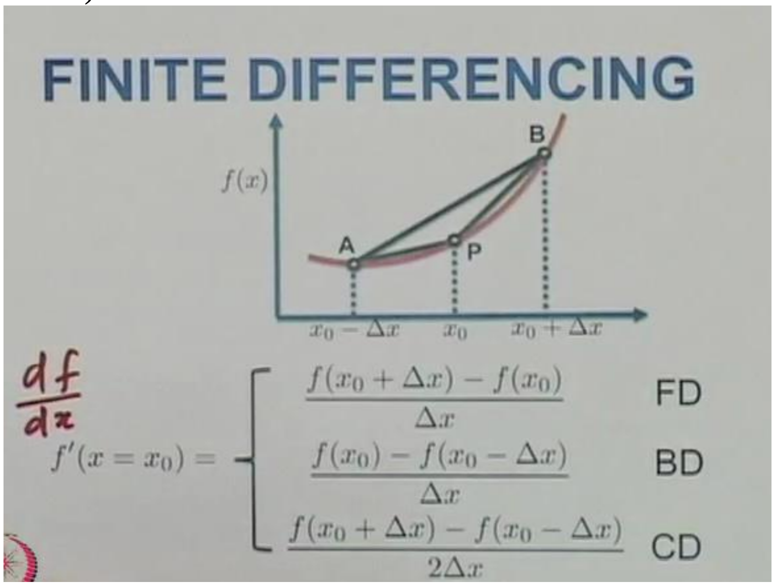
Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Lecture 02
Finite Difference Methods –1

So we have introduced the Finite Difference Method. We have talked about the very basic introduction to the method. Starting from the historical perspective and we gave also certain introduction into some of the basic Finite Differencing Techniques. So let us move forward

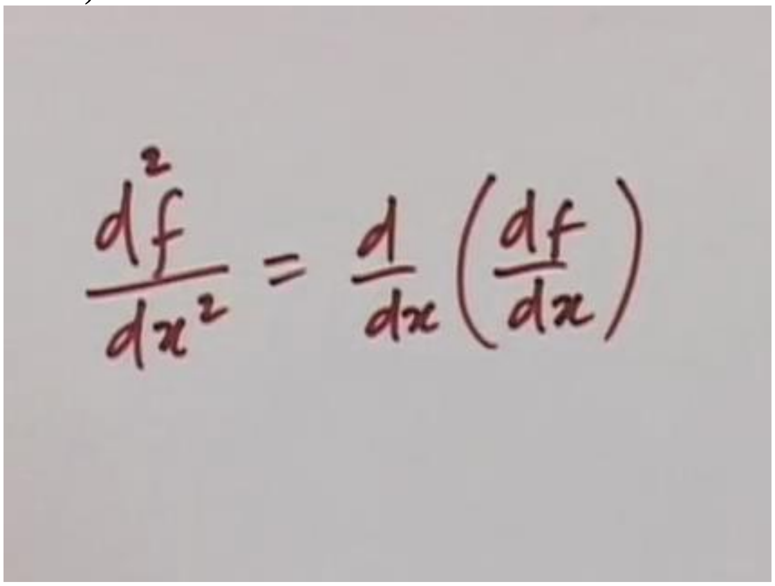
(Refer Slide Time: 00:36)



Remember in the earlier module we discussed on certain way to compute the value of df by dx, at the point x not. And we used certain differencing schemes which we called as Forward Differencing, Backward Differencing and Central Differencing.

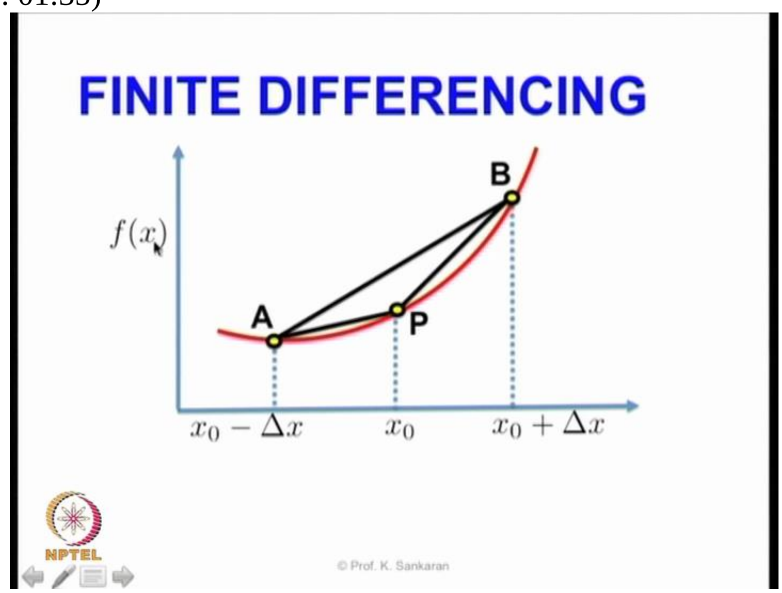
So now let us say we are interested in not only in df by dx, but let us say we are also interested in the value of second differential.

(Refer Slide Time: 01:09)



So what I mean is let us say I am interested in the second differential with respect to x. So, I am differentiating the value of df by dx once more. So this can be written as d by dx of df by dx.

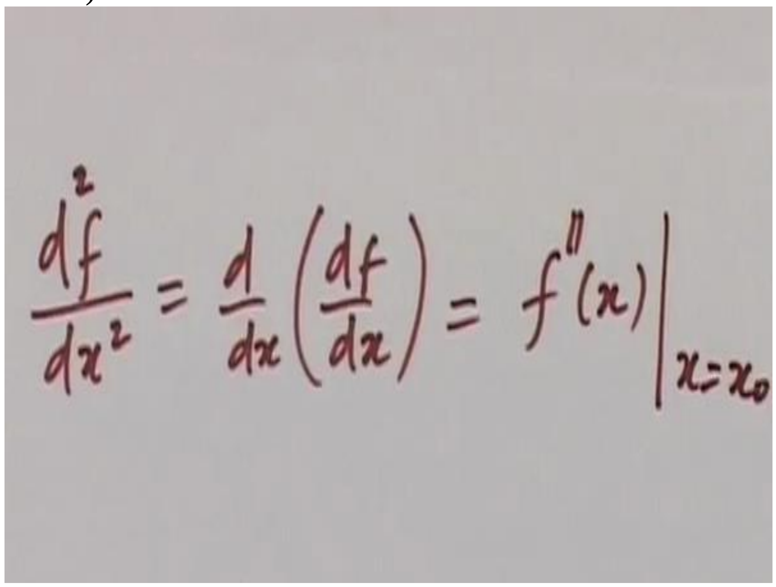
(Refer Slide Time: 01:35)



So remember that we computed the value using these particular slopes. So at one point we computed the slope like this, at one point we computed the slope like this, based on whether we are using Forward Differencing or Backward Differencing.

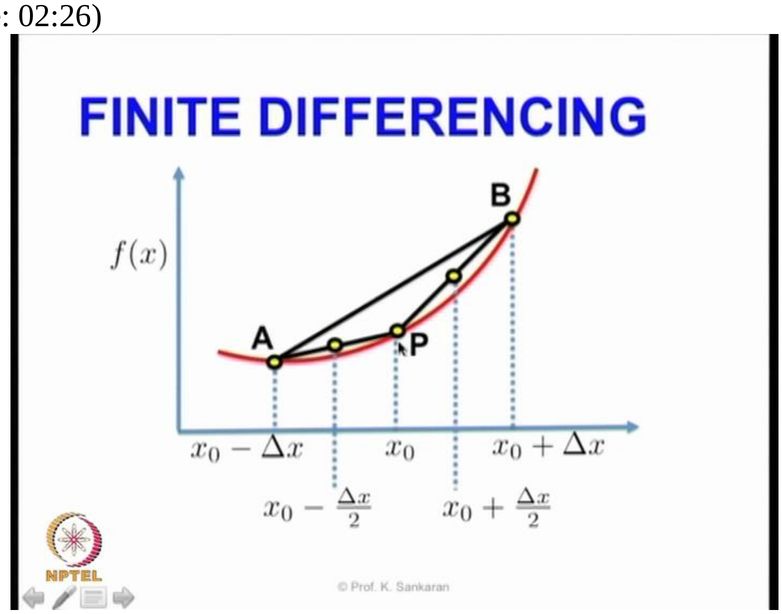
So in the case of Forward Differencing this will be the slope that is computed, which is represented by P, B; and in the case of Backward Differencing it will be P,A; and in the case of Central Differencing it will be A,B.

(Refer Slide Time: 02:13)



So let us say I am interested in computing the value of f double prime. In other words, this thing can be written as f double prime of x. So if I am interested in finding f double prime of x, at x equal to x not. So here, what I am interested is in knowing the value of f double prime of x by differentiated twice at this point at B. This I can do in very elegant manner by taking into account the points that are in between x not and x not minus delta x.

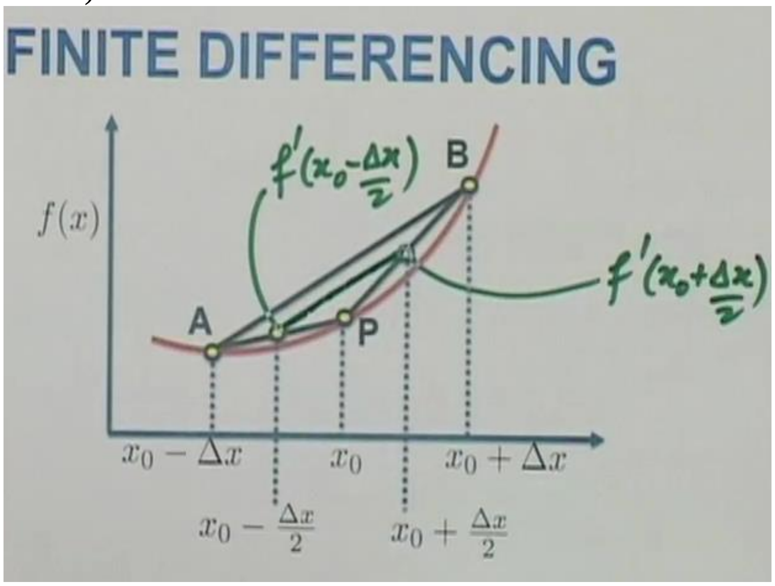
(Refer Slide Time: 02:26)



So what I am interested is in the half step points. So these are the points that are actually located at x not minus delta x by 2 and x not plus delta x by 2. Where delta x as we said before it is the step size in X direction.

So if I apply the same central differencing scheme on the value of f prime of x at this point, so in other words I am taking the value of f dash of x which is basically sitting on this A, P so I am able to compute the value.

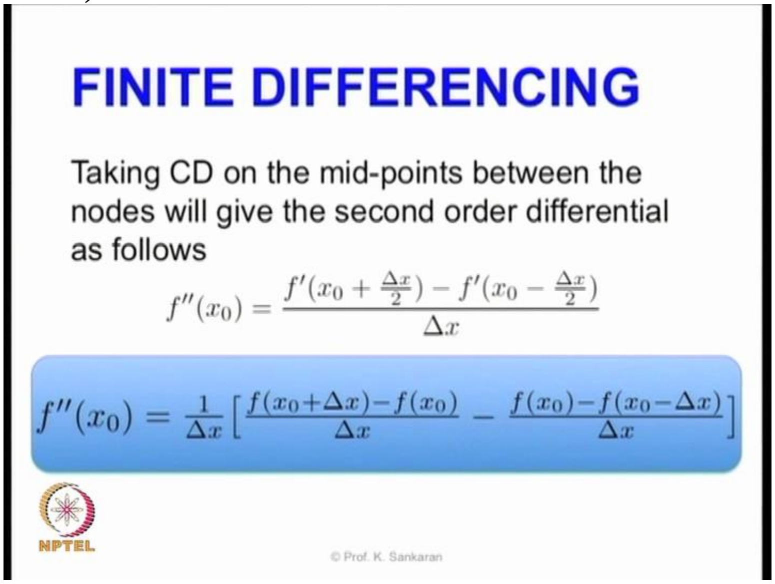
(Refer Slide Time: 03:27)



In other words as you can see, what I have in the graph here is this particular point is going to be the value of f dash at x not minus delta x by 2 and this point is going to bef dash at x not plus delta x by 2.

So by using those two values, I can basically compute the value of the central differencing between these two points. In other words, I am computing the value between these two points. So this is the thing what I am computing.

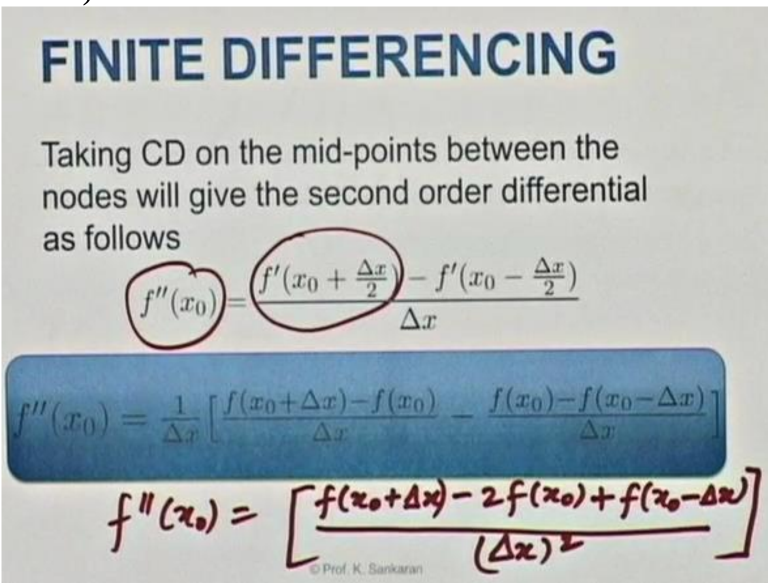
(Refer Slide Time: 04:08)



So this is what we will get if we use the Central values and the value of f prime of x at those points which are the mid points. And again the step size will be delta x because we are exactly in the middle of those two points.

If you apply the value of those already first differentials what you will essentially get is second order differential with respect to the value given here. In other words we can express them in more simplified notation.

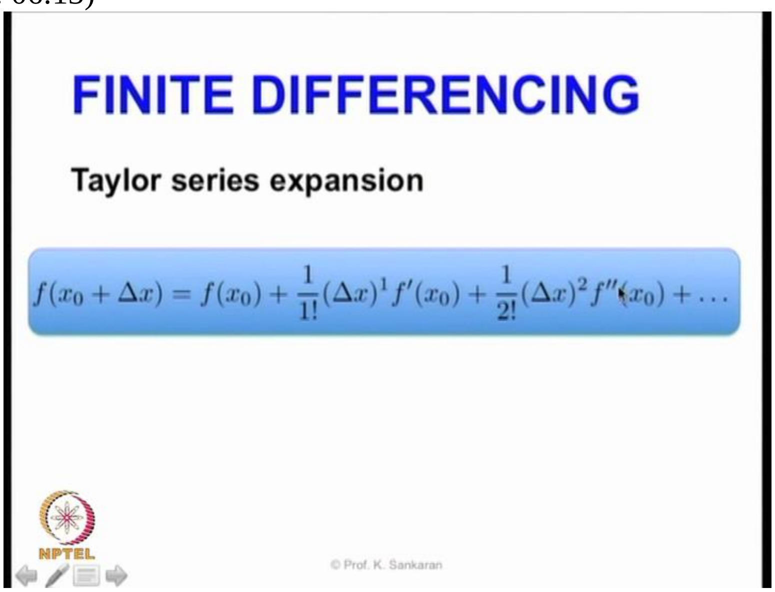
(Refer Slide Time: 04:42)



Because the value of f at x not will get doubled so what we will get is f double prime at x not is equal to so what we will have in the numerator will be the value of f x not plus delta x and twice the x not value will get added. So we will get 2f at x not minus and minus will become plus so f x not minus this will be the numerator and denominator will be delta x and then we have a delta x also at the outside so it will be delta x to the power 2.

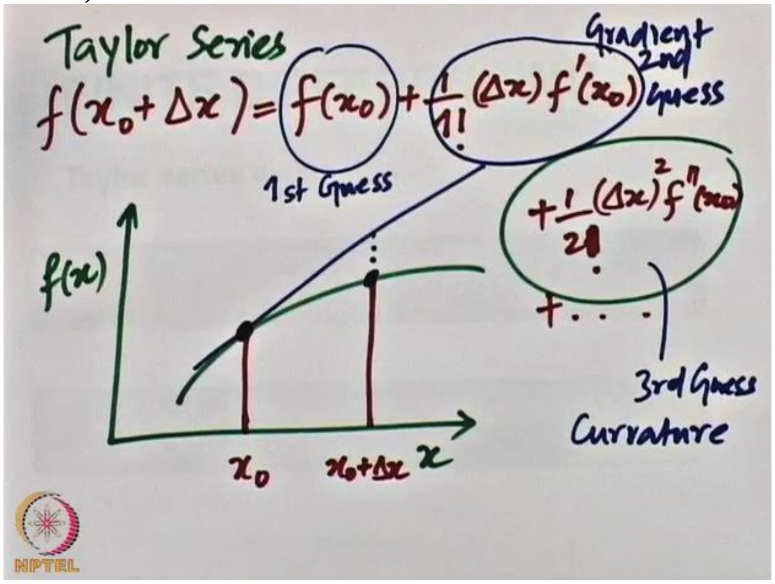
So this is a more compact notation, so we are able to compute the value of various differentials, whether we are f double prime of x not or in the previous case we were interested in first prime of x not. We are able to use only the discrete values at different points to get an expression of this sort. But the question still remains what is the accuracy of this approximation?

(Refer Slide Time: 06:13)



This we can understand when we use the Taylor series. As we can see the Taylor series expansion gives us a very nice way to compute the value at x not plus delta x purely based on the value at x not using certain basic terms. And what are these terms let me write on these terms in a much more easier form.

(Refer Slide Time: 06:30)



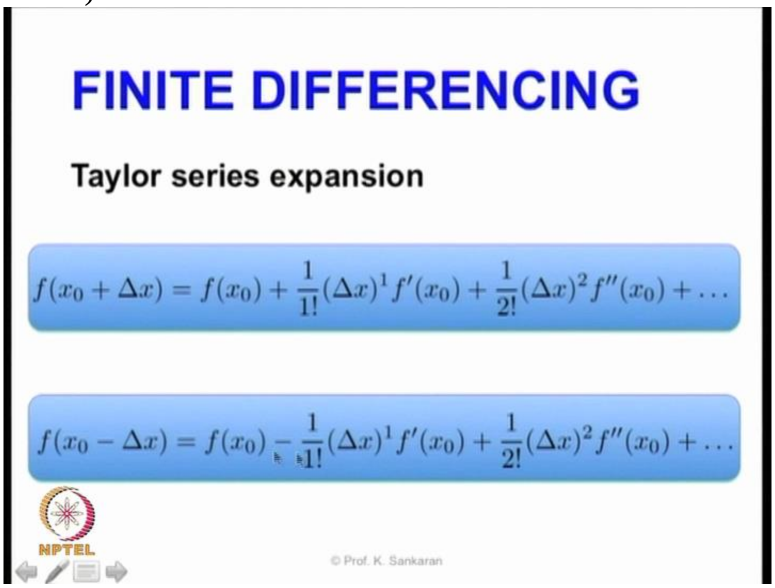
So we are talking about x not plus delta x is equal to certain value at x not plus 1 by 1 factorial delta x f dash x not plus 1 by 2 factorial delta x square f double dash of x not plus so on and so forth. So, what is important to know is, let us say I know this is the function where this is the independent axis and this is the dependent axis, and I know the value of the function let us say at the point given as x not and I am interested in this value at let us say the value at x not plus delta x.

So when I have this what I am doing here is, I am saying the value is nothing but the combination of certain other values. So, what are those certain other values? So, the first guess is I say so this is the first guess, I say the value at let us say x not plus delta x is the same value as that of what is in x not itself. So I am getting a starting point. So I say the value here is exactly same as the value here. So that is the first term.

The second term is basically the second guess, so I say the second guess is the value depends on the gradient also. And then I say the third guess is, it is here it says not only that it also depends on the curvature at the value x not. So this term is the third guess. And it is the curvature and this is the gradient. And this is the pure approximation. So what we are saying that here is the value takes into also account the curvature of this particular line at this particular point.

So what we are getting now is a kind of a physical meaning of the Taylor series. So when you talk about a Taylor series, of a particular equation. A Taylor series is nothing but a series of approximations based on certain guesses what we make. The Taylor series can be close to the analytical solution if we keep on adding more and more terms. Obviously for practical reasons we stop at sudden finite number of terms. So in this case we are already stopping them in the second order term.What we can see is we are able to see the guesses slowly go closer and closer to the real value here. The real value here is represented by the green line and this is coming closer and closer and closer to this particular point.

(Refer Slide Time: 10:20)

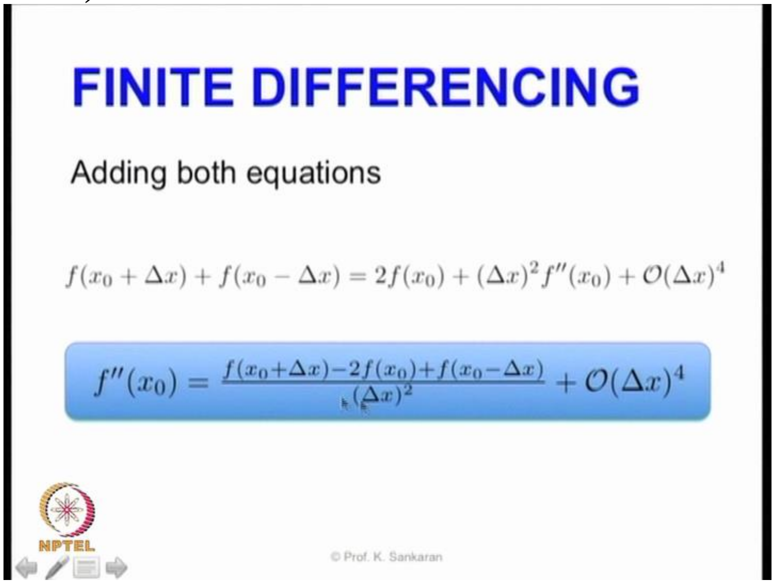


So what we see in the slide is, f of x not plus delta x is equal to the first guess, the second guess plus the third guess so on and so forth.

Similarly, you can do the Taylor series expansion for x not minus delta x which is given here. And this is the straight forward repetition of what we did but here we are doing for x not minus delta x. And you can see the terms are very similar so you are taking the first guess, you are taking the gradient, and then you are doing the curvature so and so forth.

So these two expressions are quite important because you can get quite a lot of interesting outcomes by just adding and subtracting these two terms. You might wonder why are we adding and subtracting but this will become clear when we see what is the outcome of adding these two expressions or subtracting these two expressions.

(Refer Slide Time: 11:09)

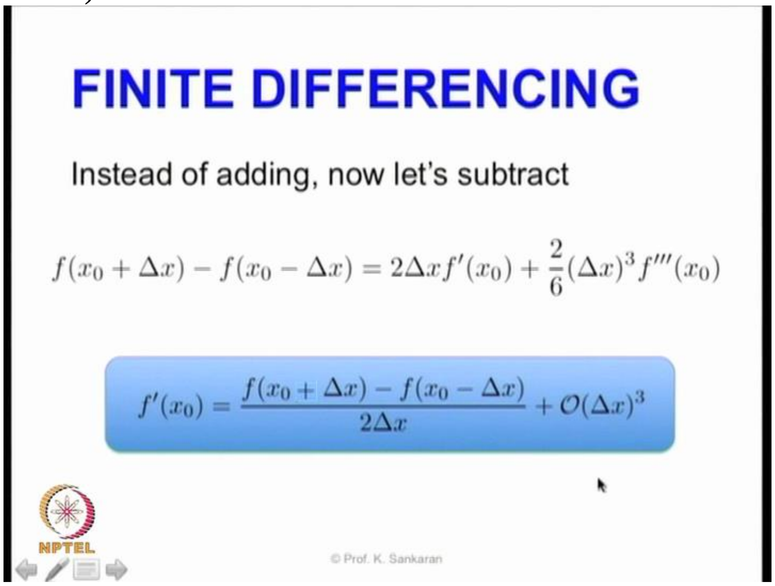


So what you are seeing here is the terms are adding up on certain cases and the terms are cancelling in certain cases. So this particular expression is good enough for us to find the value of f double dash at x not as follows.

So what we are computing is, we are computing the second differential of f at x not is equal to I am simply re arranging this term, I am bringing this term on to the left hand side and then I am having only an expression for f double prime at x not.

As you can see the order of accuracy here is in terms of delta x will be 4. Because we have terms still running but we are not interested in these higher order terms so we can say for practical purpose the second differential of f using this particular expression is by the order of delta x to the power 4.

(Refer Slide Time: 12:04)



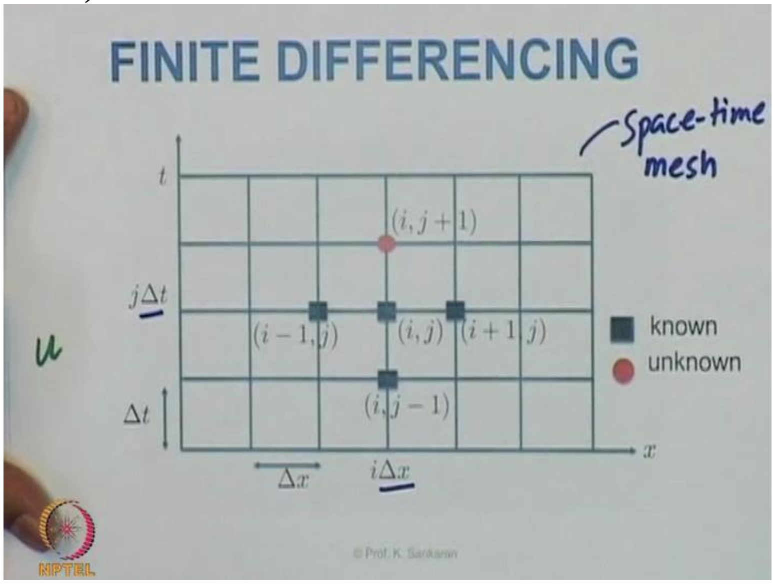
Likewise instead of adding these two equations if we subtract these two equations what we get is odd terms that got cancelled in the previous case in this case the even terms get cancelled so what you will get is an expression like this.

So as you can see this is a nice expression to compute the value of f prime at x not. We are not interested in the higher order terms. So if I re arrange the terms such that I can get an expression for f prime at x not as I have written here, with certain higher order terms as you can see which I am not interested.

So this is an accuracy of order delta x to the power 3, whereas in the previous case we got the second differential with the order delta x to the power 4. So when we are talking about accuracy and order of certain equations what we are saying about is the truncation error. So this is what we call it as a truncation error. And we will look into it much more in detail later on it is enough for you to know, when we talk about truncation error we are talking about this particular term.

So we have seen how to compute the value of the first differential and the second differential adding and subtracting certain terms of the Taylor series. With that you can look at how we can compute the value of certain functions at certain time steps or certain special steps using those basic approximations which we have seen before.

(Refer Slide Time: 13:39)



So our problem is, let us say we are interested in one dimensional problem. Let us say the grid is like this and we are having one dimensional problem in X coordinate and obviously it also has a temporal coordinate and we are discdizing the special and temporal coordinates using certain steps and we call them as space steps delta x and time steps as delta t.

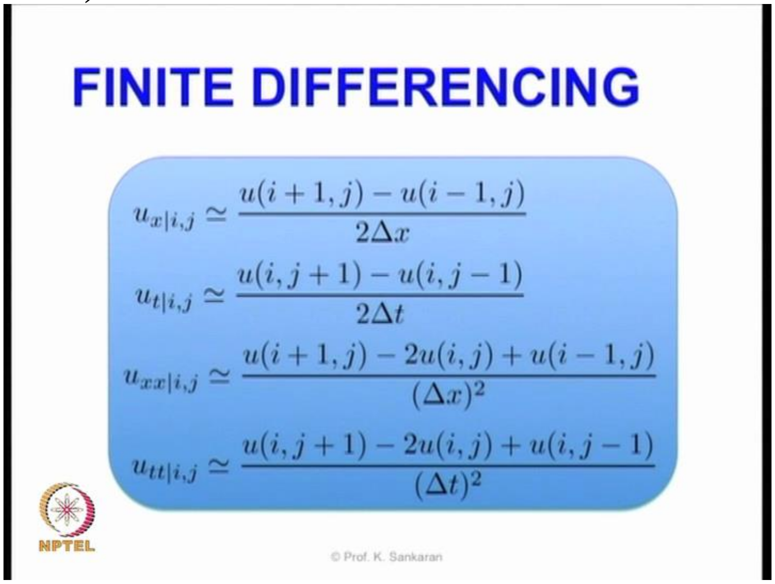
S i is the index number for the special coordinate and j is the index term for the temporal coordinate and as you can see I can go from i equal to 0 from i equal to some value and j equal to 0 to j equal to certain value, so what I have here is i,j or i,j minus 1 are nothing but temporal and special coordinates. Or in other words I used just the index number of the special coordinate and the index number of the temporal coordinate to point certain notes in the space time mesh.

So this is a space time mesh. And It has one special coordinate and one temporal coordinate. Obviously when we do higher order problems we will have more special coordinates, second special coordinates and third special coordinates.

And for this particular example let us stick with the one dimensional case. And there are certain things that we need to pay attention, the points which are marked black are points where we know the value of certain functions let us say u, our function is u and we know the value of u at certain special and temporal coordinates. So as I am proceeding upwards, I am marching in time, so I am going from one point to another point, I am going from time equal to 0 to time equal to 1,2,3 and 4 and so on and so forth.

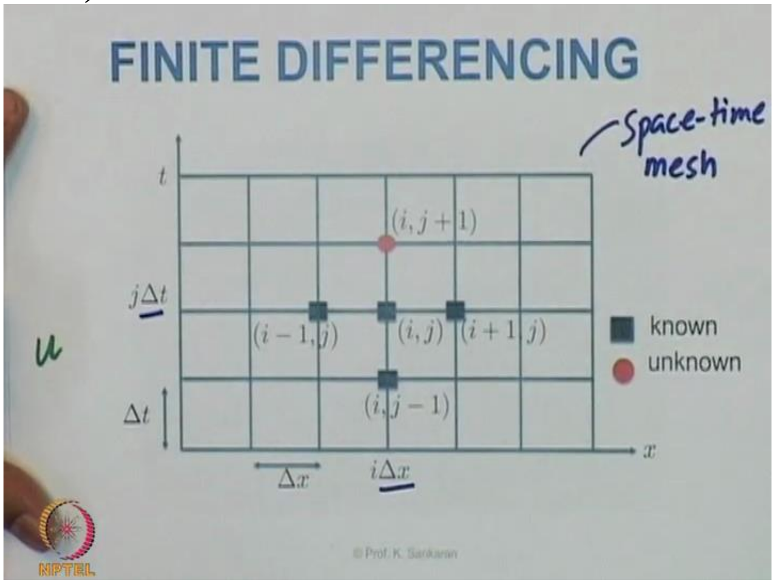
So what I am saying here is I can compute the value at j plus 1 using only the values at time before j plus 1. This is a explicit formulation. So I will define what is the explicit formulation at a later stage.

(Refer Slide Time: 15:59)



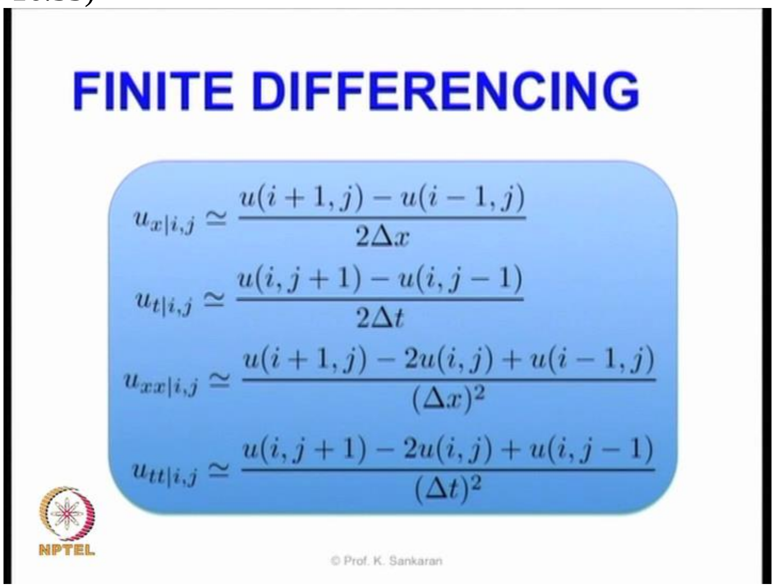
For now let us stick to a simple case. So based on this particular mesh, we can compute the value of the differentiation of u with respect to x at the point i , j so what I mean here is, I am computing the value of u in this particular point i,j.

(Refer Slide Time: 16:17)



So what I am computing here is I am computing the first differential with respect to x; I can also compute the second differential with respect to x; similarly, I can compute the first differential with respect to the time, and the second differential with respect to time.

(Refer Slide Time: 16:33)



So these particular expressions here are the Central differencing, I know it is Central differencing because I have here 2 delta x and 2 delta t. so this is the Central differencing with respect to x, this is the central differencing with respect to time.

So what you see here is I compute the du by dx by using the value we saw before I am computing the value at i plus 1 and i minus 1and dividing it by twice step size in X axis

Similarly, I can do the same thing for du by dt which is written here as ut is given by the expression here. And approximate signs are here in place because we have some order of truncation. Which we have neglected and that is why we say this is approximately equal to.

Similarly we can do the second differential with respect to x using the Taylor series expansion we know it will be an expression here similarly we have the second differentiation with respect to time as given by the value here.

As you can see in the case of second differential we are going from i plus 1 to i to i minus 1. Similarly, in the case of second differential with respect to time the i does not change but the j term changes; j goes from j plus 1 , j to j minus 1.

So with this being said what we have done so far is we have got certain approximations and it will become clear that we are going to use these approximations to model certain problems. And we will look into those problems in our next module.

So that being said we will come back again and we will focus on the further techniques in Finite Differencing. Thank you!