**Computational Electromagnetics and Applications**
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**Indian Institute of Technology Bombay**
**Lecture No. 8**
**Finite Difference Methods - III**

Where are we now we have done the finite difference time domain algorithm for Maxwell system. I said that we will look into the frequency domain approach also. Although this lecture my main focus will be on the time domain simulations for certain reasons. I will explain you why the time domain simulation is good. In which case we have to go for frequency domain simulations. But the entire lecture is about time domain simulations. But that being said we do not want to leave aside the frequency domain analysis as well.

Because in certain instances the frequency domain approach is quite good and we need them, so it is good to learn them. What is the basic logic behind it how do we do it. And in a more summarized manner although we will not do much with the simulations in the frequency domain. There might be here and there some simulations we might show, but the general domain approach will be on the time domain approach.

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Let us look at the frequency domain formulation of the Maxwell system.

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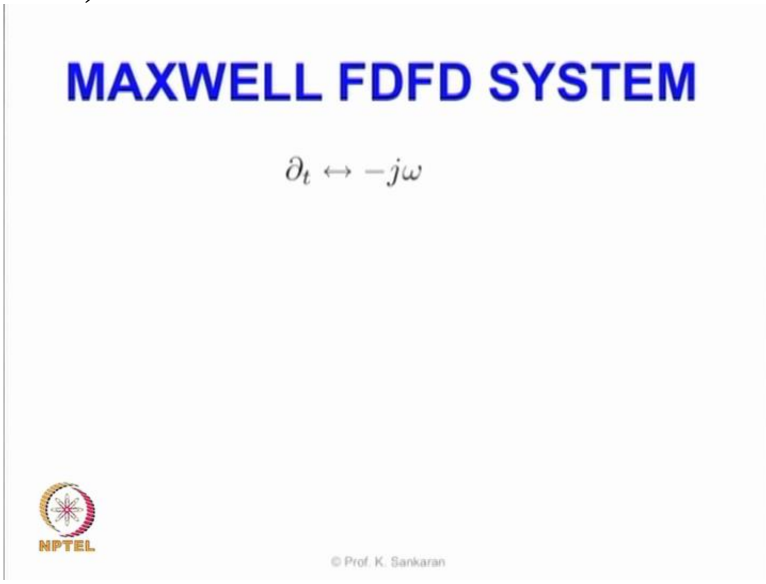


So the frequency domain simulation actually starts with the simple logic where our partial differentiation with respect to time is going to be equated to minus j omega where j is the square root of minus 1. It is a complex number and omega is going to be the 2 Pi f the frequency. Frequency is the value that we are of the wavelength that we are going to look into. And this frequency is going to be the frequency any particular input source is going to have.

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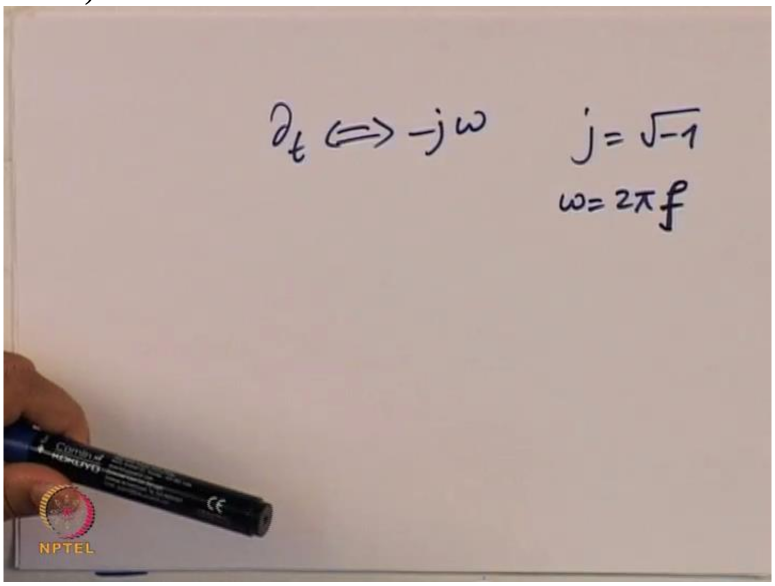


And with that being said we can basically write the Maxwell equation, the two curl equations where we have replaced the minus time derivative with respect to time on the one of the curl equation with j omega and also the plus time derivative with respect to E which is here as minus j omega. And it is a general equation so you also have the current density term. So when you take the Curl.

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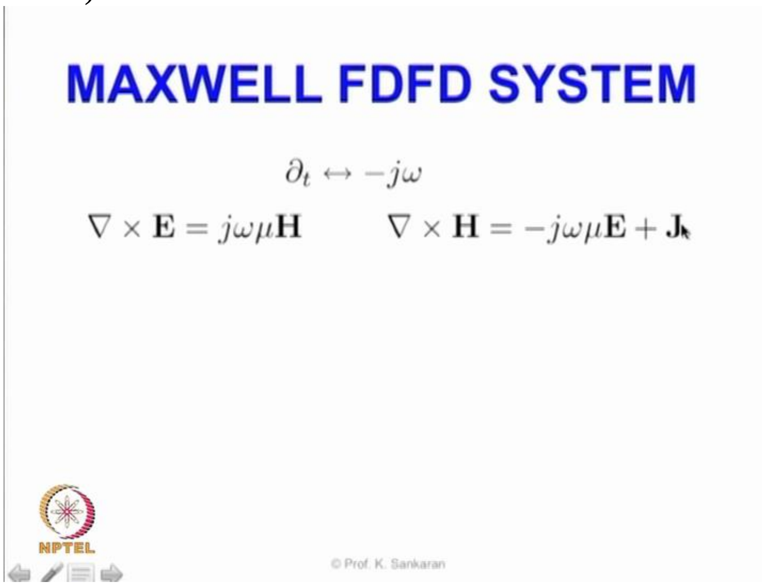


For example I have the equation given by the first equation and I take the Curl of the first equation. And what I essentially get is the (Curl of E) is equal to j omega Mu (Curl of H) and then I can substitute the value of Curl of H in this equation and I can only get an equation which is represented in the form of E or H.

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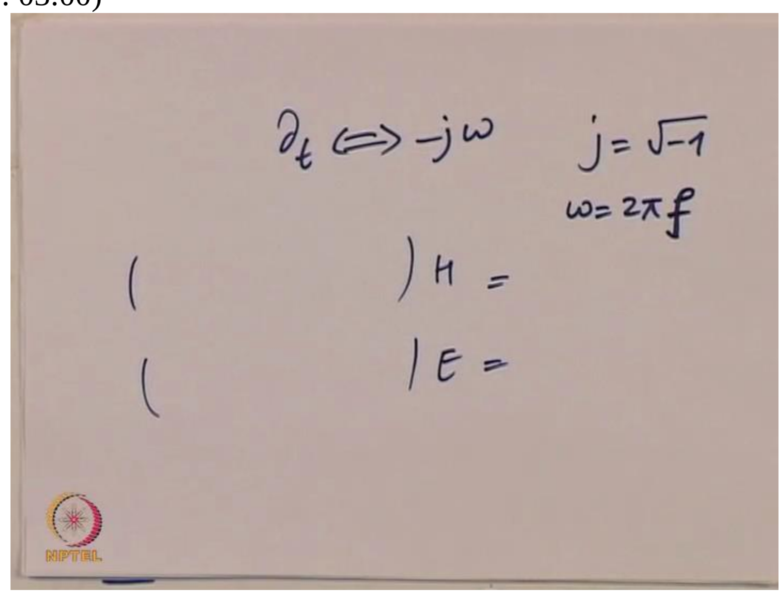

So for example what I am doing here is I wanted to write an equation which is only as certain function times H is equal to certain value or only as a function of E as a function of certain value. So when I wanted to do that I do this trick.

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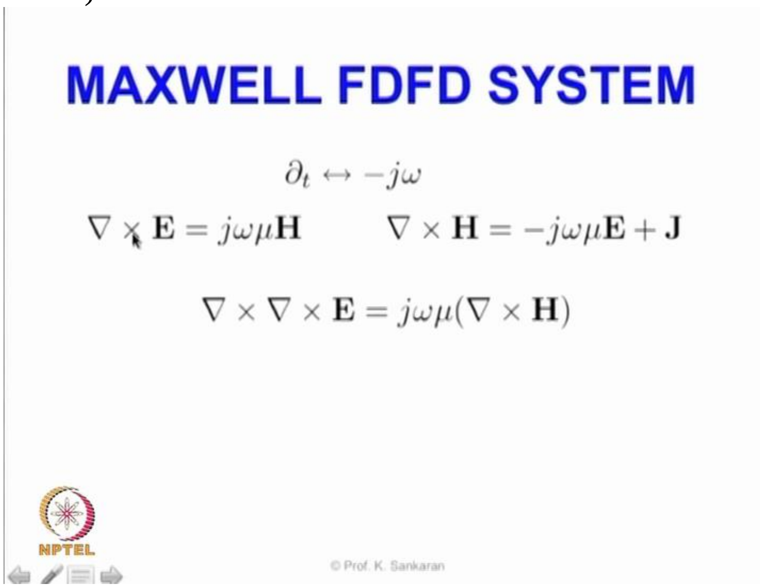


And this trick is what I have done here, I take the curl of curl of E times curl of H will be on the right hand side and I substitute it for the curl of H, I can get an expression that is only in terms of H or E. So let us say I am interested in the expression which is only in terms of H, so we take the curl of the curl of the equation what we have, so let us say I am interested in writing an equation that is only in terms of H.

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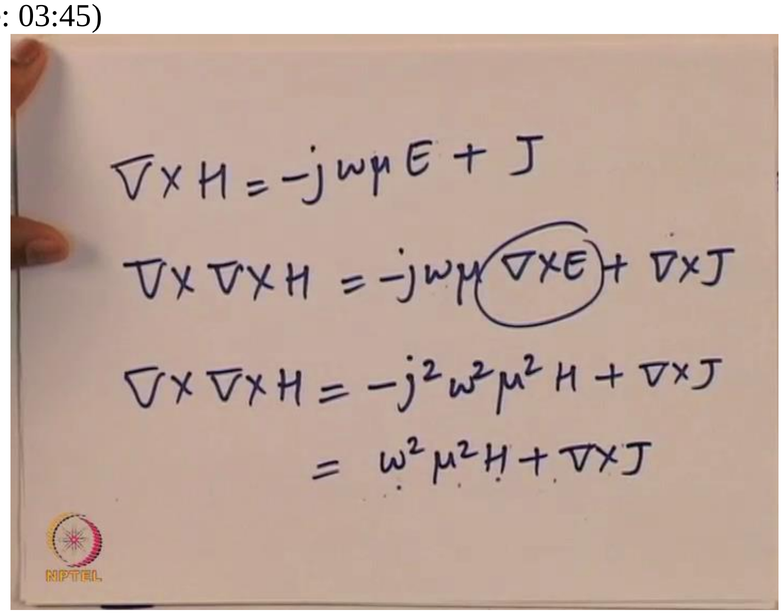

So I have the curl of H is equal to minus j omega Mu E plus J. And I take curl of curl of H this is equal to minus j omega Mu curl of E plus curl of J. And I know this value because it is given that this value is J omega Mu H. So I can substitute this curl of curl of H is equal to minus j square omega square Mu square H plus curl of J. And j square will be minus 1 and minus of j square will be plus 1. So you can write this one as omega square Mu square H plus Curl of J.

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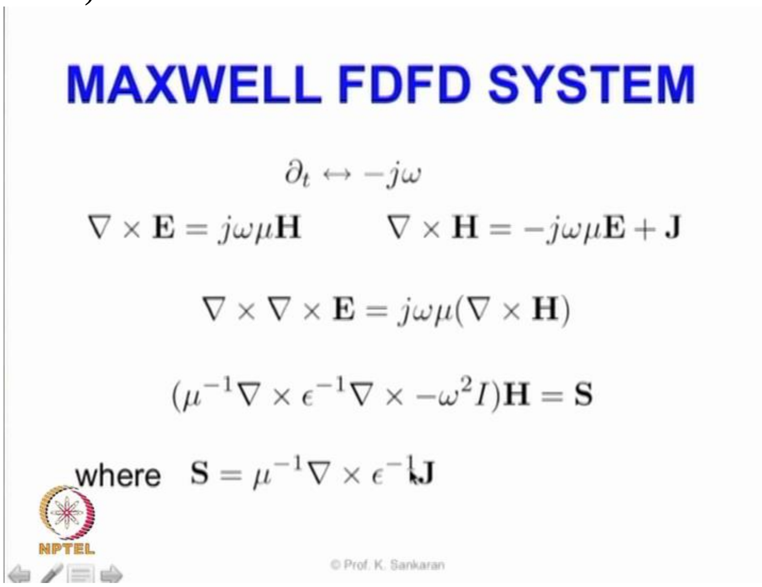


And when you rearrange the term what you essentially get is the equation that is represented here where the s term is the term that contains the source term and the j term is contained within this s term. So essentially what we have done is we have made this entire equation as equation in one variable H with certain source contribution.

So we can basically take this and try to symmetrize this particular matrix. So the matrix on the left hand side what you see here is purely consisting of the material component the frequency component and the spatial component.

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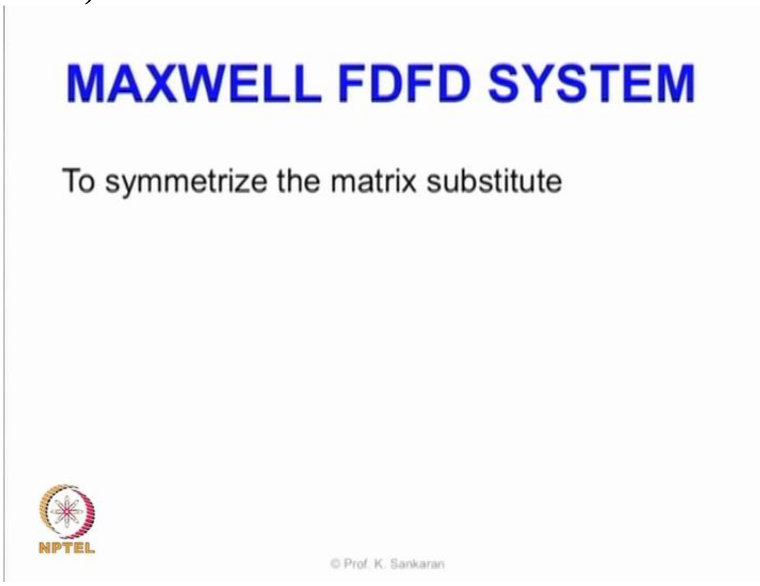


So when you try to symmetrize this matrix using certain tricks what you can do is you can substitute certain value for H and certain values for S.

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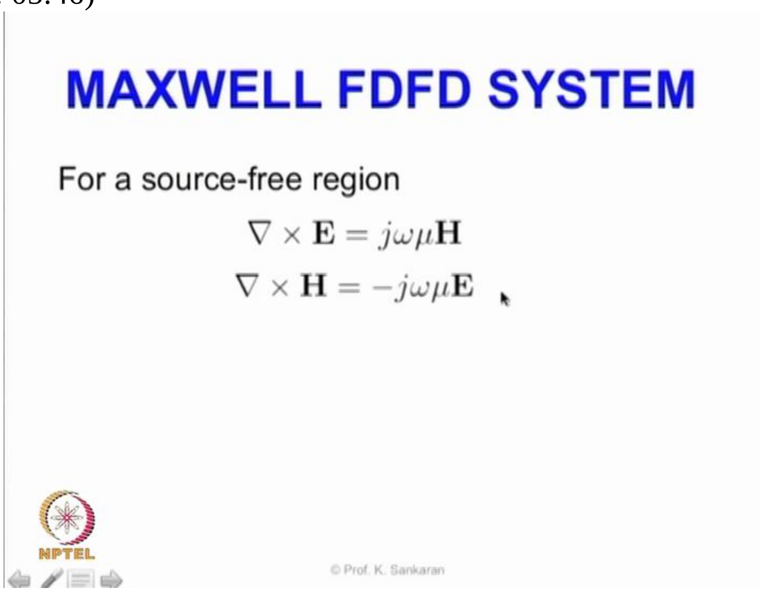


We are not going to go into the detail of S component here so we can we take a source free condition. So we assume that the source component which comes in the equation we have taken it out. So when we do that what we get is essentially an eigenvalue problem in omega.

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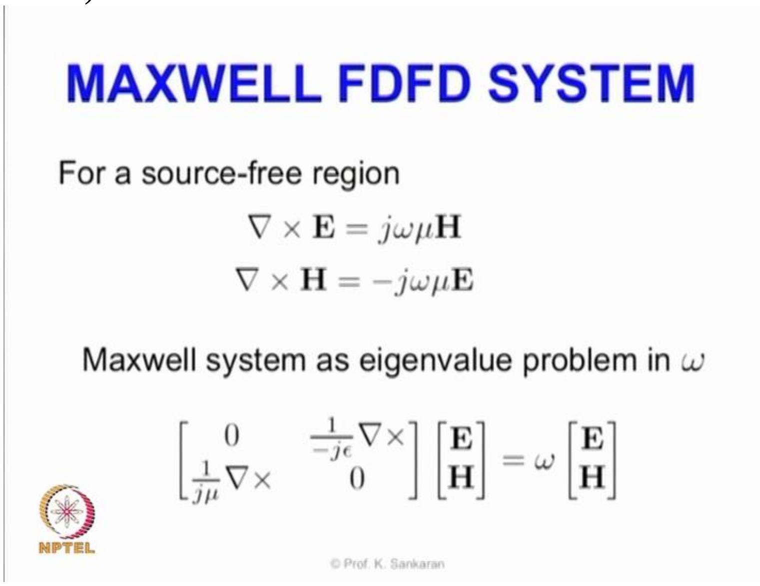


And this will be given by this particular term, and obviously this is the matrix that contains the material parameter and also the j value. And you can basically combine these two equation into one equation the way we wrote before.

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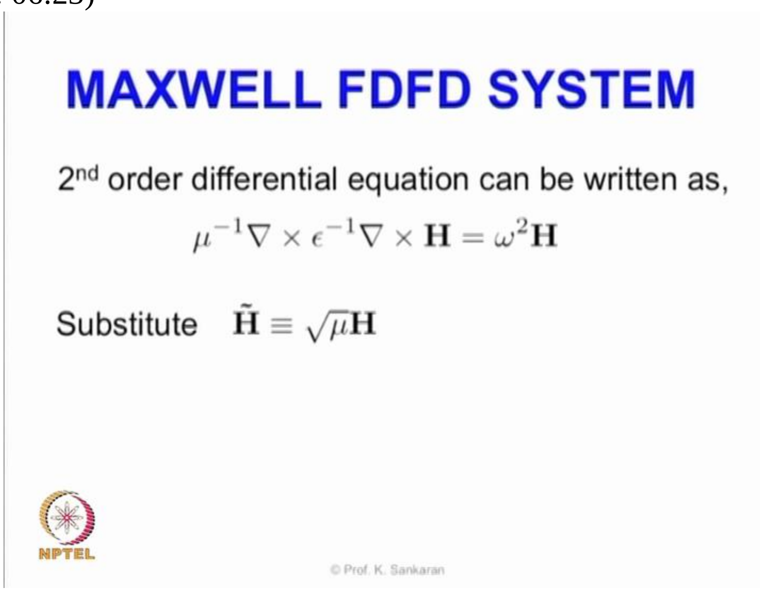


And you will get an eigenvalue problem for which you have to compute for the value of the frequency and the wave number and omega so on and so forth. So you can substitute the value which is H is equal to h times square root of Mu in this particular equation.

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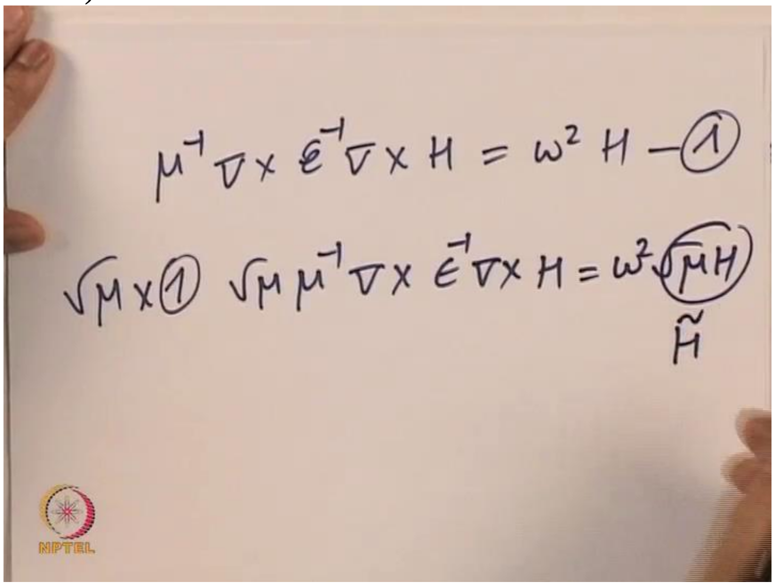


So we have an equation which is given by Mu  to the power of minus 1the curl component multiplied by Epsilon to the power of minus 1 the curl component times H is equal to omega square H. So when I multiply this equation with, let us say call this as 1, I multiply this on 1 what I will get is under root Mu Mu to the of power of minus 1 Curl of Epsilon to the power of Minus 1of curl multiplied by H equal to omega square under root Mu H. And I see this value H tilde.

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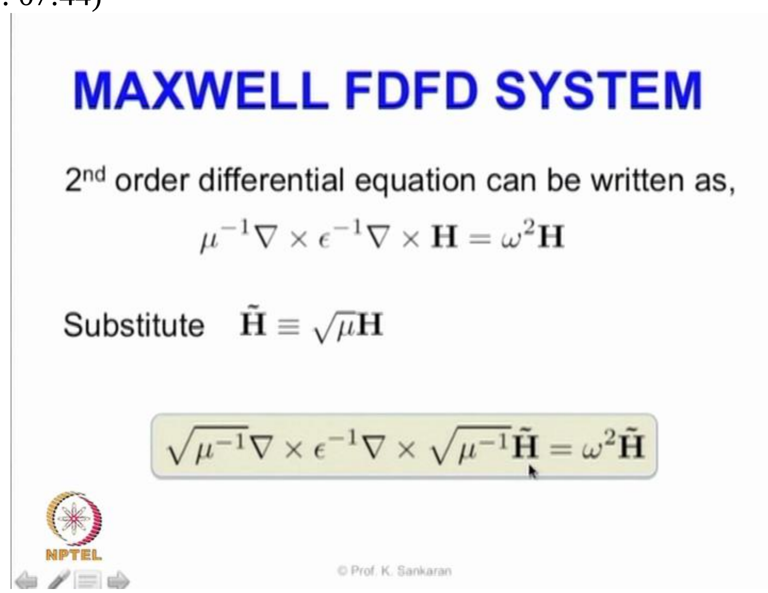


And that is what I have done here, so what I will essentially get is an equation that is given by H tilde.

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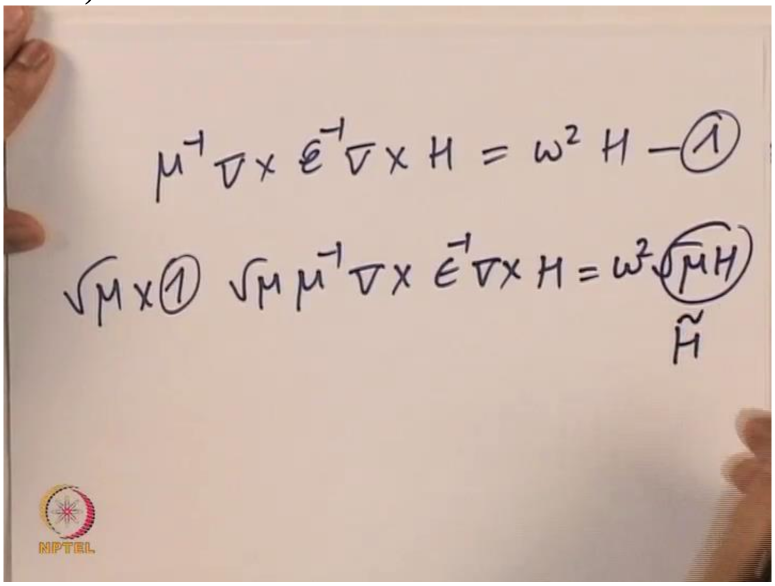


So I can basically write this value as H tilde and I can get the value correspondingly.

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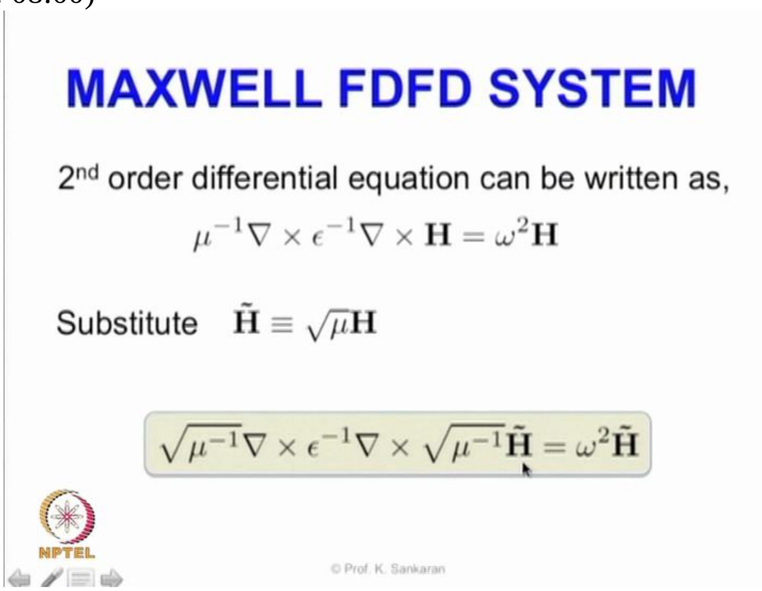


So this is a simple Eigen mode formulation where I am computing for the frequency and this is the matrix where I have the values of the material components which are sitting here. So what you get essentially is a system which is given by this equation. Obviously it has the H component here what you are solving is for particular omega and this is a simple straight forward formulation. The moment you are going and modeling this on a general system where you are also having complex boundary conditions the value of permeability and permittivity will turn to be complex numbers. And it is a bit complicated to model them in a more generalized form.

However for a simple system where the domain where Mu and Epsilon are constants, it is a straight forward approach.

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So I said that we should have a word about the frequency domain simulation, so what I wanted to say here is when you are doing a frequency domain simulation, what you are interested is in a series of frequencies where you are interested in simulating your problem. It could be one frequency you know it could be a group of frequencies where you are interested in modeling the problem. And when the simulation itself is quite simple your domain is quite simple it is not a big issue.

Whereas when you have a large simulation where the number of cells are going to be quite large simulating it for individuals frequencies is going to be very difficult. So in that kind of cases what I would suggest is doing a time domain simulation with a broadband pulse. Where you can pretty much have at the broadband information while running in a time domain simulation and doing the frequency domain transformation using a fast Fourier transform or any of the FFT proteins what you have.

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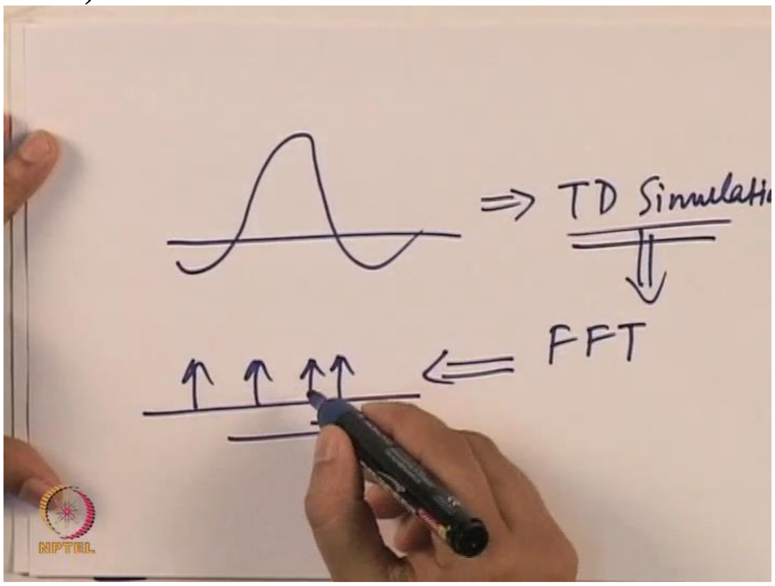


So in that sense what you do is when you wanted to run a simulation in time domain for multiple frequency. So what you do is you take a sync function with certain bandwidth and you do a time domain simulation. And then you get the results in time domain and you do the fast Fourier transform and you get the entire bandwidth of frequencies you are interested in. So let us say I am interested in the series of frequencies. So this is the frequency I am interested in so I can get the frequency domain response as well.

So this way what we are doing is we are running a time domain simulation and getting multiple frequency responses in one single run.

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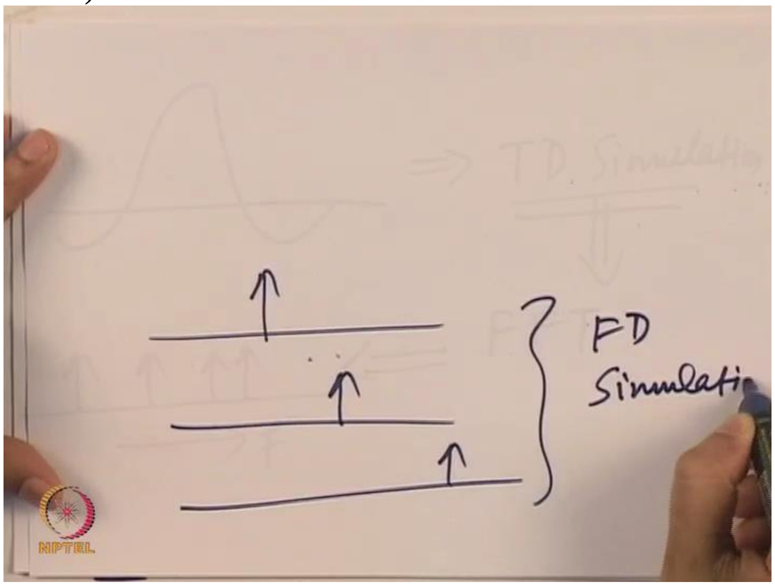


Whereas if you are doing a frequency domain simulation you have to individually run the simulation for each of those frequencies. So I have frequency simulation at 1, 2, 3. I have to run three different simulations to get the frequency domain response.

So we have seen now how we do frequency domain approach. Of course we did not go more into the detail of the frequency domain simulation. In the case of the finite difference approach our entire approach is going to be a time domain approach and I also gave the reason for why time domain approach might be useful for your case and when also the frequency domain might be useful for your case. You are interested in only one particular frequency you are interested in Eigen mode analysis you are interested in certain aspects of let us say one or two frequency regions then probably doing it in a frequency domain approach is less time consuming and more efficient.

But if you are interested in a broad band simulation and you are interested in knowing the broadband response of your system. It is better to go for a time domain simulation with a broadband pulse and then doing a fast Fourier transform and getting the frequency response accordingly.

With that being said we will come to the end of this module. We have covered quite a bit in this lecture. And it is important that you get an overview of the finite difference time domain and the finite difference frequency domain approach. In the next modules what we are going to look into is the domain truncation technique itself. We have not looked into the domain truncation so far. They are very important because we cannot simulate in an infinite space we have to somehow truncate the space. So how do we do it accurately the problem is more efficiently and at the same time more accurately solved. So that will be the focus on the next modules. So with that being said I am sure you are following the earlier lecture series, there is more to come in the next modules as well. So follow it through and let us get back in the next module.

Thank you!