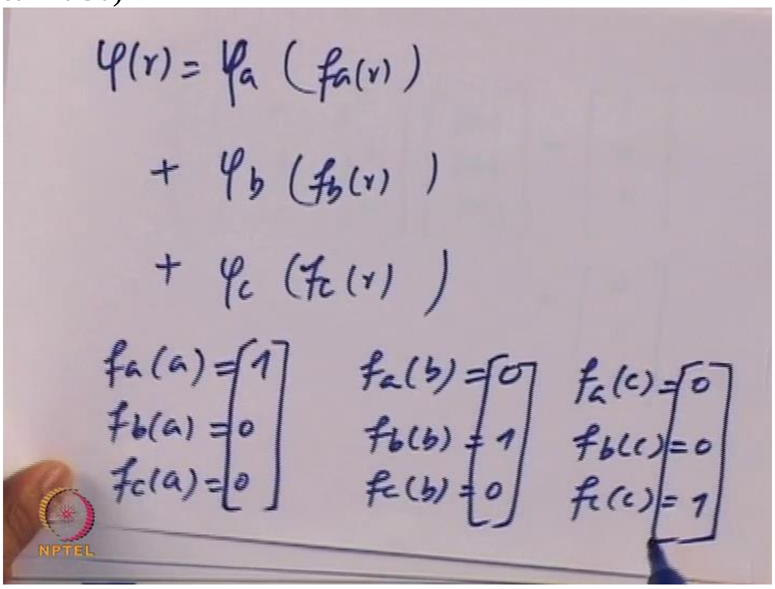

As we have described here. So this is the first right hand side. So the second right hand side will be like this and the third right hand side will be like this.

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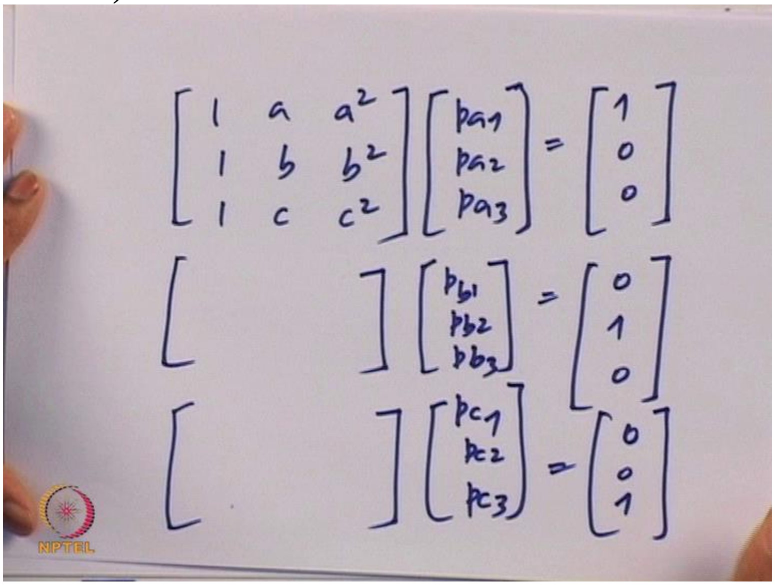

That is what we are trying to write here. So the first term the first matrix will be the same for all the three. The only thing that is going to change is instead of p a 1 p a 2 p a 3, it will be [p b 1 p b 2 and p b 3]. Similarly [p c 1 p c 2 p c 3] equal to [0 0 1]. So solving these three sets of equation for a equal to 1 and b equal to 100. So we set initially in the program we put a equal to 1 and b equal to 100 in the simulation and we tried to compute the values. So we can do the same things and we can get the results.

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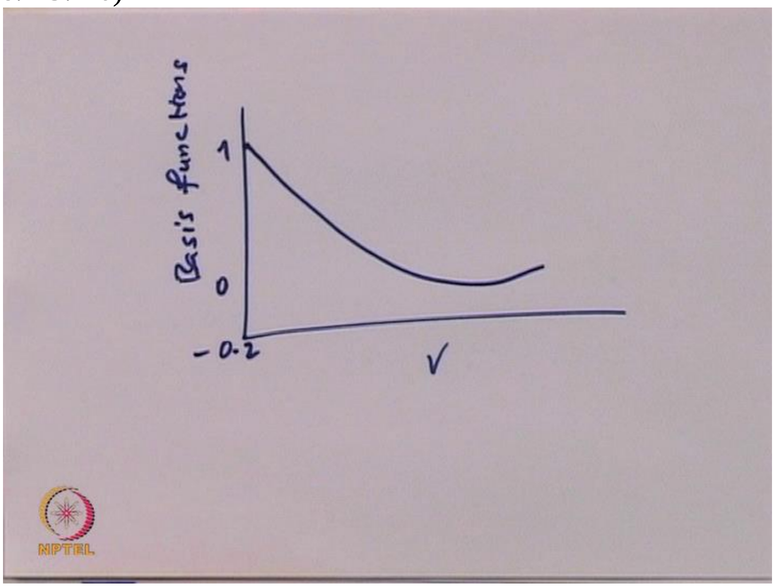

Before that let me plot the basis functions and the r coordinates. So what we have is r and then we have the basis functions. So the basis functions are going to have certain behaviour. I said it is going to go from let us say this is 1 this is 0. So let us the 0 be here and this is 0. Something let us say minus 0.2. It is going to go from here to here. So let us go into the code itself and let us see the way we have computed various parameters for the quadratic case and let us understand how we can simulate the higher order basis functions and compute the value for our problem that we have been discussing so far.

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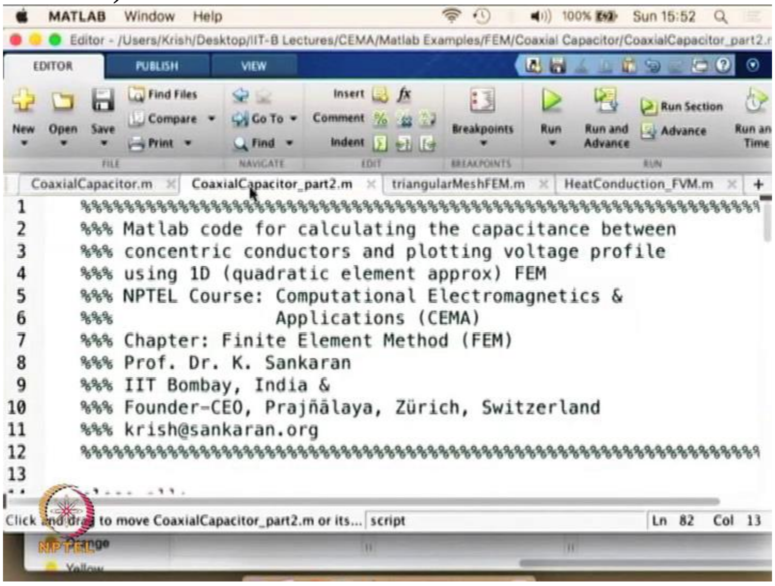

So let us go into the code. So we are going into the second order approximation, So Matlab code for calculating capacitance, using quadratic element.

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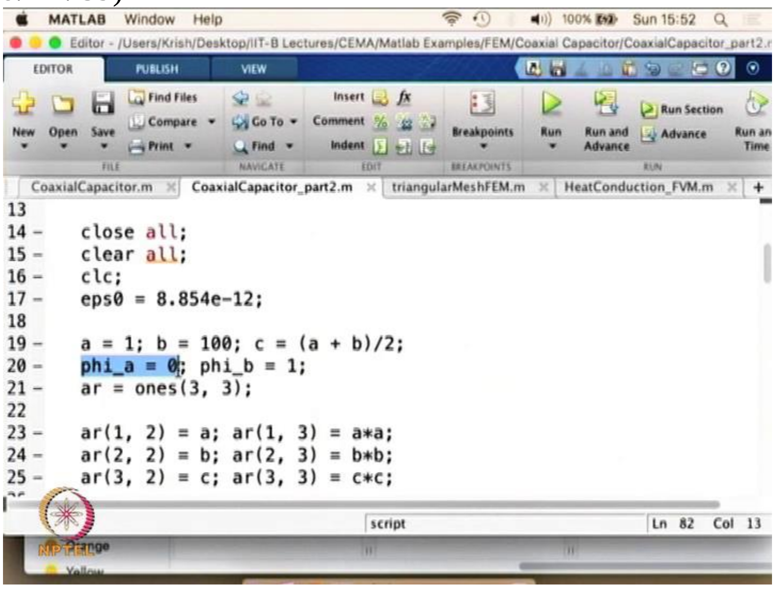

So we are going to like before set the values for epsilon 0 so we have a equal to 1 and b equal to 100. And like before our initial values for potential internally for the inside conductor and the outside conductor are given by this.

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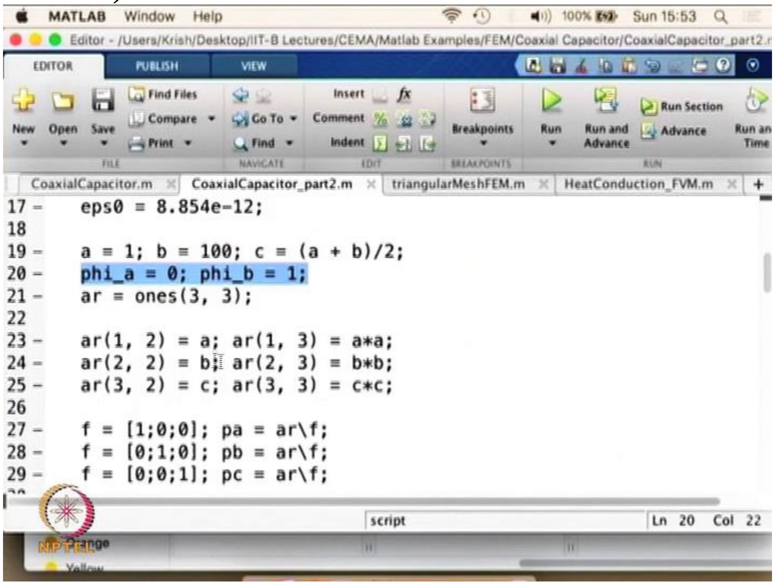

And these are the ar values that we are setting. So it is going to be a square b square and c square. For ar (1, 3) ar (2, 3) ar (3, 3). and ar(1,2), ar(2,2), and ar(3,2) are going to be a,b,c so on and so forth.

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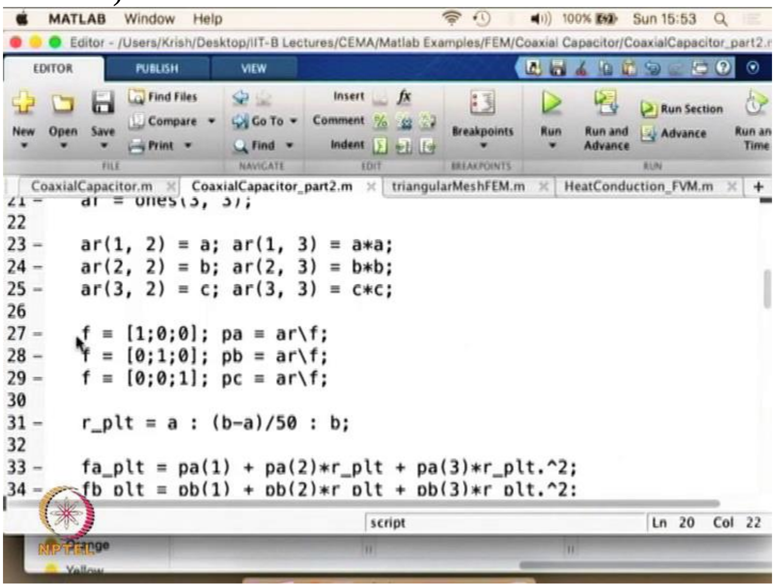

And the value of f, the right hand side of the equation that we had before, so these are the equations that we had here on the right hand side.

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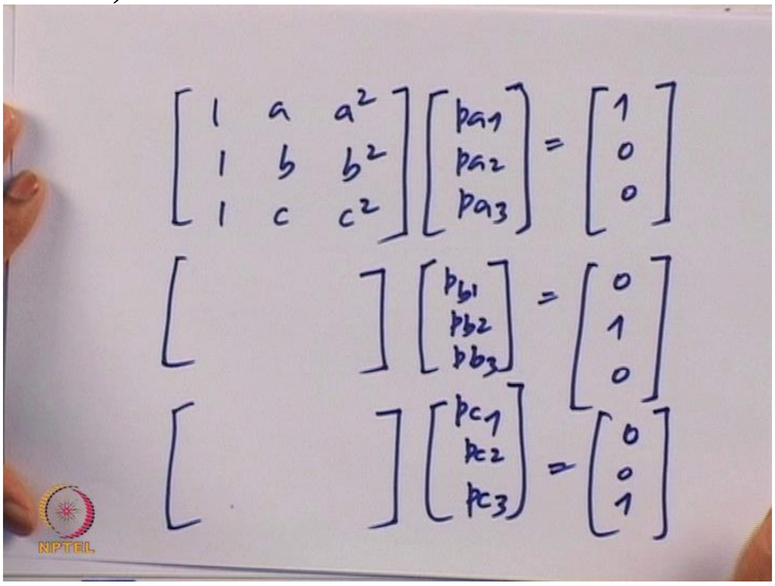

And this is the value that we have put it as f.

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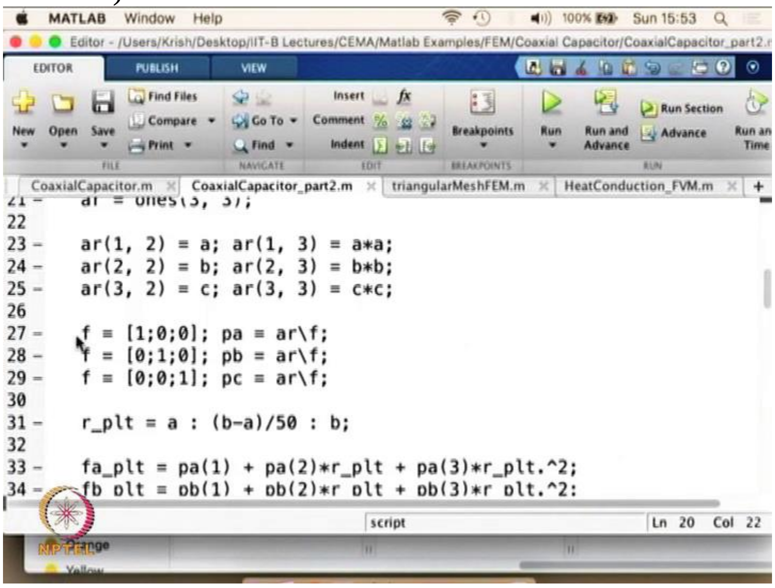

So it is [1,0,0]; [0,1,1];[0,0,1]. And once we have that we can compute the values of pa, pb and pc by inverting the ar and multiplying it with f.

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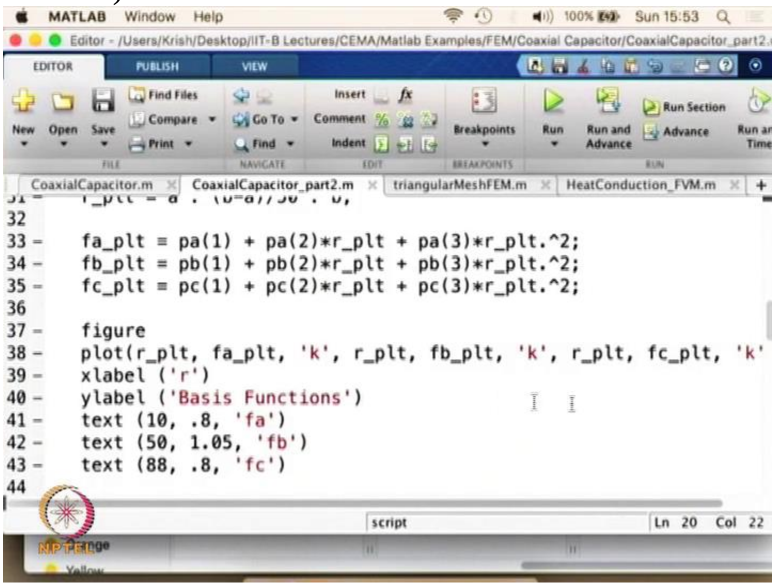

And we are able to plot the values of various basis functions. The f a, f b and f c will do that.

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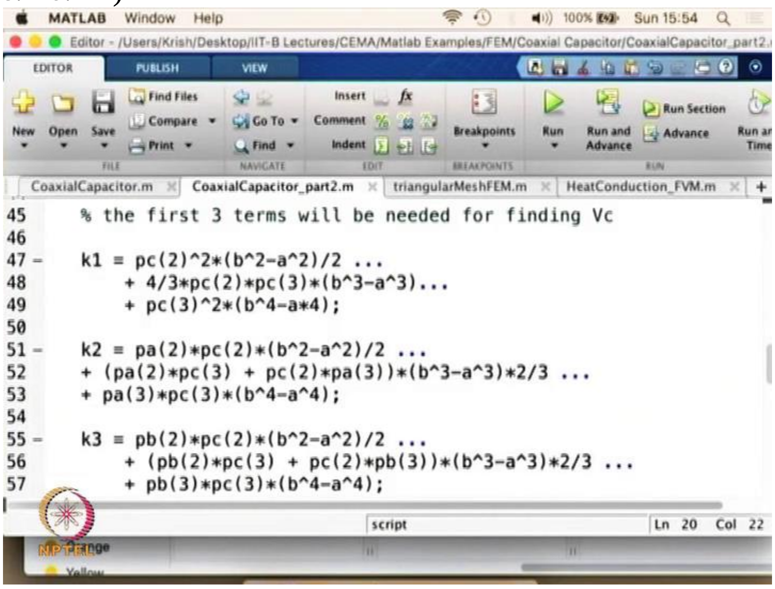

And we are now going into the program where we are computing the values of various parameters that we are going to use in computation. So we calculate k1, k2 and k3, and these values are going to be used as a weighting function values later on.

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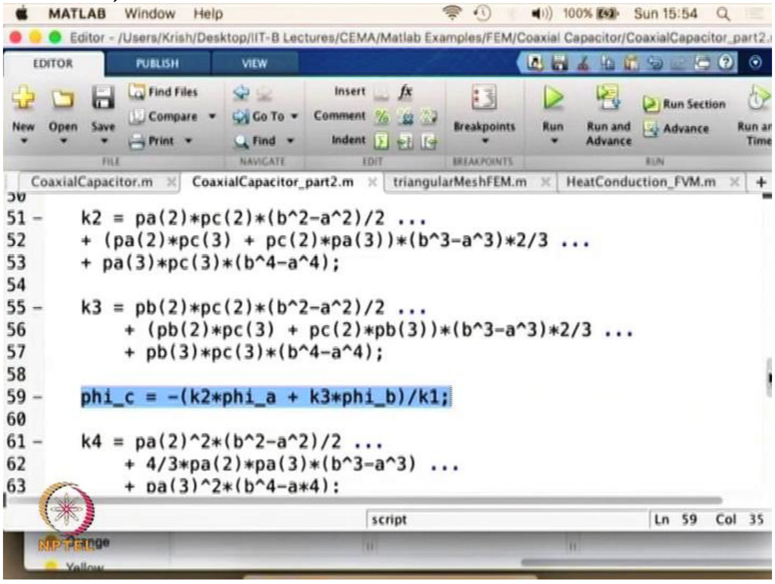

And we are computing the value of Phi c, remember Phi a is given Phi b is given Phi c is not given, so we have to compute the value of Phi c, and we are using this equation to do that.

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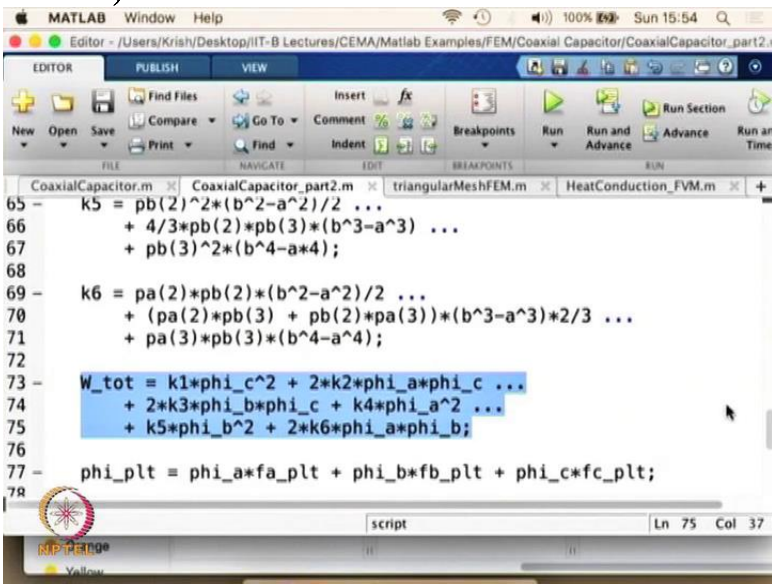

And based on that we are going forward to compute like before the total energy that is being stored in the capacitor.

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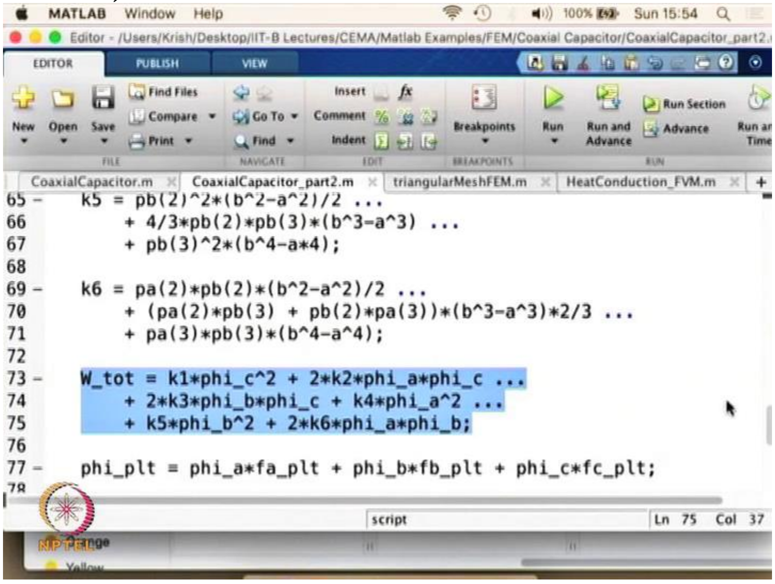

Once we get that we are able to compute the value of exact and the theoretical value and we are able to plot the error. So once we run this for the problem on hand we will be able to see how the code is behaving.

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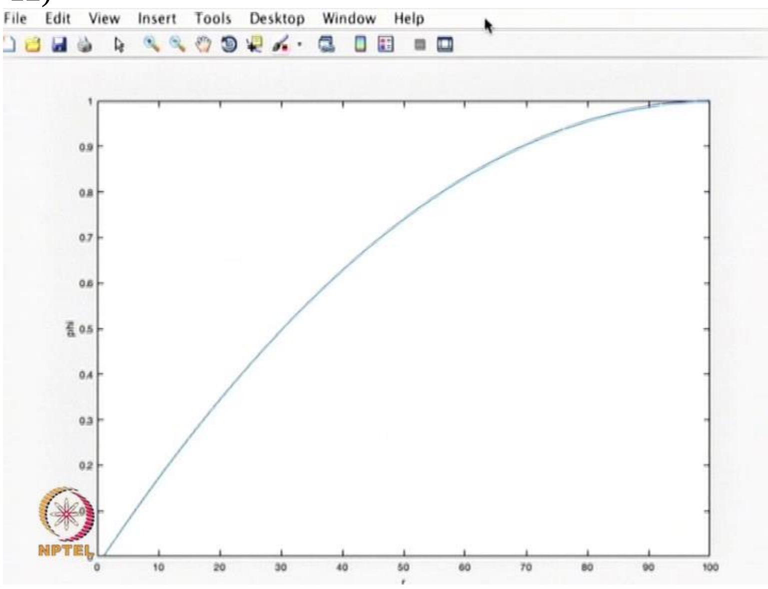

So what you see here is the potential goes from 0 to 1 and it follows a kind of a curve it is not linear like before. And the value is kind of also computed for the geometry that we have chosen.

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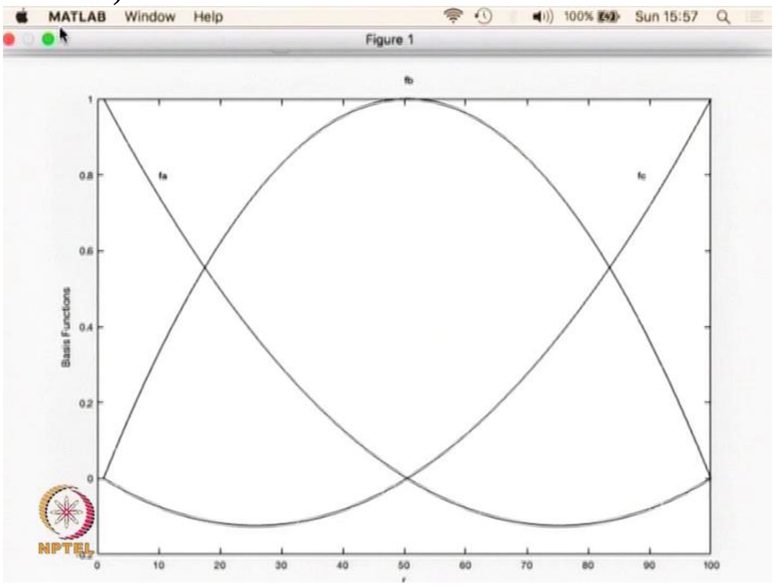

There is also another graph that chose the value of the basis function so the basis functions are going to have values for example in the domain what we had, we had the entire domain basis function. So the basis function for the entire domain is going to have this behaviour. So what you see is f(a) is going to have the maximum value at a and it will have 0 values in both c and b. So this is the centre value so both at c and b it will be having 0 value.

Whereas it goes to some 0 level in between c to b. Similarly the value for b will be 0 in both a and b whereas it will have the maximum value at the centre. And the c value is going to have the maximum value at the end. Whereas it is going to be 0 in between. So in this case a is this point, b is this point, and c is this point. And if you are changing the locations also the basis function should change accordingly. So what we have got is the three basis functions for the problem what we have chosen.

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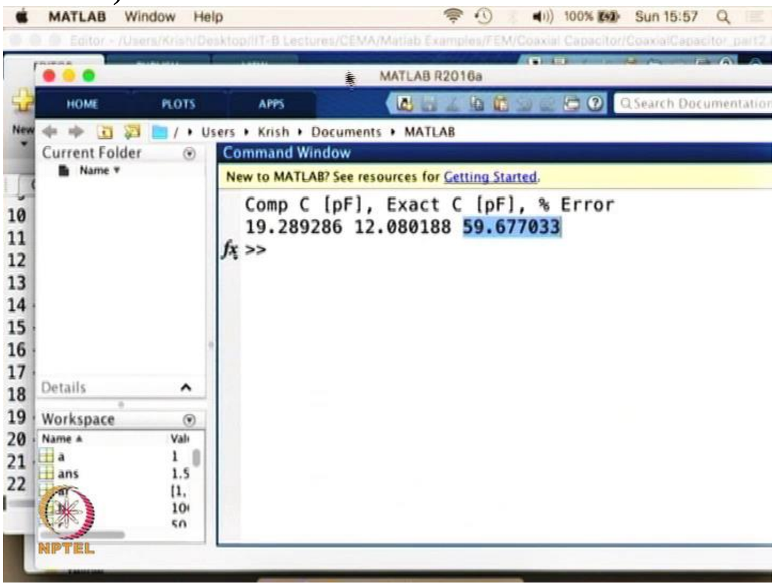

And we are able to simulate the problem in such a manner that we are able to compute the value for the potential and let us see now what is the error function what we are getting. So when you compute this result and see the error function. What you see is the exact value what we are computing is here. It is 12.0818; the computed value here is 19.289 whereas the error here is 59 percent. So if you see this program where you have three points and you have simulated it using second order quadratic basis function.

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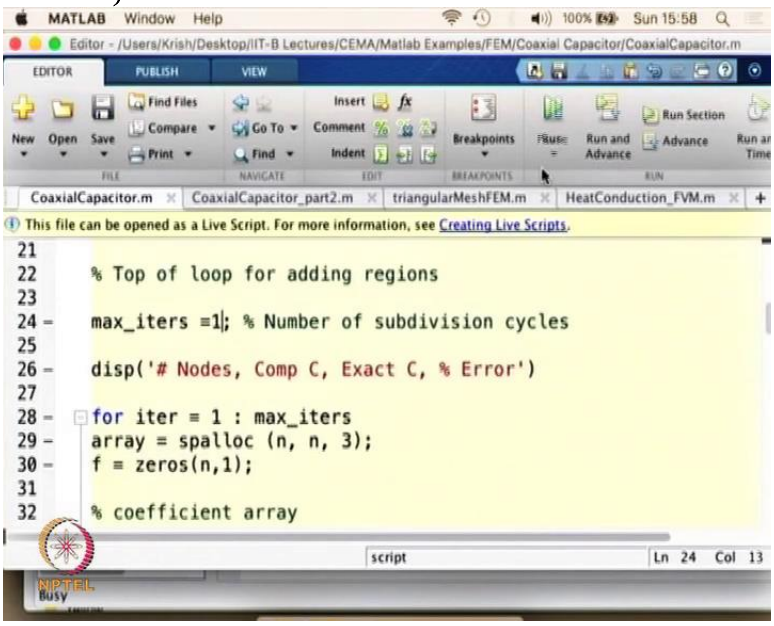

If you go back into the old code where we had a linear element, this is a coaxial capacitor with a linear element; we have two points let us increase it to three points. And let us simulate it for only one iteration and we see the result also.

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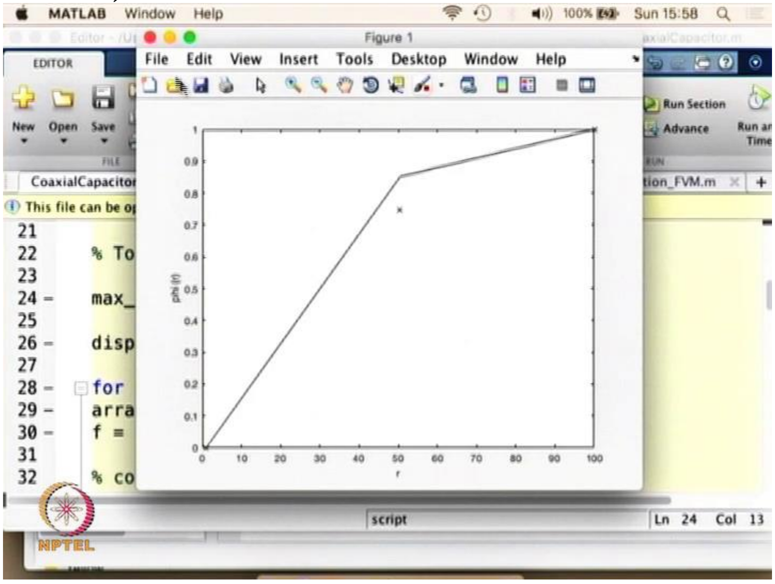

So it is giving us not nice curved line what we got whereas we got a two lines which are having certain errors.

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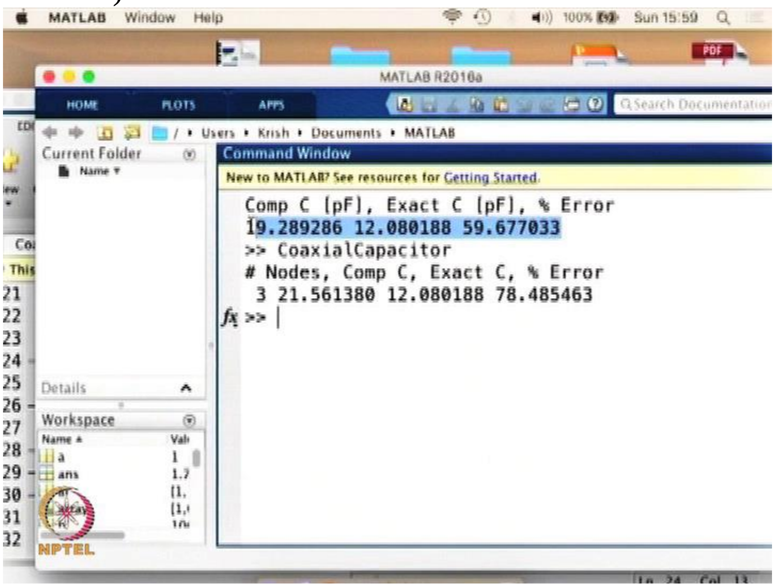

What is interesting to see is we can see the value of the error that is being computed. So we did this calculation using the quadratic element. And this one is using the linear element. So here also you have three points and here also you have three points. And what you see is the computed value here is 21.56 whereas the computed value with a quadratic element is 19.28. And the exact value in both the cases are the same 12.08 and the error in the quadratic element is 59 percent whereas the error in the linear element is 78 percent which is quite high compared to what we get with the quadratic element.

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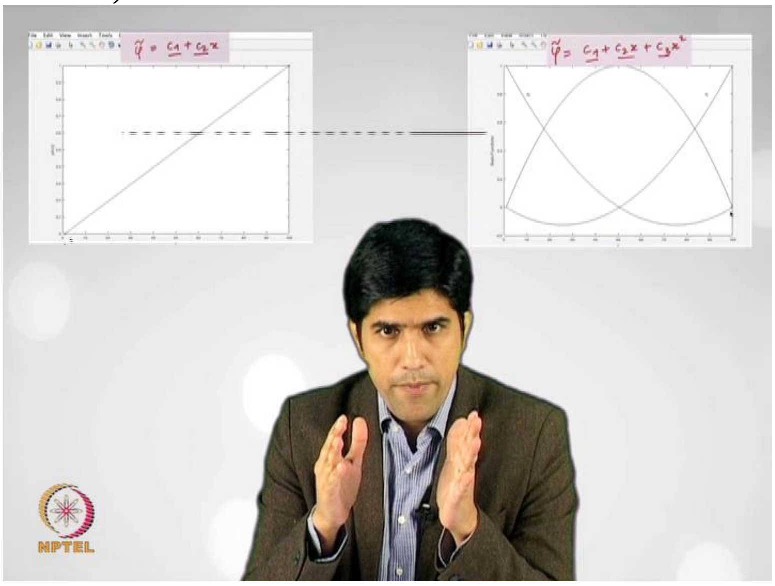

So what we have done here essentially is we have shown you side by side both the linear approximation and the quadratic approximation. How the two basis functions are going to give me different kinds of calculated value and what is going to be the exact value and what is going to be the error function. So it will enable you to see how one can compare a linear element approximation versus quadratic element approximation. And also get a very clear error percentage. That is what this code is helping you to do.

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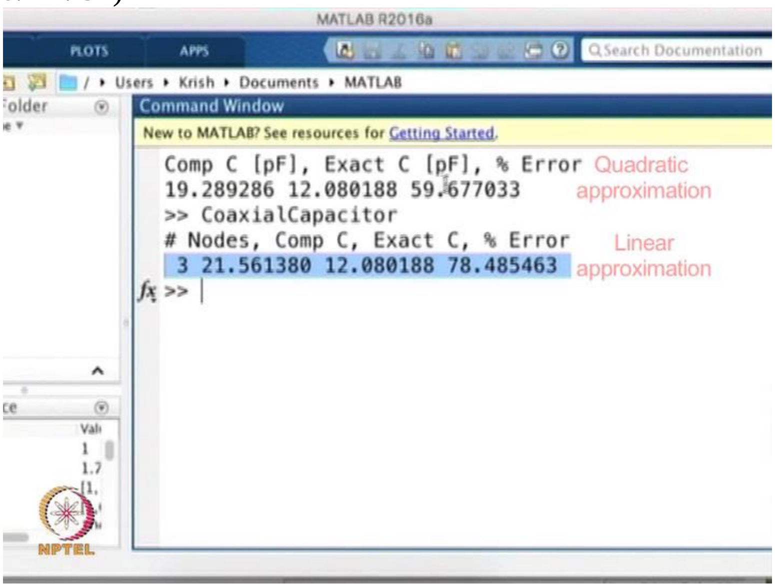

And I encourage you to always compare results in this manner so that you can see when I go higher up in the basis function order. So going from a linear to a quadratic or from quadratic to third order or fourth order so on and so forth. What is the gain I am getting?

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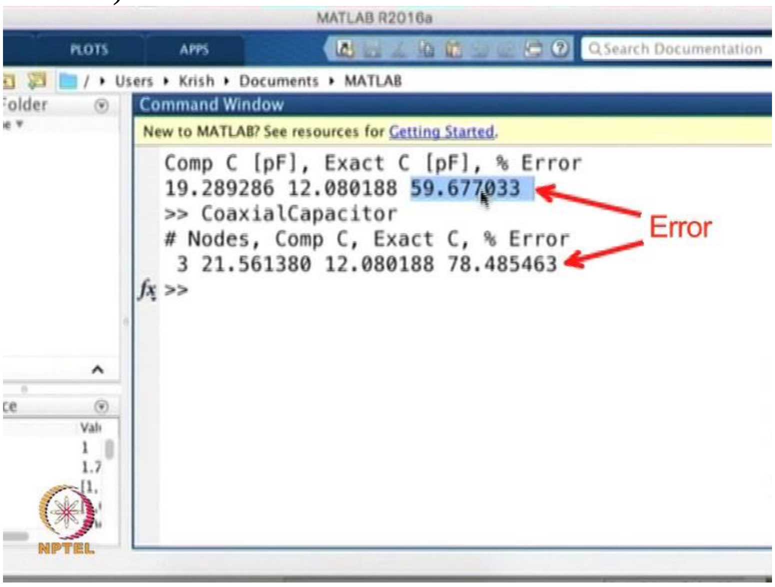

What is the gain I am getting in terms of the percentage error is going to be a very important aspect that you should learn.

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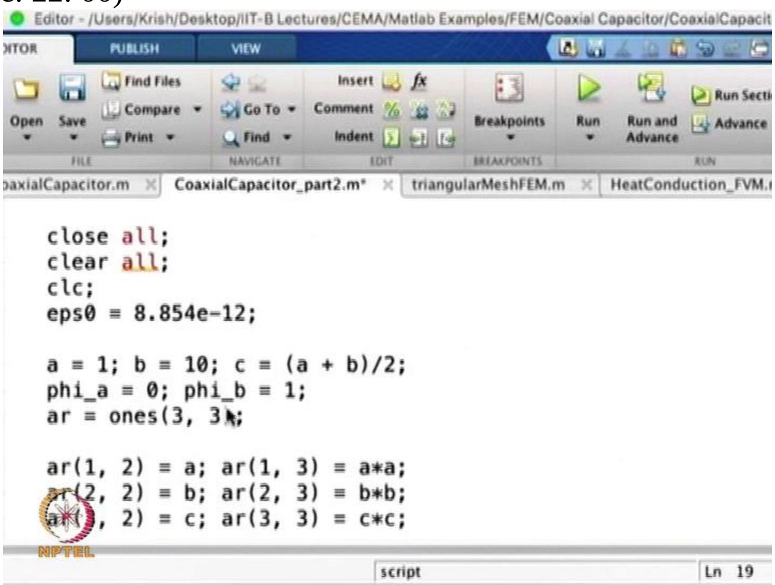

And that being said what you can practice is you can practice playing with certain parameters, so instead of going from a equal to 1 to b equal to 100. What will happen if my b value reduces to 10?

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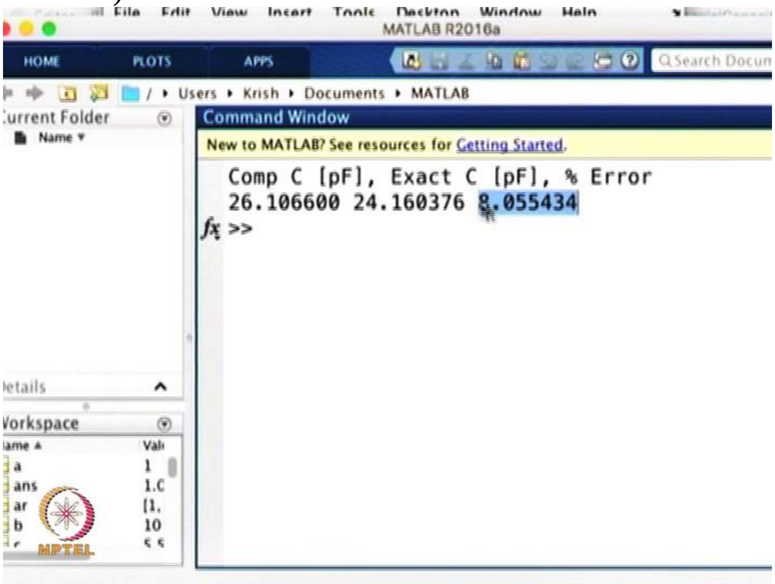

So when I simulate it I can see certain values and I can also see what is going to be my error. The error is substantially reducing. The previous case what you had is error was 59 percent and now you see the error for the same quadratic three point approximation the error has reduced.

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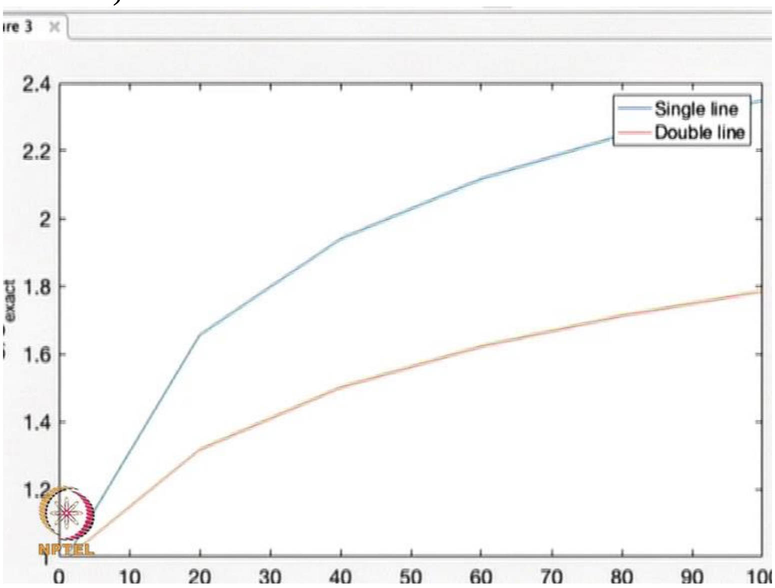

And this is exactly the graph which I showed you before where we are able to see when the value of b by a is going to come down so when b is comparable to a the error is going to also reduce. This is what you are seeing.

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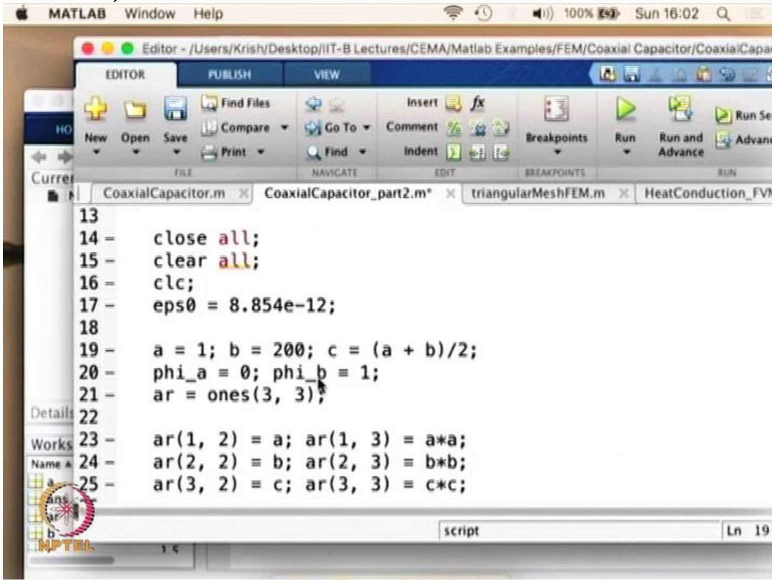

So if you do the problem even for lower b, so for example if I run the same program for let us say I am reducing the value of b even further. And so let me put 200and simulate it, the value is here the basis functions are here what is interesting is the value that we are computing.

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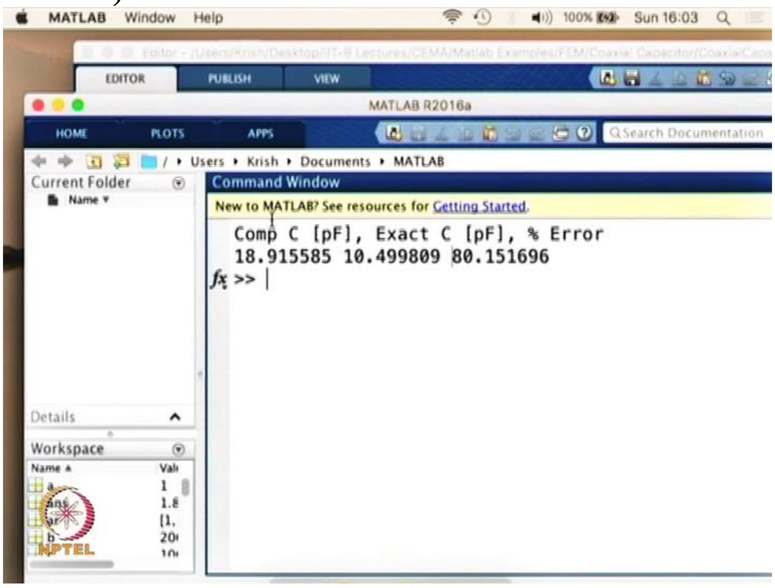

Initially we had for b equal to 100 and a equal to 1, we had 58 percent or so is the error. And for the same problem with the same basis function and same number of points when I change the geometry the error is also increasing. So what is important here is what we are doing in terms of finite element is the number of points that we need.

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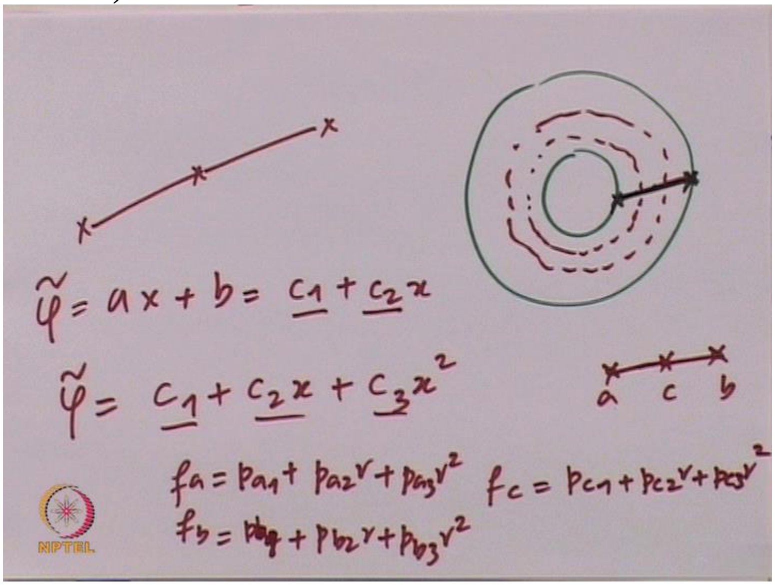

Second thing is the quadratic or the linear basis function, whatever basis function we are going to use is going to impact the result. And this being said these are the most important parameters that you can tweak in modelling the finite element problem.

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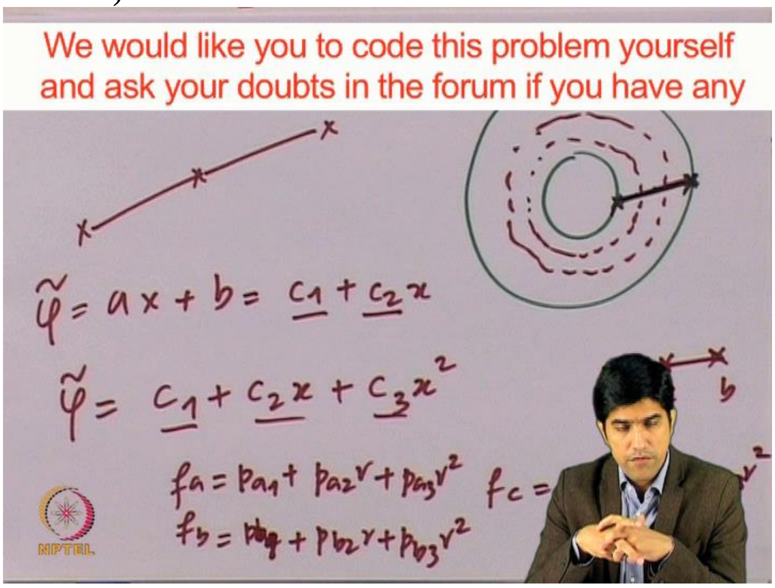

Please practice it for yourself and change various parameters I encourage you to try it out for yourself and learn how we have modelled it what we can learn an how we can extend this idea to other problems. We have done coaxial cable for the reason because easiest problem will be a parallel plate capacitor, we will definitely do also a parallel plate capacitor later module, and we will also complicate the parallel plate capacitor having certain sharp edges so that we see that singularity effect is coming into play. These are the things that we will definitely try. But already at this stage using the coaxial cable example what we had you can learn a lot about finite element method and please try it out and experiment for yourself.

Thank you!