Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture 04/Exercise 01
Finite Difference Methods –1

So we will look into some exercises in the coming modules.What we will start with is a very simple exercise, dealing with Laplace Equation.So Laplace equation is something that you will come across in every part of the electromagnetic lectures as fundamental equation that deals with potential and how the potential is disputed at given certain boundary conditions.So to understand any differencing method,it is good to start with Laplace Equation as a starting point, and then extend it to Poissonequation which is having certain right hand side term.

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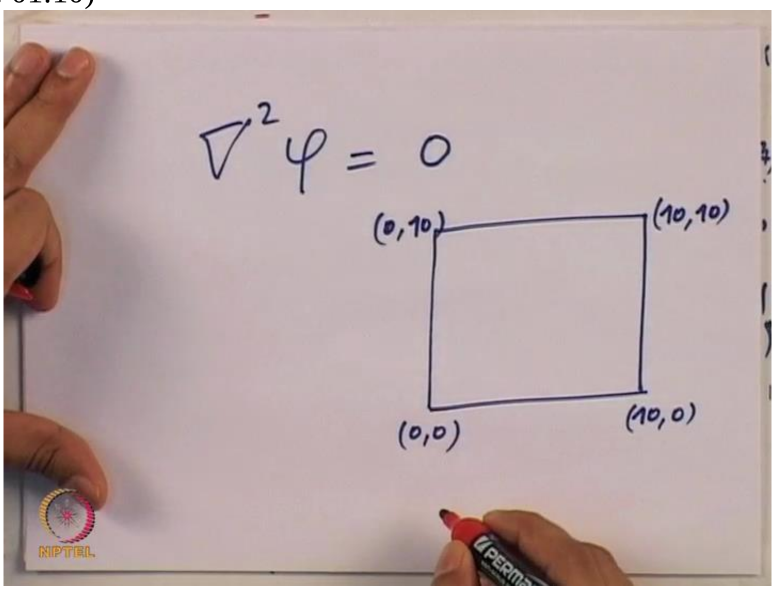

So we will explain this while we solve the problem.So let us look into the basic equation of Laplace.So what we have is we have the del square phi is equal to 0, and we have certain boundary conditions given.So let us define our domain, our domain is going to be a square with points (0, 0)(10, 0)(10, 10) and (0, 10).

Again I am just defining it for simplicity you can define different domain, dimensions.It need not be a square it can also be a circle so for the simple case let us start with the square domain

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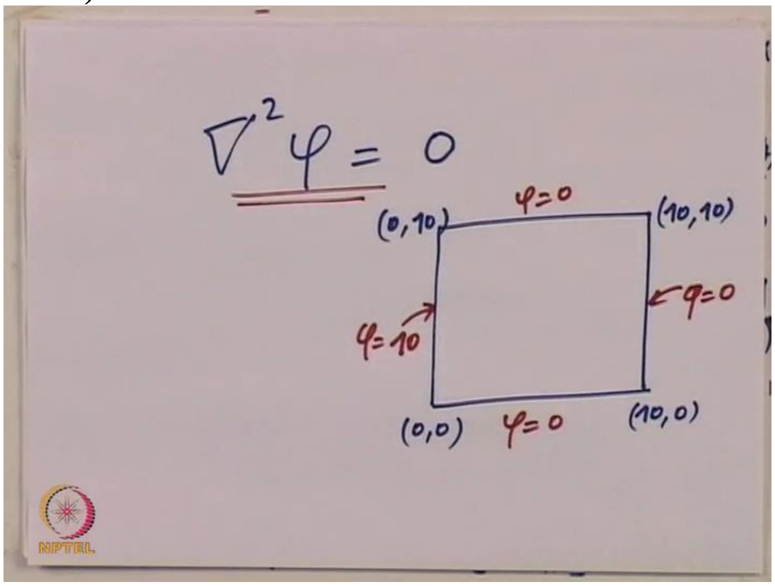

And we are going to define certain boundary conditions the boundary conditions as we discussed before are going to be the definition of potential along these 4 sides.So if we say the potential along these 4 sides are given by certain value.

Let us say we are going to give potential on this particular boundary to be 10.And the potential on this side is going to be zero,so phi equal to zero and the potential this side is also zero potential on this side is also zero.So let us take very simple example we see that the potential is going from 10 to 0 and it has to have certain behaviour.Physically we understand that it is going to be a linear drop from 10 to 0.And let us see how numerically it behaves.

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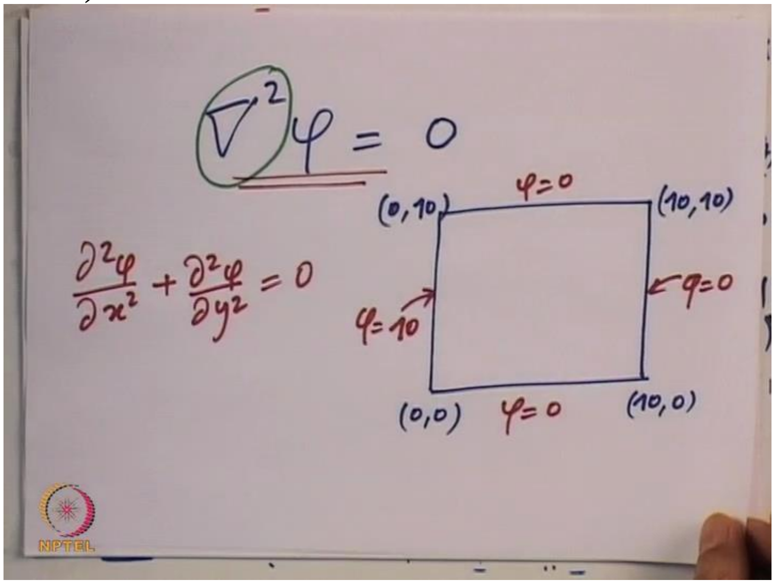

So for that we have to start with this particular equation and we have to find difference analogue of this continuous del square.So what we have learnt in the earlier module is we can basically use Central differencing method.So this particular case we can write them as (do square phi by do x square)Plus (do square phi by do y square) is going to be equal to zero.So we have the basic equation in the continuous form written as (do square phi by do x square)plus(do square phi by do y square) equal to zero.

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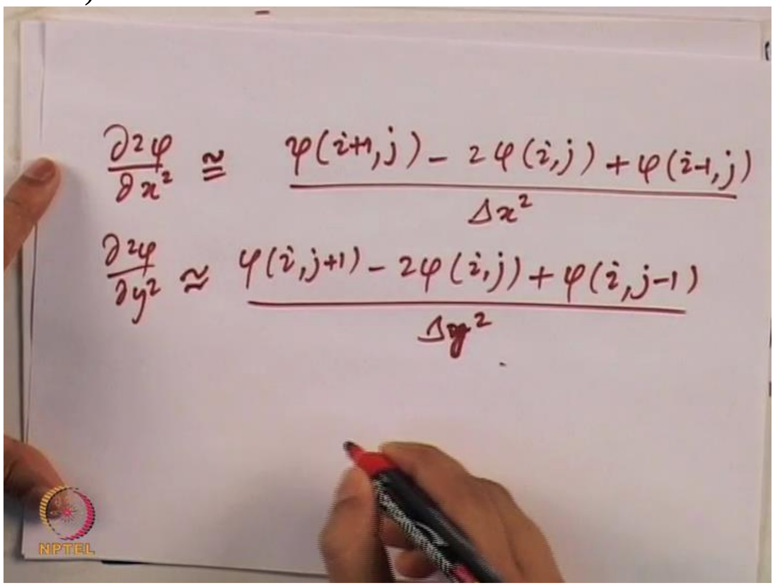

What we know from the earlier lectures on finite differencing method, we can approximate the term (do square phi) by (do x square)using the Central differencing method as phi (i plus 1, j)minus 2 phi (i, j) plus phi (i minus 1, j) divided by (del x square).So what we are doing here is we are doing the central differencing and we are getting the value in this form.Again this is only approximately equal to you have certain truncation errors.

And what we can do similarly is also the other part of the equation which is this one.So we can write it approximately equal to phi of (i,j plus 1) minus 2 phi (i,j) plus phi (i,,j minus 1)divided by (del y square).So this is (del x square) and we have (del y square) and putting this together what he will get it is the 5 pointdifferencing method where we are using the stencil to compute the value at i, j based on neighbouring points.

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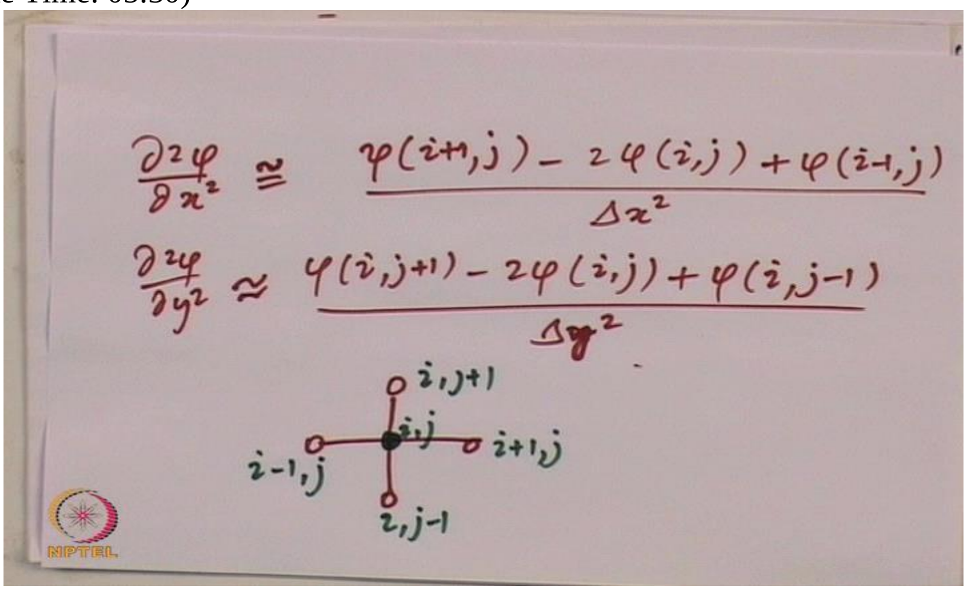

So we can write down in this form.So if this is (i, j) this is going to be (i plus 1, j) this is going to be (i, j minus1) this is going to be (i, j plus 1) this is going to be (i -1,j).And we are computing the value and the center based on certain values at the neighbouring points.So this is what we're going to do in this exercise.We are going to use this 5 point Difference in method to take the Laplace Equation from that continuous space to the finite difference space.

So let us look into the code itself and let us see how the MATLAB program can be used for simply solving this problem.So let us start with the basic MATLAB program.

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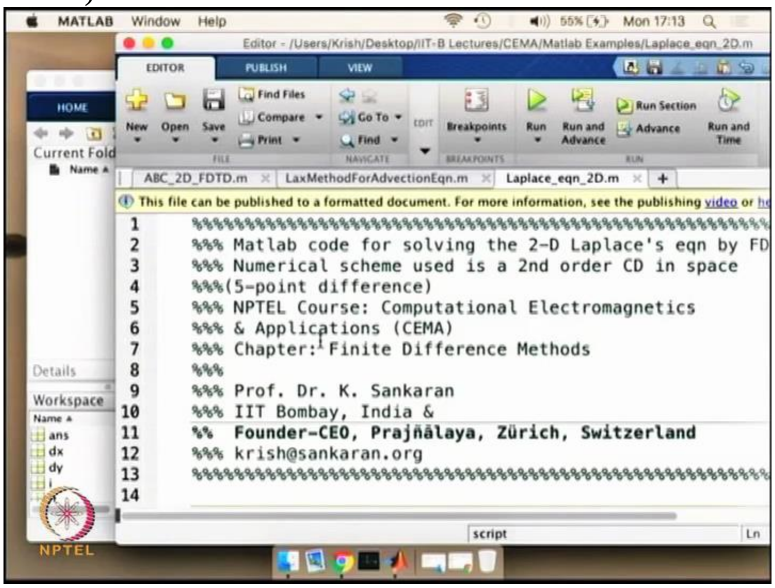

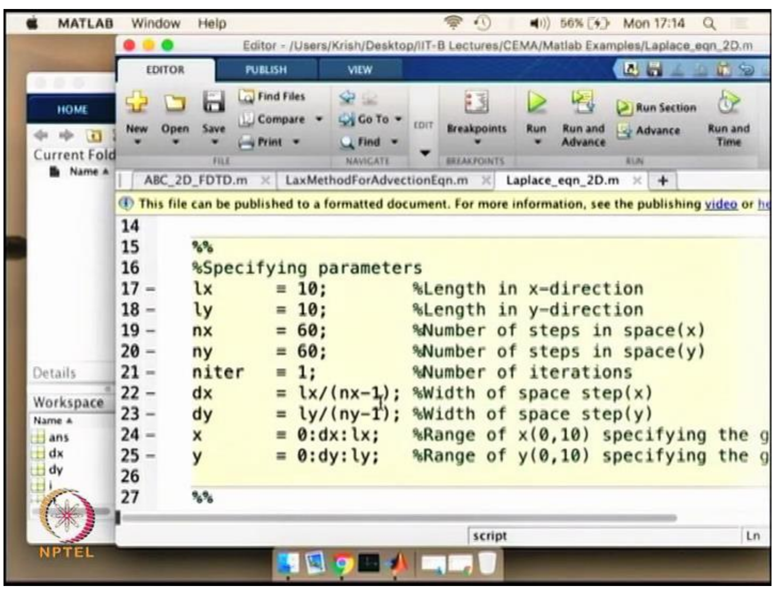

So now let us look into the code itself.So we are setting certain specifications.So these are the specifications which we will use to set the dimensions and the parameters of the problem.So the first one is the lx which is the length in the X direction and the second one is the ly which is the length in Y direction, which is very similar to the problem which we have in the paper.

As you can see, so the length in x direction is 10, the length in Y direction is also 10 and that is what we have set here.And nx and ny are going to be the number of steps in X and Y direction.So we have 60 steps in x, and 60 steps in y.And the number of iterations are going to be given by N theta and as you can see I have given here wantedly.The reason is one will be a very very crude approximation and we will see how to increase and why we have to increase to get better resolution.

Ndx is going to be the width of the step in X and Y direction will be dy.So dx and dy are correspondingly the width of the steps, or stepping size in X and Y direction.And the value of various X component and Y component are given by the X and Y parameters.And they are going to go from 0 to lx with a steps size of dx in the direction of x, similarly they are going to go from 0 todly with step size of dy in the direction of y.

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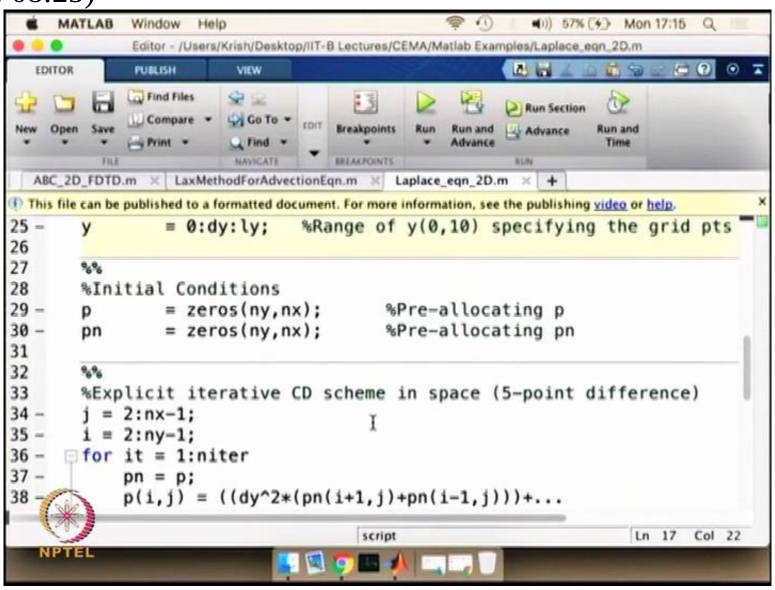

So now we are initializing certain parameters for example potential is given by p here, we are preallocating them to be 0 and p n is also a variable we are going to use and we are preallocating it to 0.And we are going to go into the iteration the one which I explained you in the paper.

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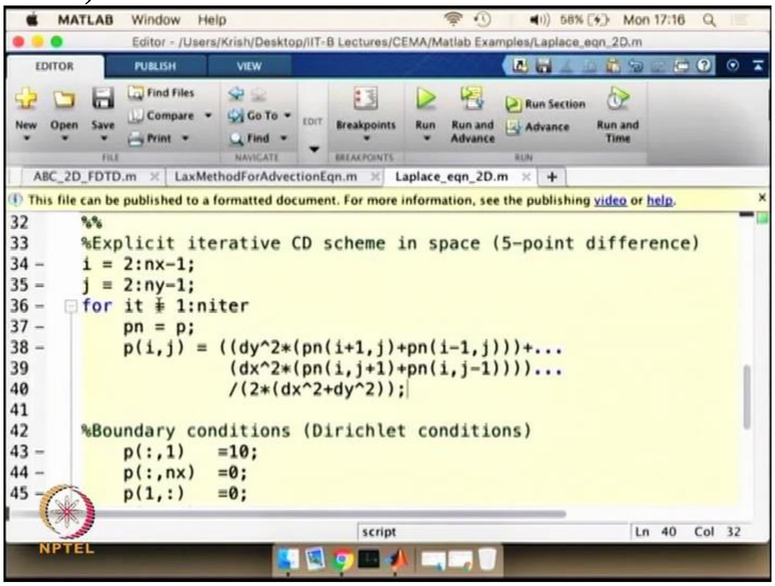

So we see that we are going to use a5 point Central differencing scheme which I explained to you just now in the paper.We are going to go with certain index which is i for the X co-ordinate and j for the Y coordinate.And the iteration is going to be the for loop and it goes from 1 to number of iterations.In our case initially we have chosen niteris equal to 1.

And you and we'll see that it is not going to give us good result but still we will start with that. And now we are using the central differencing scheme. I just explained on the paper to compute p,i,j using the points surrounding it and we are going to use the 5 points for that.So basically we are taking the four neighbouring points to compute the value and center point that is why we are calling them 5 point Difference.

And we have set the boundary conditions as given here so we have set p (dash, 1) equal to 10. So what I say (dash, 1) is it is going to be for all x coordinates but with 1 Y coordinate, the one Y coordinate going to be Y equal to 1 so its going to be a line that is going to be at Y equal to 1.

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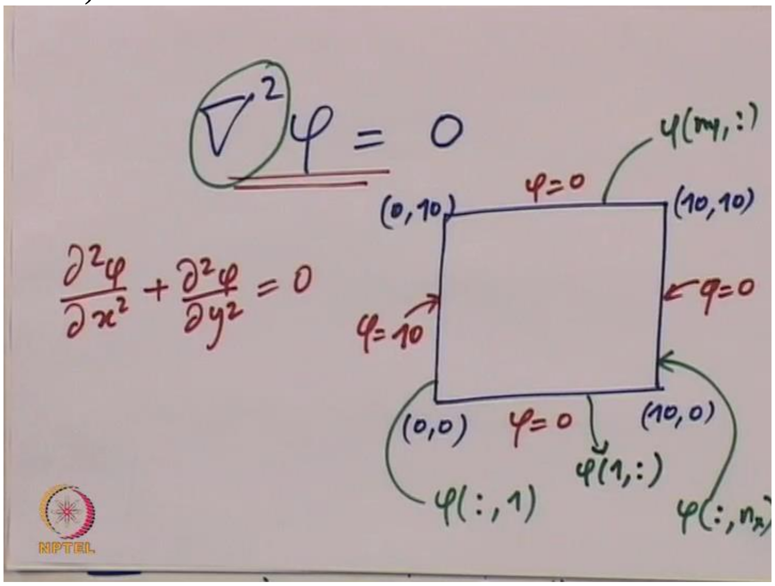

So Y equal to 1 will be the first coordinate which we have given here so this is going to be the first Y coordinates, and similarly this is going to be the last Y coordinate which is going to be ny. So let us write it down.So this is going to be the phi (dash, 1) and this is going to be phi (dash, nx), similarly we can write them also for the other two sides.So this is going to be phi (ny, dash) and similarly this is going to be the phi (1, dash).

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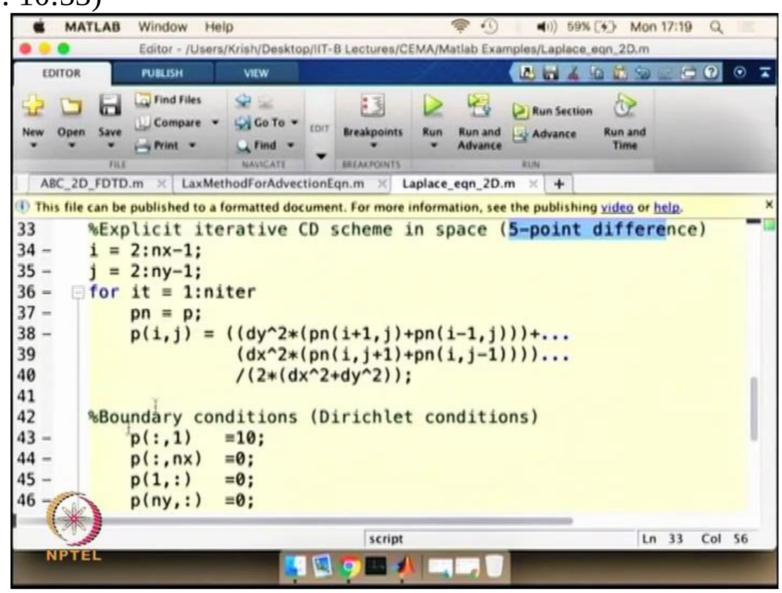

So that is what we have done here so if we see the equation we have put for the first y line as equal to 10. So this is going to be the line that is going at Y equal to zero.So Y equal to zero will be the the first coordinate since we are in the loop going from the value we are setting the value ok i, j but we are not worried about the first coordinates because we are setting the value of boundary conditions.Since the boundary conditions are going to be the first co-ordinate or the last co-ordinate for loop is only going from in between values.So the first value and the last value are actually the boundary coordinates which we are considering here if you may be wandering why we start from 2 to nx minus 1 or 2 to ny minus 1the reason is here, in the boundary coordinates.

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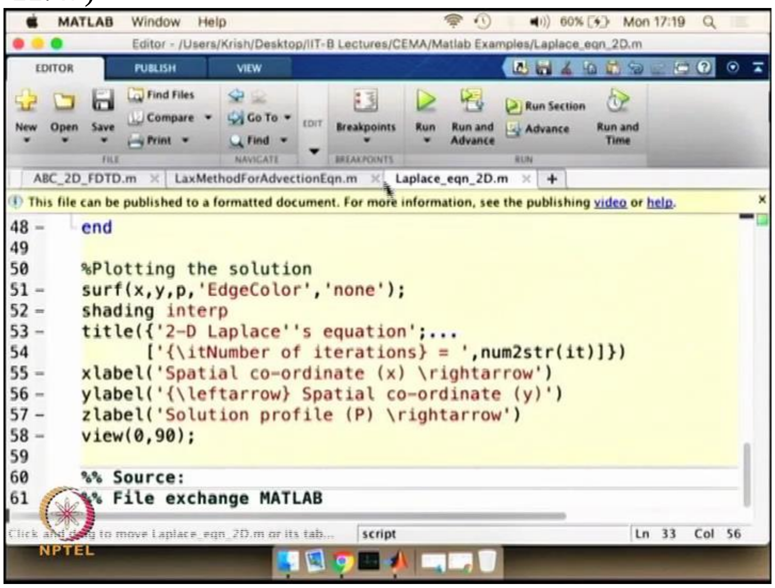

So let us run this program what we are going to plot is the value of the potential phi as a function of x,y and we are going to see it in the top view and we can rotate it and see also in the other views.

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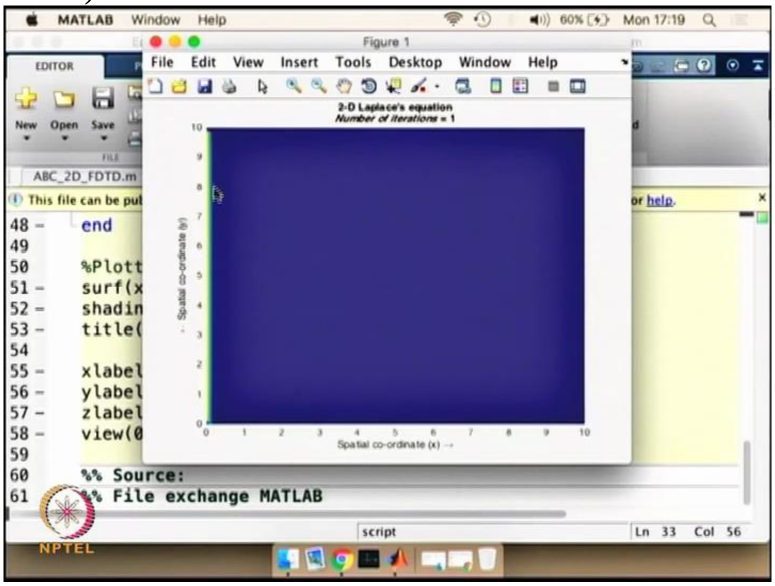

So once you run it you see that the potential on the left hand side I can rotate it so this is the top view we are on the xy plane so if we view it from xz plane you will see it is showing that the potential is 10 on the left hand side but the other potential immediately drops to 0 which is not physical.

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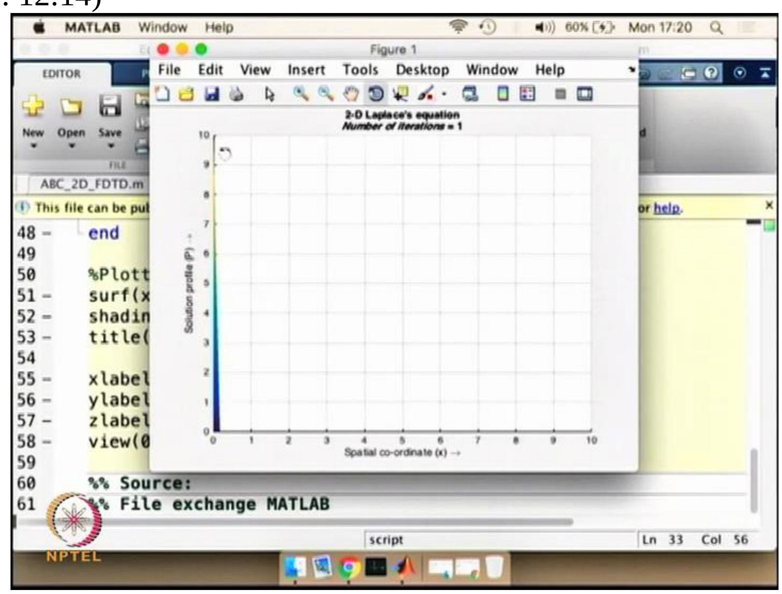

What we expect theoretically is a kind of a equipotential lines and we will see where the equipotential lines are by simulating the problem for more iteration. So let us go and iterate it for let us say 100 iterations.

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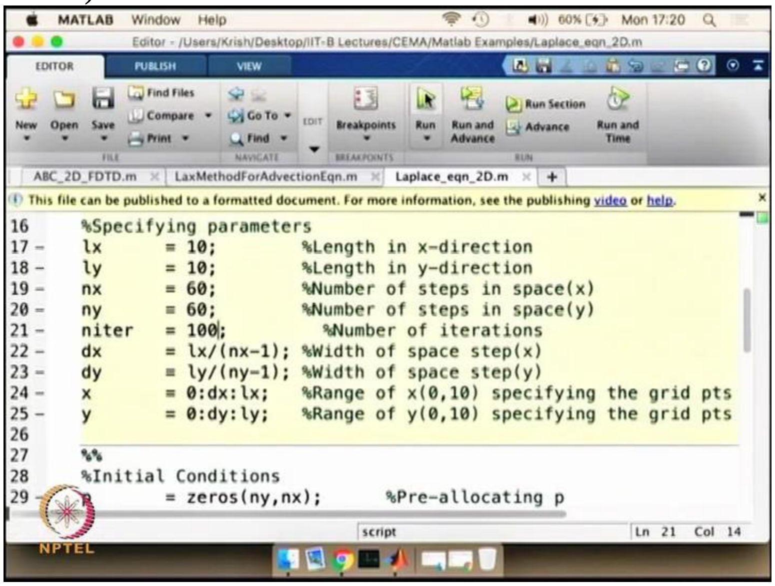

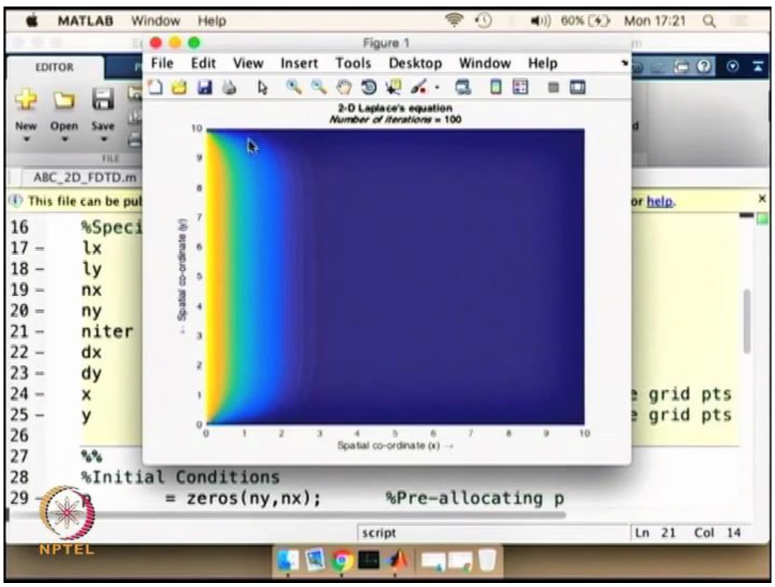

So I am putting iteration is equal to 100 and running it and you see that since this boundary and this boundary is also zero we will see a kind of a equipotential line and let us see running it for longer time let us say I am running it for 1000 iterations.

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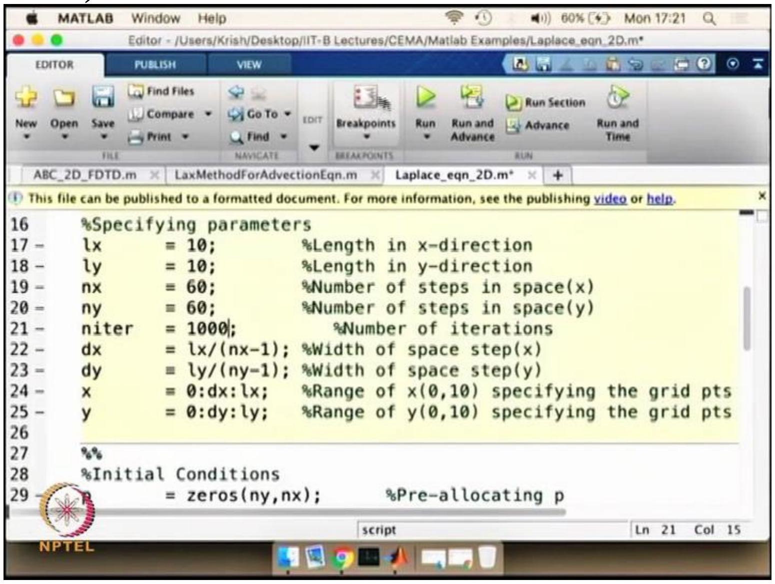

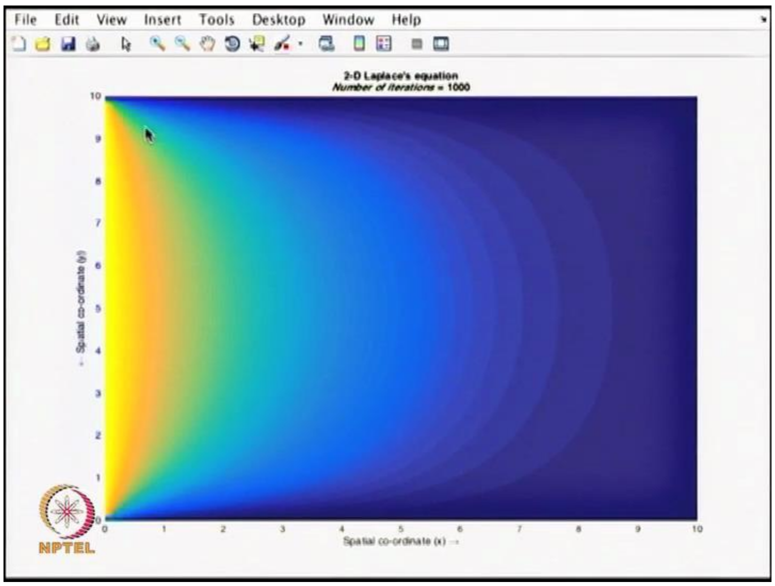

I am seeing that it is replicating but still I can see the equipotential lines. I will zoom it out, you will see the equipotential lines seen as those different shades of colors and if we rotate it in a different plane you will start to see a drop that is happening.

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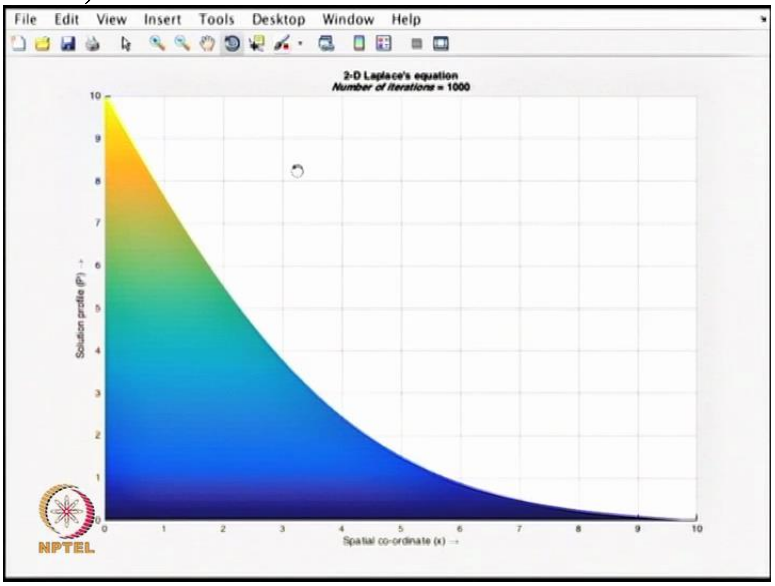

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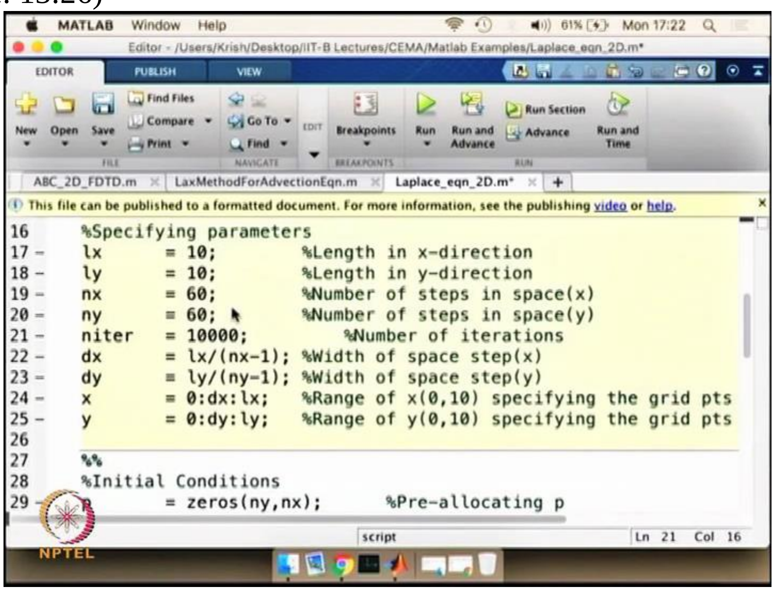

So now if you run the same thing for more number of iterations let us say we run it for 10000 iterations which should be good enough.

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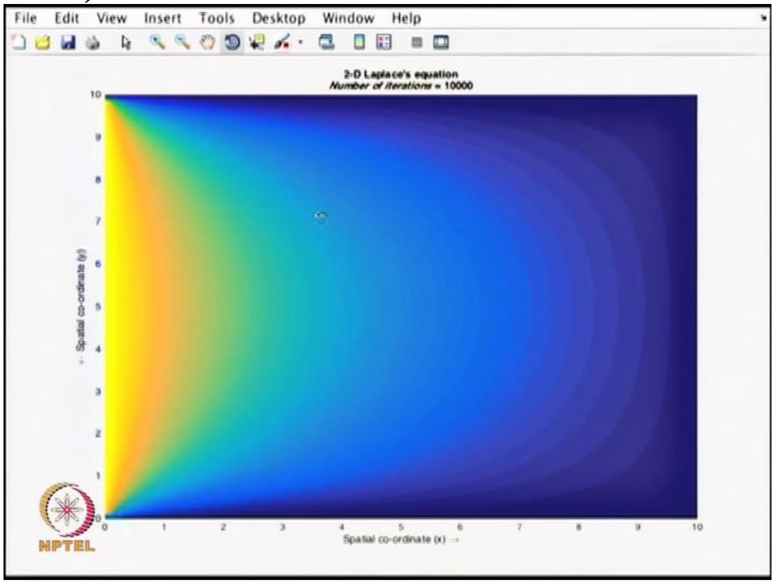

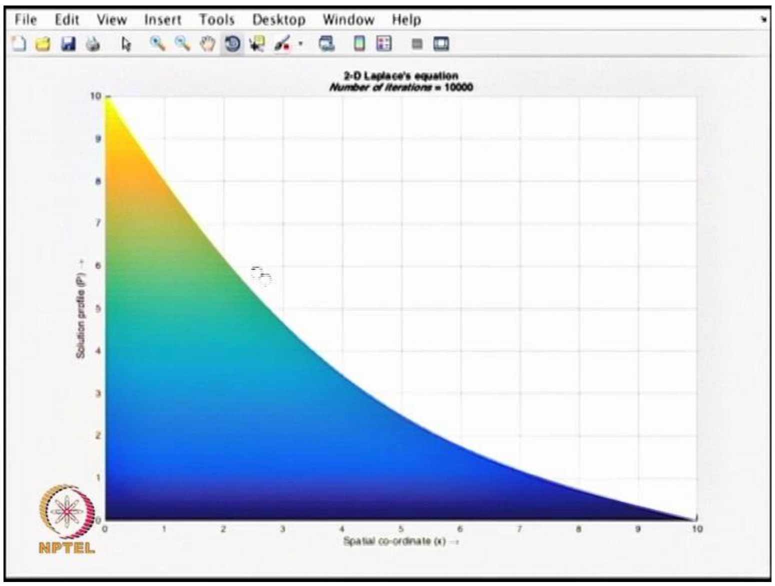

Now we can see the quality of result is improving and we are able to see a drop which is as we expect. So this is Laplace equation for this particular case what we can try doing is we can try changing the boundary conditions In order to see what will be the influence of the boundary conditions for this particular problem.So what I am going to do is instead of choosing 0 on other side I am going to choose certain values. For example let us say my two edges.

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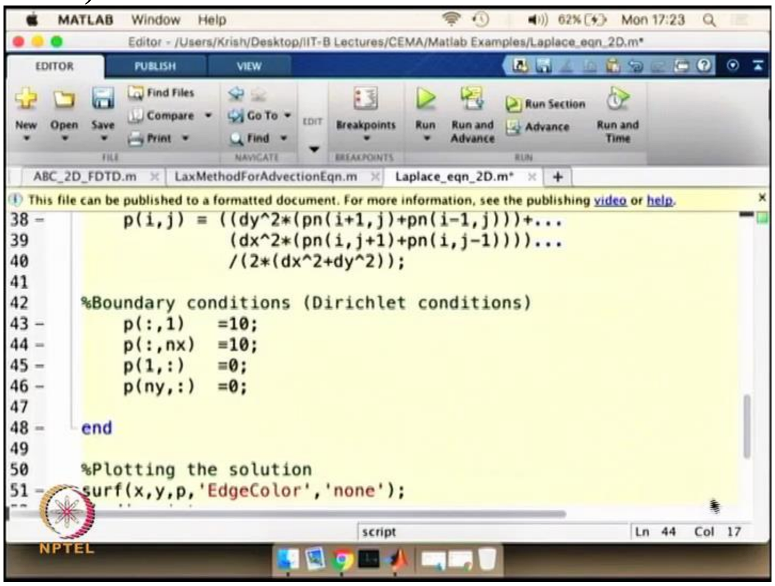

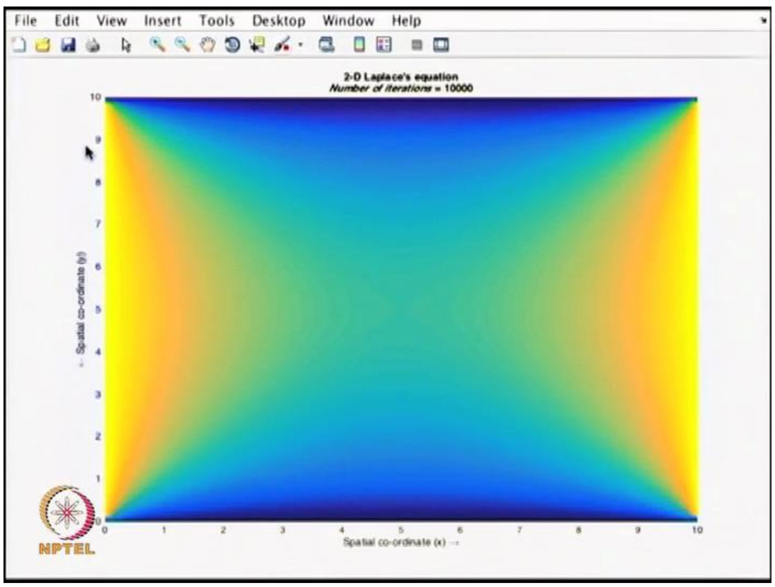

So for y equal to 0 and y equal to 10 which is these two edges I am going to put the value as 10 and the other two I keep it as 0. And I am going to run it for 10000 iterations steps and when I do that I start to see a nice curve of the equipotential lines.What we see is potential lines are following certain pattern and you will see that pattern here.

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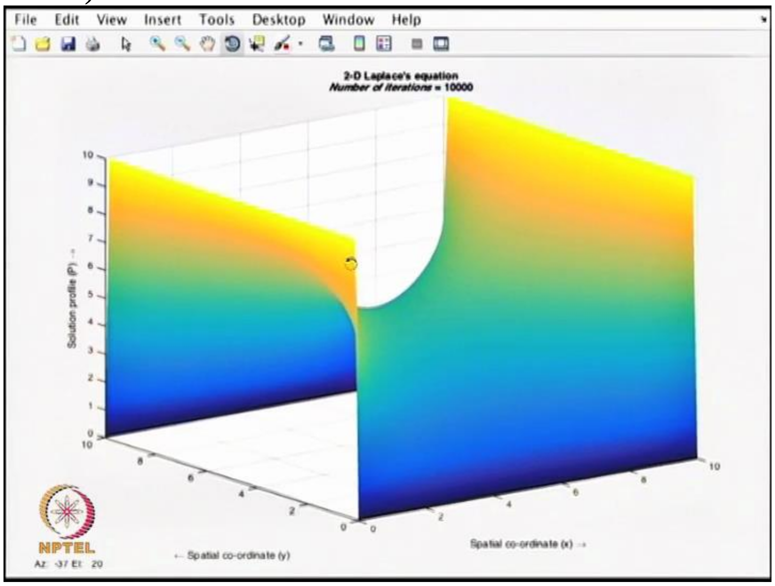

So the value here is set to 10 the value here is set to 10 whereas value on these two boundaries are set to 0 and you see a nice manifold of the potential and you see the equipotential surface when you see it from the top view.So these are the equipotential surface.

And So this is a very good example for you to start with because this gives us certain understanding of how a single equation as Laplace equation can be simulated using matlab and using certain central differencing scheme which we learnt in the previous modules and this is going to be a very important exercise for you because we are going to build on this exercise in the next step using Poisson equation.

And Poisson equation will be same as the Laplace equation except the fact that we have a right hand side term which will not be equal to 0 so we will have certain influence from the right hand side term which we will see in the next module. We can see that already in this particular case where we can define the value of dx and dy accordingly.

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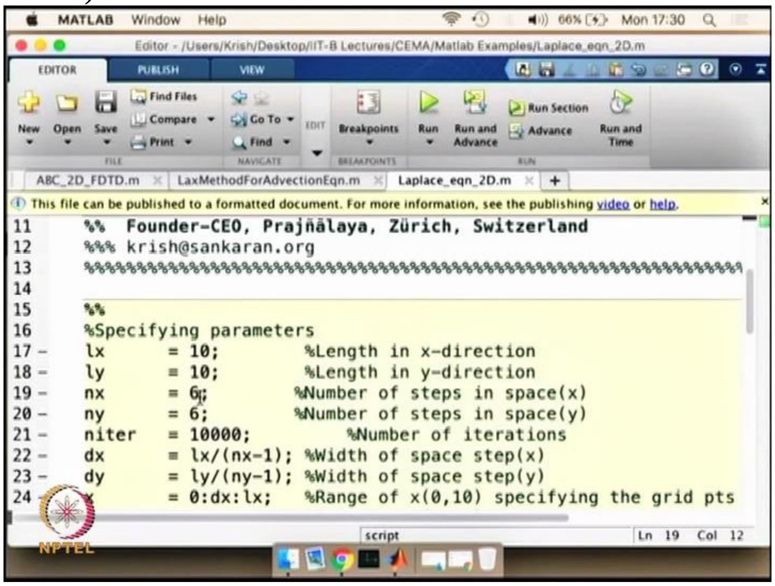

So if we choose we want the number of special discretization to be in particular manner for example if we choose instead of 60 6 points on the x and y direction we will see the influence of the problem resolution will change accordingly.Let us run this code initially we had 60 and we changed it into 6 and if we run the same program and we are giving the boundary conditions to be 10,0 and 0on other sides and if we run it you will see the result is going to be categorically different in terms of quality.

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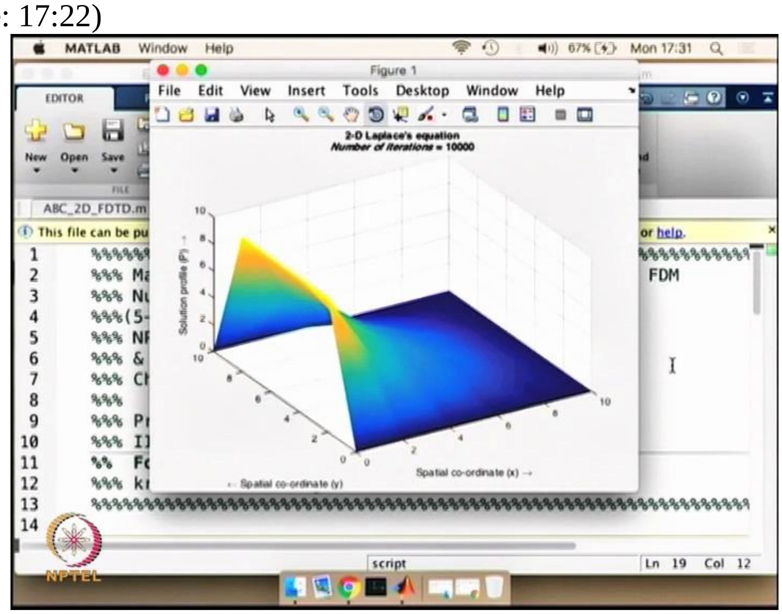

You will see that the equipotential lines are not really symmetrical. There is going to be some influence in certain directions.

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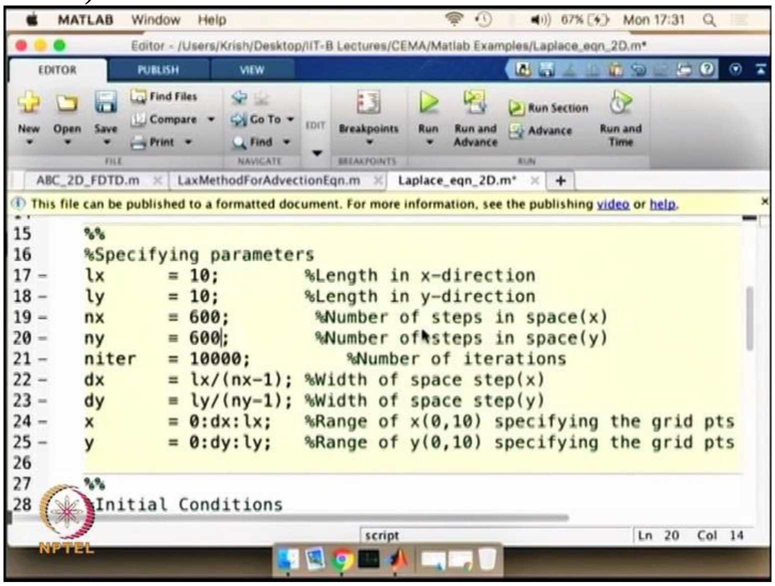

And if we keep this one aside and I am going to run the same program with higher number of discretization.Let us say I am going to increase it with 600 I am going to run the same code so what we see here is compared to the earlier problem where we had only 60 steps in the x and y direction now we have increased the number of steps to 10 times more and we can see the result is much more finer.

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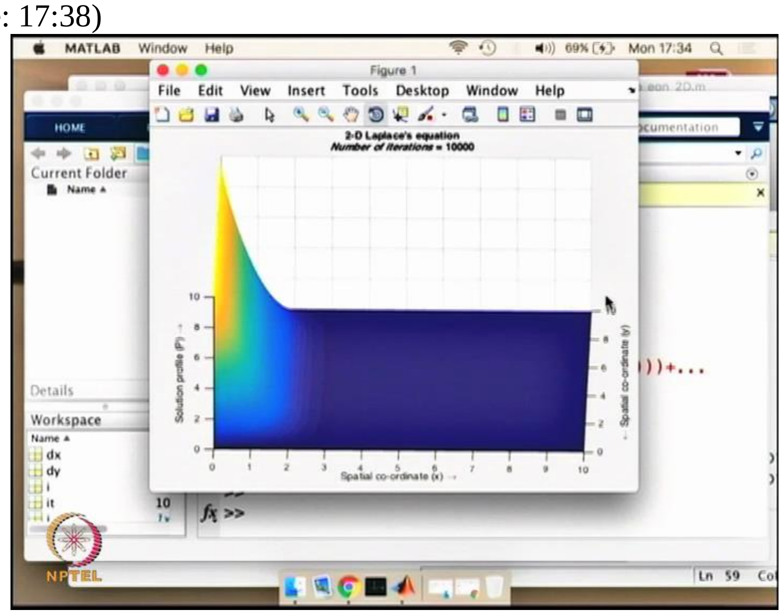

So the resolution of the result is better compared to the previous simulation. So what we are trying to illustrate here is there is going to be an impact of the special and temporal deiscdizationand also the number of iteration that you are going to run on the top of the differencing method scheme itself. So there is going to be an impact of differencing method and on the top of the impact of the differencing method the simulation parameters what we are going to use will influence the resolution or the quality of your solution.

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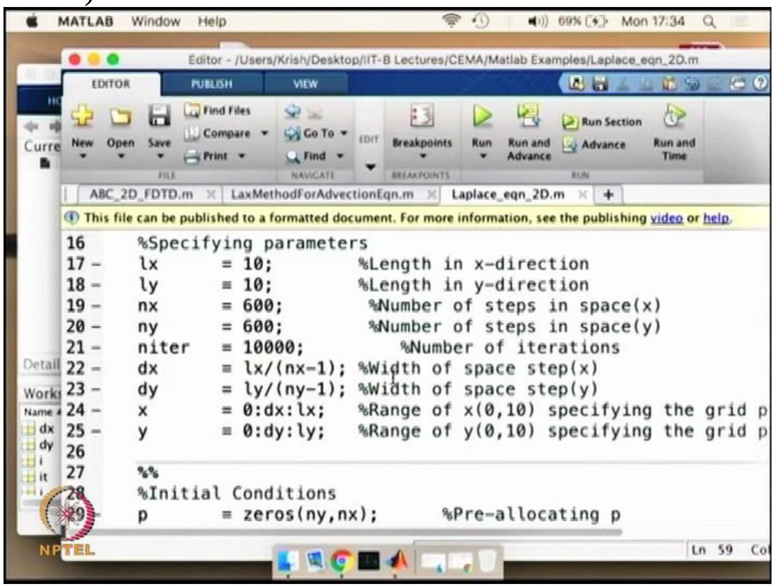

So that is a classical example for you to try, so we want you to use this particular code and try it for yourself how the central differencing scheme is implemented in Matlab for Poisson or in Laplace equation. In this case we have used for Laplace equation. We will try doing the same problem also in the case of Poisson equation to see how the right hand side can be implemented in Matlab and I would like you to see how you can manipulate the special discretization in the x and y direction and also the number of iteration that we are going to use. So that we you get a holistic understanding not only about the discretization method not only about the differencing method but alsoabout the simulation parameter and it's influence on the numerical result. So with that being said I think we will stop here and we will come back in the next example and look at Poissonequation. Thank you!