Computational Electromagnetics and Applications
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Lecture No 09
Boundary Conditions

We have looked into finite difference method quite extensively in the previous module and the method itself is discuss from the point of view of various problems whether it is advection equation Heat diffusion equation so we have covered pretty much final defence for various practical problems also for the problems we are mostly interested the Maxwell equations with that being said we cannot still go forward and implement any article problems before we discuss the domain truncation because the idea of domain truncation is integral part of any method so we are going to be discussing about boundary conditions in this module.

So this is going to be very much importance for any method since we have covered finite difference method just recently in the previous modules we are going to set the right tone for the boundary conditions using finite difference method what we are also give certain point as for extension to other methods while we discuss advanced methods or alternative methods like finite volume in the later stage of the modulus we will revisit the boundary conditions once again but let's start into the boundary conditions in this module and look what are going to be the contents of this module.
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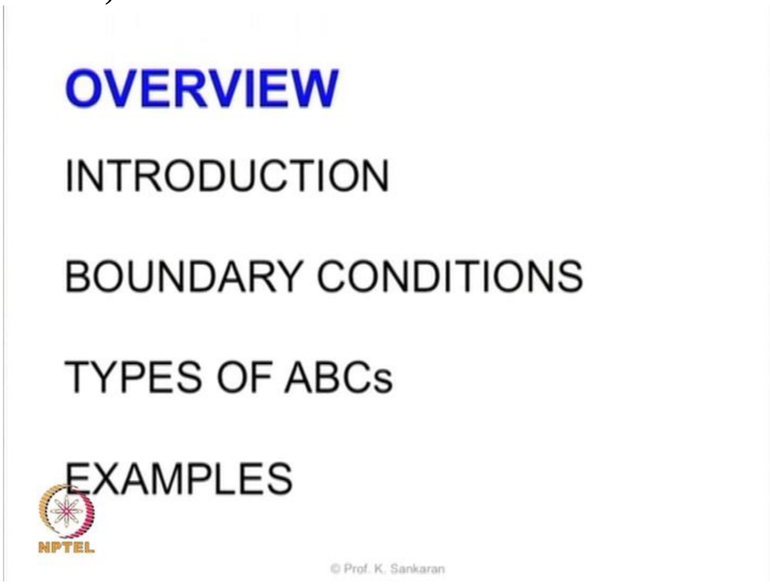

So we will start with boundary condition introduction and we will discuss about various aspects of the boundary condition and we will see certain types of absorbing boundary condition that we are going to be interested and we are going to see some examples or modelling practical electromagnetics problems in this particular exercise.

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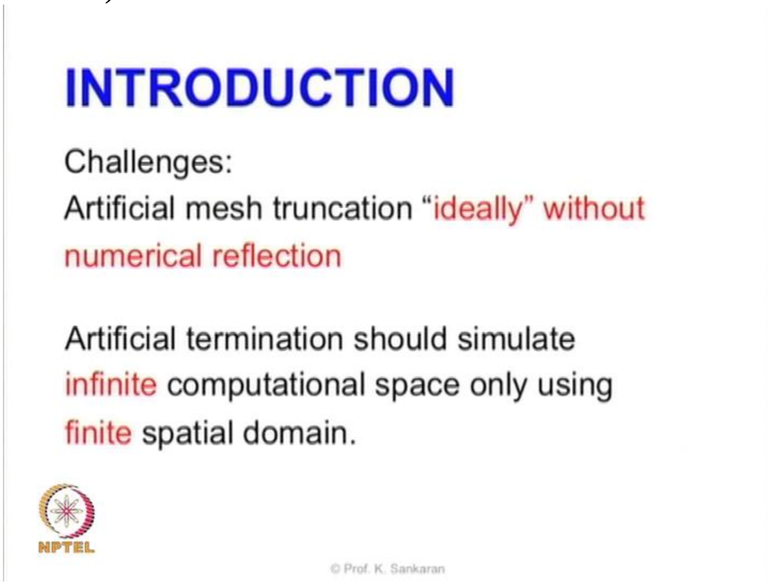

So let’s start with the introduction so one of the biggest challenges that we face when we artificially truncate domain is numerical reflection the reason we are calling it as numerical reflection because they are artificial reflection there is no reflection in the physical space are the physical problem itself but we are trying to model it in infinite space and Time we are going to get reflections and which we call it as numerical reflection the challenge for a domain truncation is to make this numerical reflection as little as possible ideally there should be no reflection and also we wanted to simulate an infinite competition and space using finite special domain this goes actually without saying that whatever we call it as finite should be small re enough so that computational effort that we are going to put in is going to be as small as possible.

Without doing this we are going to expand the computational domain quite big for a problem where our real interest area is going to be very small so if we make the boundary conditions perfect in that sense we can make them as close as possible to the domain of interest so that our finite space will be as Limited as possible we will see that there is no perfect boundary condition to come with certain conditions that we have to accept as a limitations for practical implementation so this could be the fact that we will see during the course of this module.

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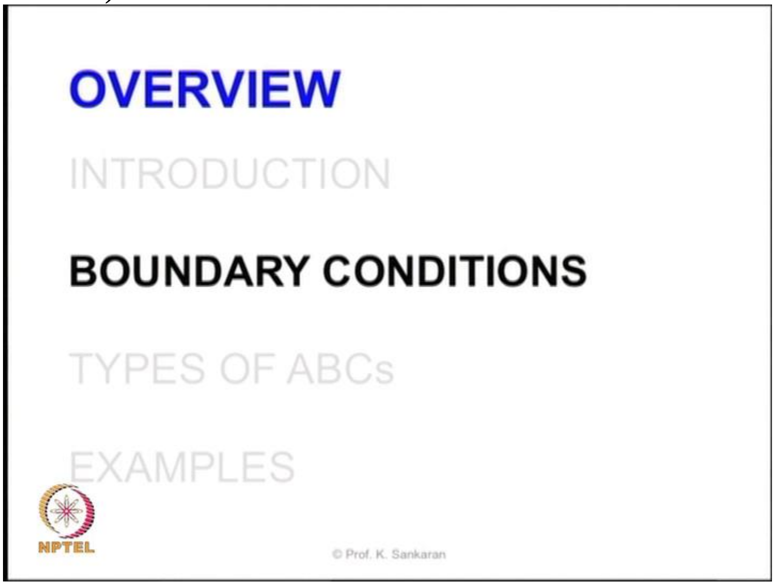

So let's look at the type of boundary conditions that we are going to look in this module

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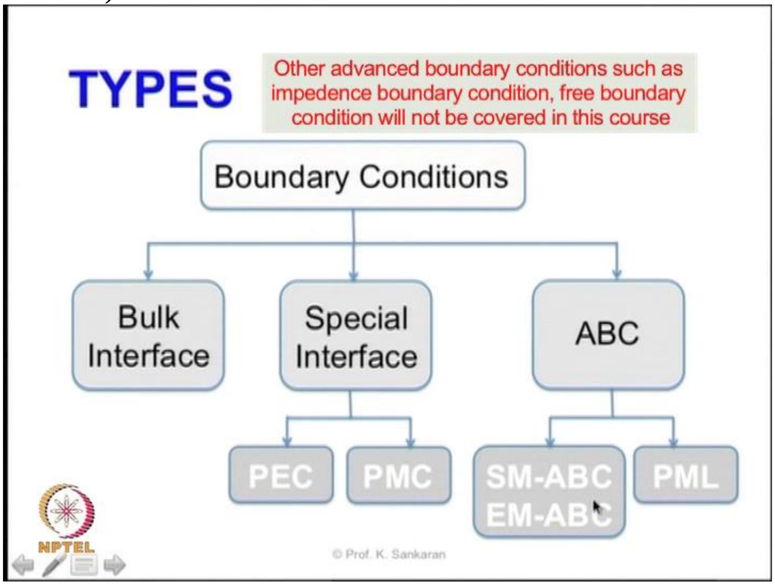

So the types of boundary conditions are of three main categories we call them as bulk interface boundary condition special interface boundary condition and ABC which is the absorbing boundary condition and the special interface boundary conditions are going to be of two types for electromagnetic problems we call them as PC perfect electric conductor and PMC which is a perfect magnetic conductor and in the case of absorbing boundary condition there are going to be two broad categories one is the absorbing boundary condition itself and the other one is going to be a layer which is called as a perfectly match layer obviously it's misnomer to call a PML as an ABC because a PML is going to be a layer not just particular boundary as in the case of silver Muller absorbing boundary condition SM ABC is a silver Muller absorbing boundary condition.

And EM ABC is Enguish Majda absorbing boundary condition these are two widely used absorbing boundary conditions which will discuss also during the course of this module so let’s look into the interface boundary condition from two different aspects one is from the aspect of bulk and the other aspect of the boundary itself.

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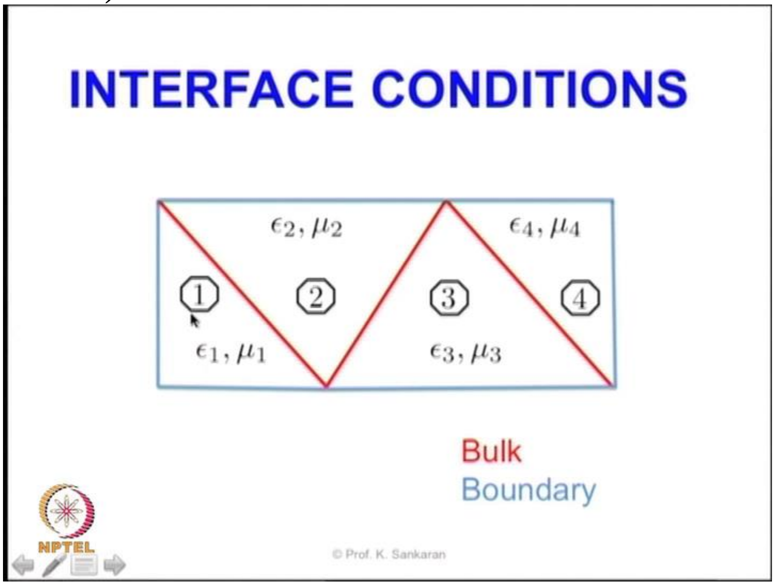

So let us start looking at this particular problem assume that this is a waveguide so let me try it here on the paper so that you get a sense of what we’re talking about.

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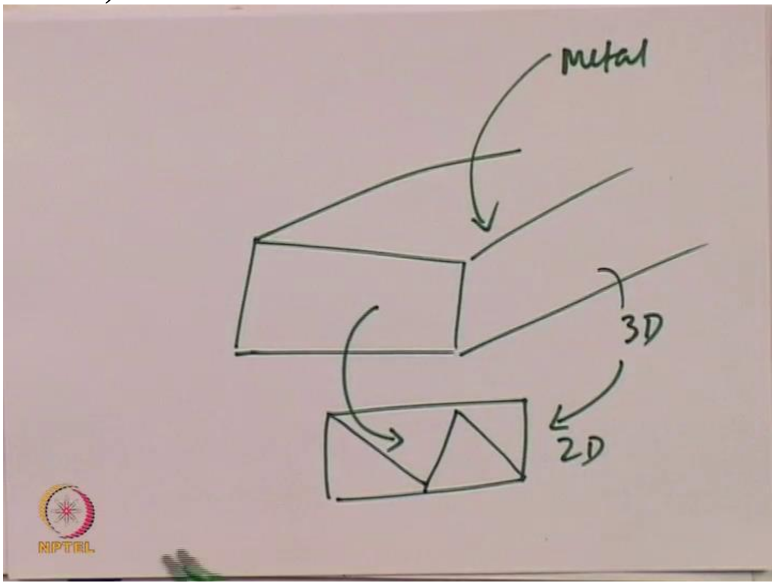

So we have a waveguide and we are trying to model this particular cross section of the waveguide so we are trying to take this particular cross-section which is nothing but this area and obviously this part are going to be the metal and this is going to be the free space where we are going to feed in certain mode and what you are trying to do here is we are going to discretise this problem so now we have converted 3D problem to a 2D problem where we are seeing only 1 cross section of this waveguide and of course this waveguide is going to be in a computational domain and we have to discretise is waveguide so let's say we are discussing this waveguide using some triangles.

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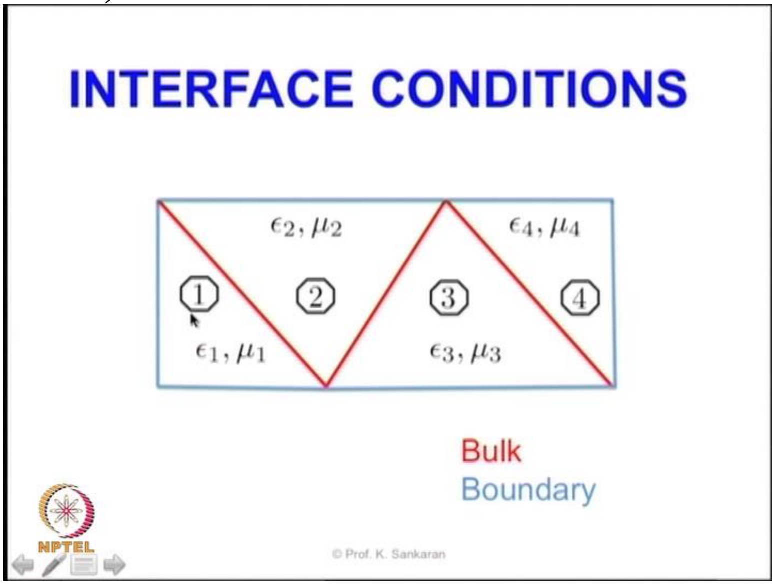

And that's what we have done in this slide. So there are going to be 4 Triangles inside this cross section as you can see if the waveguide has different materials instead of free space where all the Epsilon and Mu is going to be Epsilon 0 and Mu 0 in this case this waveguide has 1,2,3,4 different types of material obviously it's a hypothetical case just to show the extreme cases what we are going to confront so we have made it like this so you have 4 different material constituents and the waveguide cross section is going to be like this what are the different interfaces that are going to be involved in this particular problem is what we are going to look into now so what we have seen here is the red colour LINE which is talking about certain interfaces so what we have as interface is the interface between two dielectric medium so medium number 1 medium number 2 and interface between medium number 2 and medium number 3 and interface between medium number 3 and medium number 4 so what we are going to see is once we are discretizing particular problem using finite difference method so let's take this particular problem.

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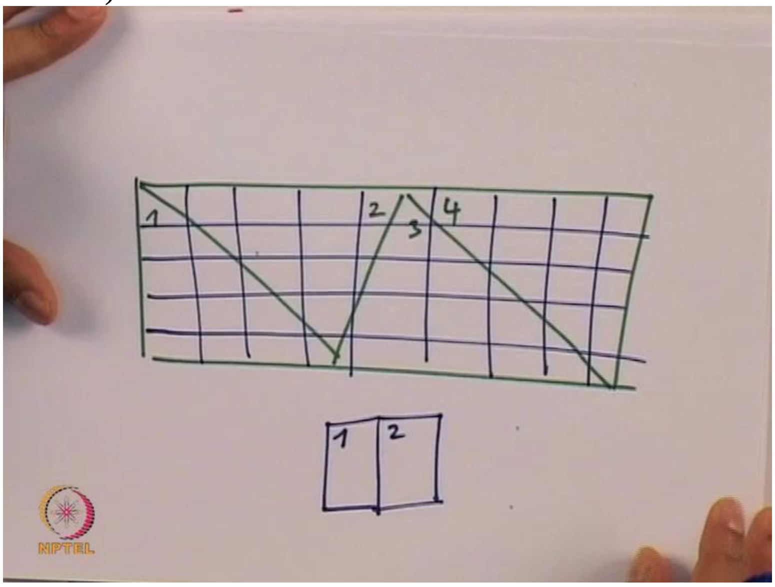

We have four different materials one two three and four so now I am going to discretize this domain by using finite difference method so when I do the finite difference method as in the case before I will have the Axe oriented grid and I will have the Y oriented grid on and so forth you have already seen in the case of the introduction of the finite difference method we said there are going to be certain stair casing error so we see that these are the staircase in error that are going to come while we are modelling because this particular cell is partly in material one and partly in material to similarly this particular cell is partly in material to and partly in material 3 so on and so forth so that is not the main important thing that I we are worried about let’s say we have an exact layer where 1 rectangle is going to be in material 1 and the other rectangle is going to be in material 2 which is not the case here because we are having a kind of a geometry where the discontinuity is not align to the grid size itself so let’s take an example where our geometry itself is going to be aligned with the grid so that we will not have the star gazing error.

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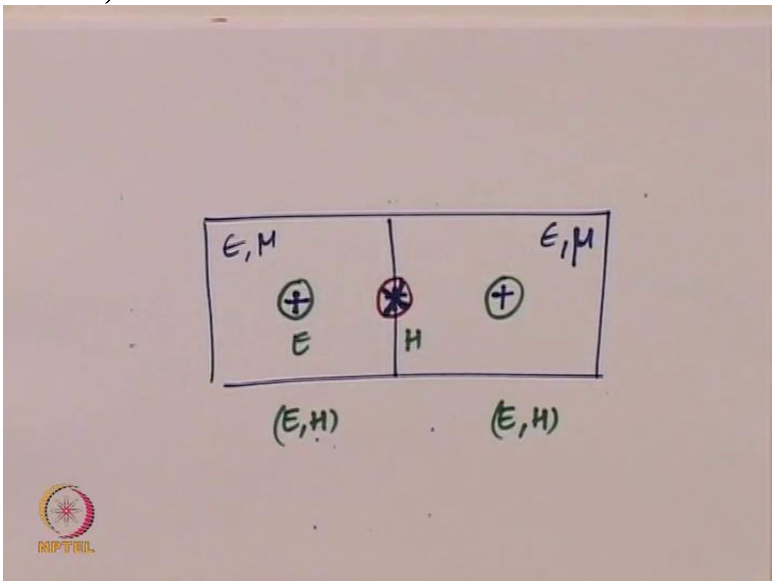

So what we are interested is we have a particular geometry and assume that this is a cell and this is another cell and one cell is in Epsilon Mu and other cell is also in excellent view if it is going to be a homogeneous medium it is going to be different if it is a inhomogeneous medium where one will be Epsilon 1 Mu 1 and the other one will be Epsilon 2 Mu 2 assuming now for the simple case both the mediums are the same mediums so we have one cell that is having parameters computed at the Cell Centre and the face centre and then the other one has also similarly Cell Centre and centre so the value that I am computing in the Cell Centre I am marking them as Plus and the value that I am computing at the face centre I am marking them with certain different colour.

So let’s say I am marking them with red and let me make it a little bit different so I will colour it and make a cross and A Plus so what is going to be the cases in the case of finite difference we have a staggered e and h so he is going to be let us stay at this point and it is going to be at the Faiz centre so similarly we can see how the different cells are going to be oriented but there might be methods were both a and S will be at the same point so in that case both a and h are going to be at the same. So we have to compute the value of e and H using the values at Cell Centre and extrapolating to the face centre so you will see this will be the case in the method of finite volume which will we will discuss at the later. Is the method of finite elements which we will also discuss at the later. So here we are going to see the interface as follows

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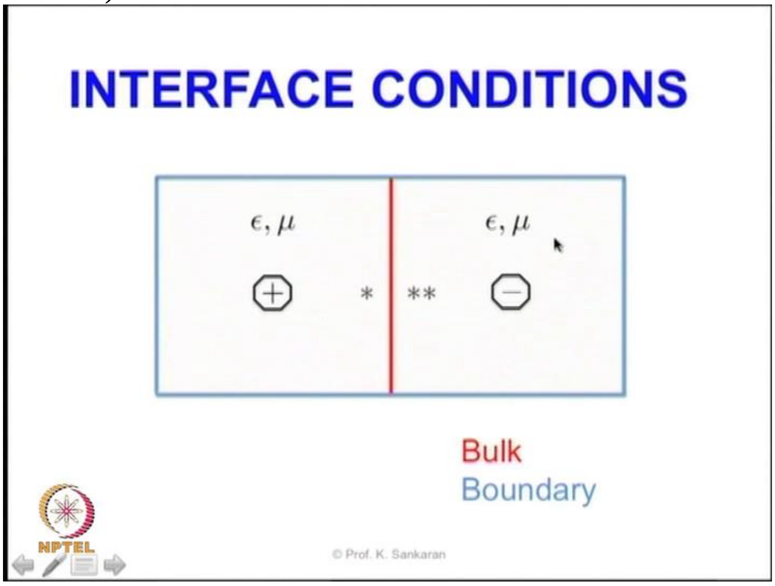

The first interface condition that we are going to look into it the tangential continuity of the fields so as you mean that these are two cells with the same permittivity and permeability we are going to have the tangential continuity satisfied according to this particular equation.

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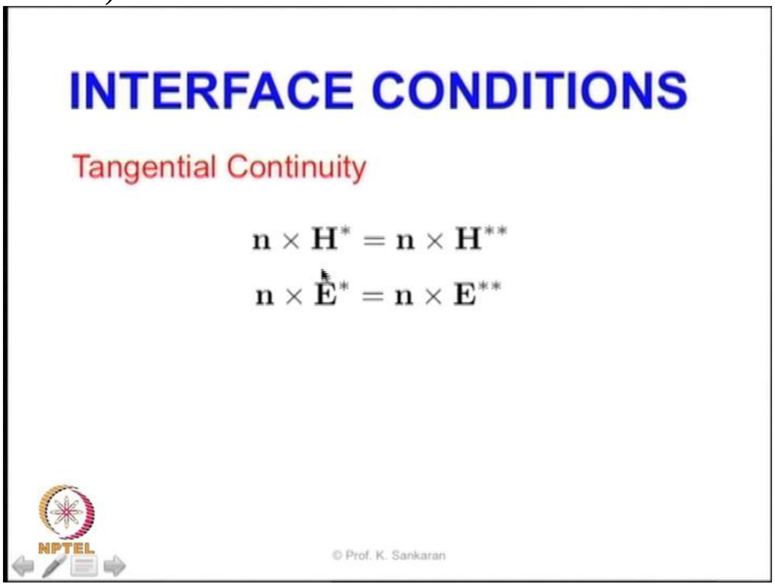

So the tangential component of the a field and H feel so in this case the left hand side and the right hand side are going to be same so this is the first interface condition that we have so in the case of special interface conditions we have to satisfy certain other conditions.

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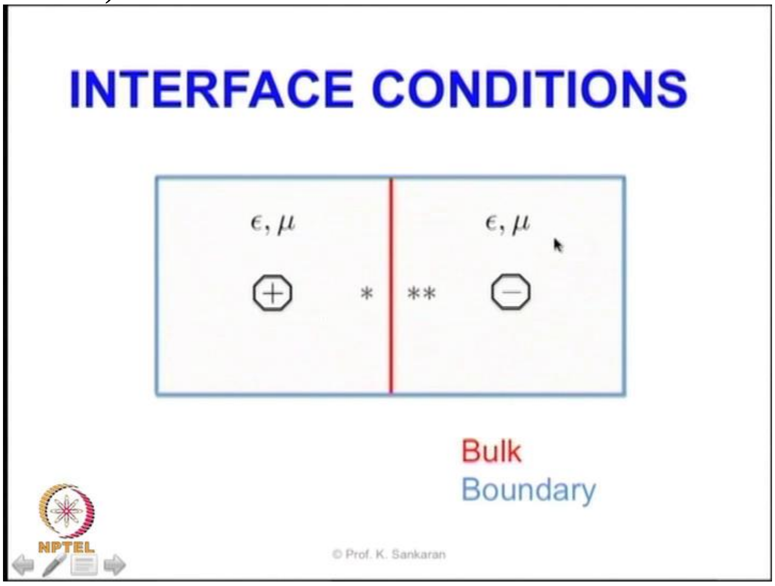

For example when you see that this particular layer the layer on the top which is a metal layer in case of a waveguide when we are modelling them for simplicity we can assume that this particular layer is going to be a perfect electric conductor.

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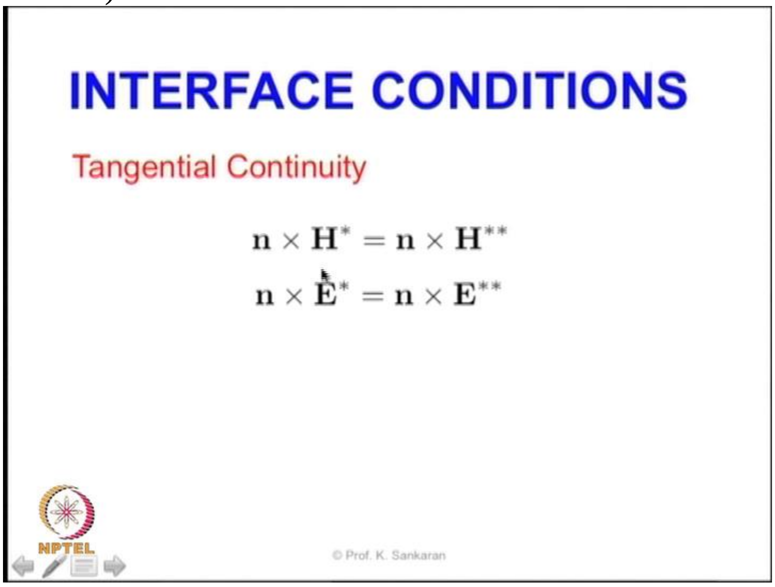

If it is a perfect electric conductor we know that the tangential component of the electric field will become equal to zero.

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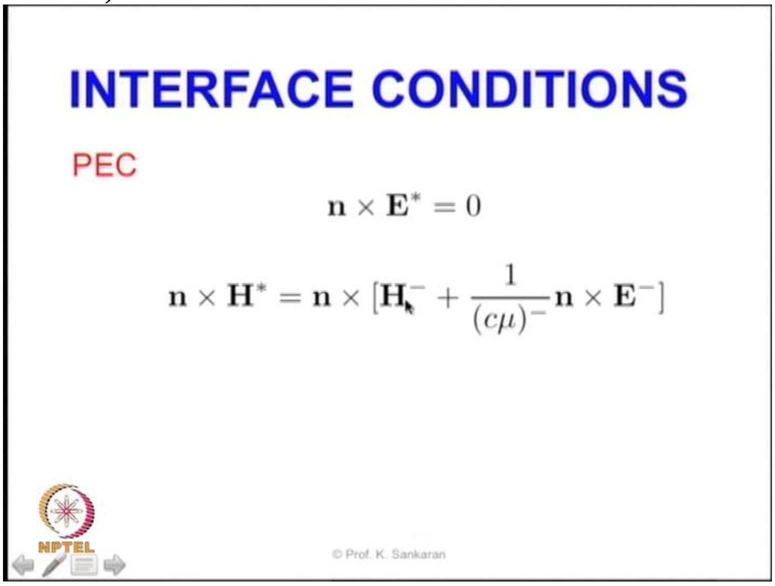

And that is what we are seeing here that is the pic which is a special interface condition we put the tangential component of the electric field equal to zero where is the tangential component of the magnetic field has to be computed and we will use this particular equation if it is staggered grid you have to adapt this particular equation according to the formulation of the method where it is non staggered grid in the case of other methods like finite element and finite volume you can directly use this particular formulation the minus sign and the Plus sign are the only written for you to know whether it is going to be left neighbour or the right neighbour.

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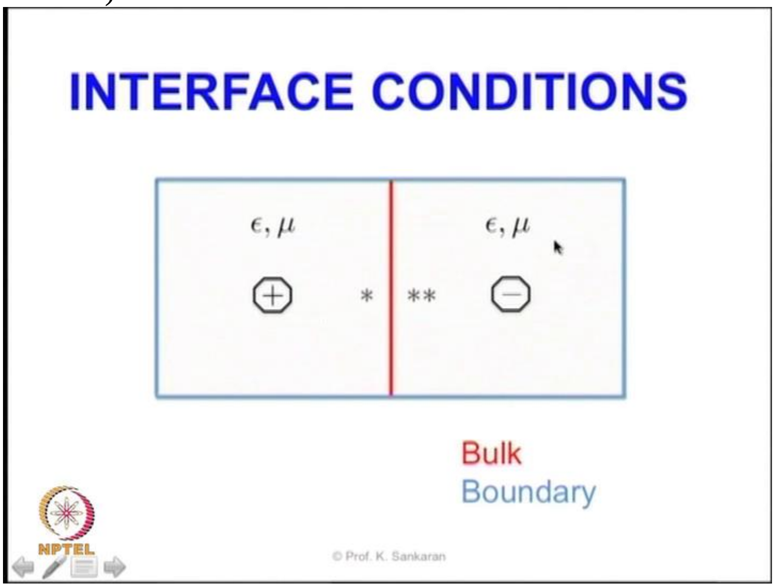

so we have set here in this particular example all the boundary edges will have the right neighbour the left neighbour will not be there we will only have the right neighbour so in this case for this particular edge the right neighbour is this one for this age we have to compute the right neighbour has this one so on and so forth.

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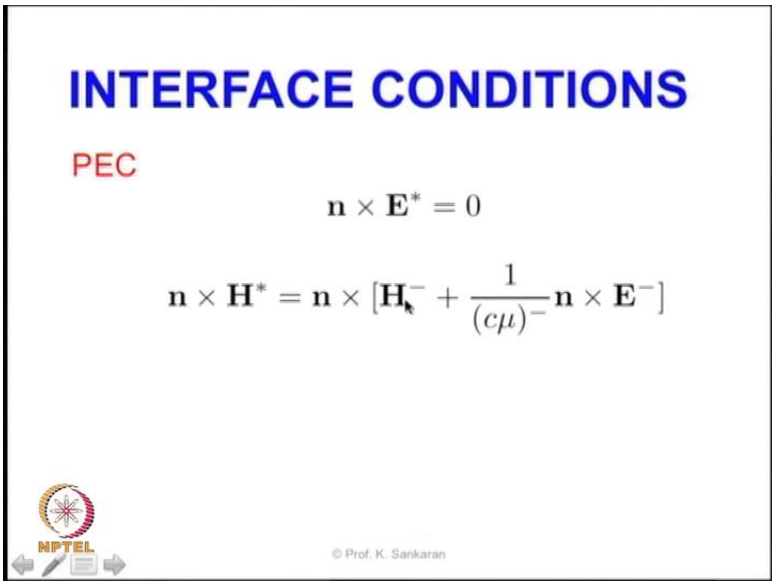

So let's look at the perfect magnetic conductor counterpart in the perfect electric conductor we have put the tangential component of a field as zero.

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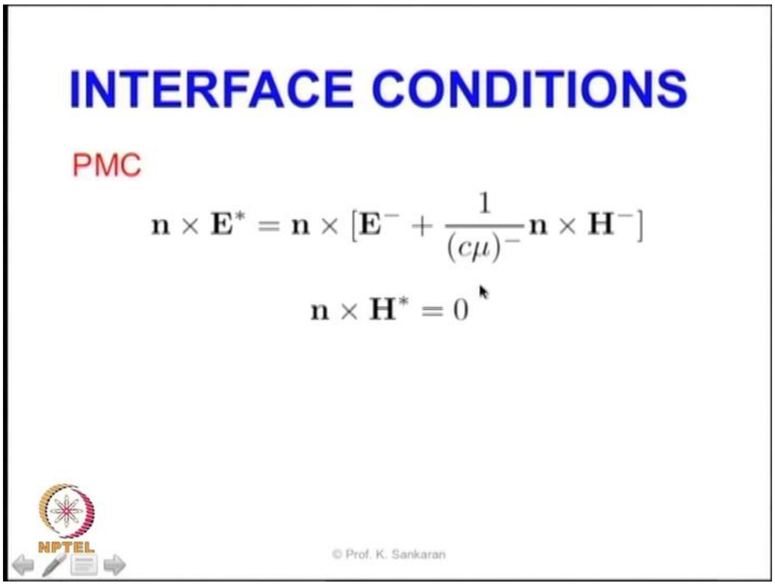

whereas in the perfect magnetic conductor you have to put the tangential component of h field as zero and you compute the tangential component of electric field using the equation which we have given here what you also see that is this particular condition When We combine both the perfect magnetic conductor equation and the perfect electric conductor equation such that the fluxes are computed for both the tangential component of electric field and magnetic field we end up in the simple first order accurate silver Muller boundary condition.

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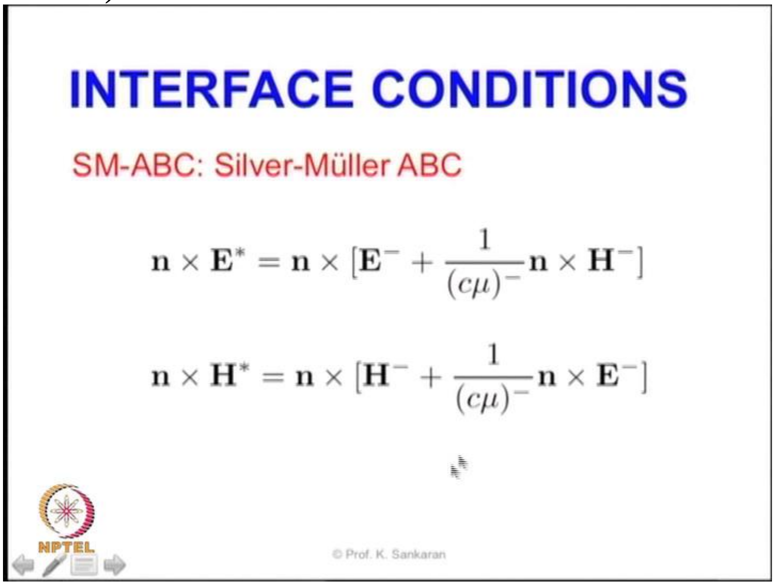

That is what we have see in here we will not be using the Silver molar boundary condition in the case of the finite difference simulation that we are going to show later in this part of module just for you to know the Silver moon boundary condition is expressed by this equation we will revisit the Silver Muller boundary condition when we talk about the method of finite volumes which is an alternative method for now it's enough for you to know there is a boundary condition called a silver Muller boundary condition we will see this at a later stage so with that being said that's going to the special class of boundary condition.

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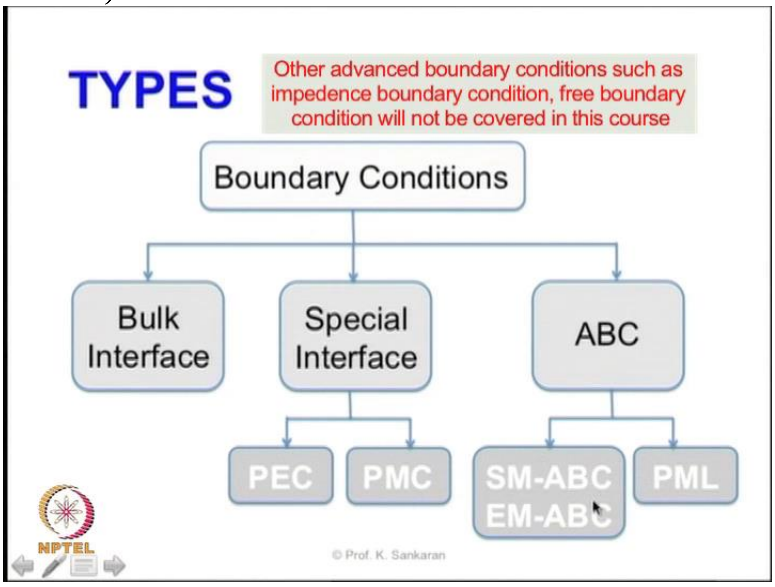

we have looked into three categories of boundary condition in that specific case we have specially looked into the bulk interface boundary condition where we talked about the tangential continuity conditions and we had looked into the special interface condition which are the perfect electric and the perfect magnetic conductor what is missing is the absorbing boundary condition itself now we are going to look into the absorbing boundary condition.

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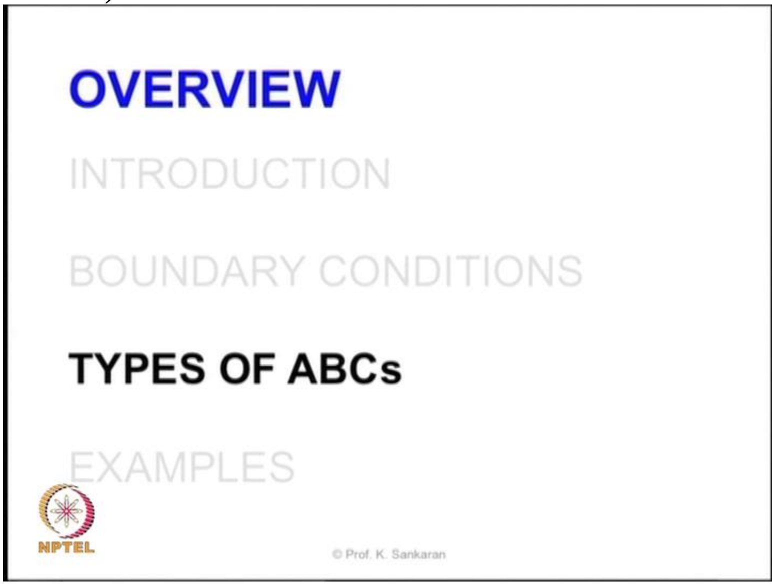

There are different classes of absorbing boundary condition as we have introduced in the introduction slide so we are interested in two categories first one is the pure ABC in that category we have already introduced the Silver molar absorbing boundary condition but he will also see a special kind of absorbing boundary condition which is the anguished master boundary condition we will derive the equation for angriest master boundary condition now so we are going to also introduce a very important boundary condition which is as I said in the part of the introduction wrong to call it as a boundary condition rather we should call it it has boundary layer which is called as the perfectly matched layer we will see that in more detail in this lecture series as well at a later stage. But we will introduce the perfectly matched layer for finite difference problem now and also extend it for other methods later on so let’s start with the simple one dimensional absorbing boundary condition.

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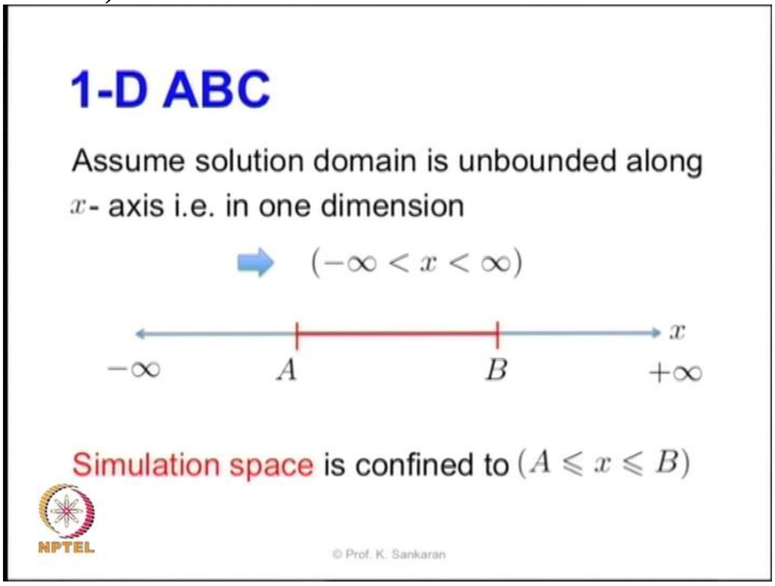

Assume that you have a one dimensional unbounded computational space which we assume that it's an x axis so we say it is looking like this and what we are interested is to model this one dimensional unbounded space using finite one dimensional space so what we are going to do now is we are going to truncate this one dimensional unbounded space into a finite one dimensional space so here the simulations face is from a to b and our goal is to make it simulate as if the spaces unbounded in other words what we are doing is what you want if we don't want any reflection to come while we truncate the space.

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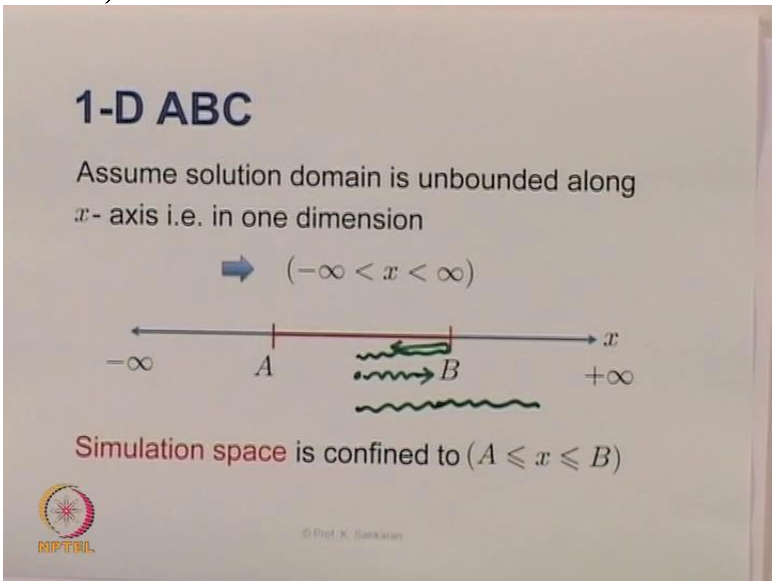

Show the ideal spaces largest space when we truncate we do not want any reflection that is going to come what I mean by reflection is assume that there is a way that is going from here to here and what happens when it reaches B is it gets reflected back so the wave that is going goes like this and gets reflected back because there is a truncation here this is a kind of adhere but the goal is to make it look as if that there is no reflection and this layer doesn't exist it's going to be replicated as if it's an infinite in space in other words what we want is the wave that is going here should not see this be existing should travel as if it's travelling in an infinite unbounded space that is the goal of this particular challenge.

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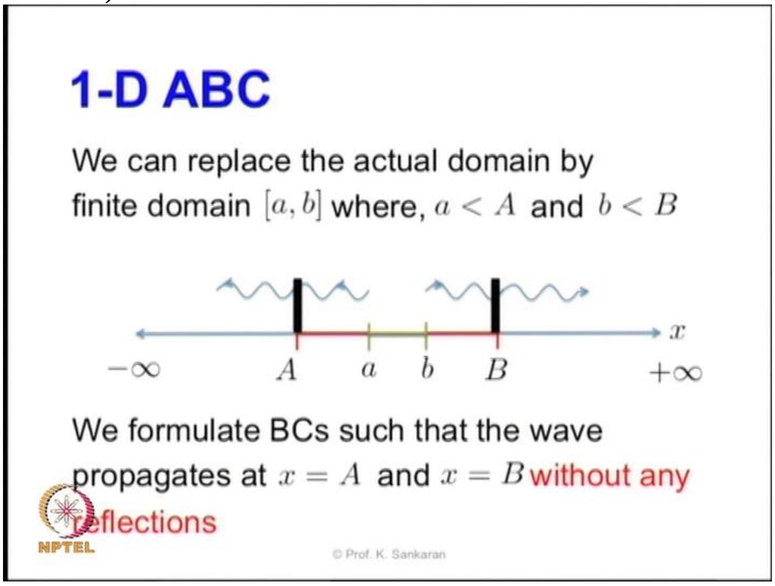

So what we are doing now is we are replacing this actual domain by a finite space so the finite space is going to be a, b and the idea is to make sure that the reflection is going to be zero at this point A and B.

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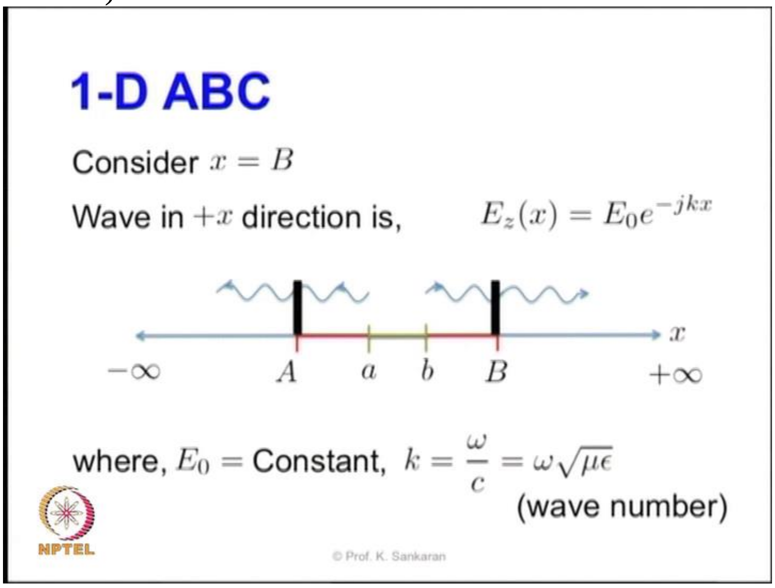

So consider that x is equal to B and now what we are doing is we are assuming that the wave propagating in the plus x direction once we know that the wave propagating in the plus x direction we can give the analytical value of the solution so let us say we are interested in computing the value of E Z and it's reckon getting along the value of x and we are saying that it has a certain magnitude or not and it is going to have a kind of certain dependence on the K using this equation and this is analytical value E 0 is amplitude and E power minus jkx is going to be the special dependence so what is going to happen is when the wave is propagating we have certain truncation that is going to happen and what you are interested is to compute the truncation error in the form of the reflection that we are going to see at the boundary a and b so let us assume that a is equal to omega by c which is going to be the wave number. And now we are able to substitute the value of a into that value of E z (x).

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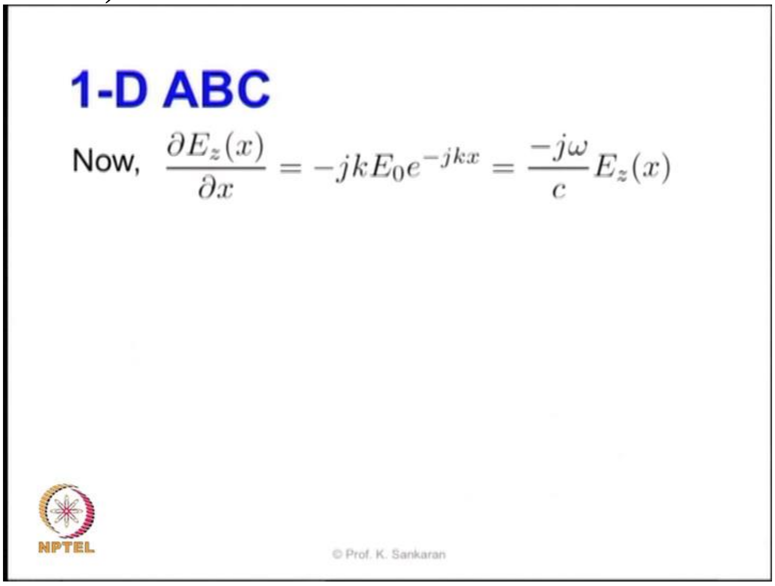

And that is what we are going to do here and we are going to differentiate that so what we are going to do is we are substituting the value of k which is omega by c.

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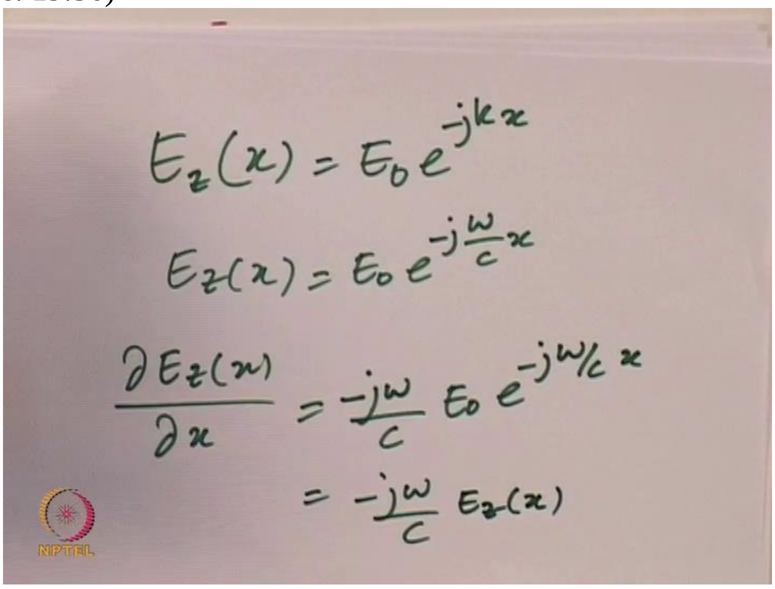

So E z(x) equal to E 0 e power minus j omega by c x. And now I am going to differentiate EZ with respect to X is a partial differentiation so what I will get is I will get minus J Omega by see and I will get or not and then I will get the same equation and this particular Time is already E z (x) so I can write minus J Omega by c E z(x).

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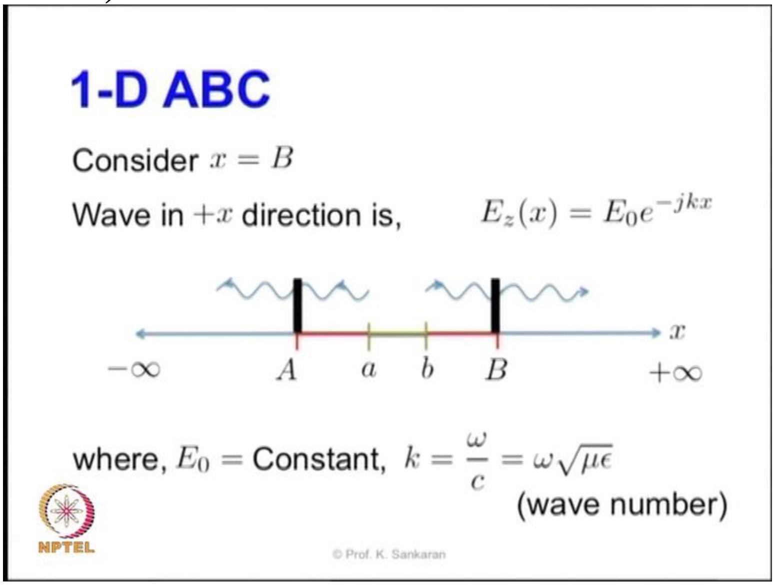

So this is going to be the value of the partial differentiation of e z with respect to X

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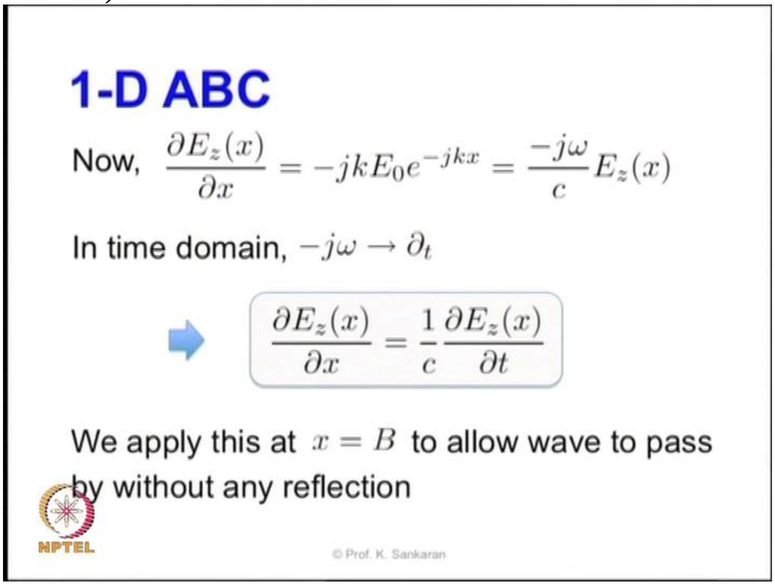

Now what we are doing is we are substituting the value for minors j omega dt and writing this particular expression in the form of the Time differentiation which you have done here so once you have done we can see what happens when we apply at the point X equal to b we allow the wave to pass we apply this condition at X equal to be that’s what we are going to do so once we do that we are able to compute the value of the reflection that is going to come out. X equal to be in the case of the one dimensional problem the reflection will be zero because it will always be a normal incidence.

So this is a very specific case because we don’t have a one dimensional case and three dimensional problem because we have a proper getting in Pretty much all direction so we cannot have the same perfect absorbing compound recondition like in the case of one dimensional problem we have seen so what we will do now is in the next part of this module we will see how we can extend this analysis for two dimensional case we will stop here he will come in the next module and see how this absorbing on a condition can be extended for a two dimensional case so we will look back in the next module until then Goodbye!