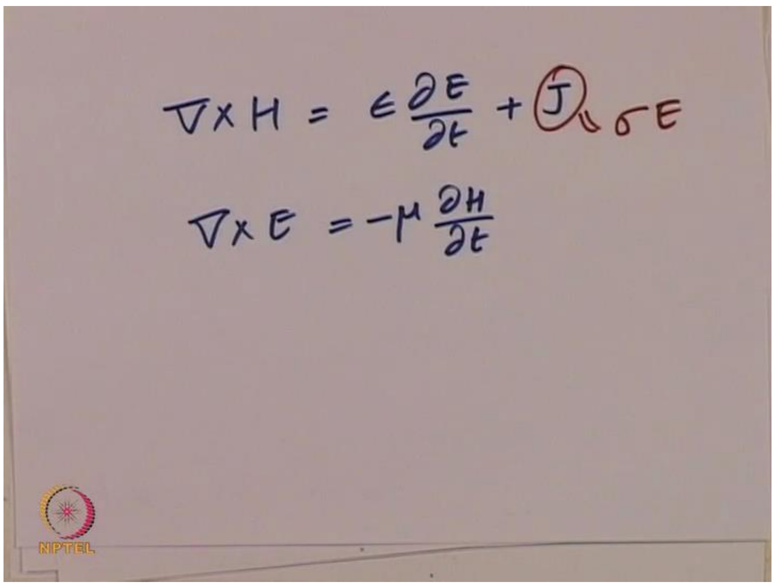
Computational Electromagnetics and Applications
Professor Krish Sankaran
Indian Institute of Technology Bombay
Exsercise No. 7
Finite Difference Methods - III

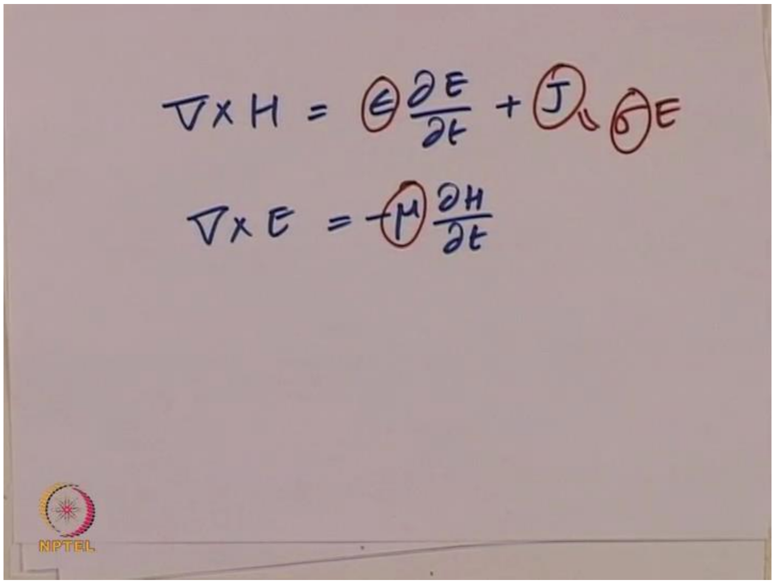
We are now going to look into an interesting problem which is going to show lot of interesting insights about Maxwell equations. So for doing that we are going to simplify the problem into one dimensional problem in order to get much more insight about various parameters that are involved in the Maxwell equation. So let us look into the Maxwell equation itself and then start going into the problem step by step.

(Refer Slide Time: 00:44)



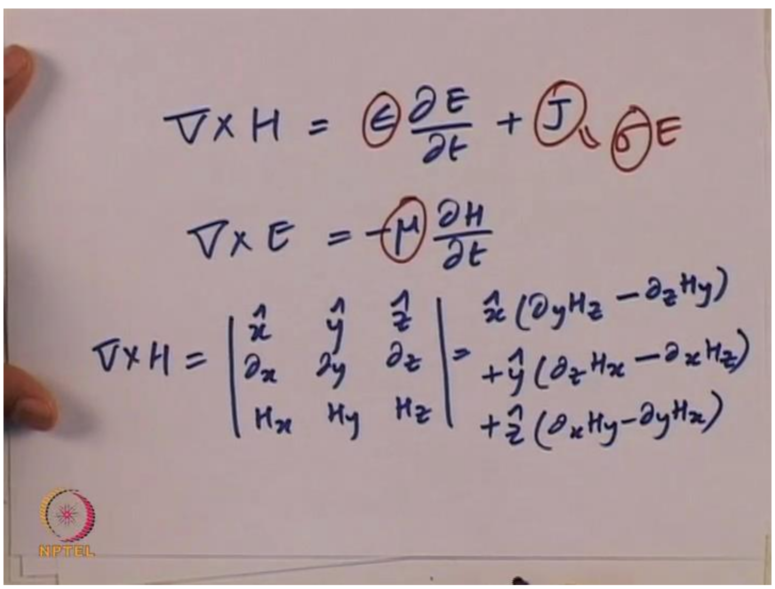
So now let us start with the standard Maxwell equation which is written in the curl form Curl of H is equal to Epsilon Doe E by Doe t plus J and the curl of E is equal to minus Mu Doe H by Doe t. And here the value of J is the source term which is equal to Sigma E. And since we do not have magnetic charges we do not have any additional term in this case

(Refer Slide Time: 01:47)



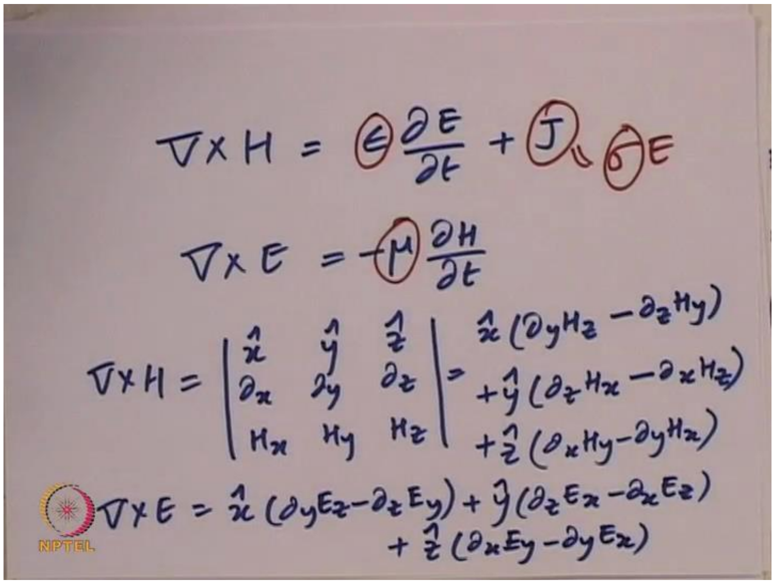
So we are going to stick with this particular form and we are going to understand the impact of various terms namely permittivity, permeability and conductivity. So these terms are going to impact the way the weight is going to propagate.

(Refer Slide Time: 01:59)



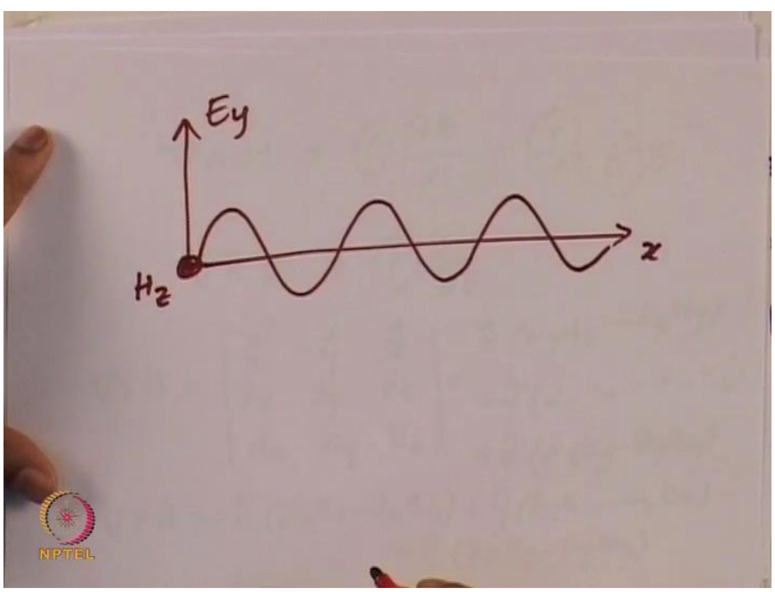
So if you are going to write this expression in the expanded form what we are going to have is the following: the curl of H is going to be written x y and z; Doe x Doe y Doe z; H x, H y, H z. So it is going to have three components, So the x component is going to be written as (Doe y H z minus Doe z H y) and similarly the y component is going to be written as plus y and I am going to switch the sign, so it is going to be (Doe z H x minus Doe x H z) and the z component is going to be written as Doe x H y minus Doe y H x).

(Refer Slide Time: 03:14)



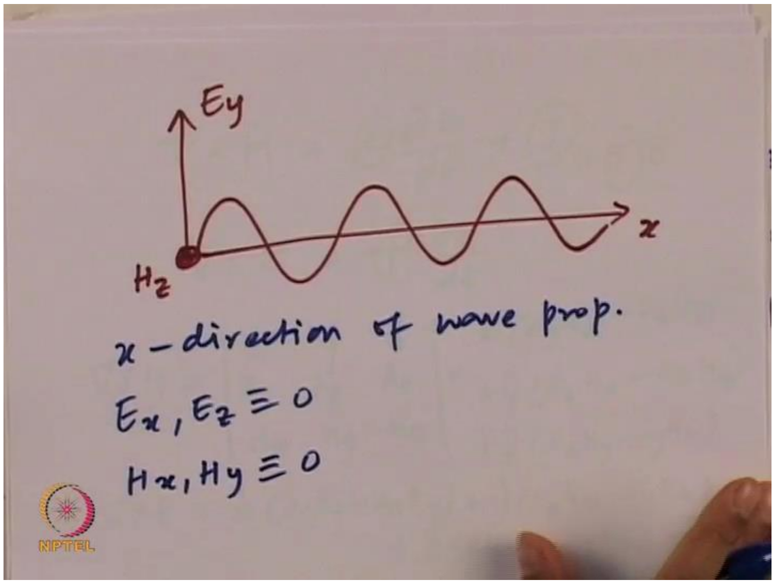
Similarly we will also have the components for del cross E which is going to be expanded directly as the x component is going to be Doe y E z minus Doe z E y) wherever there is H I am going to convert it into E so that makes the equation a much straight forward plus y component (Doe z E x minus Doe x E z) plus z (Doe x E y minus Doe y E x). Or in order to catch this problem in one dimension so we are going to make certain assumptions.

(Refer Slide Time: 04:07)



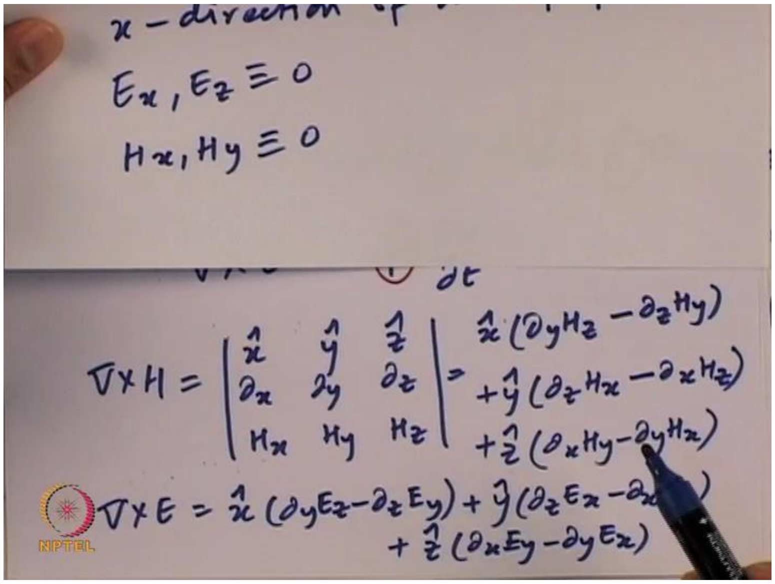
So the assumptions are our E component is going to have value only in the y direction, so E in going to be only in the y direction. And z component of magnetic field is going to be present so assuming that z is coming out of this particular paper, so we are going to have the H z component. So that means the wave is going to propagate in the x direction so we are going to have the wave propagation of this sort.

(Refer Slide Time: 04:56)



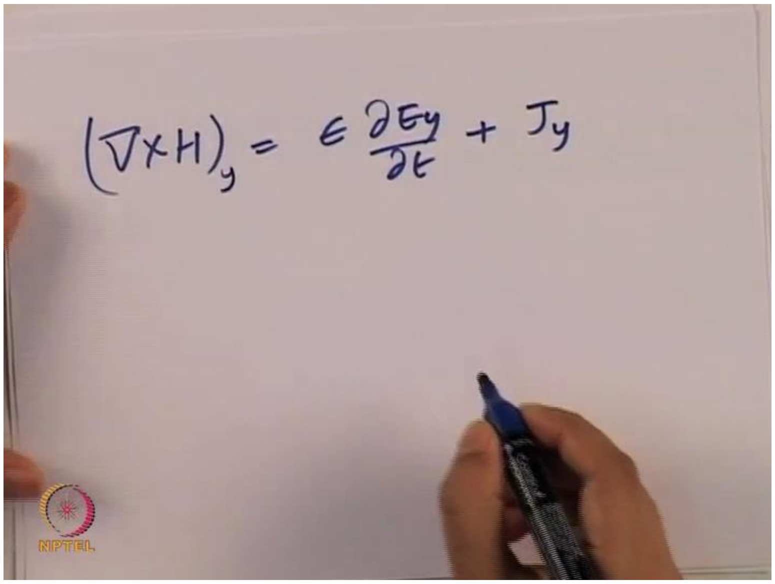
So what we have is x direction of wave propagation. And we assume the E x, E z is equal to 0 and H x, H y is equal to 0.

(Refer Slide Time: 05:26)



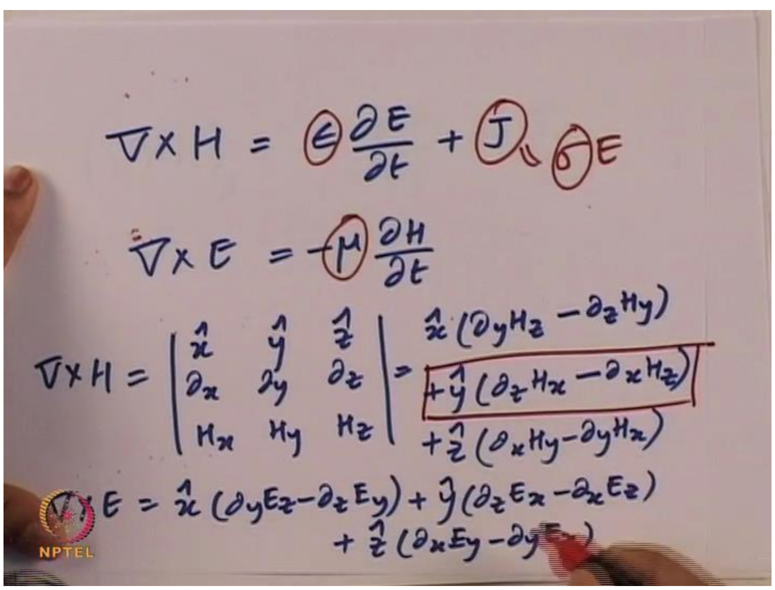
So if we go back to this equation what we have and simplify this form putting these values what we will get is a one dimensional wave equation or one dimensional Maxwell equation.

(Refer Slide Time: 05:42)



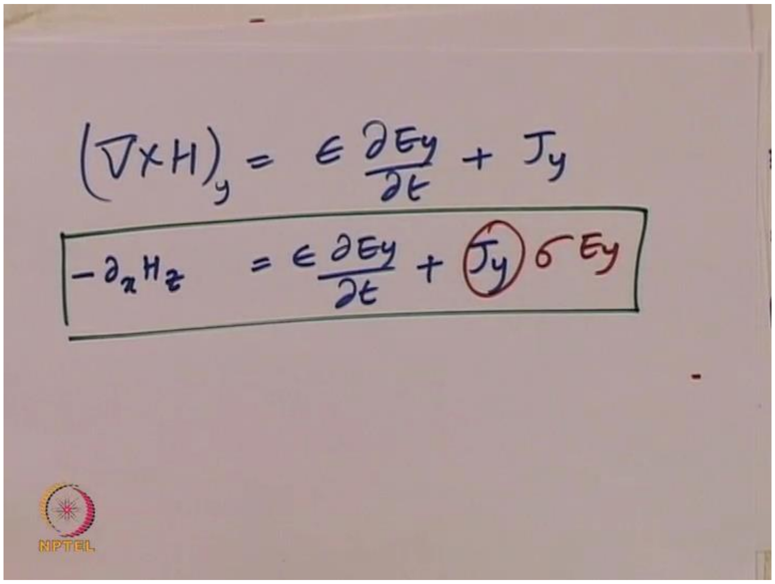
So let us write it down as follows. So let us look at the Maxwell equation (()) 05:46 in one dimensional form. So curl of H is equal to Epsilon Doe E y by Doe tplus J y. Because we are assuming that E and J are going to be in the y direction. And now we can expand the value of the curl of H and we are interested in the y component.

(Refer Slide Time: 06:20)



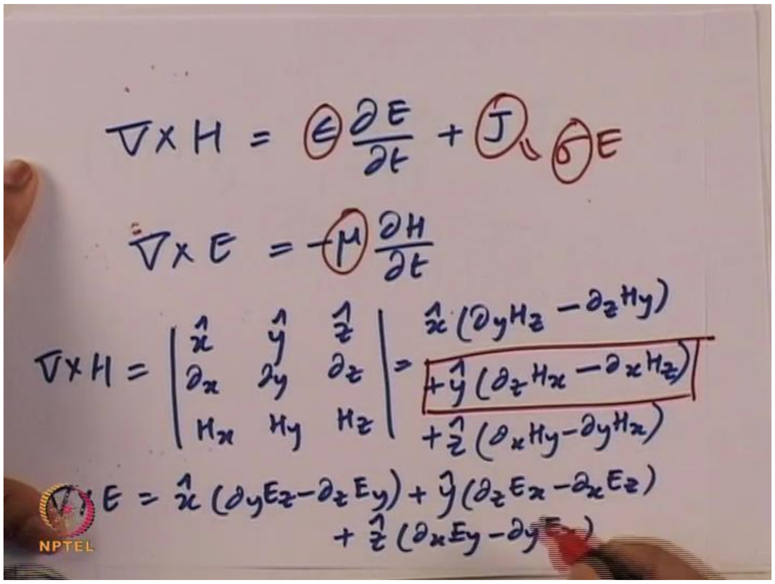
And if we look at this particular expansion of the y component what we see is this particular term is a term what we are interested in. And of course in this particular term the H x value is going to be 0 which we have given in the assumption and we will have only this particular term.

(Refer Slide Time: 06:42)



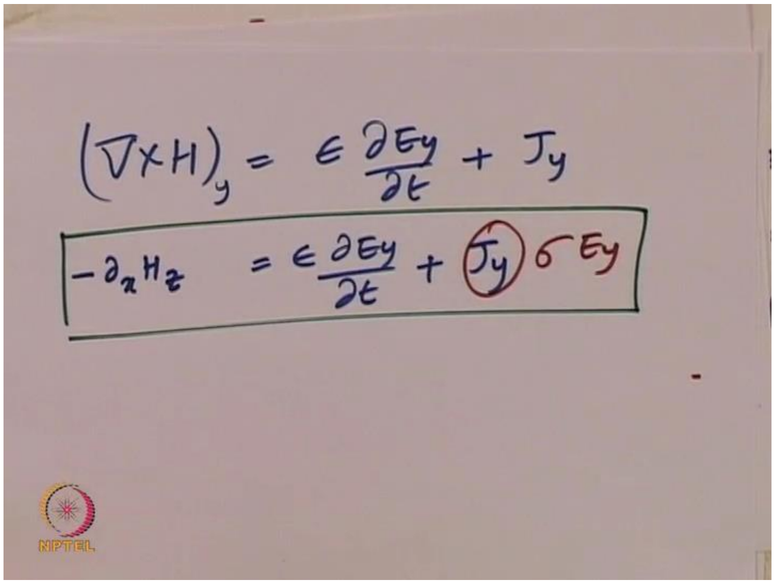
So let us write it down so what we have is Epsilon Doe E y by Doe t. So now we will write the term that we have from the curl term. So it is going to be minus Doe x H z.

(Refer Slide Time: 07:08)



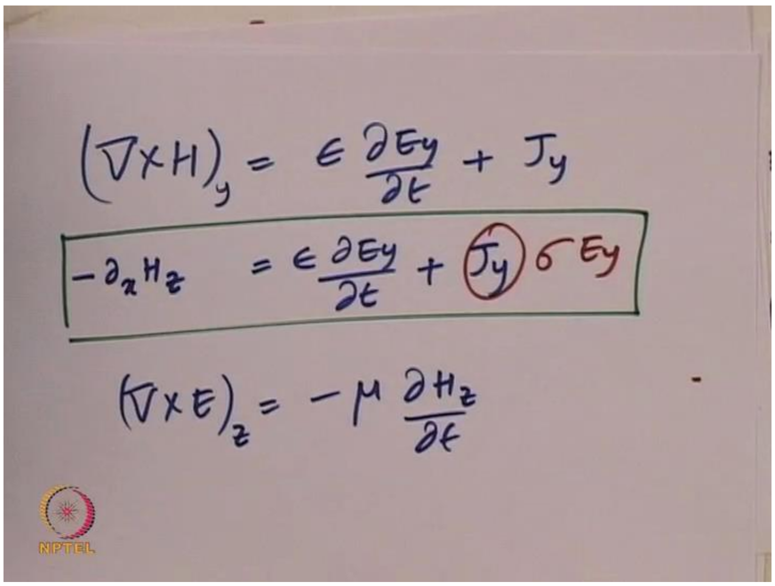
This is the term I am taking it from here

(Refer Slide Time: 07:10)



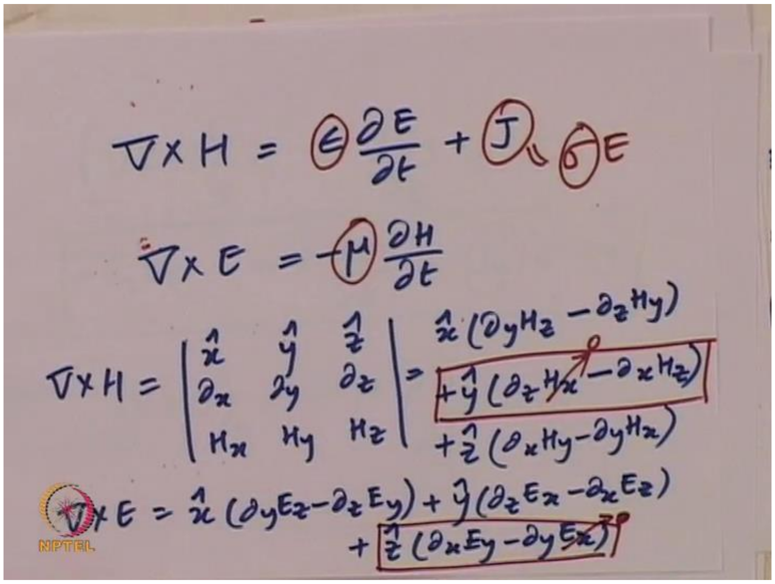
And putting it here plus J y and I can write J y as Sigma E y. So this is going to be the first curl equation.

(Refer Slide Time: 07:34)



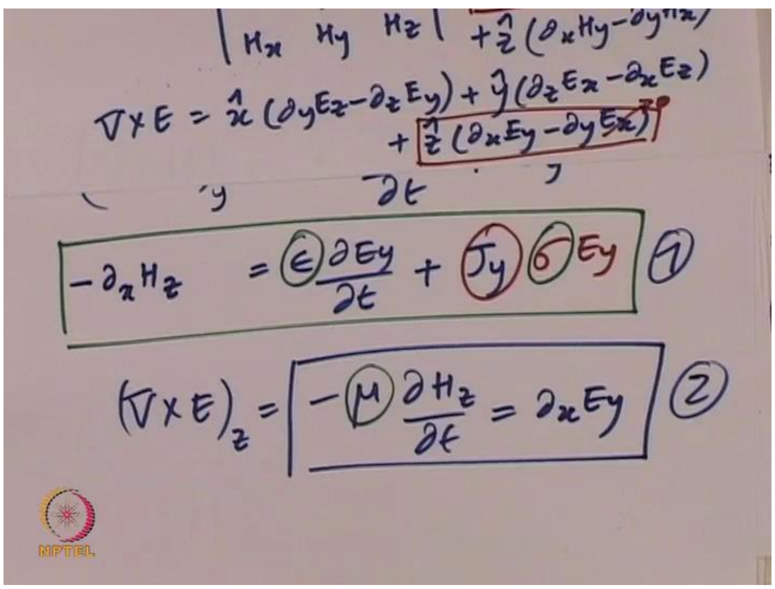
Similarly let us write down the second curl equation; curl of E is equal to minus Mu Doe H z by Doe t. And I am interested in the z component.

(Refer Slide Time: 07:50)



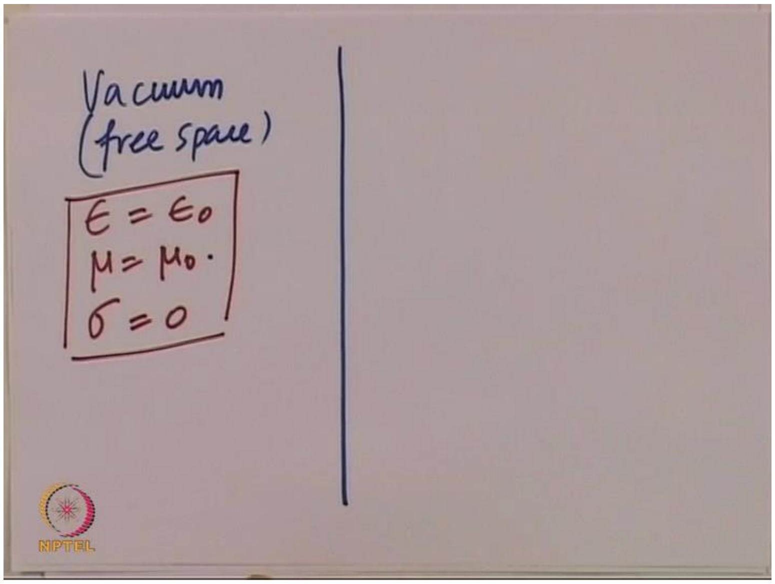
Let us go back into the equation and we see that we have this particular equation. And we know this term E x term is equal to 0.

(Refer Slide Time: 08:17)



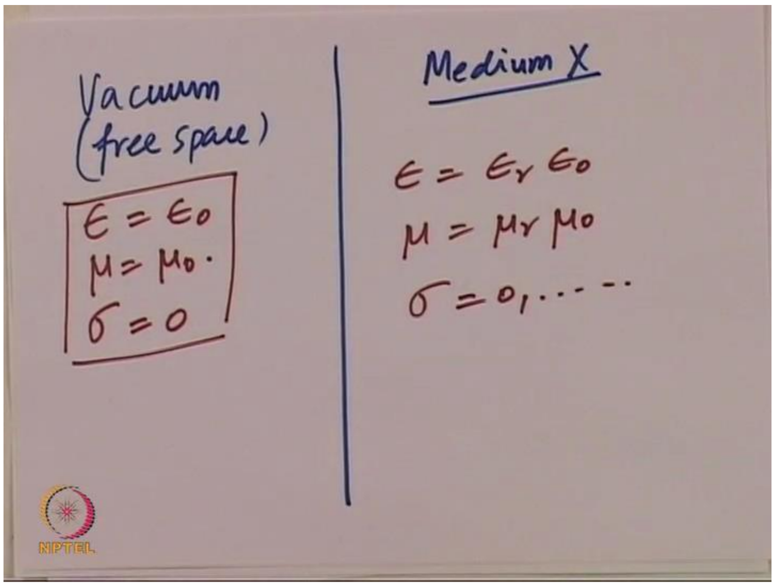
So we will have only the first term which we can write directly into our one dimensional equation. That is equal to Doe x by E y. So this is going to be the second equation. So this is the first curl equation this is the second curl equation. And in this second curl and first curl equation what you have is those important parameter of permittivity, the parameter of permeability, the parameter of conductivity. We are going to vary all these parameters and we are going to see a simple simulation that is going to help us understand the way the electromagnetic field is going to behave when it is entering into a different medium from free space.

(Refer Slide Time: 08:54)



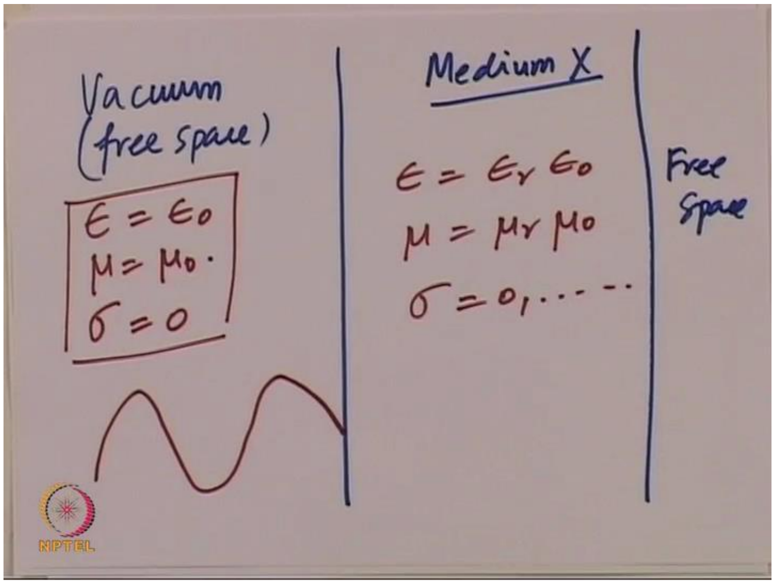
So let us say this is the vacuum free space and we have certain values of permittivity, permeability so on and so forth. So Epsilon is going to be Epsilon 0, Mu is equal to Mu 0and Sigma is equal to 0. So these are the free space counter part

(Refer Slide Time: 09:28)



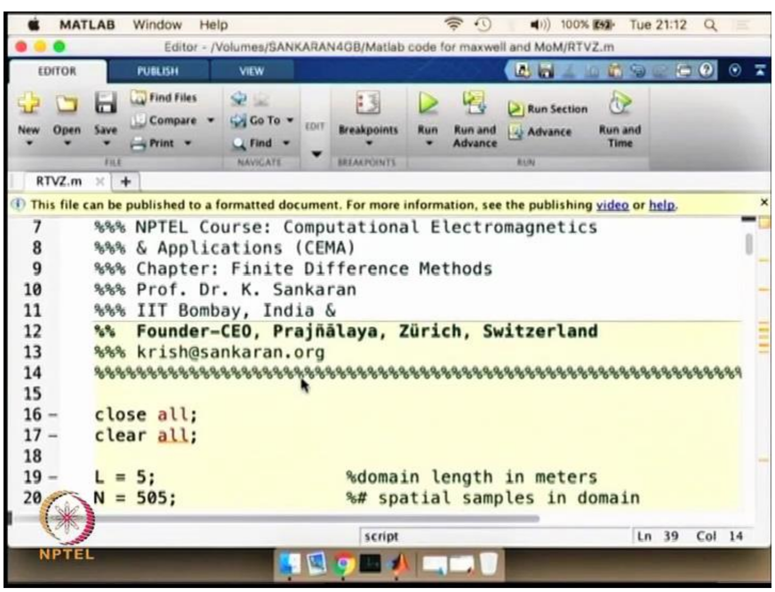
So now we are going to enter a medium, Let us say medium with certain parameters that are going to change. So this is going to be medium X. And the value of Epsilon is going to be Epsilon r multiplied by Epsilon 0. And Mu is going to be Mu r multiplied by Mu 0. And Sigma is going to vary so it can any value, so it can be 0 So on and So forth. And we are going to simulate this particular medium.

(Refer Slide Time: 10:13)



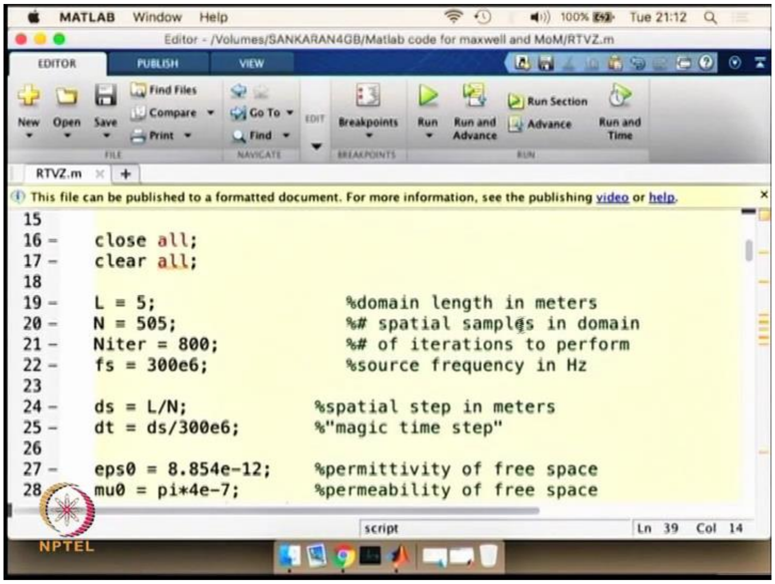
And what we are interested in knowing is when the wave is going from free space to this particular medium we wanted to see how this medium is going to affect and of course we can think of a scenario where the medium is going to be a bounded medium and we are going to go into the free space here. So we can see what is happening in the in between area so as to get a good sense of the impact of permittivity permeability and Sigma on the wave propagation. So let us go into the code and take a look at various parameters that are going to affect the propagation.

(Refer Slide Time: 10:57)



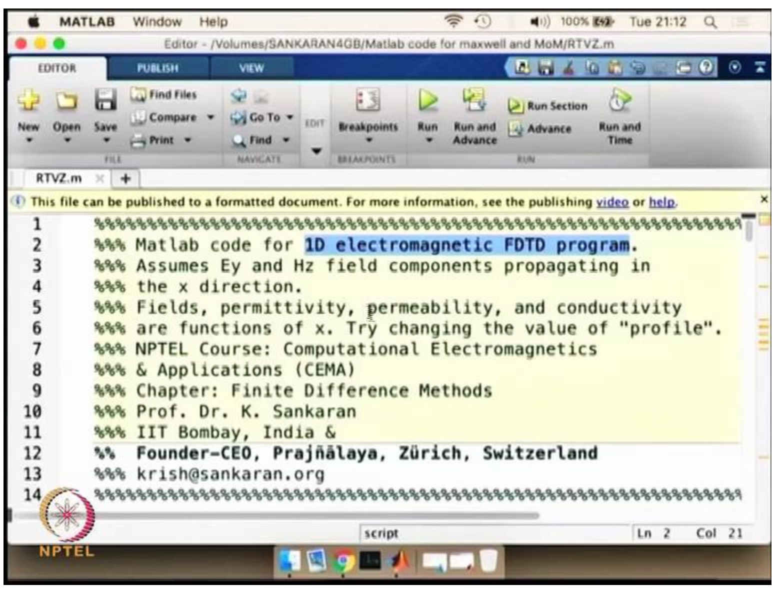
So this is the code what we are going to use for this particular problem.

(Refer Slide Time: 11:08)



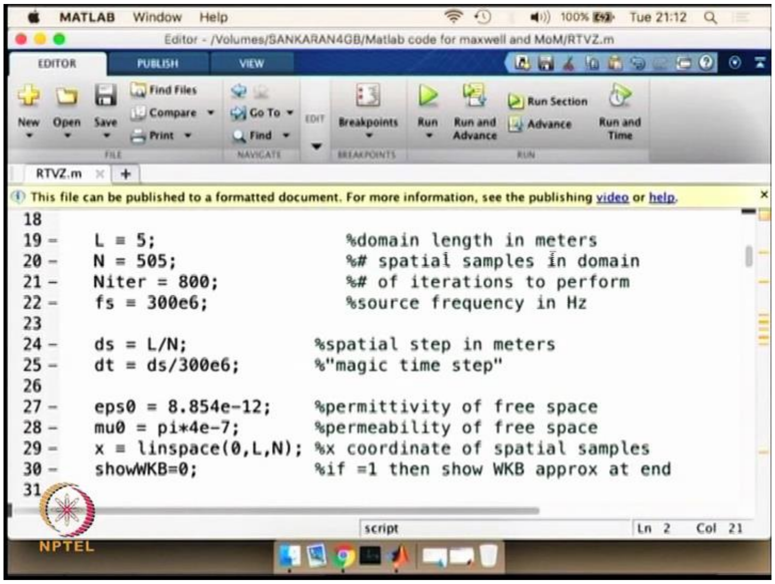
And we have set the domain length to be 5 and we are interested in discretizing this particular domain using finite difference time domain method.

(Refer Slide Time: 11:22)



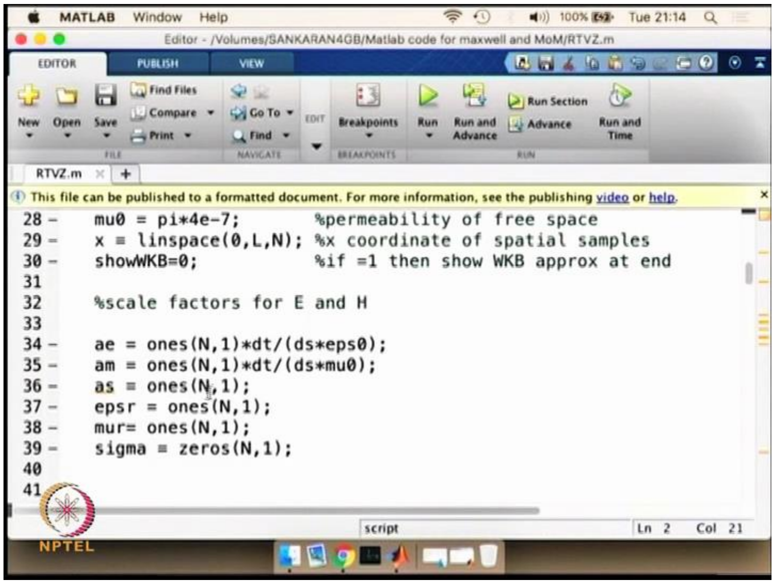
So our code is going to be aFDTD code but in one dimension.

(Refer Slide Time: 11:28)



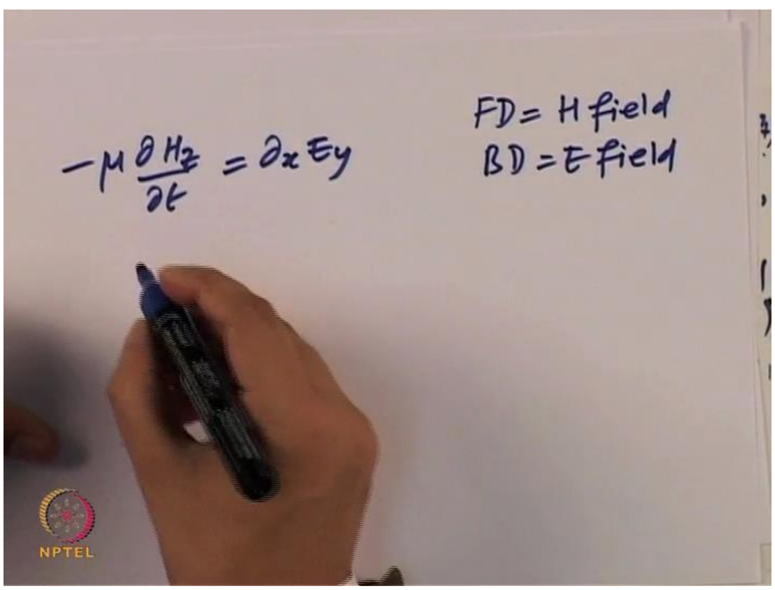
The spatial samples in this one dimension of 5 meter is going to be 505 samples. Of course by changing this I can change the accuracy of my method and my simulation time is going to be 800 steps and the source frequency. The frequency of the incoming wave is going to be 300 Mega Hertz. The spatial steps in meters is given by ds that is going to be L divided by N.So this is going to be 5 divided by 505 and dt is going to be the value of the spatial step divided by frequency. And Epsilon 0 is going to be the value of the free space permittivity which is 8.854 10 power minus 12. The free space permeability is going to be 4Pi multiplied by 10 to the power minus 7. And we are going to create a value for x which is basically the space of the simulation which is going to be from 0 to n

(Refer Slide Time: 12:45)



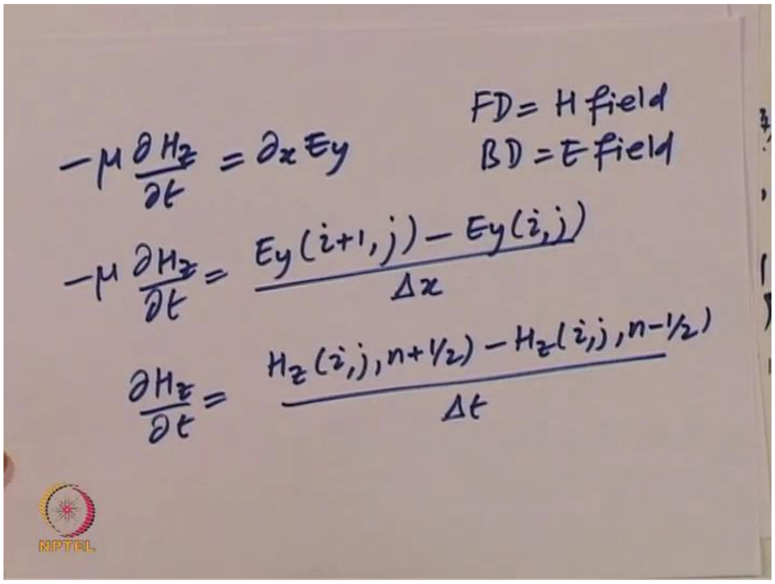
And these are the scaling factors that we are going to use for our problem so ae,am and as are going to be the terms that are going to be used in the update equation. And they follow the standard discretization where we have dt, ds and Epsilon 0 depending on the equation what we are interested in these terms are going to change.

(Refer Slide Time: 13:41)



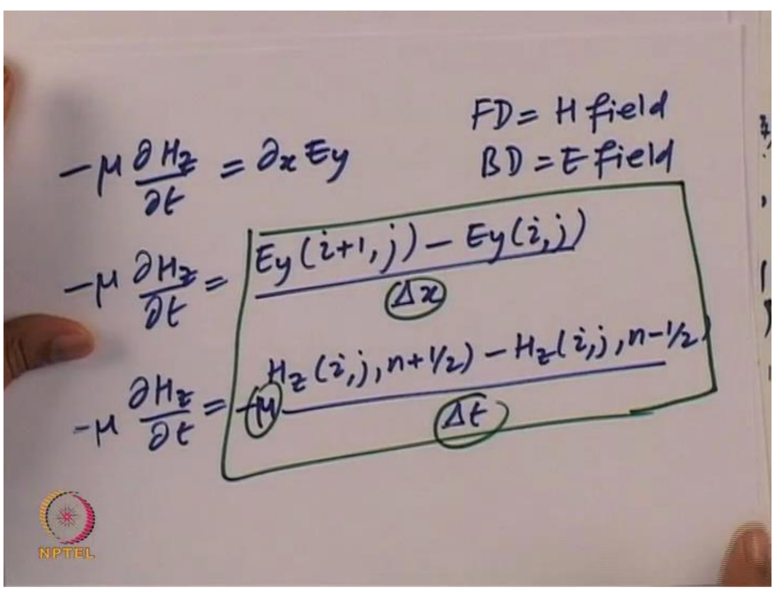
So let us look into one simple case where so the update equation here is let us take the second curl equation we have minus Mu Doe H z by Doe t equal to Doe x E y. And if you are doing forward differencing for the problem for one of the field and backward differencing for one of the field. So we will do forward differencing for H field and we will do backward differencing for the E field.

(Refer Slide Time: 13:55)



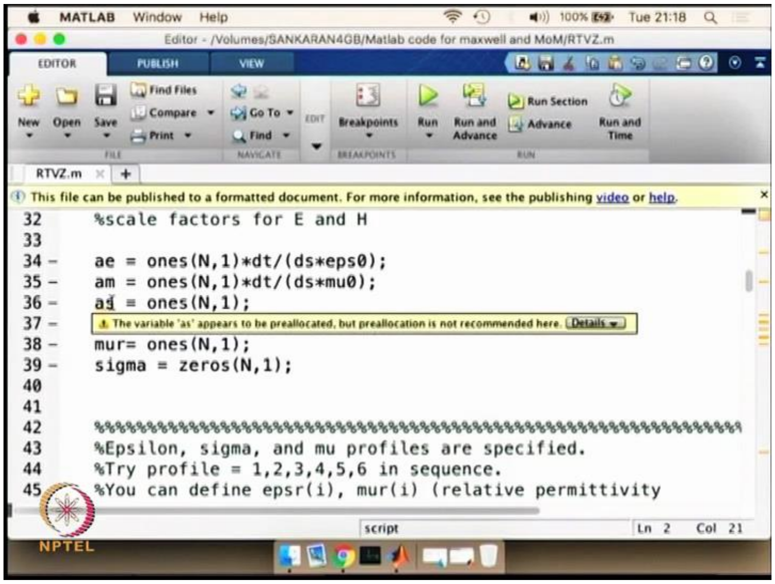
So in this case for the H field we will do forward differencing. So what we have is minus Mu Doe H z by Doe t is written as E y (i plus 1, j minus E y (i, j) divided by delta x and the time stepping is going to be given by Doe H z by Doe t can be written as H z (i, j, n plus 1 by 2) minus H z i, j, n minus1 by 2 divided by delta t. So we are doing half time stepping leave frogging for H field and E field in the time stepping and we are doing forward differencing and backward differencing for the spatial domain as we have explained here.

(Refer Slide Time: 15:25)



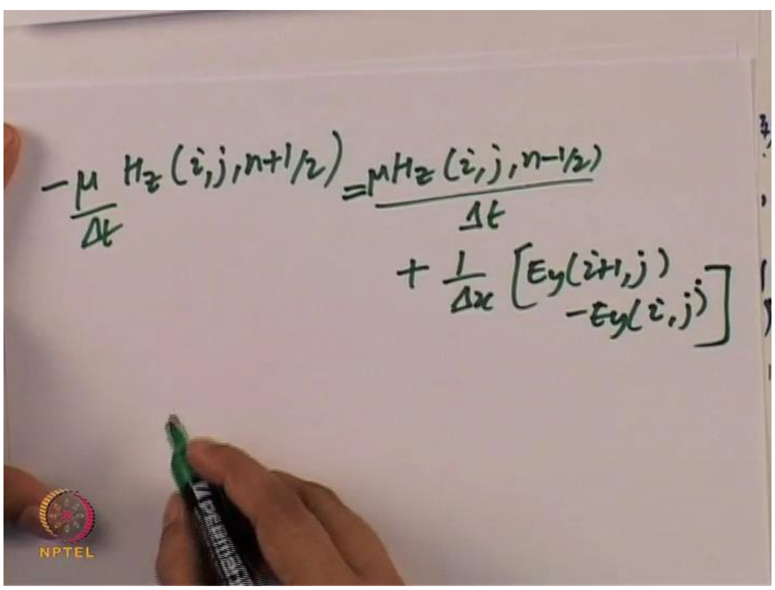
So this particular equation now if I have to multiply it with minus Mu, this will be minus Mu. So this will be one of the curl equations. And these terms that are going to be here are going to be the term that we are going to use in the ae, am and as.

(Refer Slide Time: 15:57)



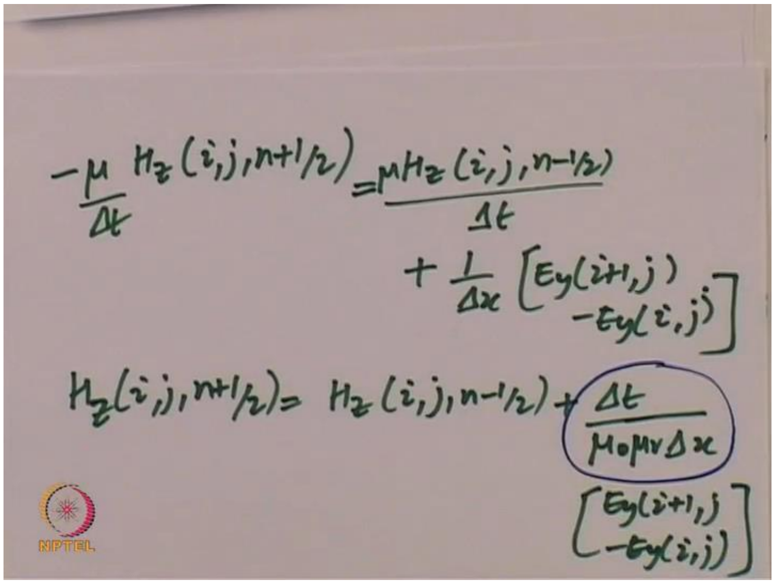
So if we expand this one little bit, what we will get is the update equation for H z at n plus 1 by 2.

(Refer Slide Time: 16:05)



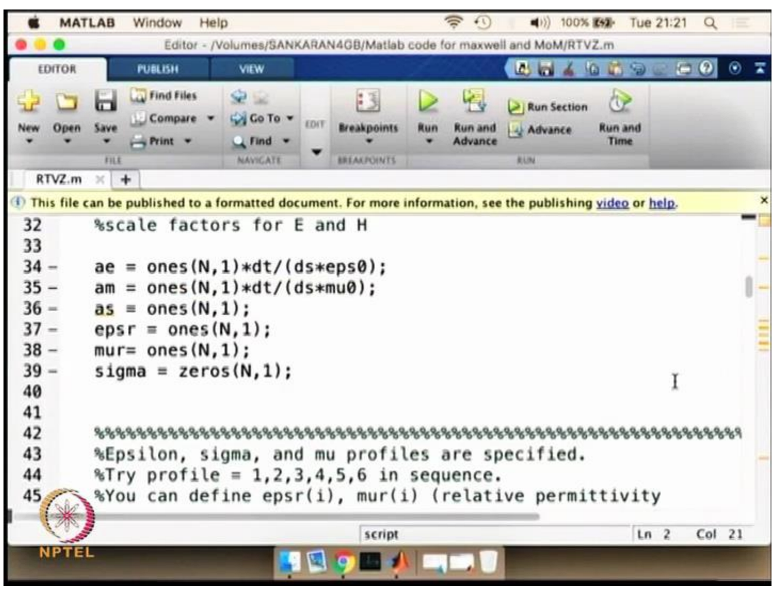
So let us write it down. We have minus Mu H z i,j, n plus 1 by 2 divided by delta t is equal to Mu multiplied by Hz(i,j, n minus 1 by 2) divided by delta t plus 1 by delta x multiplied by [E y(i plus 1,j) minus E y(i,j)]. And if I take the Mu divided by delta t on the other side what I will get is.

(Refer Slide Time: 17:09)



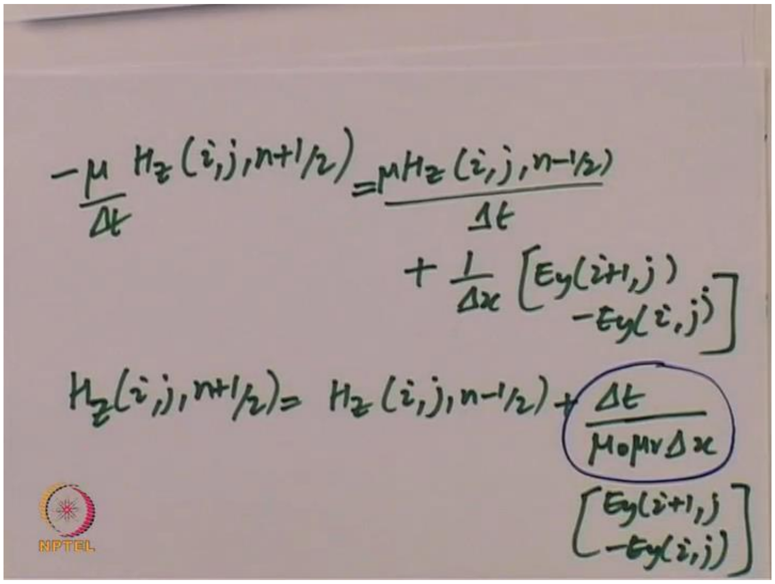
H z (i, j, n plus 1 by 2) is equal to so delta t by Mu will go away. So this will be H z (i,j, n minus 1 by 2) plus delta t divided by Mu 0Mu rmultiplied by deltax and then the curl term will be here. So [E y (i plus 1, j) minus E y (i, j)]. And this is what you see as a term that we are going to use in these equations.

(Refer Slide Time: 18:10)



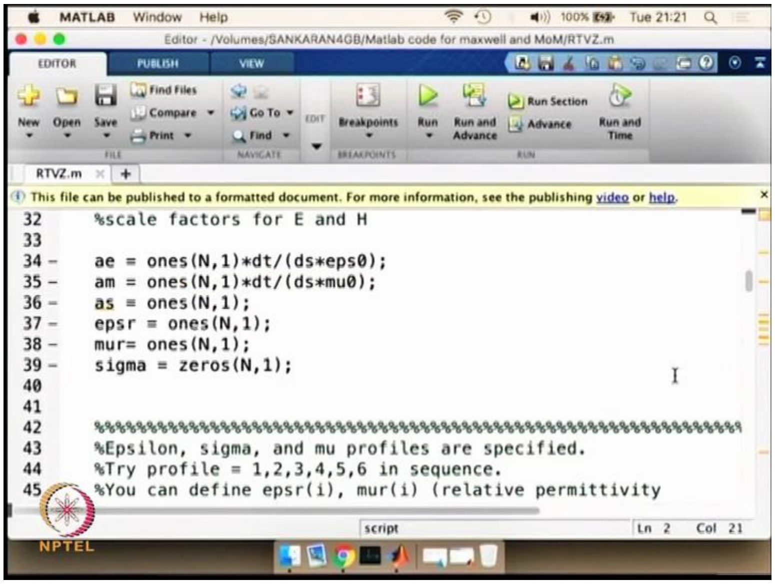
So for the am this is the magnetic term that you have delta t divided by delta s Mu 0.

(Refer Slide Time: 18:25)



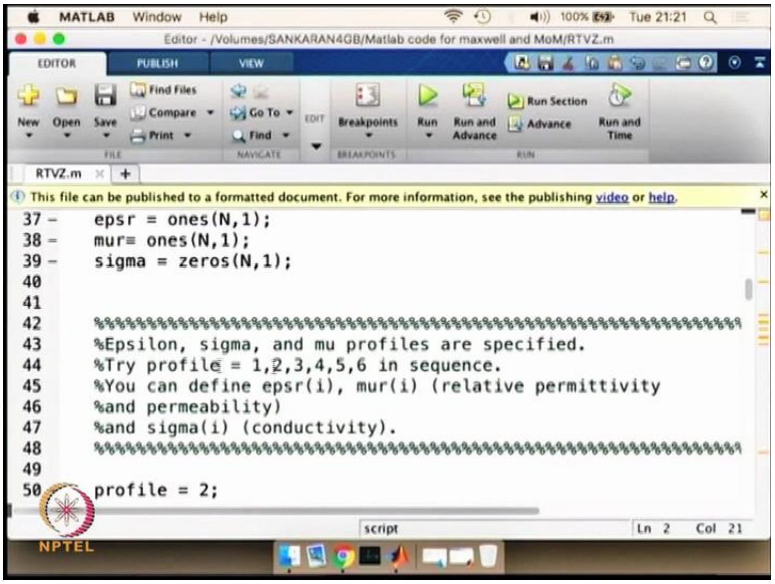
As you can see here in the paper, this is the term that is the am term. Of course we have used Mu r here we have directly taken Mu 0 and delta x is the ds term here.

(Refer Slide Time: 18:37)



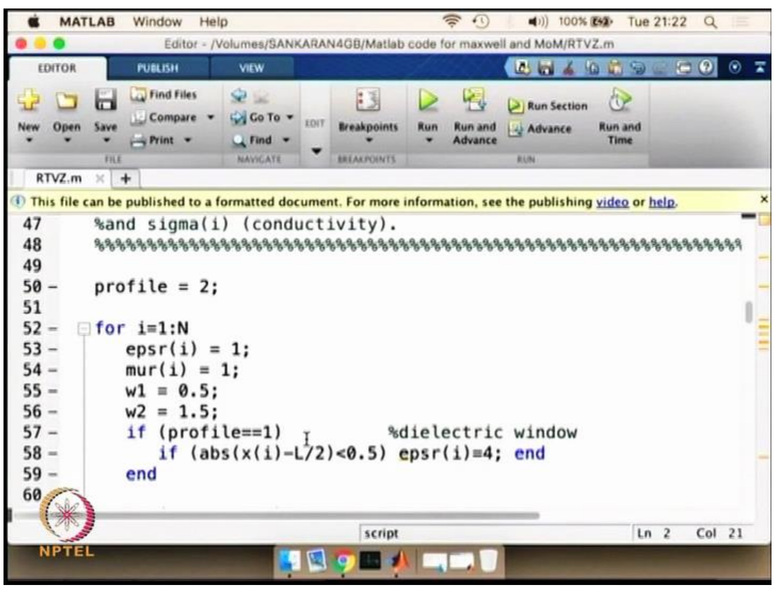
Similarly you can also derive the way for ae which is the electric term which will have Epsilon instead of Mu. And the other ones are 0 Mu r Epsilon r they are all kept to 1. Because we are talking about free space and Sigma is going to be 0 as well.

(Refer Slide Time: 19:00)



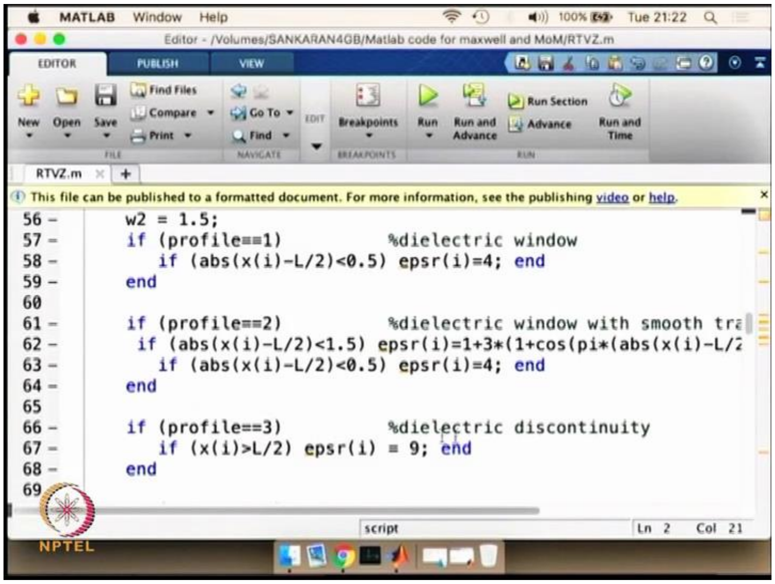
And we can change the value of Epsilon, sigma, Mu profiles as specified here. You can try different profiles for example we have tried a profile for 2.

(Refer Slide Time: 19:21)



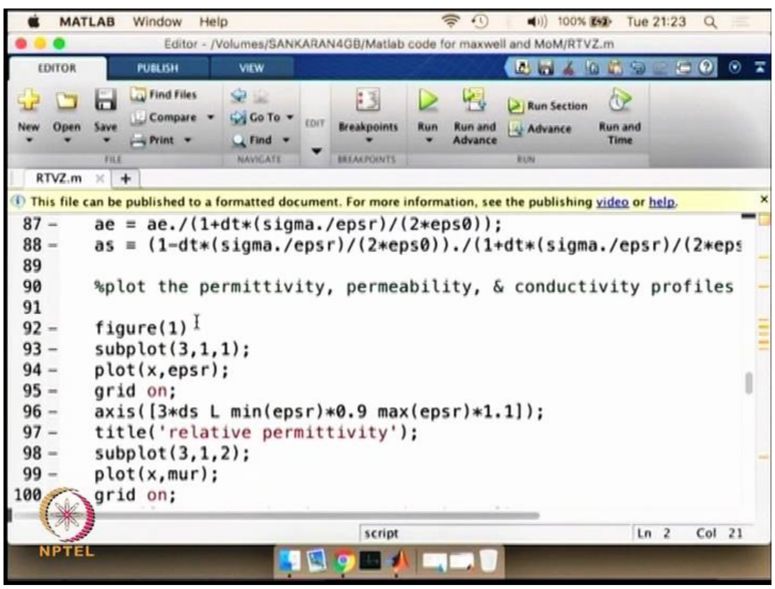
And if the profile is 2 the value of the Epsilon r Mu rinside the medium is going to change.

(Refer Slide Time: 19:24)



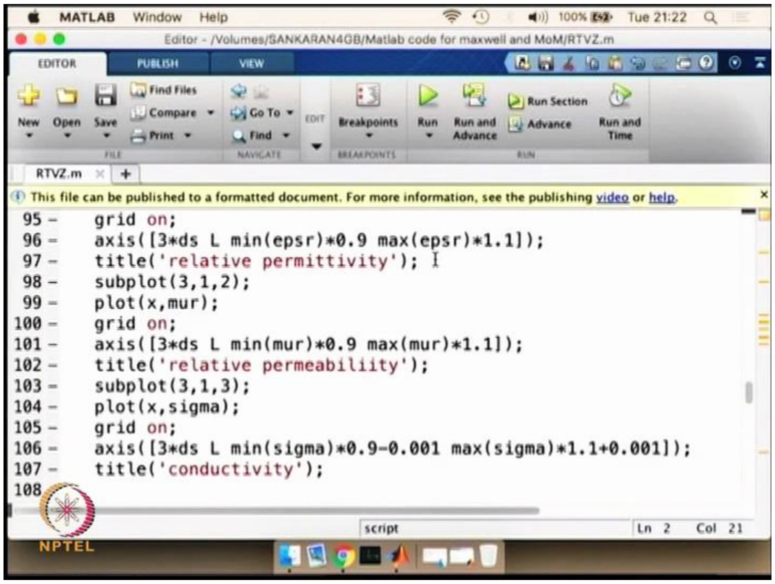
And the dielectric constants are going to also change.And the window is going to have different discontinuity. So we are going to use the value profile is equal to 2 initially and then we are going to see for other profile as well.

(Refer Slide Time: 19:40)



And we are plotting here is the permittivity, permeability, conductivity profiles.

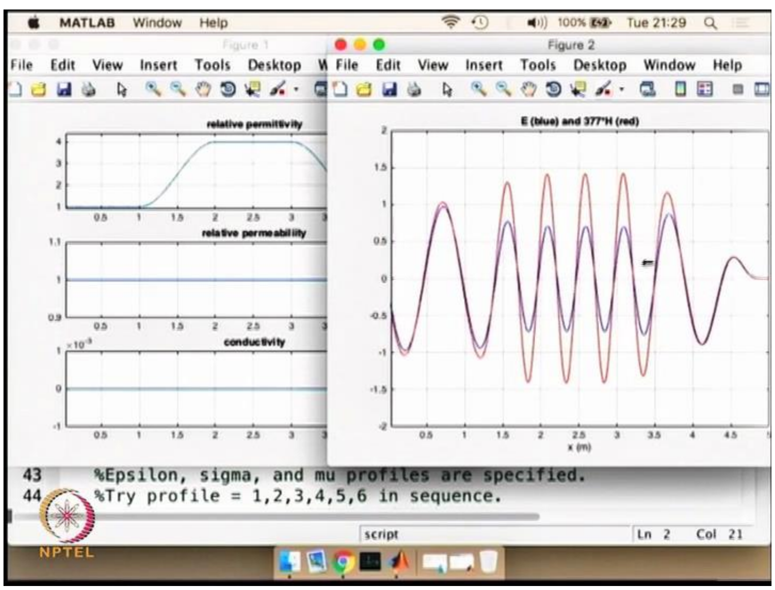
(Refer Slide Time: 19:47)



Apart from that we are also plotting the propagation of the wave itself into the new medium.

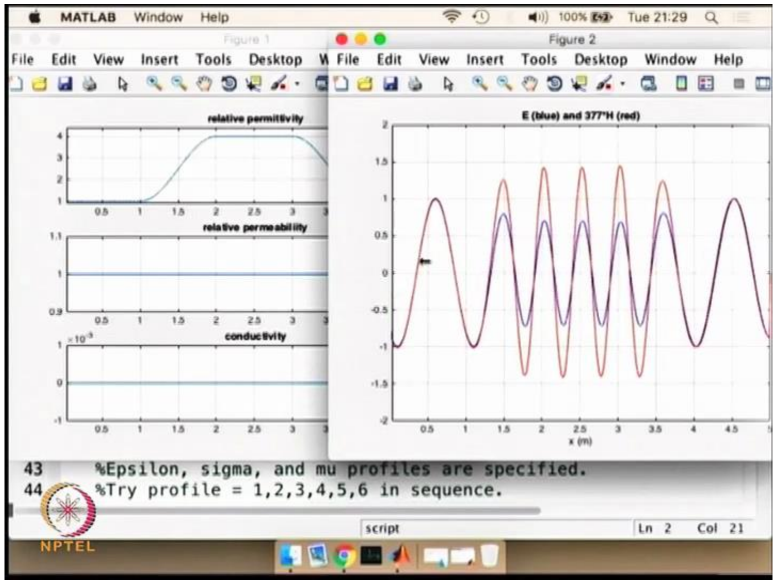
So let us start simulating this problem for a simple case and see how this particular setup is reflected in the physical movement of the wave inside a new medium. Basically looking at those various parameters we can simulate this problem.

(Refer Slide Time: 20:14)



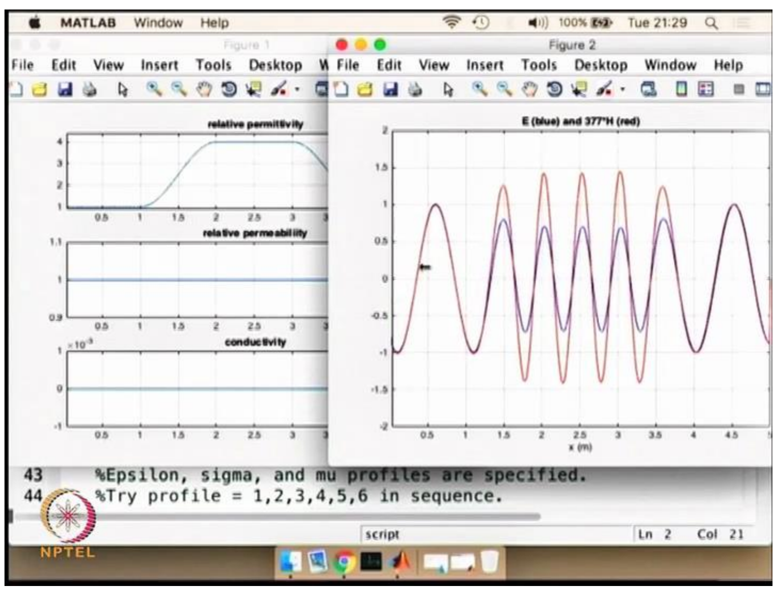
So let us start the simulation. So what we are going to see is two things. On the fig 2 we are going to see the way the wave is going to propagate.

(Refer Slide Time: 20:32)



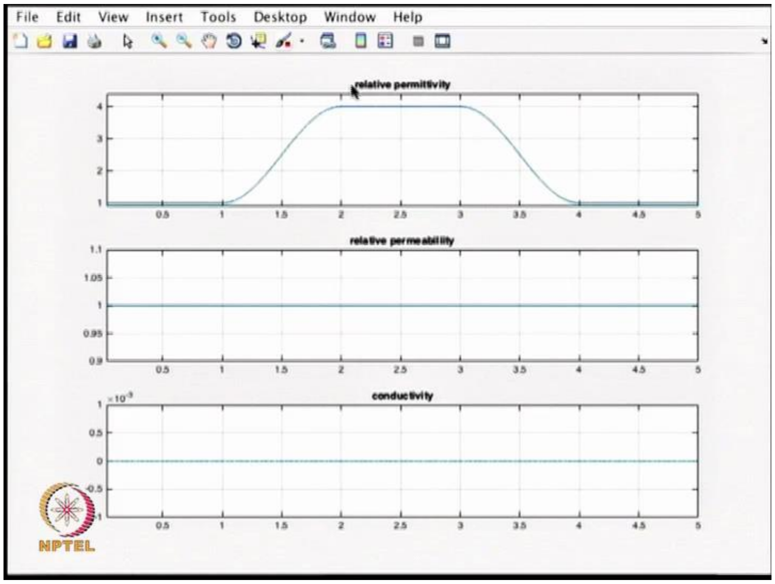
The relative permittivity and the relative permeability values are here. What we are seeing is when the relative permittivity is having certain profile as we have set here. What we see is some kind of concentration of the incoming wave.

(Refer Slide Time: 20:54)



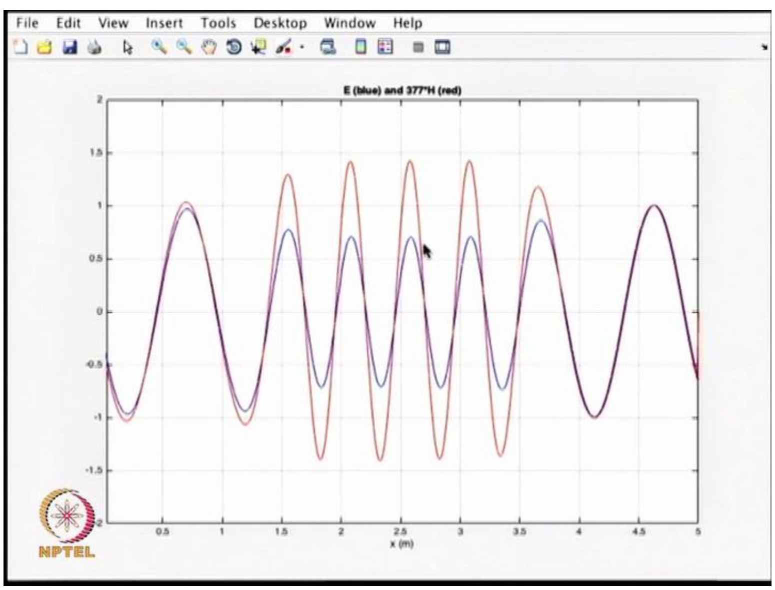
So what we see is let us simulate it again. We are going to plot both the E field and H field. And as you can see the wave is propagating the E field and H field values are given in blue and red E is in blue and H is in red. And the wave is slowing inside the medium x. And there is a kind of a focusing aspect the wave is getting compressed. And you see that the H field is increasing in magnitude. Whereas the E field is decreasing in magnitude. And there is a kind of a slowing down of the wave. And you see that as it comes out it emerges out in the way it comes in. So this is a classical example for seeing the effect of relative permittivity

(Refer Slide Time: 22:00)



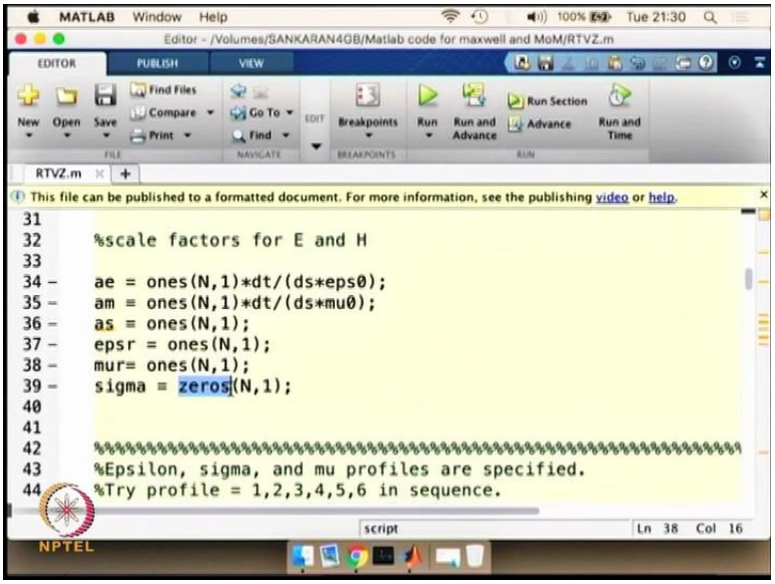
So the relative permittivity is going to be in this form and the wave when it comes inside it is getting slow down the electric field amplitude is getting increased whereas the magnetic field value is getting reduced.

(Refer Slide Time: 22:15)



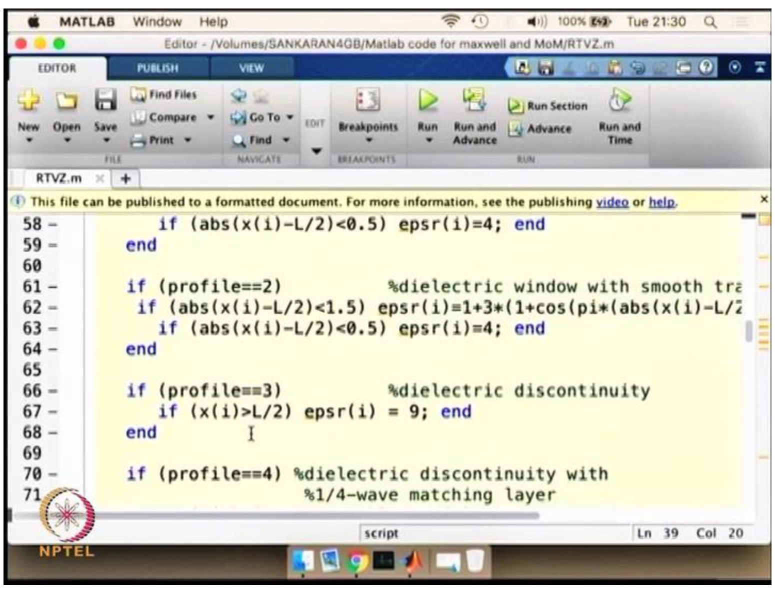
And so what we see in the case of the medium is the magnetic field value is increasing whereas the electric field value is decreasing in its amplitude. And the wave is getting slowed down and it is getting compressed. The wave length is changing and the amplitude is changing. So this is the way we see the impact of the medium.

(Refer Slide Time: 22:40)



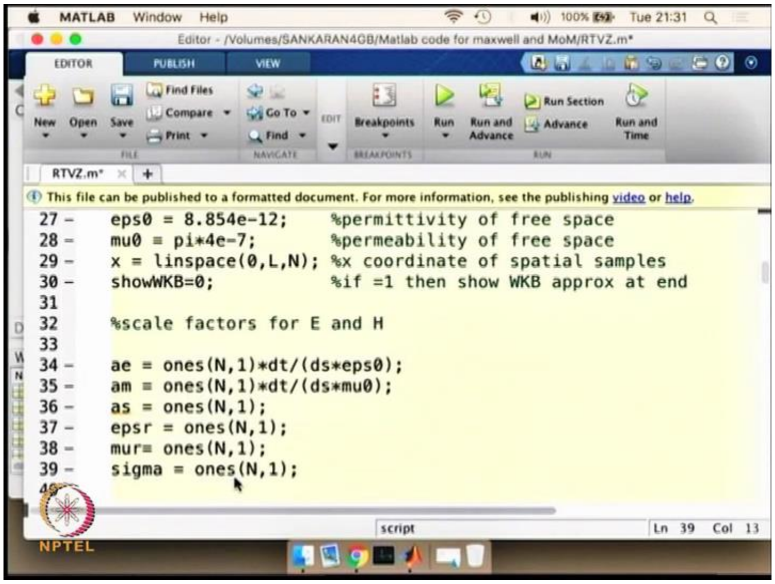
And now what we are going to do is we are going to change the value of sigma. So let us say the sigma value is going to change inside the medium that we are going to simulate.

(Refer Slide Time: 22:55)



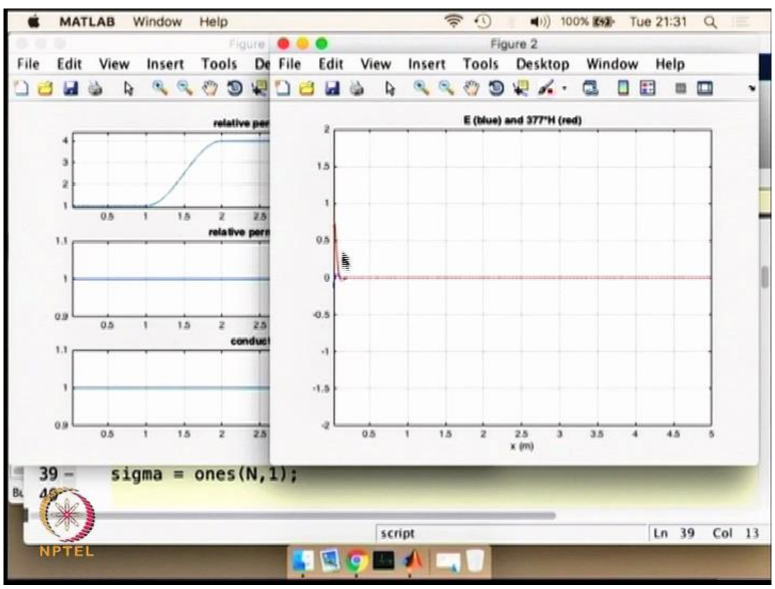
And if the profile value is going to be 2.We have set the value of Epsilon r so on and so forth.

(Refer Slide Time: 23.00)



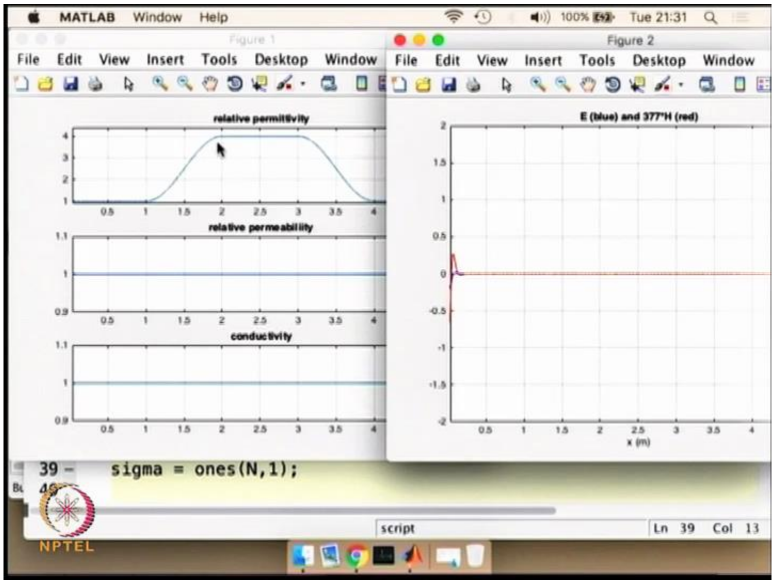
And let us say we are interested in setting certain value for sigma. So we put the value of sigma as 1.

(Refer Slide Time: 23:11)



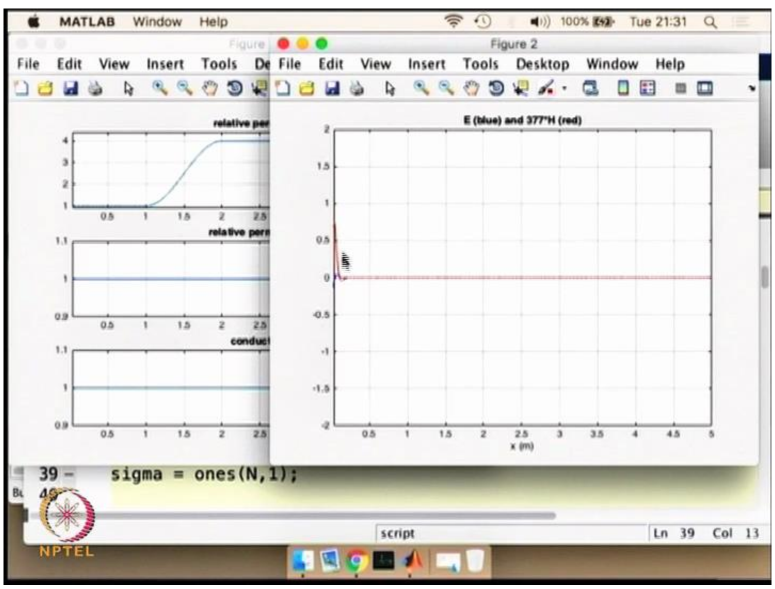
And we are trying to simulate this problem. What we are going to see is a very different kind of response. We see that the wave is trying to enter the medium. But it is not able to propagate It is getting reflected and you hardly see any impact here.

(Refer Slide Time: 23:36)



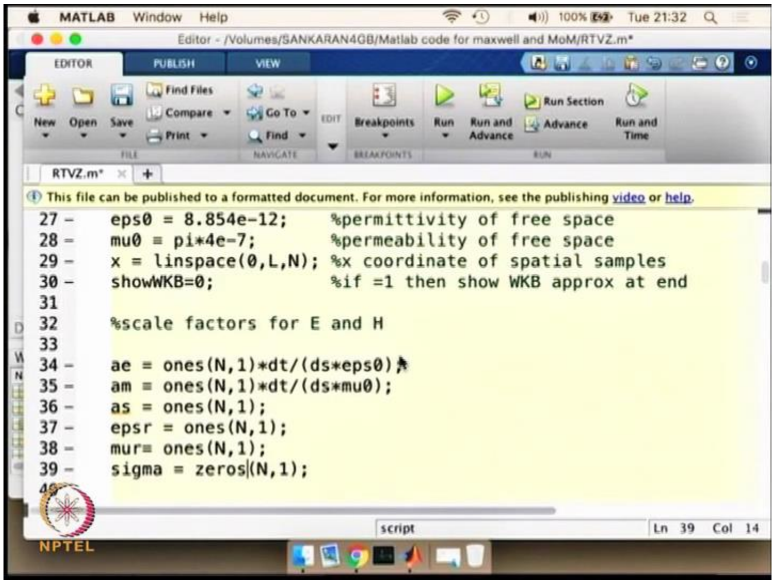
Whereas the conductivity is going to be seen in the way we have. Our relative permittivity has not changed. Our relative permeability has not changed. The only thing I have changed the conductivity from 0 to 1.

(Refer Slide Time: 23:50)



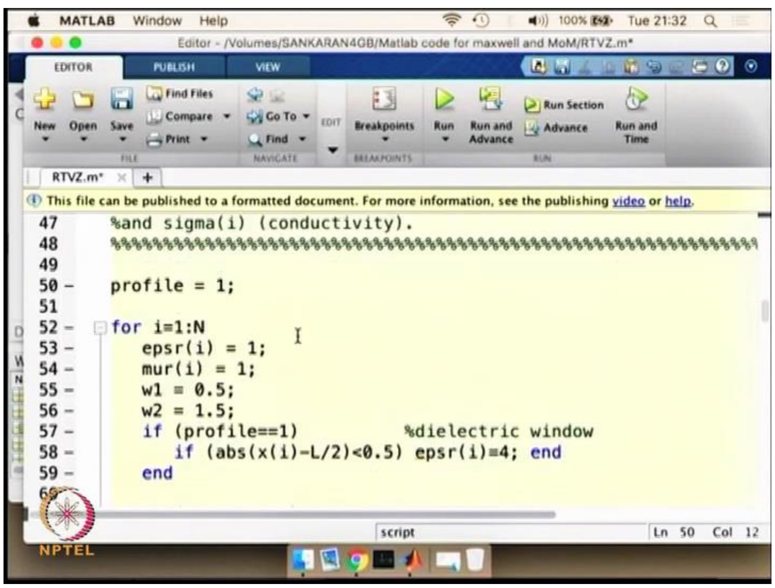
So in that case there is no propagation. There is a (()) (23:52) that is happening immediately and there is no propagation.

(Refer Slide Time: 24:01)



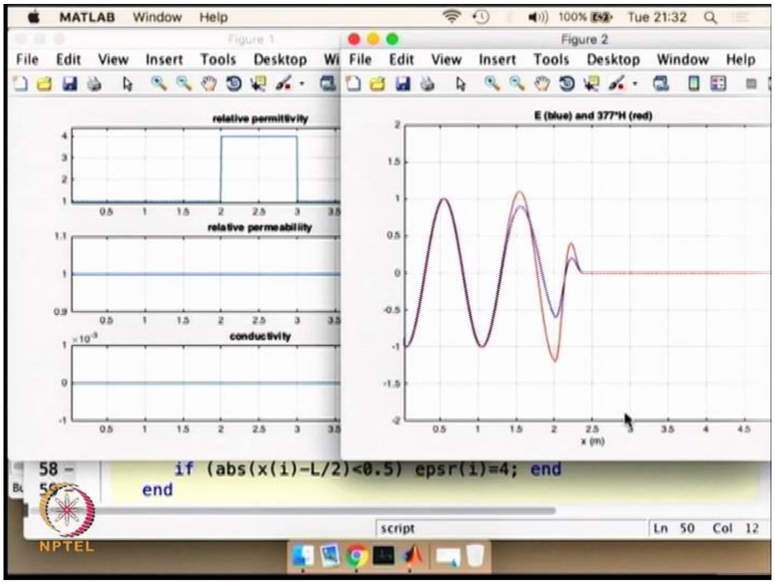
So we can now change the value of Sigma to be 0, like the way we had.

(Refer Slide Time: 24:10)



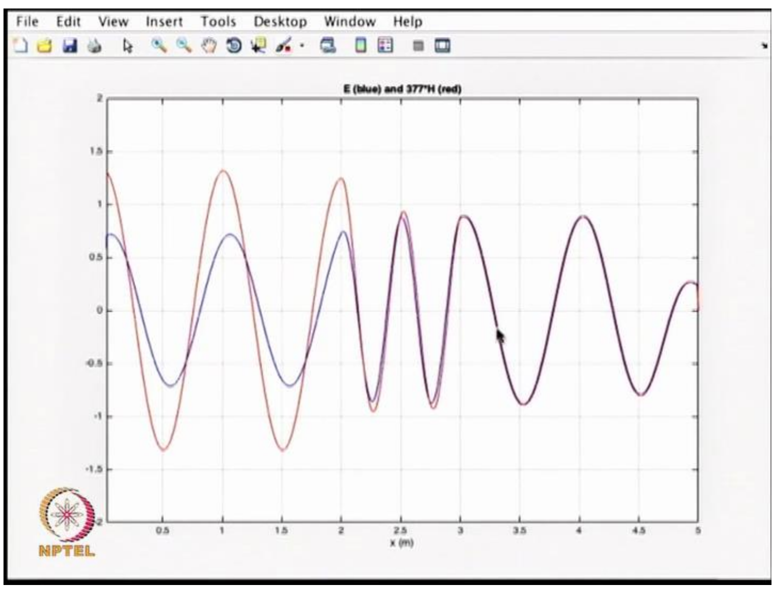
We now change the value of the profile itself let us say we are interested in profile that is 1.

(Refer Slide Time: 24:20)



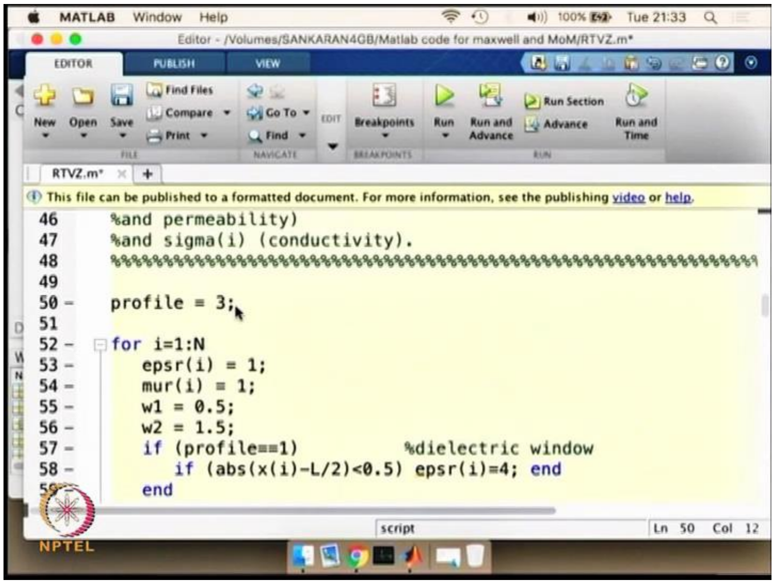
Let us run the simulation. Now the profile is a step function for relative permittivity and it starts at 2, it is a big discontinuity.

(Refer Slide Time: 24:46)



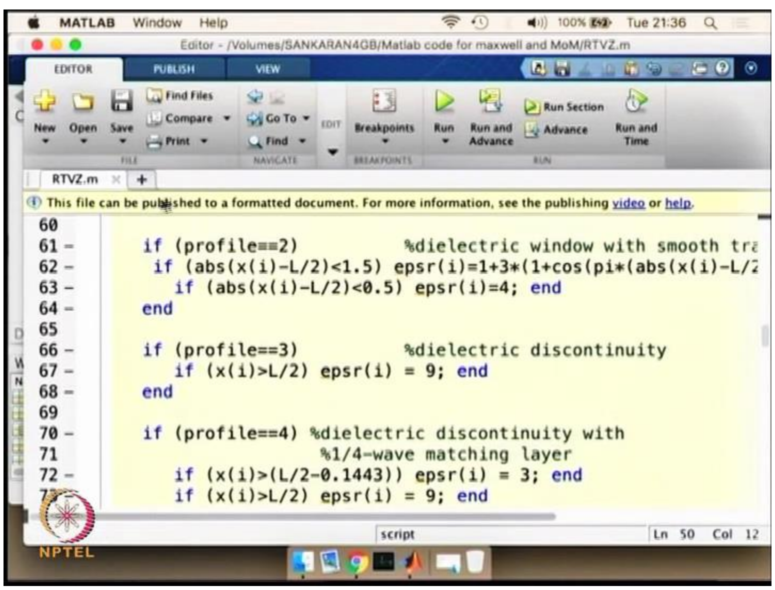
You can see the changes are quite obvious, when the wave is entering the electric field is decreasing its amplitude whereas the magnitude is increasing in its amplitude and the wavelength is changing. There is a kind of compression in the wave, and slowing down of the wave. And when it comes out it comes out in the same way.

(Refer Slide Time: 25:20)



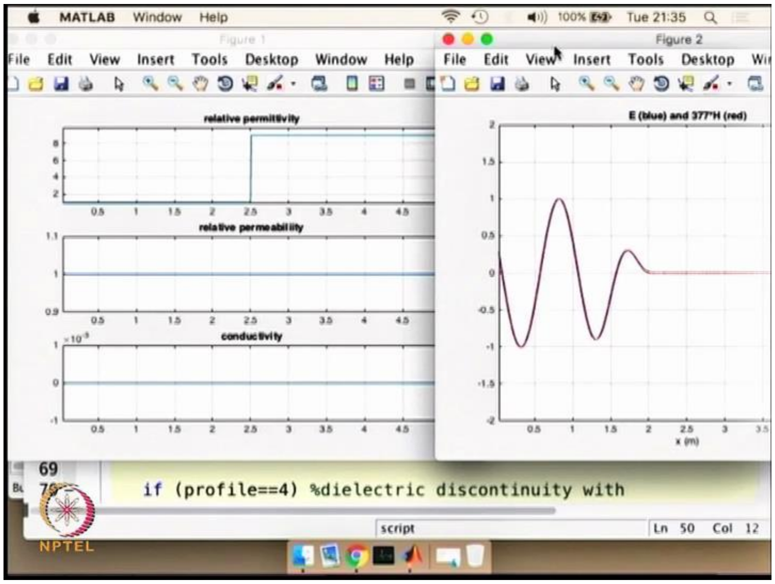
So now we can see the impact of a different profile. Maybe we will go for profile 3 a cubical profile.

(Refer Slide Time: 25:30)



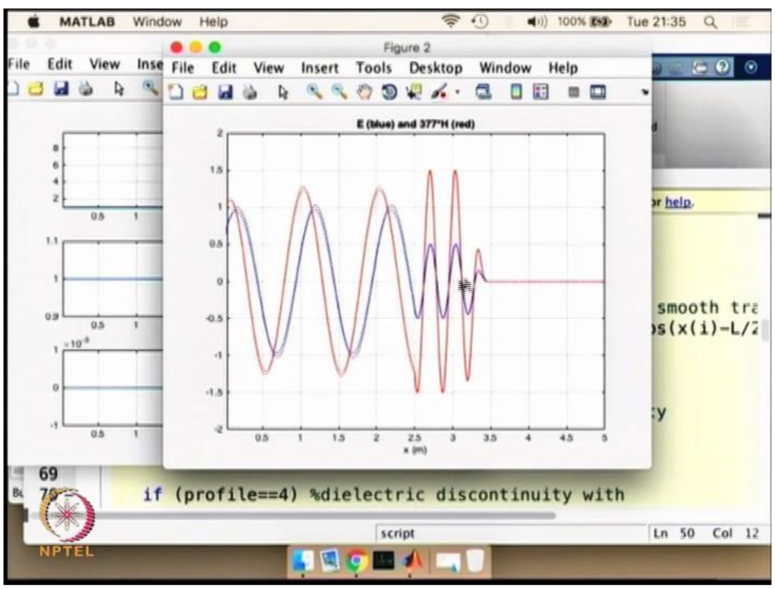
What we are seeing is a step change that is going to infinity.

(Refer Slide Time: 25:37)



So what I mean is the wave enters and stays inside this particular domain. So what you see is there is infinitely long step. So it is a step that is going forever.

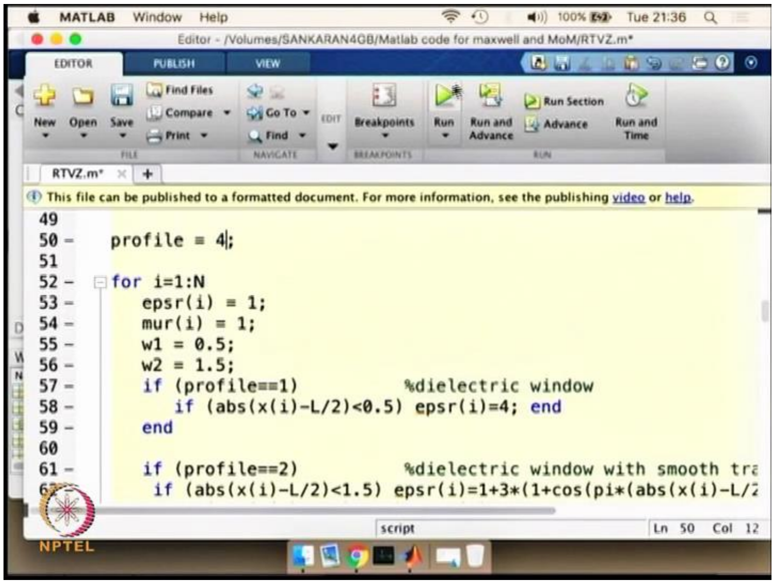
(Refer Slide Time: 25:58)



And the wave is going to stay inside the same medium x. And it is not going to emerge out in the free space. So what you see is there is a kind of a similar behaviour. The H field value is amplitude is increasing the E field amplitude is decreasing and the wave is slowing down as we expected. And it is going to slowly emerge into the medium x, but the wavelength is going to change or the wavelength is going to be reduced whereas the amplitude of H field and E field is also going to change.

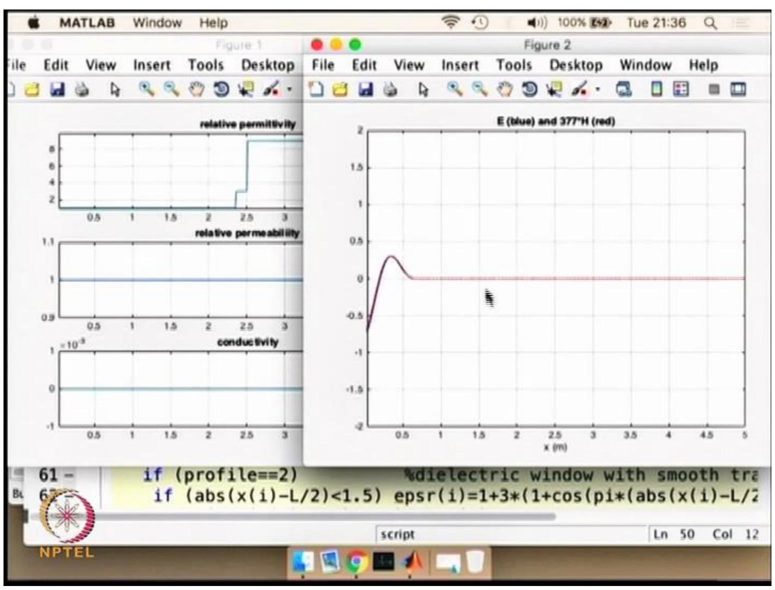
So this is a step function it is a abrupt step function and we can change this abrupt step function into a dielectric discontinuity with 1/4th wave matching layer. So that means there will be two steps instead of 1 step.

(Refer Slide Time: 26:54)



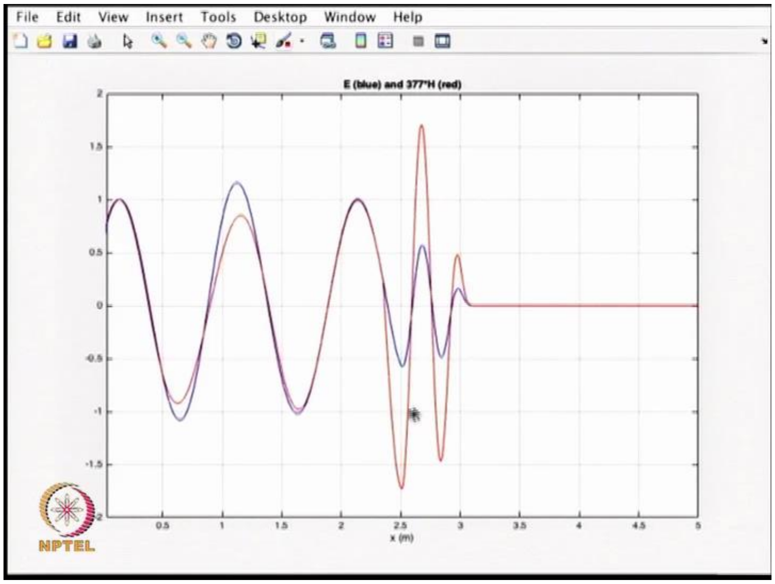
So if we put profile value equal to 4.

(Refer Slide Time: 26:58)



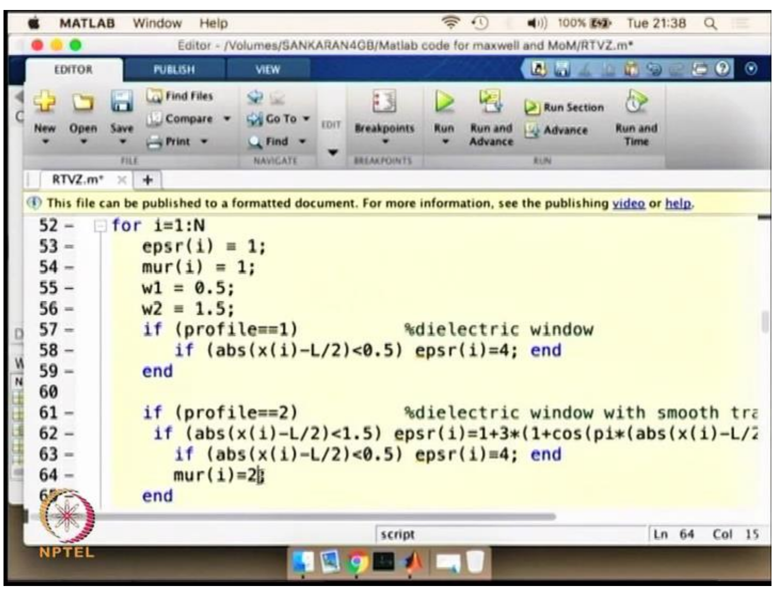
This is what we will see. What we see is the two step changes and it starts from 2.4 or something like that approximately here.

(Refer Slide Time: 27:16)



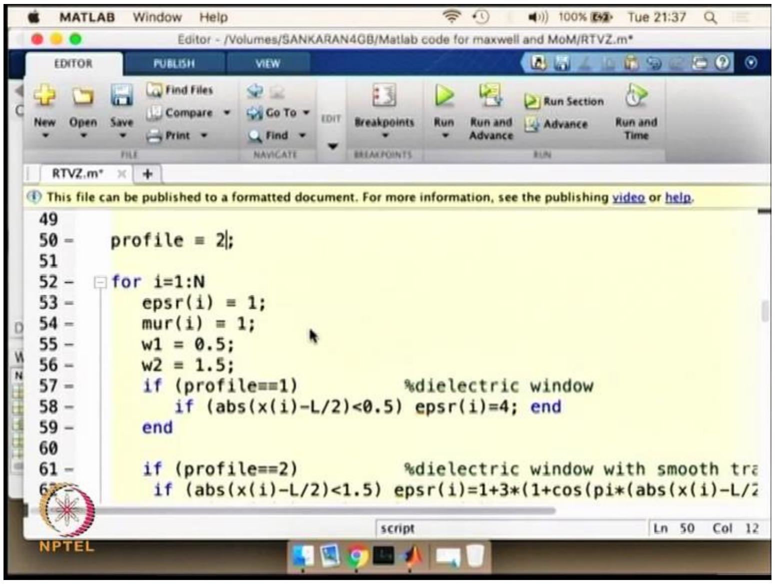
And we have to see some changes happening here and we are seeing that happening. And we have to see a new change happening at 2.5. So you see that two different types of impacts are happening within the medium x because there are two step discontinuity and this is creating the change as we expect. So what we have seen now is by changing the value of permittivity, permeability we can pretty much change the way the wave is going to behave inside that particular medium.

(Refer Slide Time: 27:55)



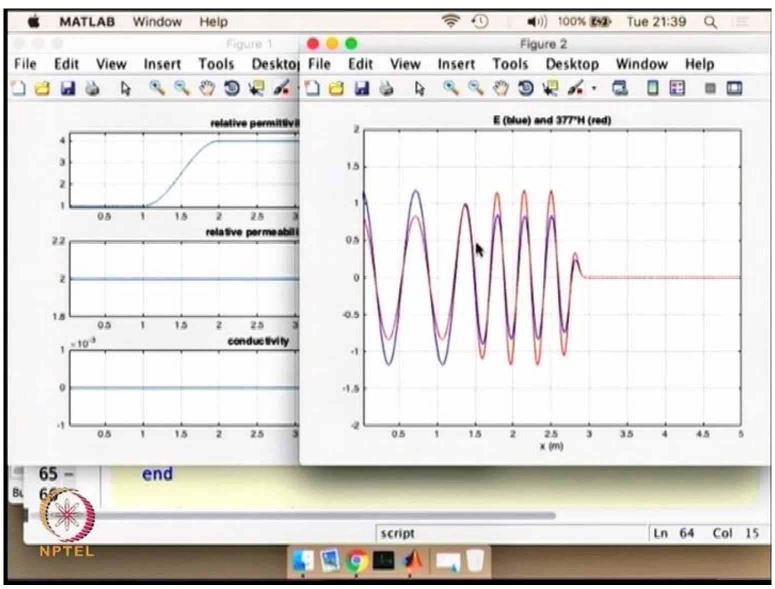
So now I have changed the value of Mu r is equal to 2.

(Refer Slide Time: 27:59)



And I am going to simulate the problem for profile equal to 2 so my Mu r will be now not 1 but 2and see what is the impact of that in the particular medium that we are going to simulate

(Refer Slide Time: 28:05)



Mu r is going to be 2 throughout the problem so it is not a free space problem here. So already coming with certain relative permeability. And the impact is going to be dramatic when it enters into the new medium. And as you can see the H field value is not changing as much as it changed when the Mu r value was 1. So that means there is definitely increase in the value of the h field but not as much as we saw before. And the decrease in the E field value is also not dramatic and we see the compression of the wavelength as we have shown in the previous case also. And the slowing down of the wave as we saw before.

So what I will encourage is to play around with this particular code, this is an excellent code for you to try out various things and change the code accordingly to see the impact of various parameters like the permittivity permeability.

So using this kind of code you can basically see various parameters impact on the wave propagation and one can learn how the electric field and magnetic field is going to respond to various Mu and Epsilon r values. And in other words how the electric fields and magnetic field is going to change in amplitude in terms of frequency so on and so forth and also the impact of conductivity inside a medium is also classically seen. Obviously the reflection here in this particular problem is quite low. Because we have truncated this problem using an a, b, c. While if you are doing a two dimensional or a three dimensional problem we have to truncate the domain using an perfectly matched layer which we will be doing at a later stage.

What we wanted to say now is please take the code, practice it for yourself test various parameters and experiment quite a bit so as to understand the various behaviors of electrical and magnetic field inside various mediums.