Computational Electromagnetics and Applications
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Exercise No. 12
Finite Element Method -II

Welcome on this module on examples using finite element method. We will start with a very complicated problem having said it is complicated I wanted to make it simple enough in the analysis. And take you step by step in the process so what we are going to look into this umm module on finite element examples we will start with coaxial cable problem.

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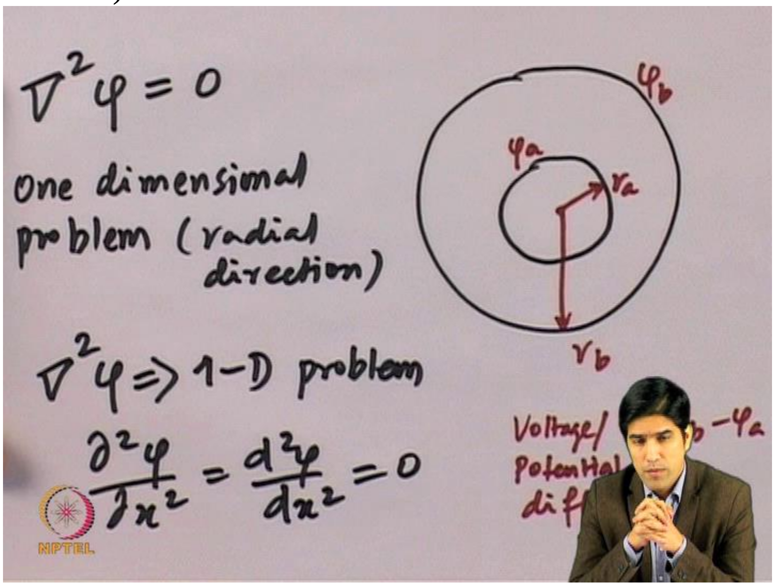

The reason why we are going into the coaxial cable problem is because we are in a cylindrical coordinate which is very different to the normal Cartesian coordinate we just got used to. So we will take the cylindrical coordinate for that reason. And secondly also that the variation that we are used to is mostly linear but in this case it is going to be different. I will explain you as we go into the problem step by step. So let us recap on some of the basic theoretical formulations that we need in order to go into this problem and simulate it. So let us define the problem geometry itself.

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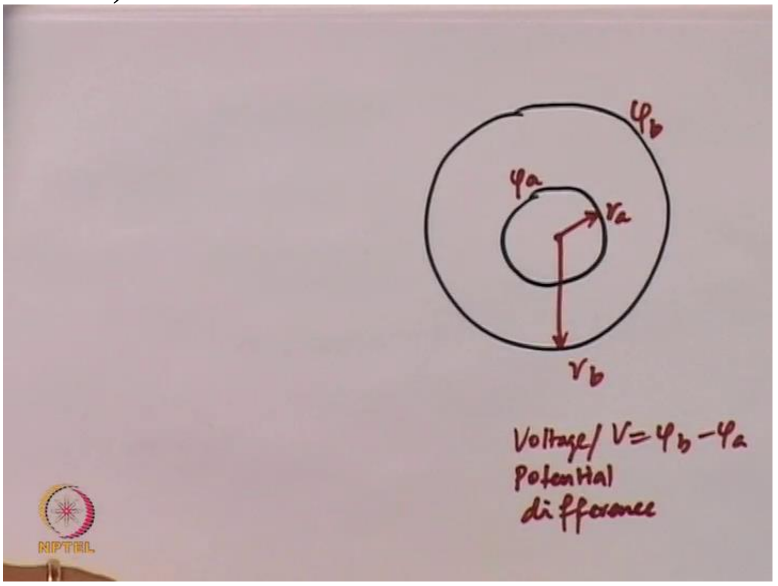

So we are going to have a coaxial cable. It has a outer conductor and it has a inner conductor. Let us say the diameter or the radius, let us say the radius of the inner conductor is going to be given by r a and the radius of the outer conductor is going to be given as r b. So this is going to be the basic geometrical aspect and on the top of it we are going to give certain electrical parameters to it. The electrical parameter is the voltage on the inner conductor is going to be given as phi a and the voltage on the outer conductor is going to be given as Phi b. So when we talk about voltage difference we are talking about v that is equal to Phi b minus Phi a. So this is the voltage or potential difference.

So this being the description of geometry that we are going to look into. So let us look into the way we are going to analyze this problem mathematically.

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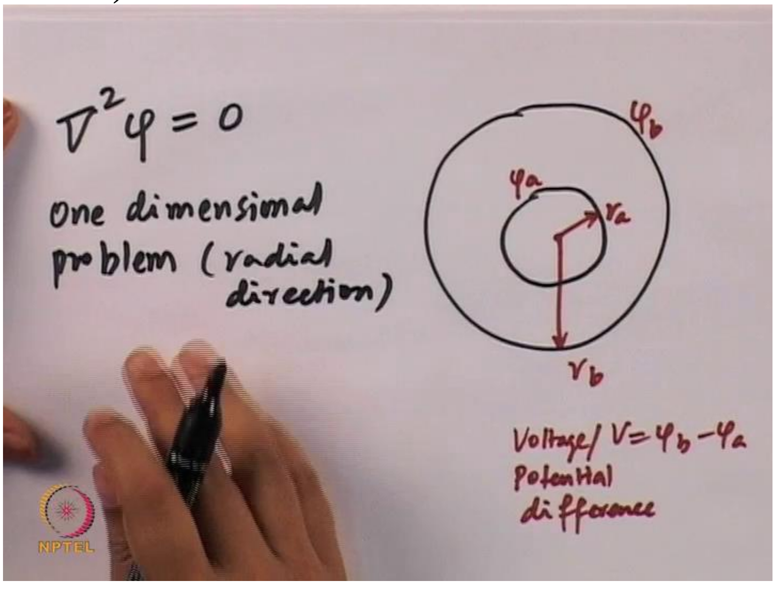

So the mathematical formulation of a problem of this sort begins with the Laplace equation which is basically written as Del square Phi is equal to 0. And of course in this case we are going to look into a one dimensional problem and the dimension is going to be in the direction of r, so we are going to have one dimensional problem. So our direction will be in the radial direction. So let us start looking at it from a simple Cartesian coordinate and then we will extend it to the radial coordinate.

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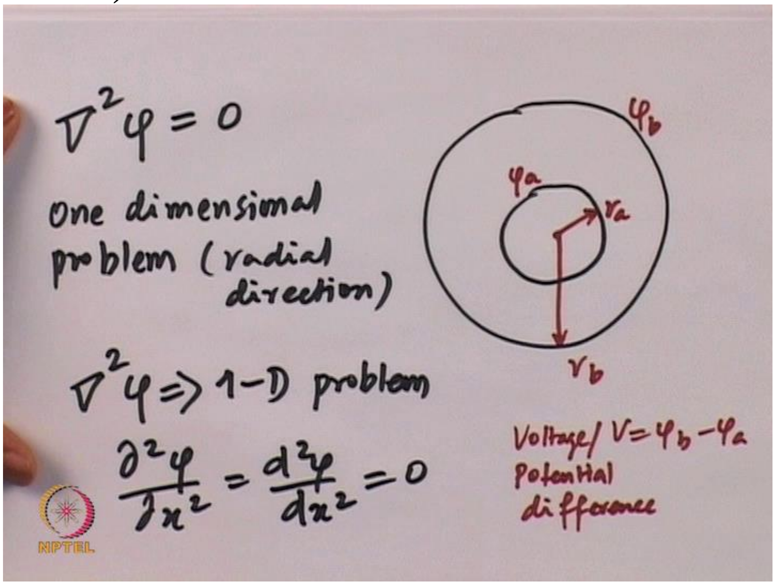

So let us assume we are having a one dimensional problem in x for the moment we will to worry about the whether it is radial or x direction. For us it is a one dimensional problem, So it is enough to write it as Del square Phi for a 1D problem, can be written as Doe square Phi by Doe x square that is equal to d square Phi by d x square, since it is only one dimension we are not worried about whether it is partial or ordinary differential is equal to 0. So this is going to be our starting point.

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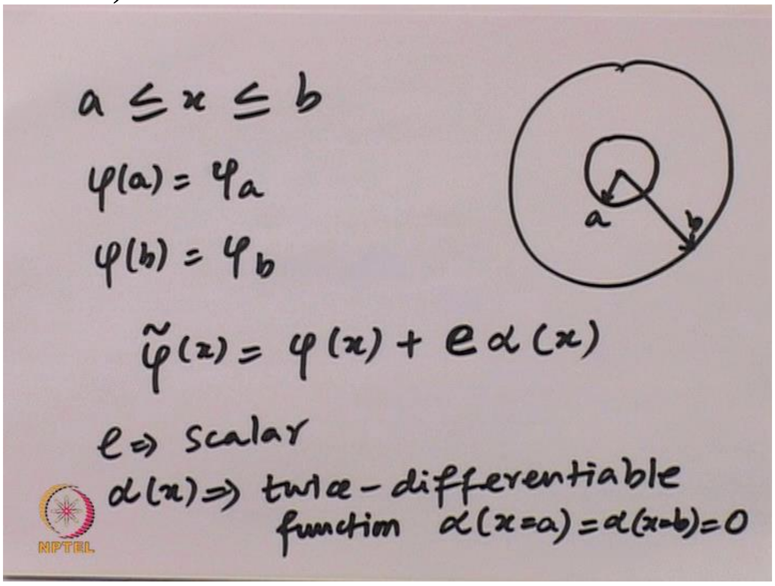

And of course we have certain things that we have declared already and these are going to be the definition of the boundary and the boundary conditions. So the region is going to be a less than or equal to x less than or equal to b. Remember we had a outer conductor and we had a inner conductor and this is a and this is going to be b. And x is any region between a and b including the point of a and point of b. And second thing is the potential Phi (a) is equal to Phi a and the potential at b is equal to phi b.

And these are the specified boundary conditions. And now in order to approximate the Phi the potential we are going to use an approximate function and this is going to be written as Phi tilde (x) is equal to Phi(x) plus certain constant e and a variable that is going to depend on x. So let us say this variable is alpha (x).So e is a small number, it is a number, it is a scalar. And alpha (x) is a twice differentiable function satisfying two conditions. The conditions are at the boundary alpha (x) equal to a is equal to alpha (x) equal to b is equal to 0. So this is a very important thing because we are satisfying the boundary condition such that at the boundaries or the domain which we are calling 1 dimensional domain from a to b the value of alpha which is a twice differentiable function goes to 0. This is a most important thing that we should have for us to proceed with. So now having this what we are going to do is we are going to follow our analysis further.

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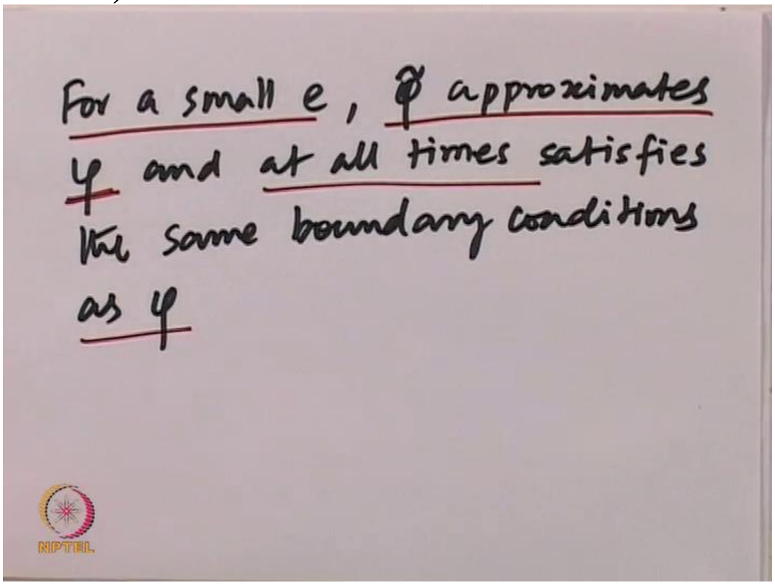

So for a small e the value of Phi tilde is going to approximate Phi and at all times satisfies this boundary condition and it at all times satisfies the same boundary conditions as Phi. So the important thing is this should be define, this should be for a small value of e the tilde the approximation satisfies or approximates Phi and at all times satisfies the same boundary condition as Phi. So once we have that we can proceed with our analysis with confidence that we can set up a function.

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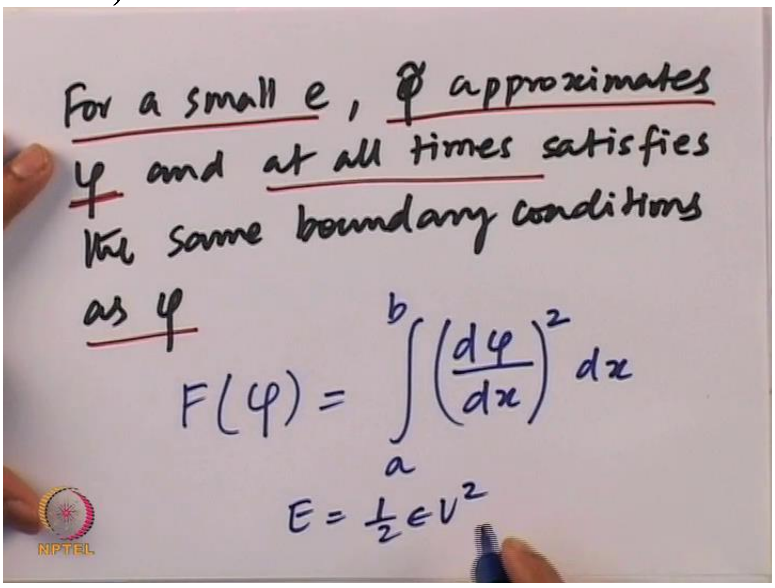

Let us say we call this function as F which is a function of Phi is equal to integration over the domain a to b (d Phi by dx) the whole square dx. You can see this as some similarity to the energy function. So the energy stored in the classical case will be e equal to half by v square. So this will be the definition of energy, where v is the voltage difference and this is the way we define the energy.

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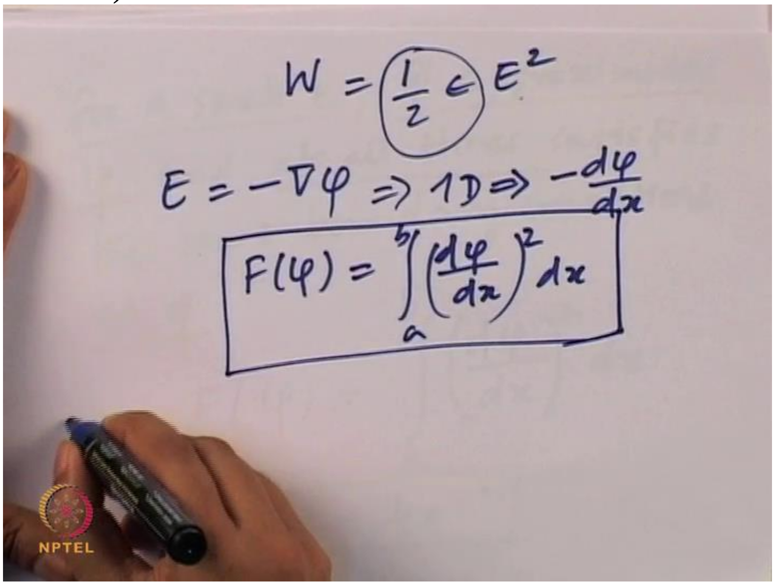

So if you look into this particular equation this has some similarity to the energy stored inside a capacitor as w is equal to half epsilon e square where e is the electric field and epsilon is the permittivity and w is the energy stored. And if you compare these two equations of course the scaling factor is missing other than that these equations are very similar because we can write E as minus gradient of Phi and in one dimension this will be written as for 1D this is nothing but minus dPhi by dx. And what we have got in our expression is F (Phi) is equal to integral a to b (dPhi by dx) the whole square dx. And you can readily see if I square this term this is nothing but E square term. The only thing that is missing is half Pi square half pi the only thing that is missing is a term half epsilon. But other than that this is very similar to the energy stored.

So now we are going to replace the value of Phi in the equation that we had before with the approximation, so we are going to replace in this equation the approximate value of Phi.

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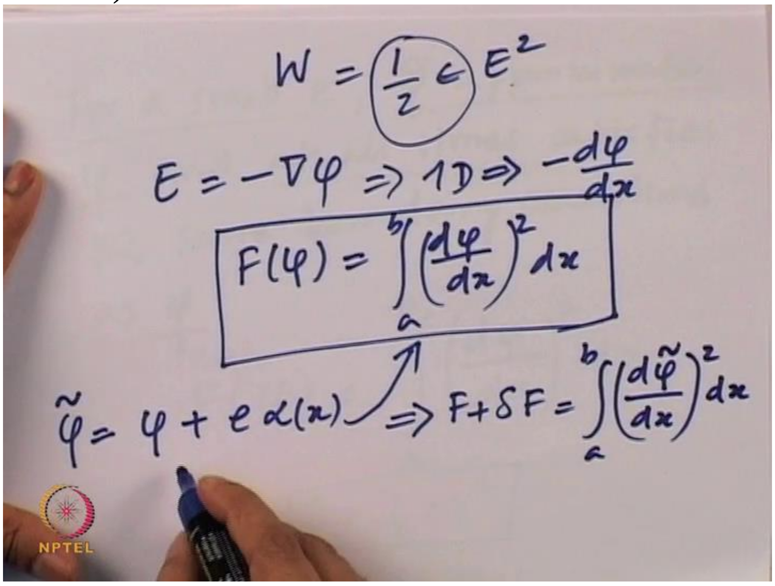

We know that Phi tilde is equal to Phi plus e alpha (x) I am going to drop the x in order to make the equation easier, so this in this will give me F plus certain error which I call it as Del F because of this approximation going into this exact equation, I will have the error term, so that is equal to integral of a to b (dPhi tilde divided by dx) the whole square dx.

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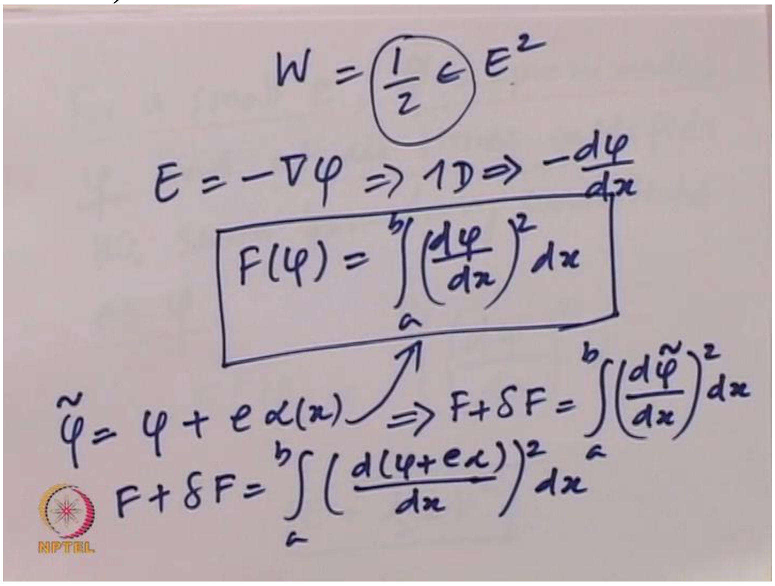

So I can expand this further using this value so F plus Del F is equal to integral a to b (d (Phi plus e alpha) divided by dx) the whole square dx. So this is going to be the starting point and now we are going to expand this one this equation further.

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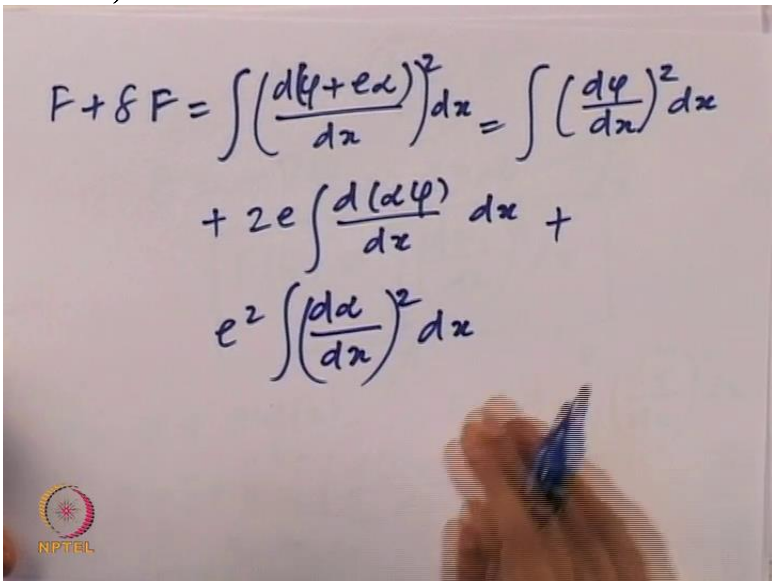

So let us expand this further. So that equation of f plus Del F is equal to integral, for the moment we will forget about the integration limit and focus on the expansion, so we will have (d (Phi plus e alpha) divided by dx) the whole square dx is equal to the first term will be dPhi by dx the whole square dx plus 2 e integral d (alpha Phi divided by dx dx plus e square integral d alpha divided by dx) the whole square dx.

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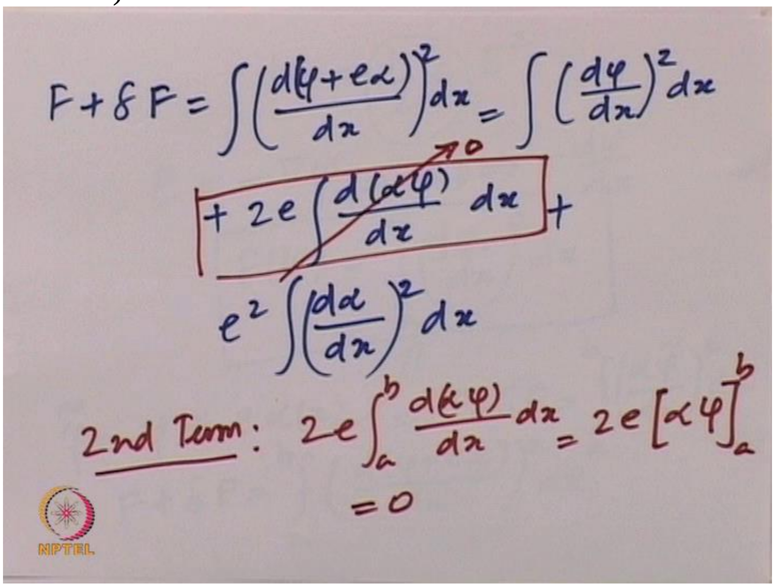

What I have done is I have just expanded this term using the integral calculus and I have got this term. What is important to know is this particular term the second term reduces to 0 because of the definition of alpha that we have chosen so let us write this one down the second term is nothing but.

Second term 2e integral a to b d alpha Phi divided by dx dx is equal to 2e [alpha Phi] a to b. Remember the value of alpha is going to be 0 on both of this end points. So our value will become 0 for the second term. That is why we can say this term reduces to 0 because of the reason that we have explained here. So the value of F plus Del f is nothing but the first term plus the third term which we can write it once more in our expanded way.

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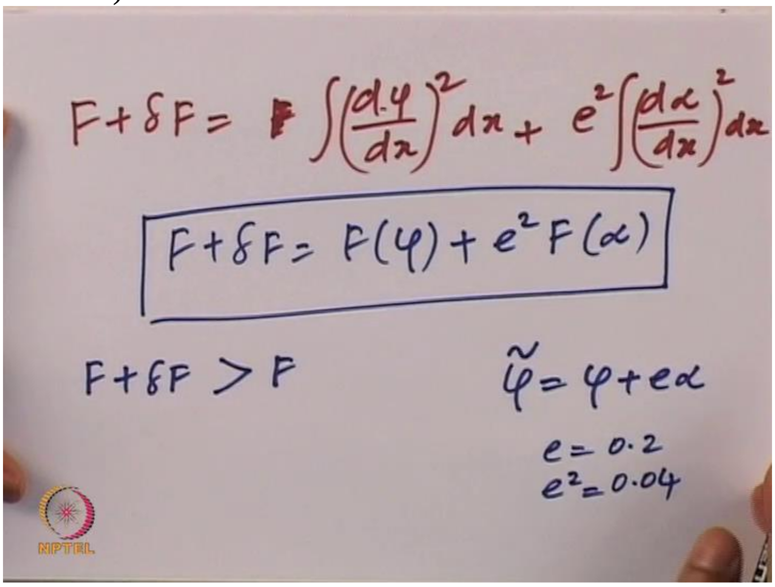

So the value will be F plus Del is equal to the first term which is nothing but integral (dPhi by dx) the whole square dx plus the small term e square integral (d alpha divided by dx) the whole square dx. If you see this is nothing but first term will be a function of only Phi so f plus Del F is equal to the first term will be the function of only Phi plus the second term will be a function of alpha. So e square F (alpha).

So there are several things that come to mind about this equation. First thing is F is always positive in other words F plus Del F is always greater than F. And the approximate expression for F has an adjustable parameter which we have set it as e here and setting these parameters to minimize the function F plus Del F gets us close to the actual value of solution that we are interested in.

So because of this dependence of the error in the energy approximation, so F is the energy approximation on e square rather than e itself the value that we get through the approximate solution for energy is much accurate that the value that we get for Phi itself. Because in the case of Phi remember the approximation for Phi is Phi plus e alpha is the weight we have defined it here the dependence is on e.

So the error will be more whereas here the error will be much more closer to the value of F that we are interested in for example if the error what we are interested in computing for let us say Phi the potential is 0.2, the value of the square of that error which will be 0.04 so e is equal to this e square will be 0.04 in the case of energy computation, so that is why we say because of this e square dependence the approximation for the energy will be much more accurate than the approximation that we have for potentials.

So these are the things that I would like to highlight before we go indirectly to the problem that we are going to solve on Matlab. With this background we will stop here and we will comeback in the next module and do a simple one dimensional problem simulated on Matlab and see what are the things we can learn from it.

Thank you!