

**Computational Electromagnetics and Applications**  
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**Summary of Week 6**

We have covered one of the most important methods namely the finite element method in this week's lectures

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## FINITE ELEMENT METHOD

Two different approaches

1. Variational/Rayleigh-Ritz Method
2. Weighted residuals/Galerkin's Method

Typically both can lead to  
**same discretization!**

FEM – Nodal or Edge Element Based Formulation



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We started with the mathematical formulation of the finite element method. We revisited some of the approaches that we introduced in the earlier lectures namely the Raleigh Ritz Method, Weighted Residual method and Galerkin method.

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## METHOD OF WEIGHTED RESIDUALS

Assuming an approximate solution  $\tilde{E}_x$

$$\partial_x^2 \tilde{E}_x + \omega^2 \epsilon_x \tilde{E}_x + i\omega J_x = Res(x) \neq 0$$

Residual is orthogonal to function space spanned  $\{w_i\}$

Project residual onto a set of test functions  $\{w_i\}$  and force the projection to vanish

$$\int_{\Omega} Res(x) \cdot w_i(x) dx = 0$$


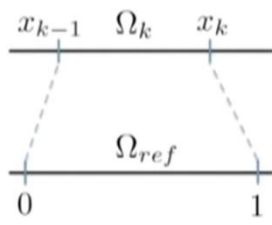

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Our attention was focused on the nodal based formulation of the finite element method using Weighted Residuals.

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### CHOICE OF BASIS FUNCTIONS

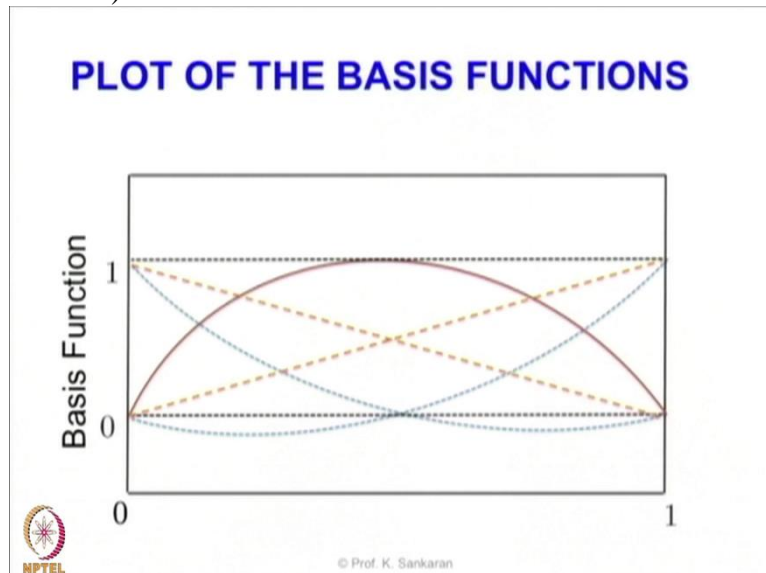
Introduce a mapping from each element  $\Omega_i$  to a reference element  $\Omega_{ref}$

$$x^k(\xi) = x_{k-1} + \xi(x_k - x_{k-1})$$
$$= x_{k-1} + \xi\Delta_k$$

$$\xi(x) = \frac{(x - x_{k-1})}{\Delta_k}$$


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Later we discussed about the choice of the basis functions.

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Showing some linear and quadratic examples.

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## GLOBAL MATRICES ASSEMBLY

Iterate over all elements and add local matrices at the right place

$$\mathbf{M} = \begin{bmatrix}
 M_{1,1}^{(1)} & M_{1,2}^{(1)} & 0 & 0 & 0 \\
 M_{2,1}^{(1)} & M_{2,2}^{(1)} + M_{1,1}^{(2)} & M_{1,2}^{(2)} & 0 & 0 \\
 0 & M_{2,1}^{(2)} & M_{2,2}^{(2)} + M_{1,1}^{(3)} & M_{1,2}^{(3)} & 0 \\
 0 & 0 & M_{2,1}^{(3)} & M_{2,2}^{(3)} + M_{1,1}^{(4)} & M_{1,2}^{(4)} \\
 0 & 0 & 0 & M_{2,1}^{(4)} & M_{2,2}^{(4)}
 \end{bmatrix}$$


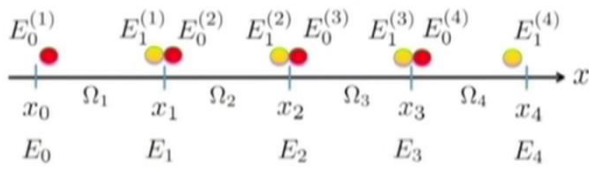

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We also showed how one can build the Global Stiffness Matrix from the local element information.

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## LOCAL TO GLOBAL MAPPING

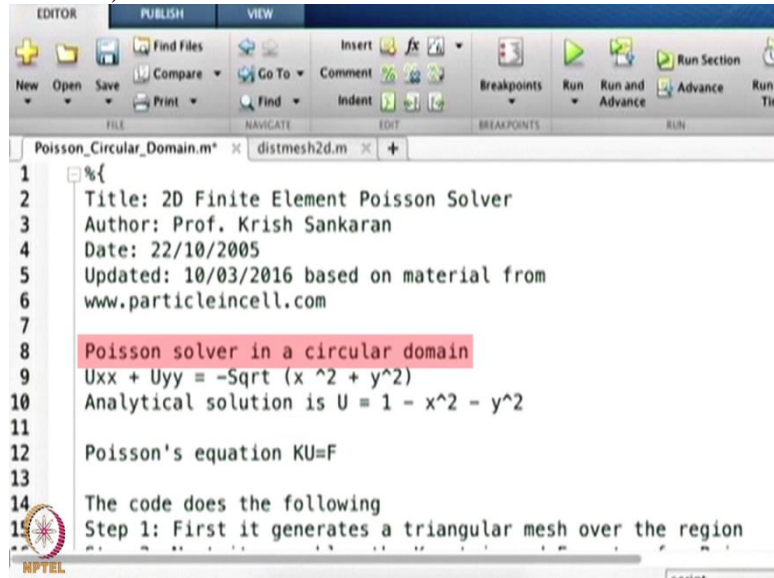
Place nodes at interface between elements

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Through the local to global mapping procedure.

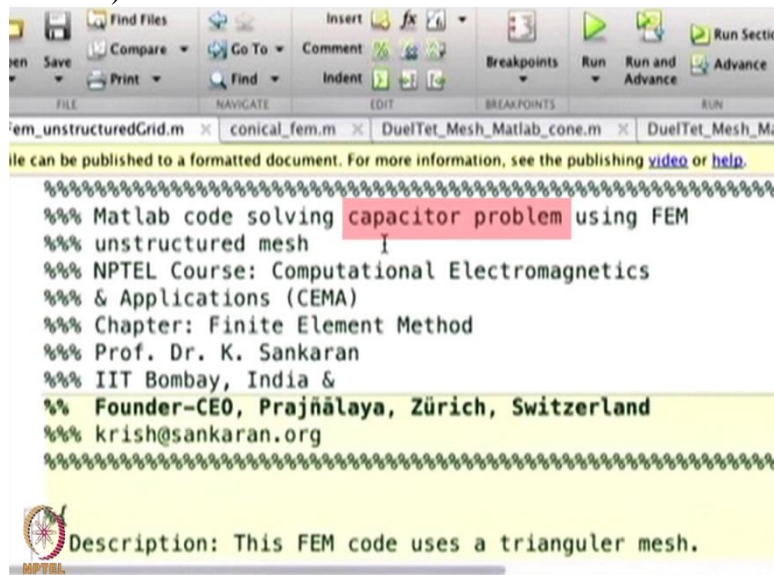
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1 %{\n2 Title: 2D Finite Element Poisson Solver\n3 Author: Prof. Krish Sankaran\n4 Date: 22/10/2005\n5 Updated: 10/03/2016 based on material from\n6 www.particleincell.com\n7\n8 Poisson solver in a circular domain\n9  $U_{xx} + U_{yy} = -\sqrt{x^2 + y^2}$ \n10 Analytical solution is  $U = 1 - x^2 - y^2$ \n11\n12 Poisson's equation  $KU=F$ \n13\n14 The code does the following\n15 Step 1: First it generates a triangular mesh over the region
```

We also demonstrated numerical FEM simulations for solving 2D Poisson Equations.

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em_unstructuredGrid.m x conical_fem.m x DuelTet_Mesh_Matlab_cone.m x DuelTet_Mesh_Ma\n\nfile can be published to a formatted document. For more information, see the publishing video or help.\n\n%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%\n%% Matlab code solving capacitor problem using FEM\n%% unstructured mesh\n%% I\n%% NPTEL Course: Computational Electromagnetics\n%% & Applications (CEMA)\n%% Chapter: Finite Element Method\n%% Prof. Dr. K. Sankaran\n%% IIT Bombay, India &\n%% Founder-CEO, Prajñālaya, Zürich, Switzerland\n%% krish@sankaran.org\n%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%\n\nDescription: This FEM code uses a triangular mesh.
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Later we also modelled a parallel plate capacitor by finding the solution to the 2D Laplace equation using finite element method.

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FETD

$$\nabla \times E = -\mu \partial_t H \rightarrow ① \quad \nabla \cdot B = 0$$

$$\nabla \times H = \epsilon \partial_t E + J \rightarrow ② \quad \nabla \cdot D = 0$$

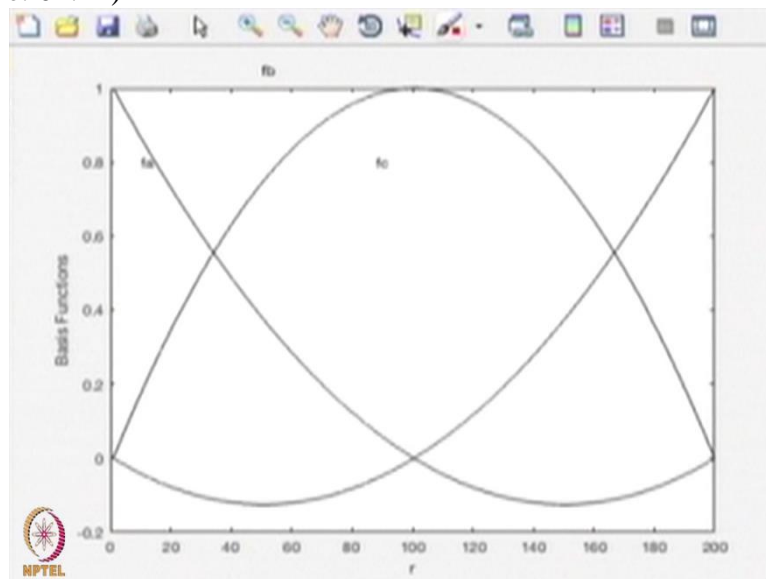
$$② \Rightarrow \frac{1}{\epsilon} \nabla \times H - \partial_t E = \frac{J}{\epsilon} \rightarrow ③$$

$$③ \times \nabla \times \Rightarrow \nabla \times \frac{1}{\epsilon} \nabla \times H - \partial_t \nabla \times E = \frac{1}{\epsilon} \nabla \times J$$

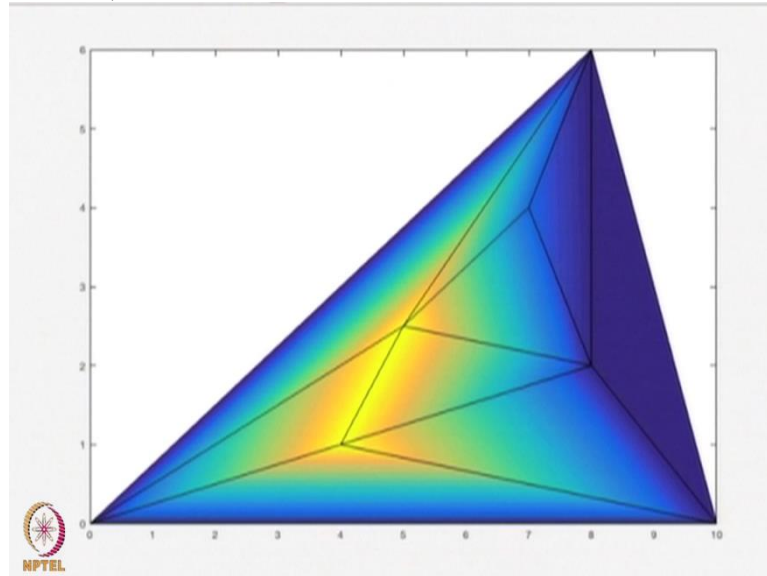
$$\nabla \times \frac{1}{\epsilon} \nabla \times H + \mu \partial_t^2 H + \frac{\mu \sigma}{\epsilon} \partial_t H = \frac{1}{\epsilon} \nabla \times J$$

We also step by step derived the FEM formulation for solving Maxwell equations in time domain. We discussed the frequency domain formulation for solving Maxwell Equation in the Finite element method.

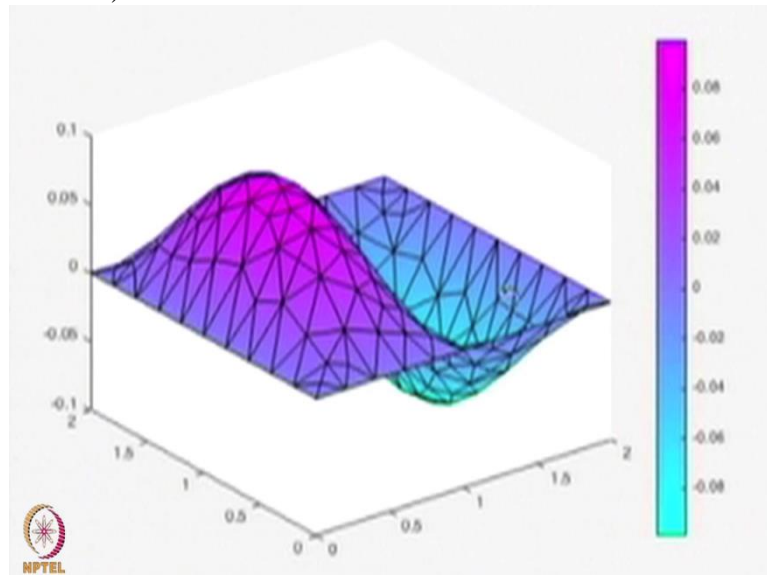
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We will discuss a lot of examples using Finite element method in the next modules. So please go through the derivations and examples that we have discussed in this module and get ready for the next week. Until then Good Bye!