

**Computational Electromagnetics and Applications**  
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**Summary of Week 3**

We have completed the course work for week 3. And we have looked into the core of electromagnetics lecture which is the Maxwell equations

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**MAXWELL PDE SYSTEM**

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$
$$\nabla \times \mathbf{H} = \partial_t \mathbf{D} + \mathbf{J}$$
$$\nabla \cdot \mathbf{D} = \rho_v$$
$$\nabla \cdot \mathbf{B} = 0$$

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We have reintroduced some of the most commonly used terms.

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**MAXWELL PDE SYSTEM**

**E, B**

Electric/Magnetic  
fields

E, B are related through Lorentz force

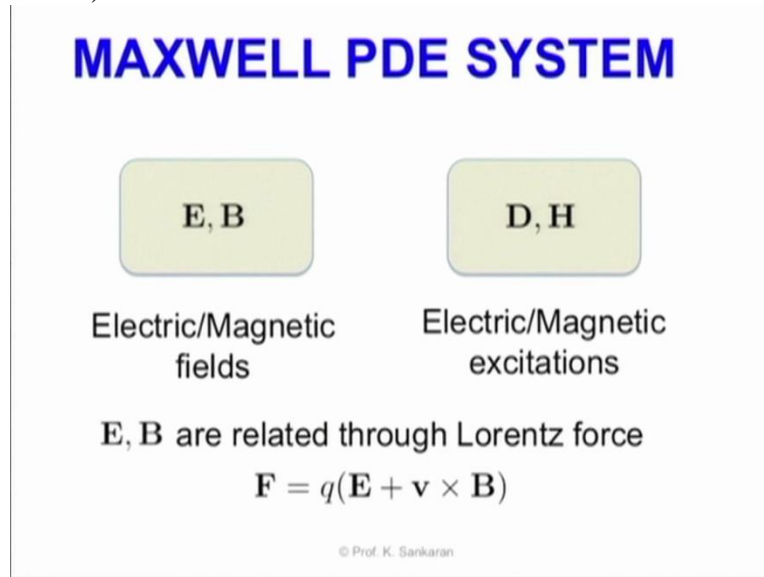
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

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Like Electric and Magnetic fields using their proper topological and physical relationships. In fact we saw that in the definition of Lorentz Force it is actually the E and B and not E and H

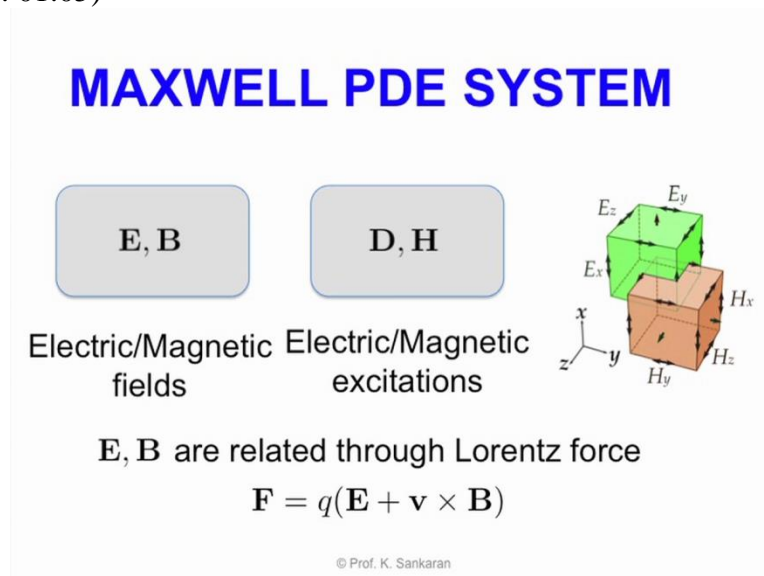
fields that are related. Hence we rightfully termed E and B fields as Electric and Magnetic fields in our lectures.

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We also reintroduced Electric and Magnetic excitations which we denoted as D and H respectively.

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The reason for coupling E and B and D and H field became also clear while we studied the E algorithm in the finite difference time domain method.

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## MAXWELL PDE SYSTEM

$$\begin{array}{ccc} \nabla \times \mathbf{E} = -\mu \partial_t \mathbf{H} & \rightarrow & \partial_t \mathbf{E} = \frac{1}{\epsilon} (\nabla \times \mathbf{H}) \\ \nabla \times \mathbf{H} = \epsilon \partial_t \mathbf{E} & & \partial_t \mathbf{H} = -\frac{1}{\mu} (\nabla \times \mathbf{E}) \\ \nabla \cdot \mathbf{E} = 0 & & \nabla \cdot \mathbf{H} = 0 \end{array}$$

Generally  $\nabla \cdot \mathbf{H}$  need not be zero **even if**  $\nabla \cdot \mathbf{B} = 0$

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We have remarked that it is the diversions of B and not that of H which is always 0. In other words even when the diversions of B is 0 we might have some value for diversions of H.

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## MAXWELL PDE SYSTEM

$$\begin{array}{ccc} \nabla \times \mathbf{E} = -\mu \partial_t \mathbf{H} & \rightarrow & \partial_t \mathbf{E} = \frac{1}{\epsilon} (\nabla \times \mathbf{H}) \\ \nabla \times \mathbf{H} = \epsilon \partial_t \mathbf{E} & & \partial_t \mathbf{H} = -\frac{1}{\mu} (\nabla \times \mathbf{E}) \\ \nabla \cdot \mathbf{E} = 0 & & \nabla \cdot \mathbf{H} = 0 \end{array}$$

Generally  $\nabla \cdot \mathbf{H}$  need not be zero **even if**  $\nabla \cdot \mathbf{B} = 0$

**$B = \mu(M+H)$**

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This comes directly from the relationships between B and H fields.

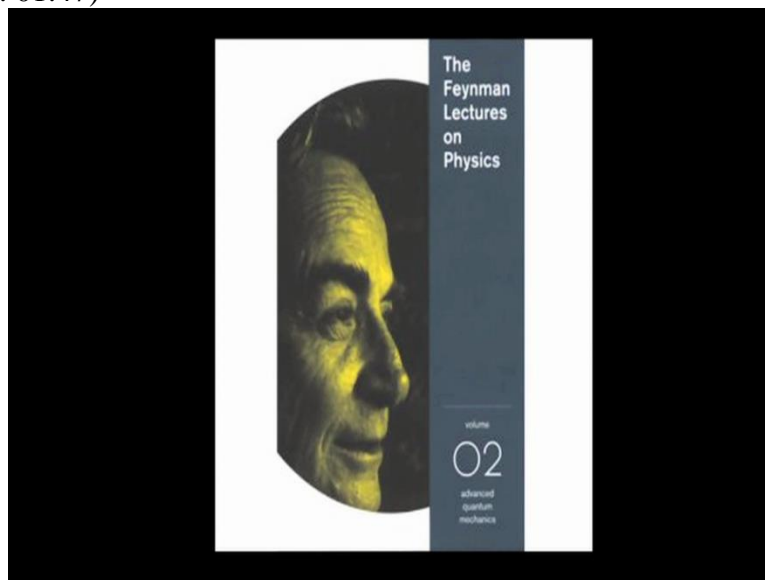
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**Use Good books for:**

- 1) firm grasp of physical concepts**
- 2) understanding the physical meaning**

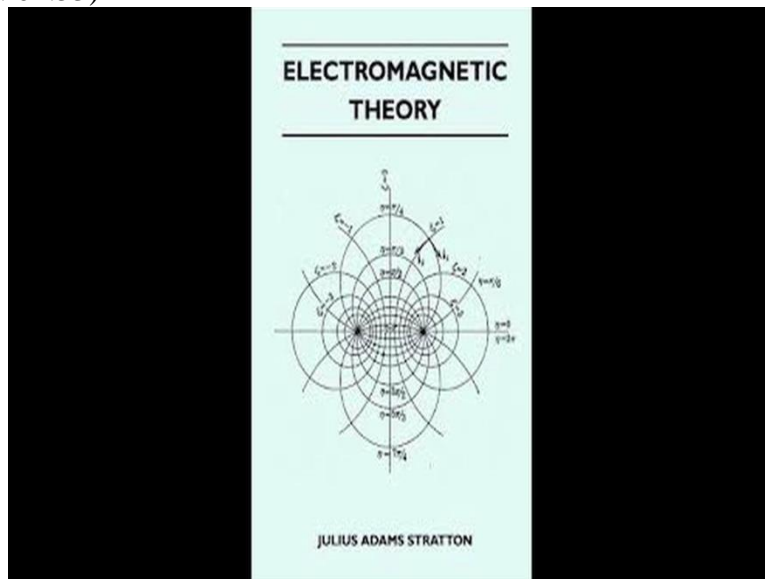
I encourage you to use good reference books to get a firm grasp of underlined physics and the physical meaning. Throughout this lecture I followed three main reference books for electromagnetic

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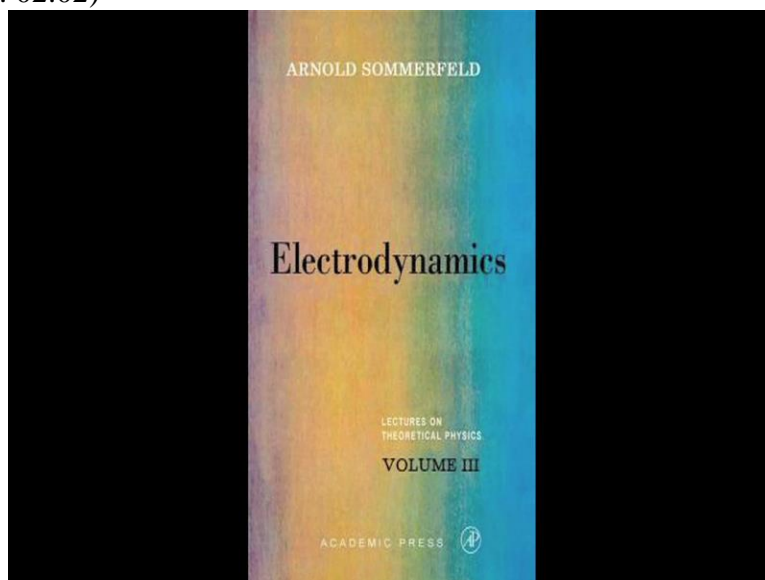
The first one is the Feynman lectures in Physics volume 2 by Richard Feynman.

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The second book is the Electromagnetic theory by Julius Adams Stratton.

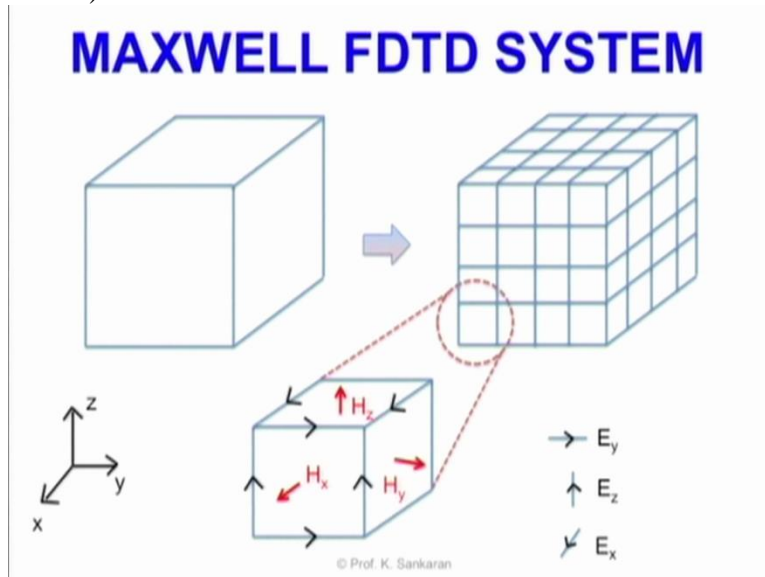
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And the third book the lectures in theoretical physics by Arnold Sommerfeld.

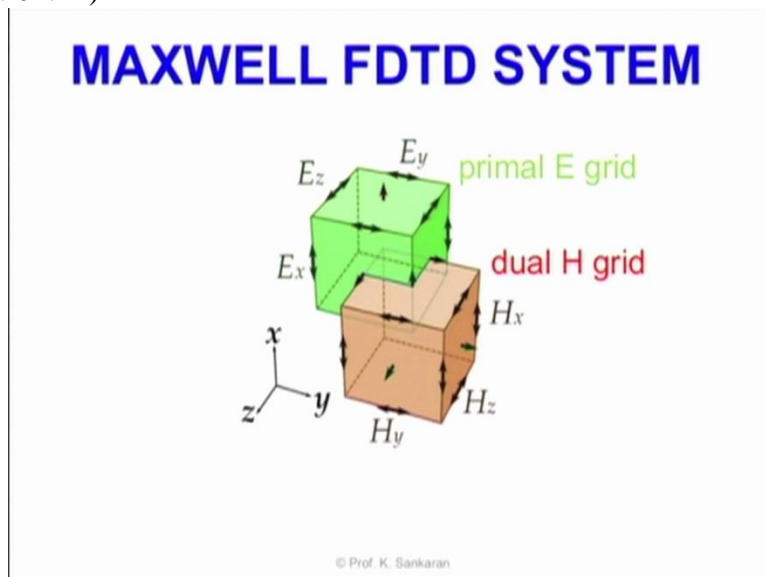
We will be giving you the links to the information about these books on our course website.

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In this week we also introduce the Finite difference time domain algorithm for the Maxwell equation which is the famous E algorithm.

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We saw how the E and H field are staggered both in space and time using this algorithm, we also notice that the PDE system is algebraic in nature and E algorithm is essentially centered in space and time scheme.

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## MAXWELL FDTD SYSTEM

Similarly can be done for other scalar equations

Maxwell PDE system in algebraic form

$$\partial_t \begin{bmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\partial_z & \partial_y \\ 0 & 0 & 0 & -\partial_z & 0 & \partial_x \\ 0 & 0 & 0 & -\partial_y & \partial_x & 0 \\ 0 & \partial_z & -\partial_y & 0 & 0 & 0 \\ \partial_z & 0 & -\partial_x & 0 & 0 & 0 \\ \partial_y & -\partial_x & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{bmatrix}$$

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We have briefly discussed Maxwell system in frequency domain. And we also compared the Pros and cons of using Finite difference time domain method and the Finite difference frequency domain method.

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## MAXWELL FDFD SYSTEM

$$\partial_t \leftrightarrow -j\omega$$

$$\nabla \times \mathbf{E} = j\omega\mu\mathbf{H} \quad \nabla \times \mathbf{H} = -j\omega\epsilon\mathbf{E} + \mathbf{J}$$

$$\nabla \times \nabla \times \mathbf{E} = j\omega\mu(\nabla \times \mathbf{H})$$

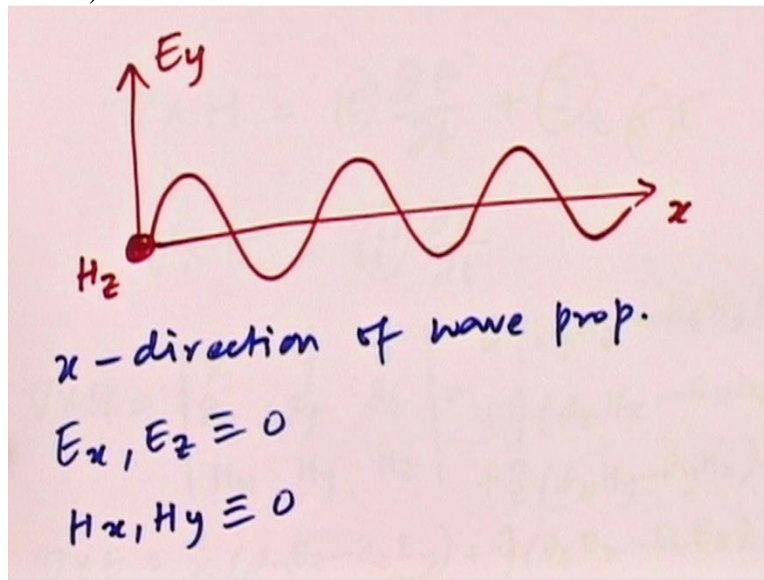
$$(\mu^{-1}\nabla \times \epsilon^{-1}\nabla \times -\omega^2 I)\mathbf{E} = \mathbf{S}$$

$$\text{where } \mathbf{S} = \mu^{-1}\nabla \times \epsilon^{-1}\mathbf{J}$$

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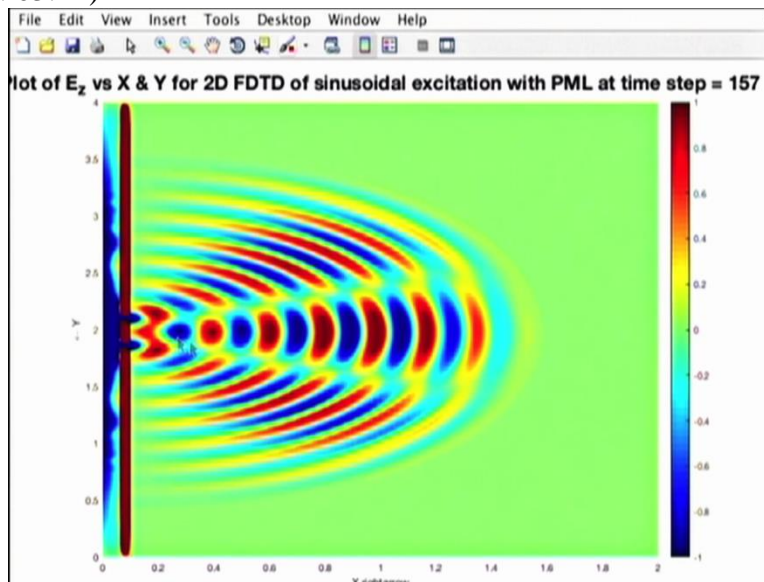
As a first exercise we looked into one dimensional electromagnetic field propagation problem.

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In the second exercise we looked into a single and double slit diffraction experiment using finite difference time domain method.

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We have also investigated the role of spatial and temporal discretizations using the Matlab code.

And this is the summary of this week. We will be looking into (03:22) boundary conditions in the upcoming lectures.

I will encourage you to practice the exercises and examples that we have studied in this week and get ready for the next week.

So until then Good Bye!