Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture No 14
Boundary Conditions

Welcome back we started with the Maxwell equation inside an anisotropic model we came to the point of driving first step of the PML equations.

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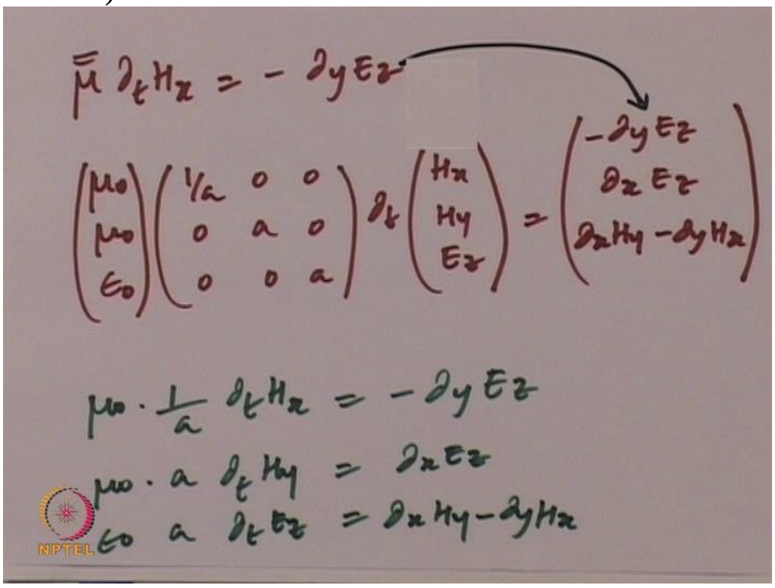

Where we are now in the step so we have got basic certifications we are in the process of deriving step by step the 3 field equations from to curl equations so we have started with the permittivity model which is going to be an isotropic in X direction we have casted it on the Standard Maxwell equation terms so we are at the point of getting the simplified equations for all the three feel terms so let's not start with the first equation of H x.

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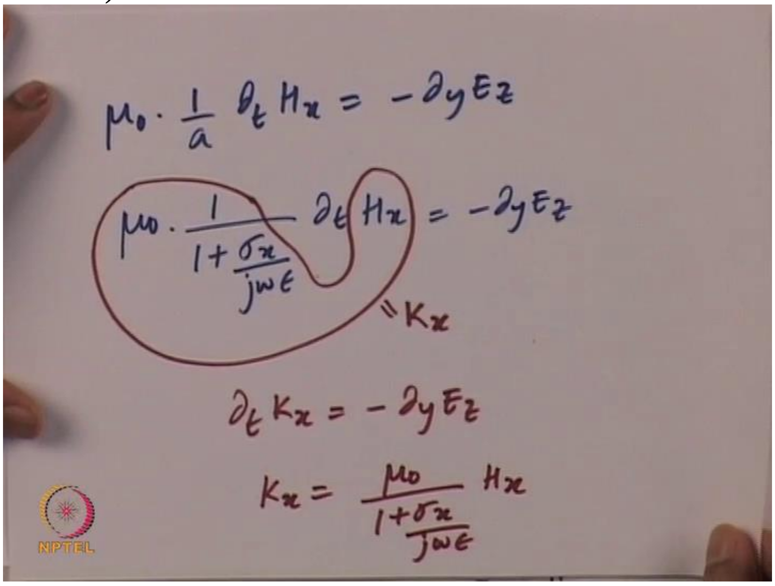

So what we have got now is the term Mu 0 x 1 by a dt H x that is equal to minus d y E z. So I am going to plug in the value for which I have given in the earlier module which is going to be 1 plus Sigma x divided by j Omega Epsilon so let us put that in. it is going to be Mu 0 multiplied by 1 by 1 plus sigma x multiplied by j omega Epsilon dt H x is equal to minus d y E z.

So what I am going to do is I am going to substitute the value Mu 0 H x this one to be equal to the value called k of x I'm using the term k because it is arbitrary variable so I am going to use kx because it corresponds to H x. So I'm going to have dt k x is equal to minus dy Ez. If I plug this value as kx I'm going to get the equation like this word of course kx itself is going to be equal to Mu 0 divided by 1 plus sigma x j omega Epsilon dt H x.

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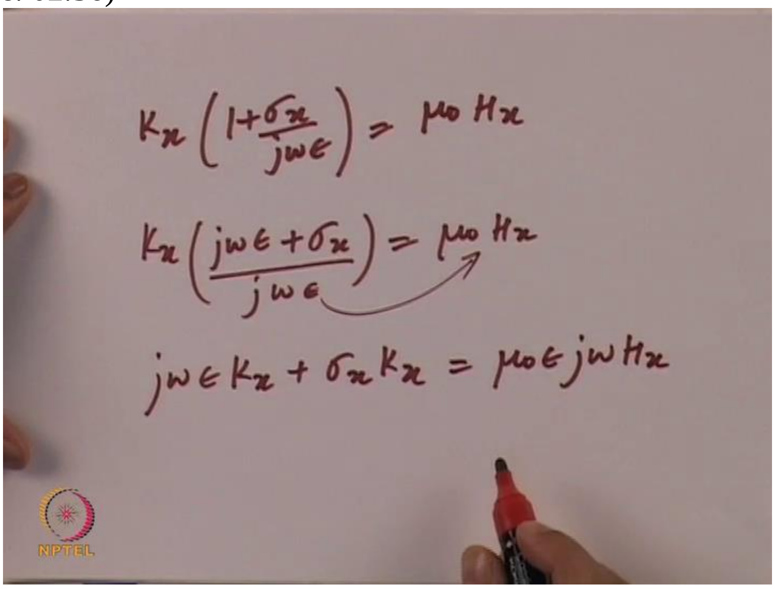

So let’s go forward in simplifying this equation so what you will get is a value for HX and kx in a much simplified form so what we will get is kx multiplied by 1 plus Sigma multiplied by j Omega Epsilon is equal to Mu 0 H x. And now we can multiply this even father it is going to be equal to Kx (j Omega Epsilon plus sigma x divided by j Omega Epsilon) equal to Mu 0 H x. I am going to take the value j Omega Epsilon on the other side so what I will get is j Omega Epsilon K x plus sigma x k x is equal to Mu 0 Epsilon j omega H x.

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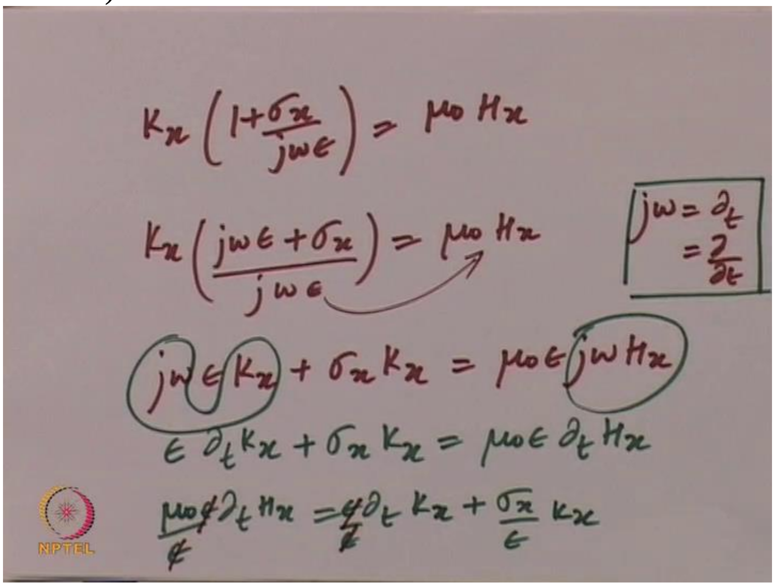

And now we can substitute the value j Omega equal to Doe by dt partial derivative with respect to time this is in the for your frequency space and then we are going back to the time space this is the frequency domain we are going to the time domain so if I do that what I will get here is Epsilon Doe t k x plus sigma x k x is equal to Mu 0 Epsilon Doe t H x. I am going to divide the entire equation by Epsilon and I will get rid of the Epsilon and I will get the denominator Epsilon for this term and I am going to rearrange the term so that I get Doe t H x on the left hand side. So Doe t H x Mu 0 divided by Epsilon is equal to Doe t k x plus sigma x by Epsilon . So here there is a Epsilon, here there is a Epsilon, so I am dividing it by an Epsilon and I have k x here. So get rid of this excellent get rid of the Epsilon and this Epsilon space.

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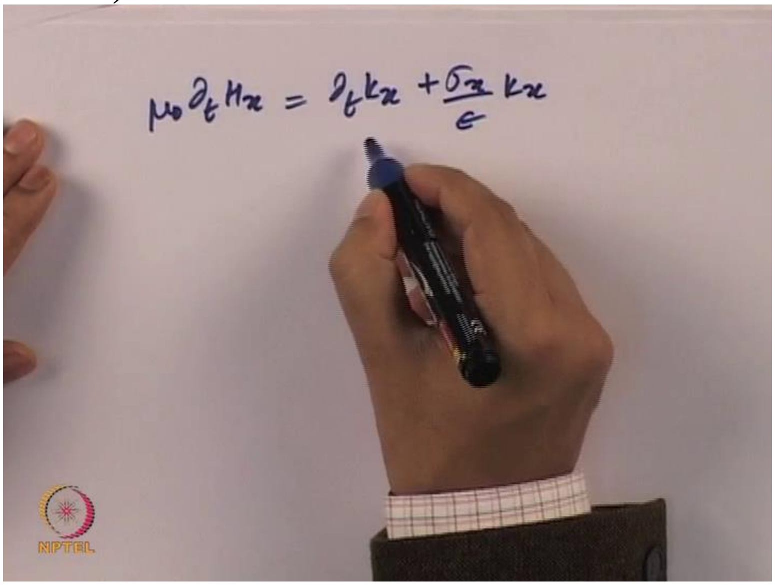

So what we will get is a simplified equation for dt H x which is given by doe t H x Mu 0 equal to Doe t k x plus sigma x by Epsilon k x.

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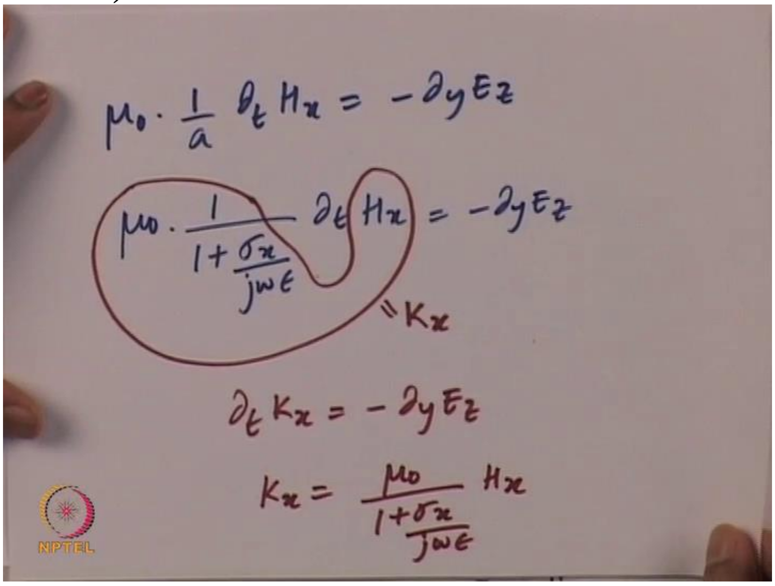

And of course we know that what is the value for Doe t k x because we got that from our earlier equation. Which is nothing but minus Doe y Ez.

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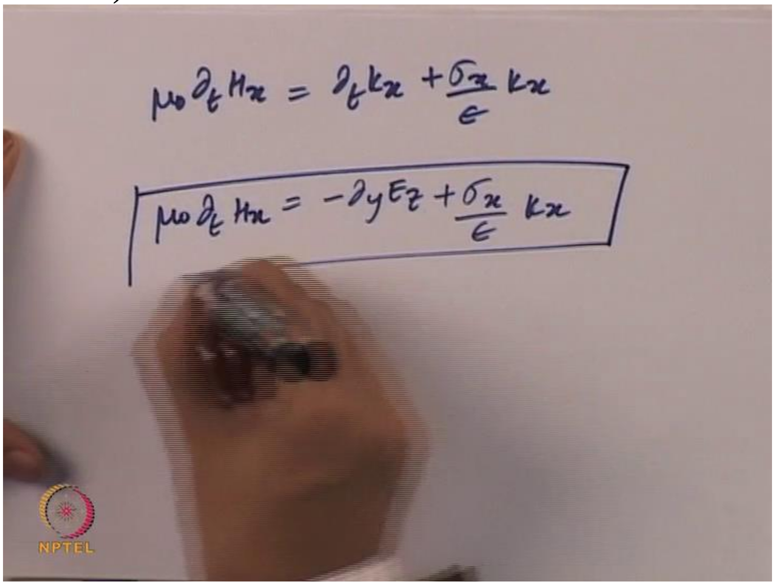

So I am going to substitute that value here size equal to minus Doe y Ez plus sigma x by Epsilon kx. And this u 0 Doe t H x. This is going to be one of the equation ;

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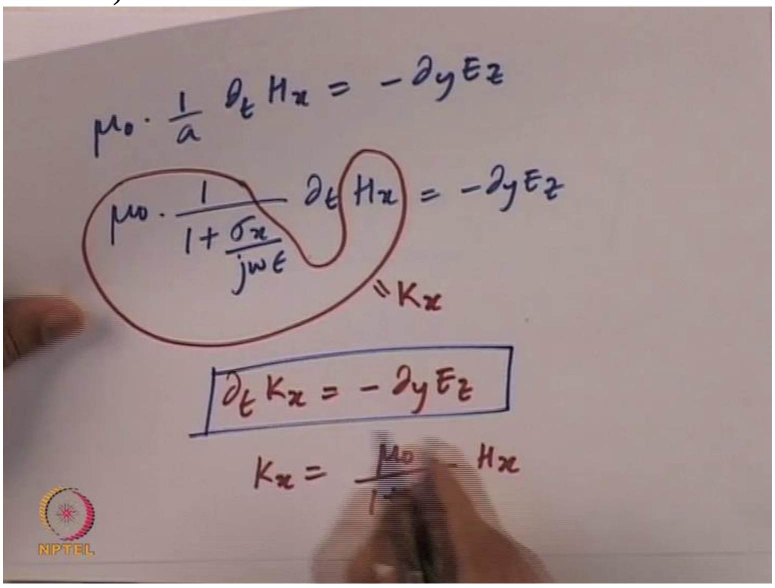

This is going to be one of the equation.

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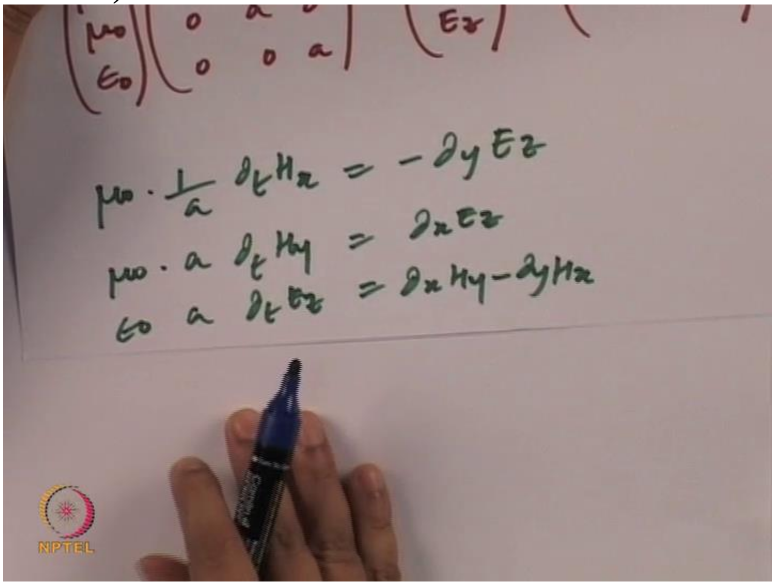

So let’s write down the second equation which is here;

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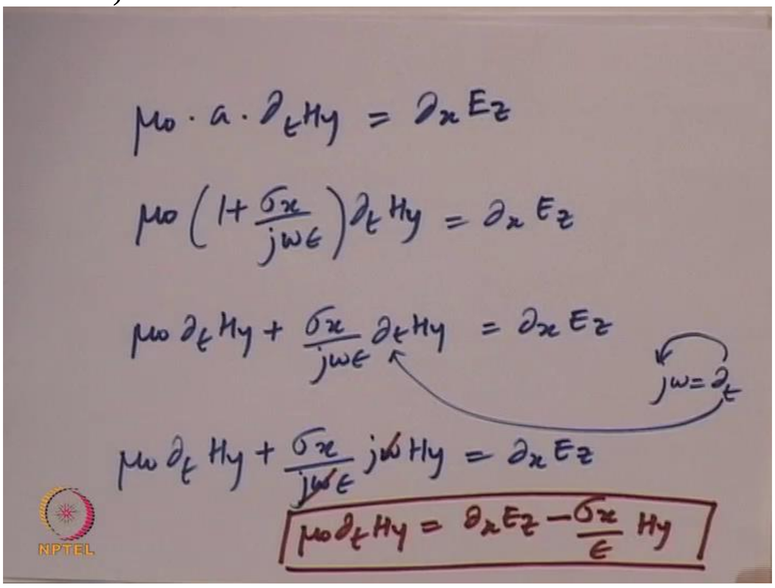

So lets write it down Mu 0 multiplied by a Doe t H y equal to Doe x E z. Again the value is going to be Mu 0 multiplied by (1 plus sigma x divided by j omega Epsilon) Doe t Hy equal to Doe x E z. So now we are going to expand this term what we will get is Mu 0 Doe t H y plus sigma x divided by j omega Epsilon Doe t H y is equal to sigma x E z.So I am going to substitute the value j omega equal to Doe t and I am going to do it in a different way. Instead of substituting j omega equal to Doe t I am going to substitute doe t equal to j omega. So this is what I am going to do here. So I will get Mu 0 Doe t H y plus sigma x divided by J omega Epsilon j omega H y equal to Doe x E z. i can cancel j omega on both sides on the numerator and denominator. What I will get is the term update equation for the H y term which is given by Mu 0 doe t H y equal to d x E z this will go on the right hand side and become minus sigma x divided by Epsilon H y. So I am going to put them in the bracket. So this is going to be my third equation.

So the first equation is going to be the one for H x which we have just derived.

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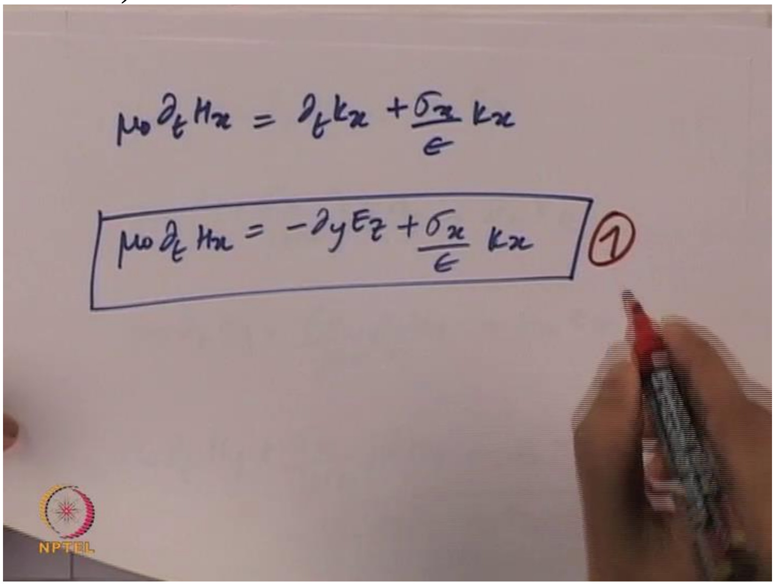

So this is going to be our first equation.

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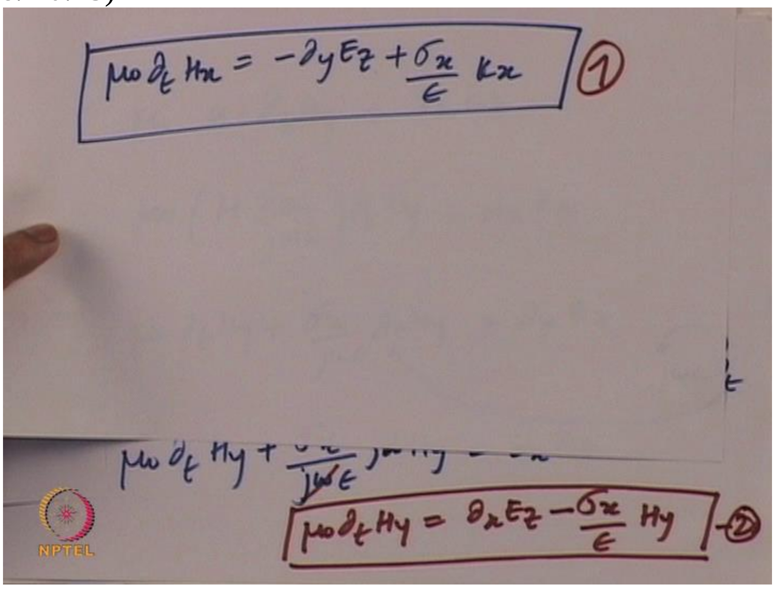

So this is going to be our second equation. And we are now going to look for the third field equation which is the E z.

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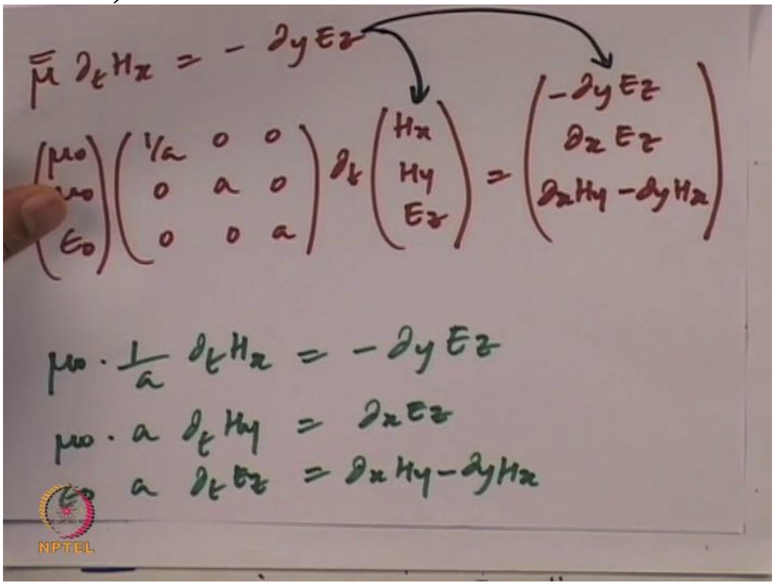

So we are now starting with the third equation here. So we will derive the equation for E z as well.

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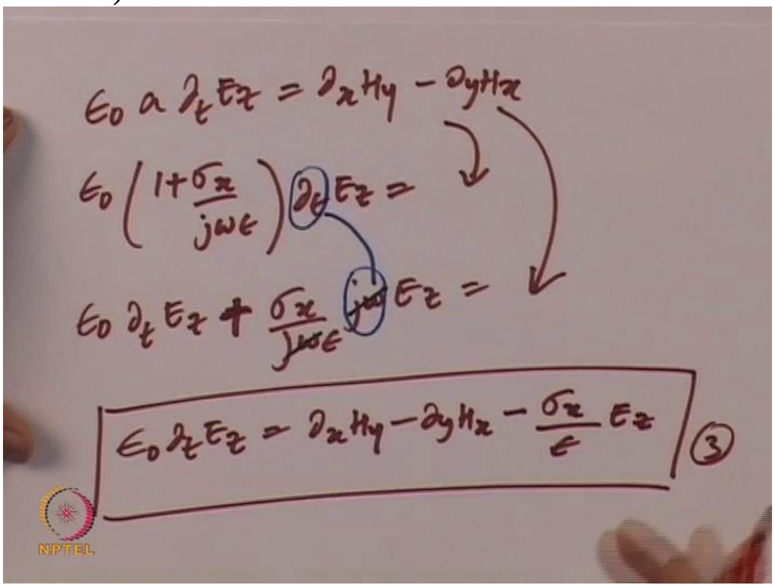

So what we have got is Epsilon 0 a Doe t Ez is equal to Doe x H y minus Doe y H x. I am going to substitute the value for a Epsilon 0 (i plus sigma x divided by j omega Epsilon) multiplied by Doe t E z equal to the right hand side. I am expanding it so you get Epsilon 0 Doe t E z plus sigma x j omega Epsilon. And I am going to do the same thing what I did for the H y term. I am going to write dt as J omega E z. That is going to be equal to my right hand side. And I wanted to make sure that you know what I have done here. So I have substituted the value for dt as j omega.

And now I can get rid of these two things and write that in a simplified form by taking it on the right hand side. And hence we will have the third equation that we are looking for Epsilon dt E z is equal to Doe x Hy minus Doe y H x. And in the right hand side this term will come to the right hand side. We will have minus sigma x divided by Epsilon E z. This is going to be our third equation.

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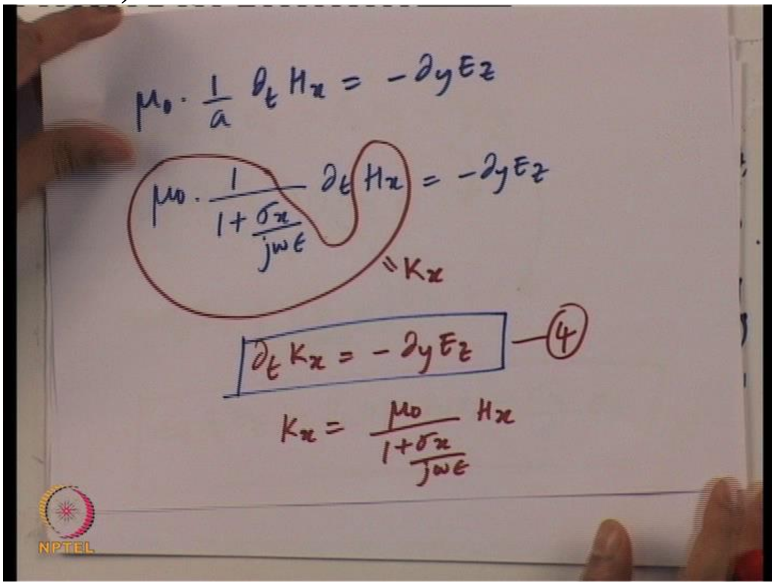

Of course there is a fourth equation that we will have which is going to be given by this equation for K x. So I am going to write down all the four equations one by one so that you can have a quick view of all of them in one sheet.

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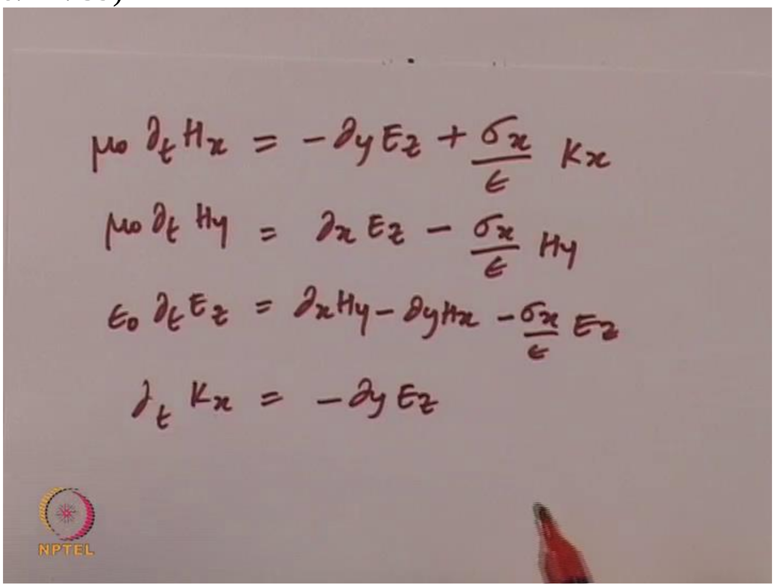

So the first equation is going to be for H x, so it is going to be Mu 0 Doe t H x equal to minus dy E z plus sigma x Epsilon K x. So the second equation is for H y so it is going to be Mu 0 Doe t H y is equal to Doe x Ez minus sgma x Epsilon H y. And the third equation is going to be epsilon 0 Doe t E z is equal to Doe x H y minus Doe y H xminus Sigma x by Epsilon E z.

And the fourth equation which is te equation that we are going to use for k x is going to be given by the term so the fourth equation is going to be Doe t k x is equal to minus Doe y E z. So as you can see there is going to be few remarks one has to give about this equation.

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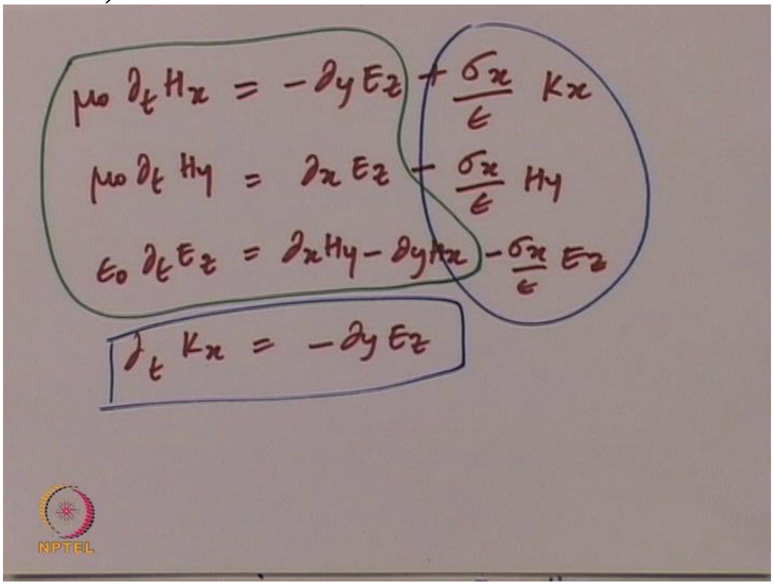

The equations that are marked within the green line is nothing but a standard Maxwell equation for 2D transverse magnetic mode.

The terms that are within the blue colour or the loss terms that we are going to consider inside the PML and of course the loss terms are going to come because of the sigma x value. So there is a fourth term an auxiliary equation that we have to consider because we are going to have the loss terms. And these loss terms are to be updated as well. So there is going to be a fourth equation and its k x 0 k y because we are considering an x oriented PML.

So with that being said what will be the analogy or what will be the counter part of the x PML when we consider the Y PML so when we consider the Y PML you will have a very similar equation and I am going to write it down so we don’t need to go step by step deriving it one more time but the logic is very similar you can drive it by yourself but I am going to explain the set of equations.

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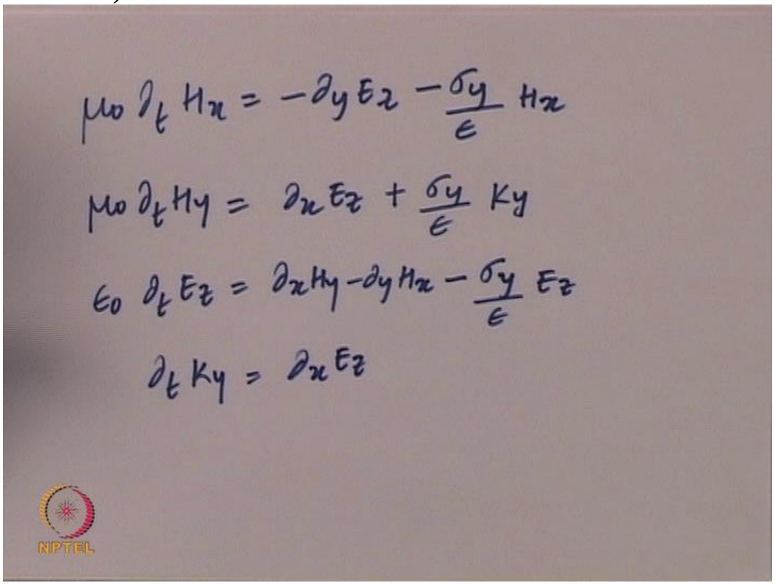

So what we will have is Mu 0 Doe t H x equal to d y E z minus sigma y divided by Epsilon H x. And the second equation is going to be Mu 0 Doe t H y is equal to Doe x E z and we are going to have a plus sigma y divided by Epsilon k y. And the third equation is going to be Epsilon 0 Doe t E z is equal to Doe x H y minus Doe y Hx minus sigma y divided by Epsilon E z. And the fourth equation is going to be because of the k y so it is going to be Doe t k y is equal to Doe x E z. So if you compare these two equations side by side there is a lot of similarities that you can see

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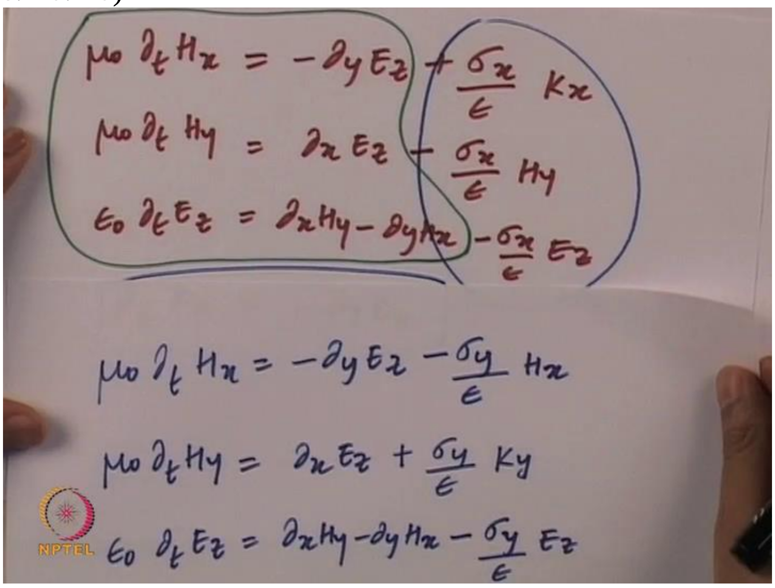

Let us look at them one by one. So what I have go in the first three equations here you see that the plus sign is going to be there for loss term. Whenever the PML is oriented in that particular direction when we are considering a x oriented PML the plus sign is going to be on the H x term. When we are considering the y oriented PML there will be a plus sign for the loss term for y oriented PML on H y.

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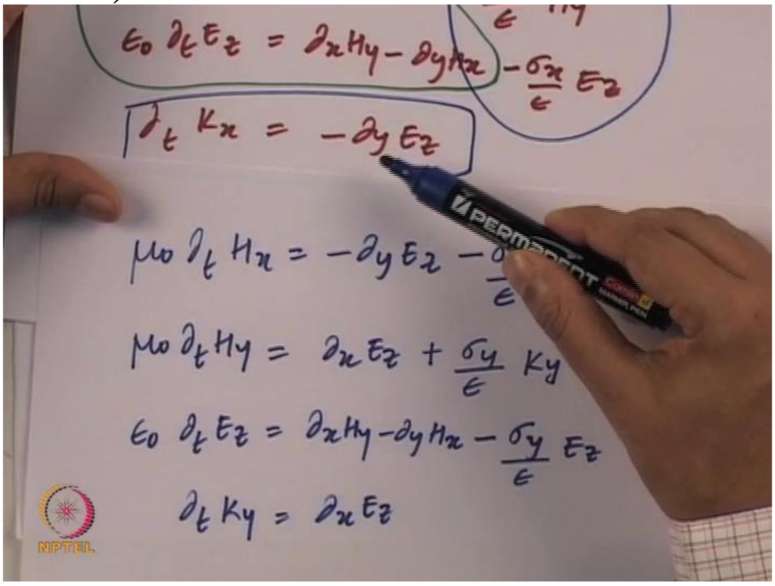

Similarly the K x term will be there for x oriented PML whereas Ky term is going to be there for a y oriented PML. And the Loss terms are going to be very symmetric since there are not going to be any variations in the z direction E z direction will be very symmetric. Of course in the case of y oriented PML we will have sigma y and in the x oriented PML we will have sigma x.

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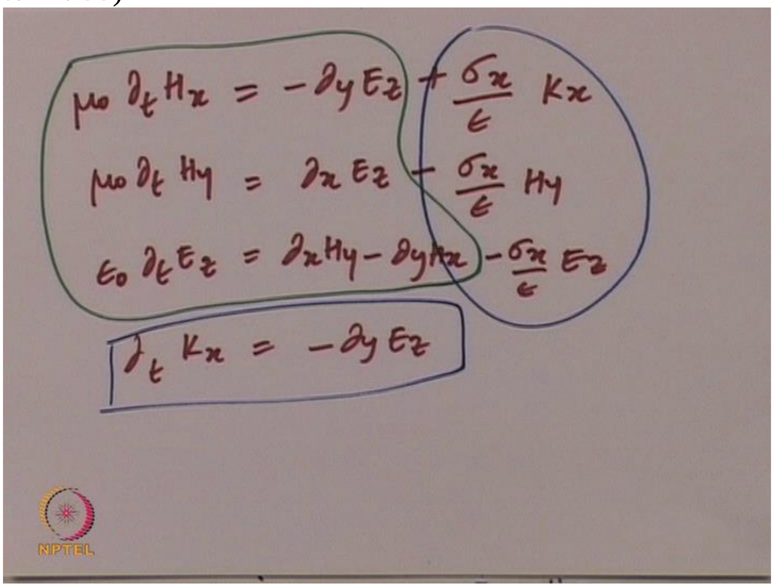

So this is going to be the structure and also you will have a dt ky expression which is going to be the exactly same flux that we are computing the same field the curl term that we are computing that is going to be sitting here. Similarly the dt kx what you see is the field term that we are computing is going to be the same as the field term that we will compute for a standard Maxwell equation without PML for H x. So these two terms are the same

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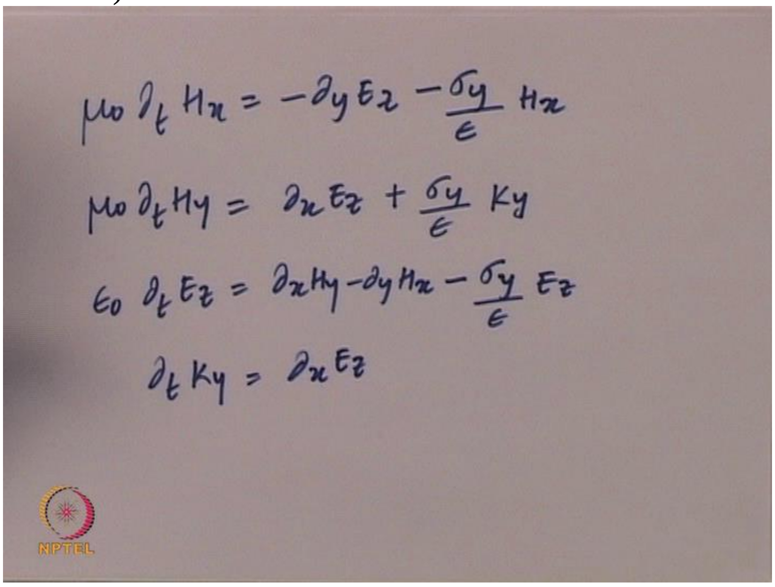

Similarly these two terms are the same both are the analogies that we have to put in place. And now we can see how we can combine this to be ml in a single set of equation that will have regardless of which orientation we are considering and that can be done in a very simple manner so what we are going to do is we are going to write the standard Maxwell equation as we have returned and we are going to add the Sigma x divided by Epsilon, Sigma x divided by Epsilon, Sigma x divided by Epsilon in such a manner that will be for the x oriented PML. And we will also have Sigma y divided by Epsilon and Sigma y divided by Epsilon and Sigma y divided by Epsilon for those terms that that are going to come so we are going to combine them in a very simple manner and I am going to write the equation directly then you can check it for yourself at a later stage.

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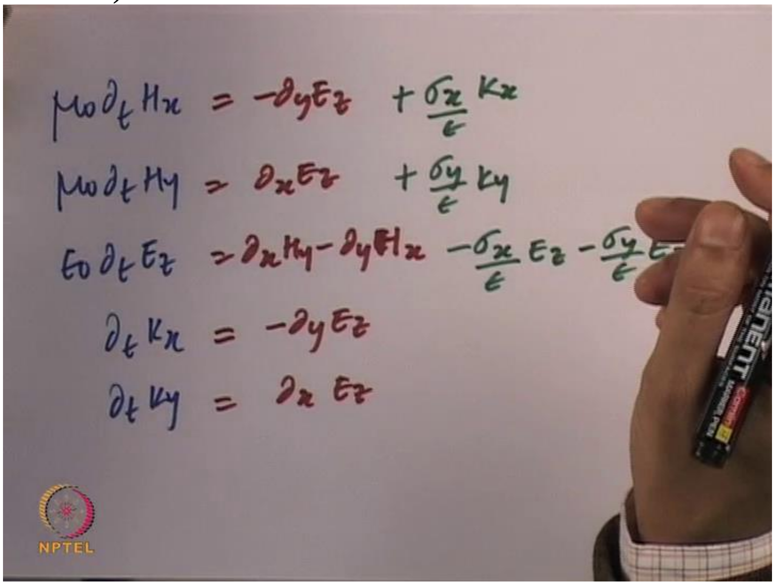

So what he will have here is Mu 0 Doe t H x Mu 0 Doe t H y Epsilon 0 Doe t E z and we will have Doe t K x Doe t k y. Doe t k y and Doe t k x are very straight forward they are the fields that come from the Maxwell equation itself so we can write them down directly it is going to be minus dy Ez dx E z. And the standard fluxes are minus dy E z d x E z d x H y minus d y H x. So these are the loss terms and now I am going to add flux terms as I said.

So it's going to be plus sigma x k x divided by Epsilon for the x oriented PML. And similarly it's going to be plus sigma y divided by Epsilon k y for the y oriented PML and we are going to have both the loss terms for sigma x and sigma y so its going to be sigma x divided by Epsilon E z minus sigma y divided by Epsilon E z. And we are going to have the counterpart terms which are the H x and H y terms.

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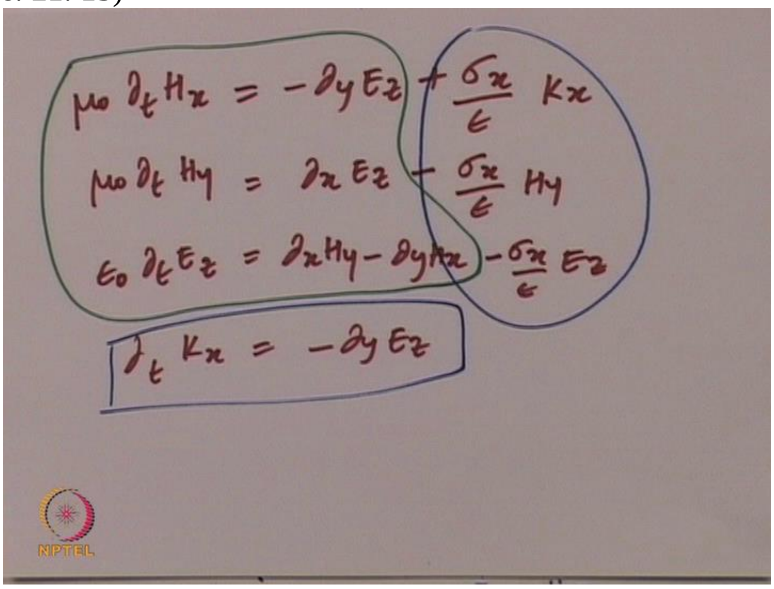

Like we have got here this is an x oriented PML we will have sigma x divided by epsilon h y

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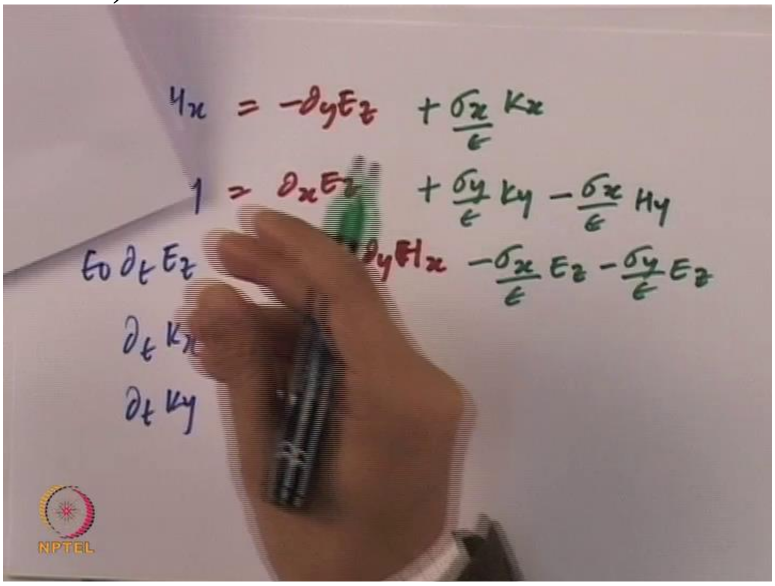

So I am going to write it down here, so its going to minus sigma x divided by epsilon H y.

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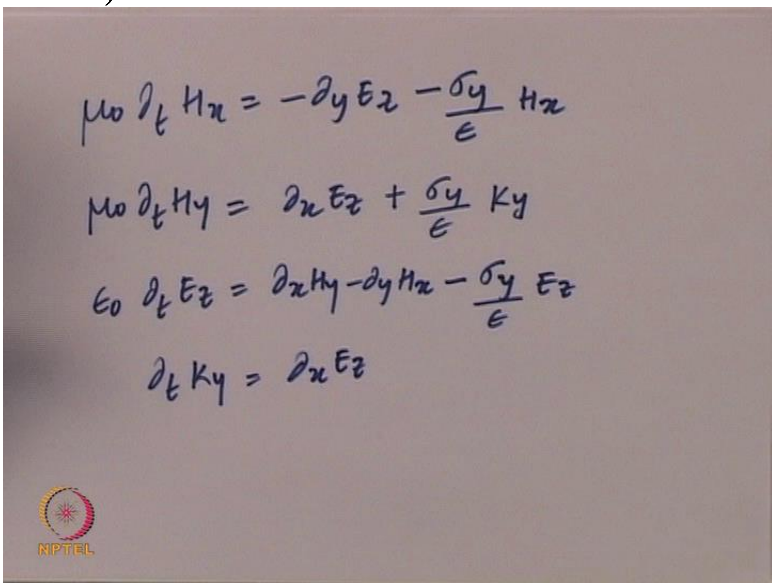

Similarly for the y oriented PML we will have minus sigma y divided by epsilon h x that is going to be minus sigma y divided by epsilon H.

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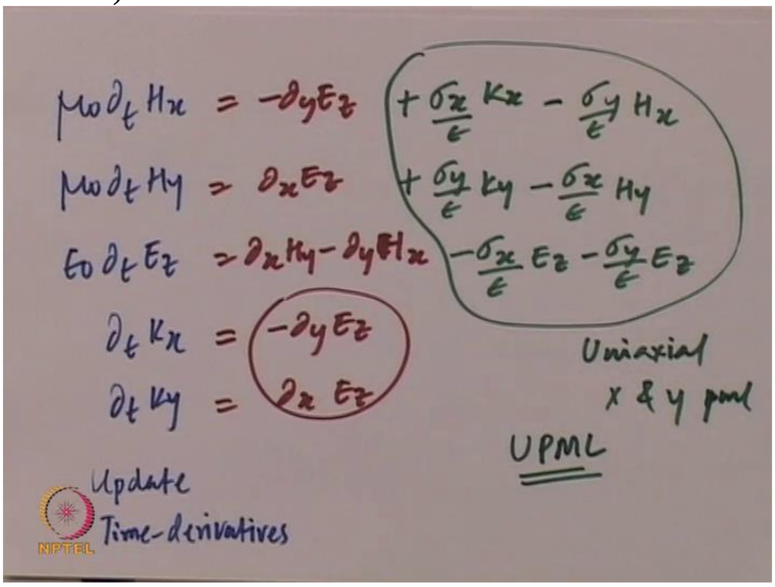

So that's it and we have got now generalized equation for both the Axe oriented and why oriented uniaxial anisotropic PML I have written specifically the equation terms in different colours so the blue colour turns are the update terms with time derivatives the atoms are the standard Maxwell equation except for these two terms they are coming because of the PML X and Y oriented PML and the green ones are the Lost terms that are coming because of the uniaxial PM else so x and y p m else normally we also called them as you PML but one has to be careful we are having too many abbreviations here u p m l means uniaxial PML perfectly match there.

So this is going to be our starting point for us to model the PML using finite difference method and we are going to do that in Matlab environment and we are going to take the test case where we will have a rectangular domain we are going to truncate this domain using AP ml and obviously there are certain things which I have not mentioned about the PML itself that we will see when we looked at numerical simulation and we are going to simulate a point source which is sinusoidal in nature and we are going to see how this point source is going to come and impinge on a scatter which is a model as a perfect electric conductor and we'll see how the scattering gets reflected in all directions and how these X and Y oriented PML is going to absorb those incoming radiation so this is going to be a modelling exercise we will do in the next module.