Computational Electromagnetics and Applications
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Lecture 07
Finite Difference Methods-III

So welcome back! So where are we now so we said that we are going to do Finite differencing time domain as a starting step. We got the nice partial differential equation of the Maxwell system that we are interested to model. So let us go into the topic directly.

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We are going to look into the Finite differencing system of the Maxwell

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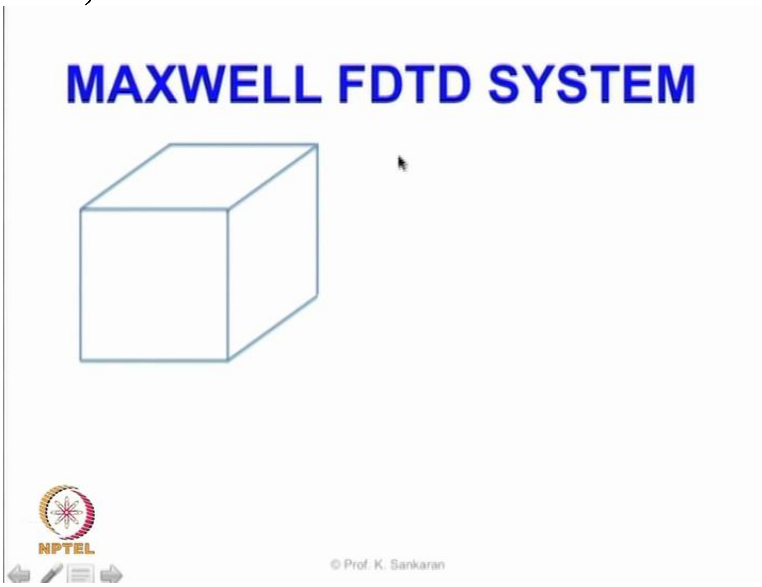

So let us take a computational domain for simplicity let us say it is a cubical structure. It is a big mass of some cubical structure for which we are interested in knowing certain phenomena that we are interested in modeling. So what we will do is we will discretize these three dimensional solid space.

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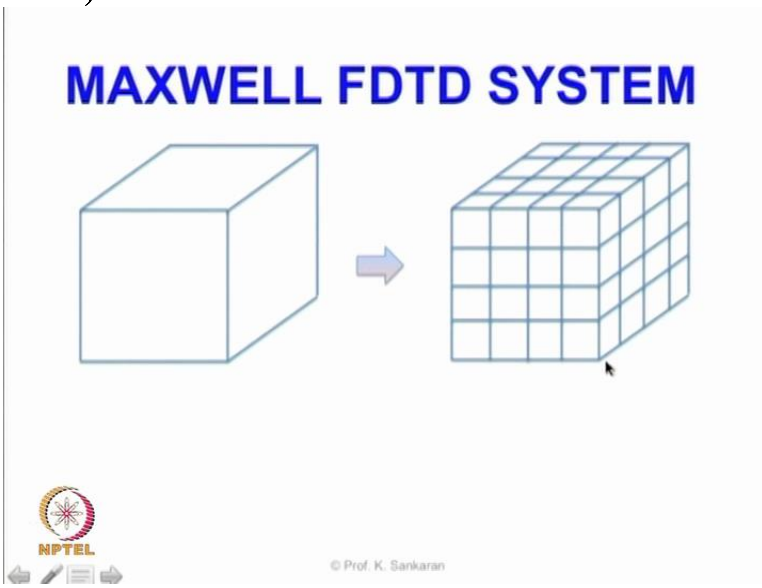

You can think of this as small building block bricks for a particular volume. You are putting all those bricks of same size in all the three directions and or did not need them in all three directions but if you wanted to make simple analysis let us say all the x and y and z direction are equal. So these are small cubes that are going to be assembled one next to other. And there are no gaps between them and this is going to completely fill the computational domain.

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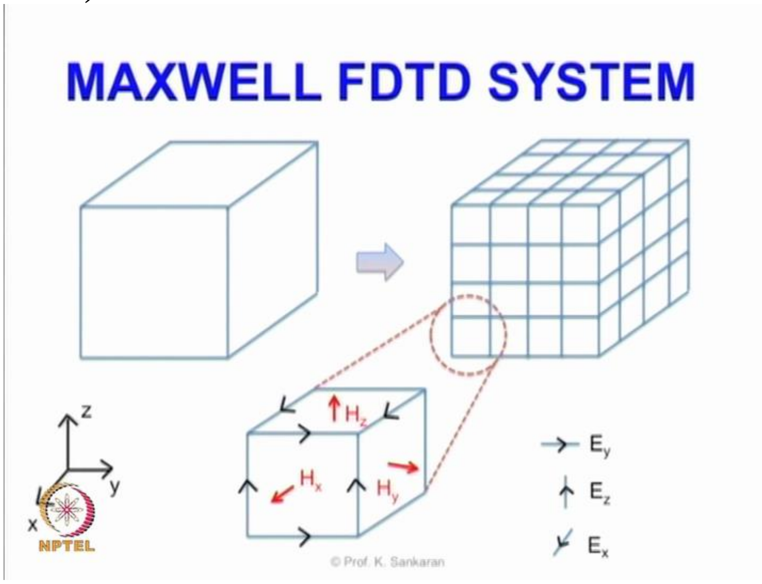

And obviously what is interesting to know is what is inside this one single cube. So if you take the one single cube.

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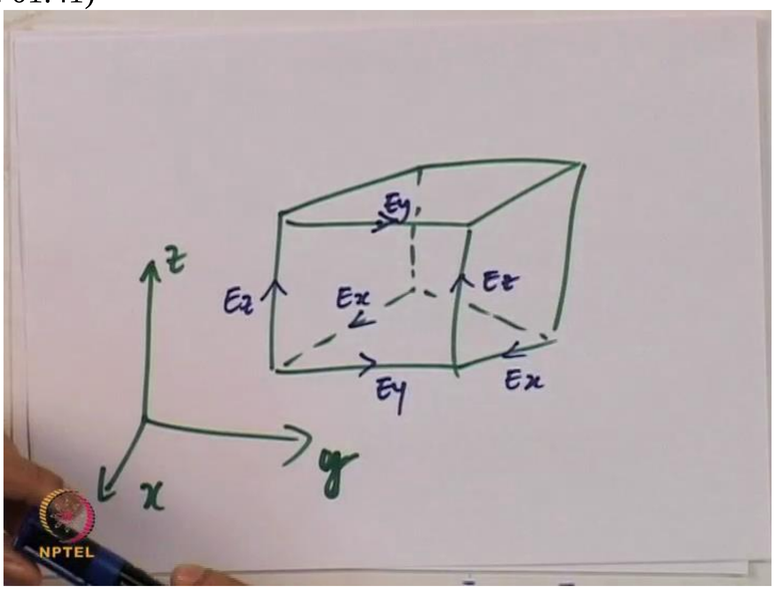

So what you have is let us say we are interested in knowing what is there inside this one different single cube. So if you see that what is happening is let us say this is the x, this is the z, this is the y. We have certain quantities that we are going to define. So what we are going to say is the electric field are going to be on the edges of the cube. So the electric field components so this is going to be Ez, this is going to be also the Ez component, this is going to be the Ey component this is going to be the Ey component and likewise this is going to be the Ex component and this is going to be also the Ex component.

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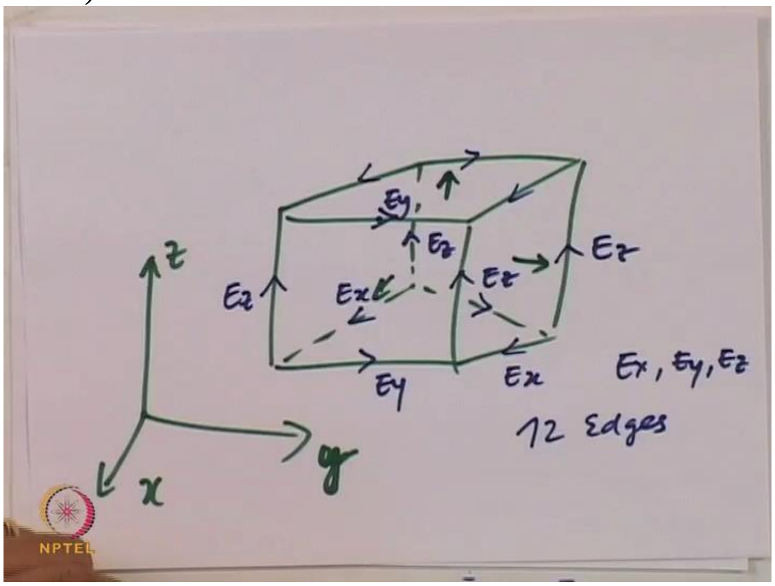

So what you are seeing is there is going to be the definition of the electric field that is going to sit on the edges of those cube. And as you have totally 6 and on the top you are going to have certain edges, so we have 1,2,3,4 in the bottom you have 1,2,3,4 and on the sides 1,2,3,4. So totally you are going to have 4 times 3 edges. And this is going to be 12 edges. And you have Ex component, Ey component, and Ez component that are going to be defined in all those edges. So you will have Ez components that are sitting on all the edges but in the vertical direction and the components are going to be in the direction of y on the top and bottom. And similarly you are going to have the Ex component which are going to be in the top and bottom.

So what you essentially see is in a particular cube in a finite volume you are going to have Ex within a particular cube you will have 1,2,3,4 4 of the edges pointing in the x direction you will have Ex, similarly 4 of the edges that are pointing the y direction you will have and the 4 of the edges that are pointing in the z direction you will have Ez.

The question comes what is the about the h direction, the h field. So the h field are actually going to be in the direction that are in the center and it is not physically intuitive to know why it is coming in the center.

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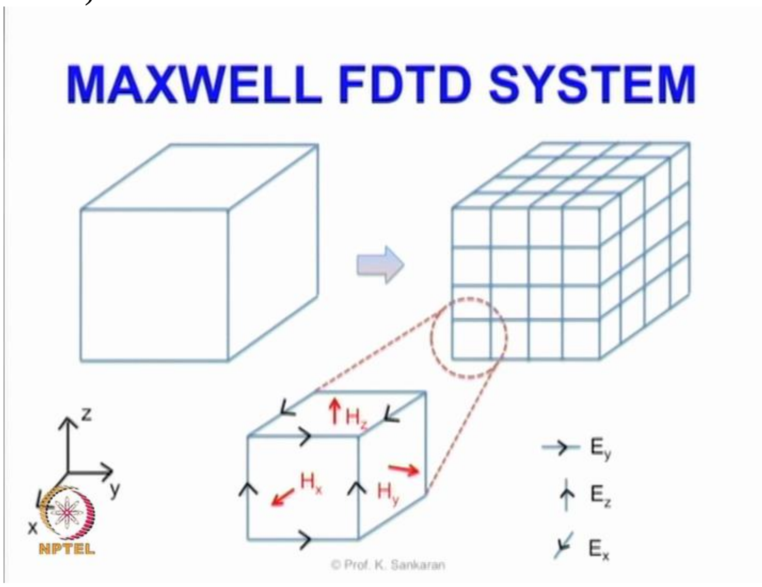

So what we are going to say is if you see in this slide what you have is the Ex component which are going to be on the edges as we said. And the Hx components are coming from the center, the reason for this will become clear in the next slides. But for now it is enough to know the Ex components are sitting on the edges and the Hz components are in the middle for a particular cube.We will see why it is so in the next slides.

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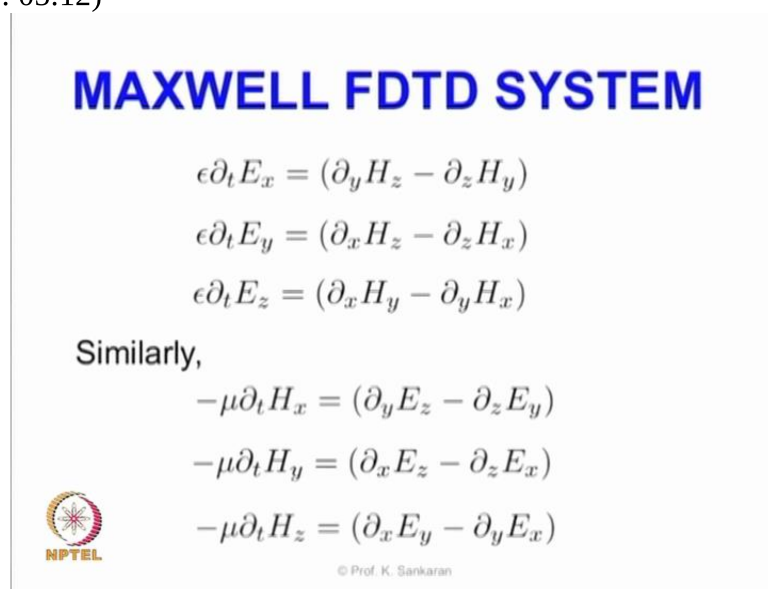

So if we have the basic equation which we derived in the earlier module so these are the two set of curl equations and their time evolutions. And we will see the reason why the Hz component is in the center if we remodel the grid using dual grids.

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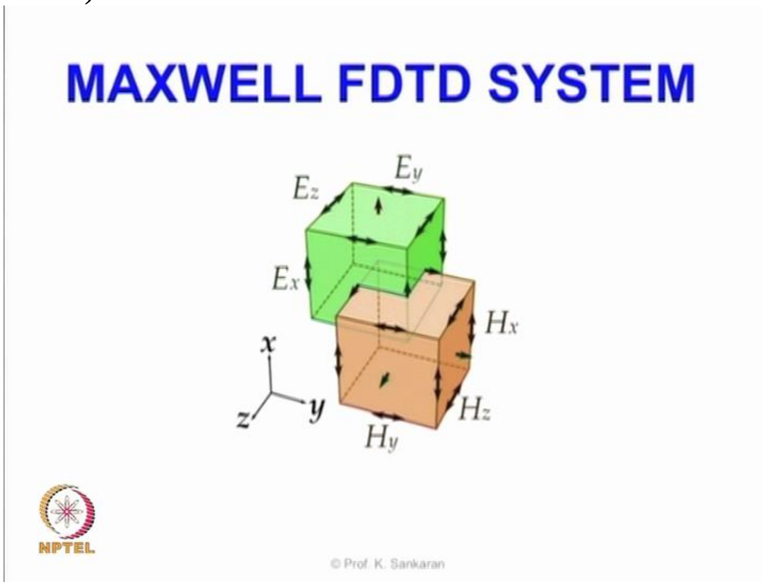

So what you have seen in the previous case, in this particular thing you see only one set of grids. Actually there are two sets of grids, and the second set of grid is something that you donot see

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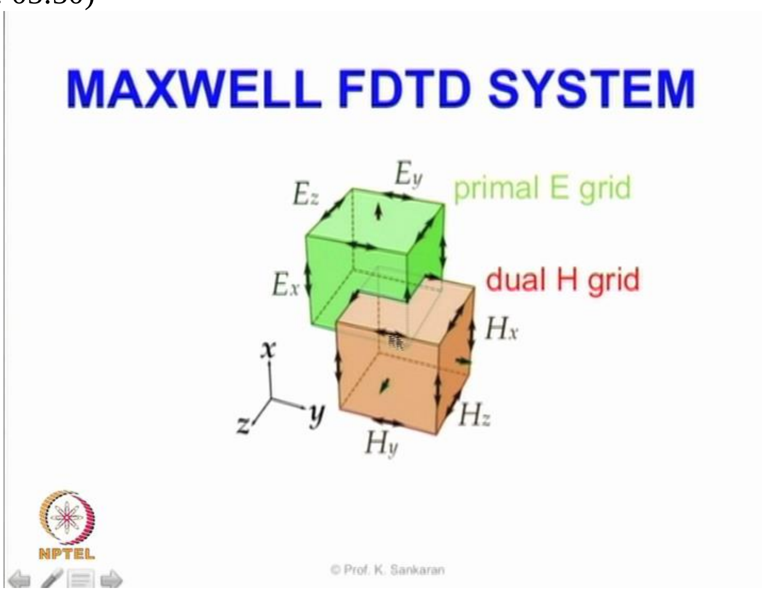

And that is what we are seeing in this particular slide here. So if you call the green grid which is the primal grid, these are the grids on which the ease components are defined. So the edges on which the electric field components are defined is called as the primal E grid and the cube in which the H field are defined is going to be staggered half time in space. So what is happening is when you see the edges of this particular grid it is going to be aligned exactly at the middle face.

So what is happening is we are staggering them in space exactly half the dimension. For this will be more clear when I explain you how this staggering is done.

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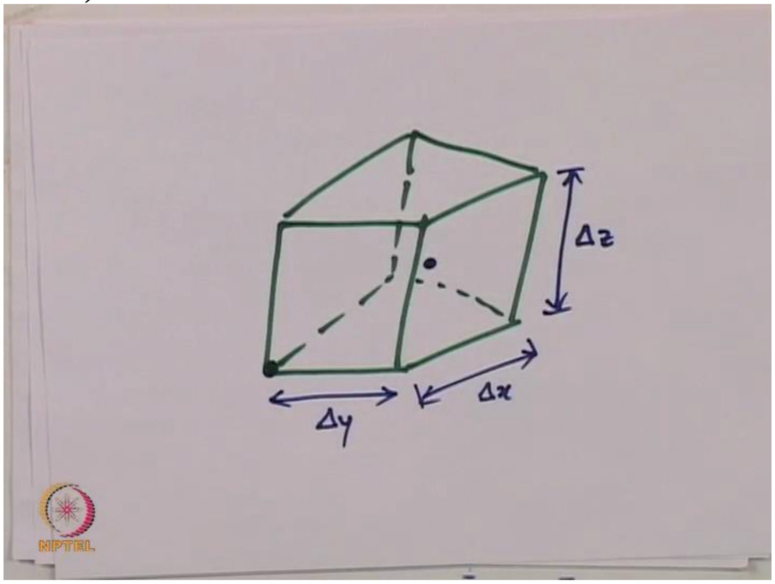

So what you have is you have the electric field grid which is marked here in green. And let us say this is the dimension in x axis. This is the dimension in y axis; this is the dimension in z axis. So when I come half in y, half in x and half in z. My middle point will be the exact Bari sector of this particular volume. So what will be the starting point so if you say the starting point of the primal grid is this node, the starting point of the dual grid is going to be the barycenter of this volume.

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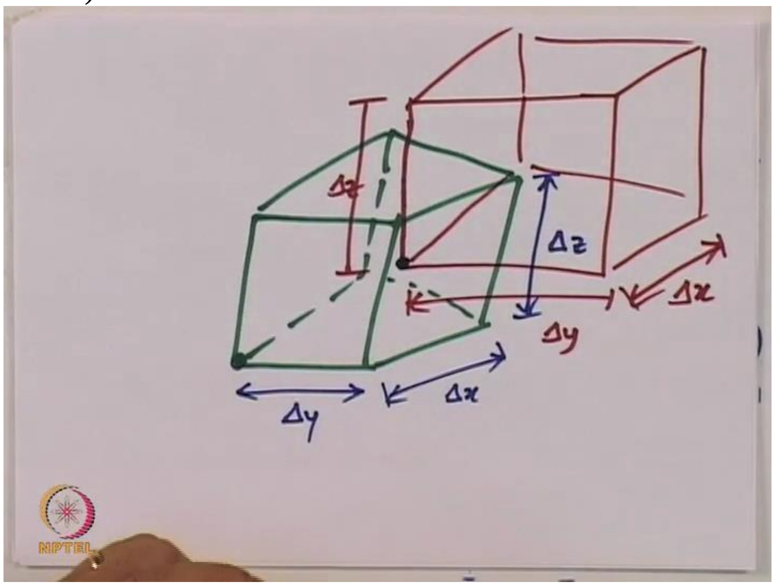

So if I take this so I will be basically creating a one cell exactly the same dimension as this particular cell. So I will have the dual grid coming into play. So basically this distance what we have here is also delta z this distance which is here which is also equal to delta y and the distance what we have for the dual grid which is going to be here this distance is also going to be delta x, so we have moved this particular starting point exactly half the distance in all the three coordinate, and when we do that the staggered grid will be having the starting node exactly at the barycenter and will be the same primal but moved half the distance in all the three direction.

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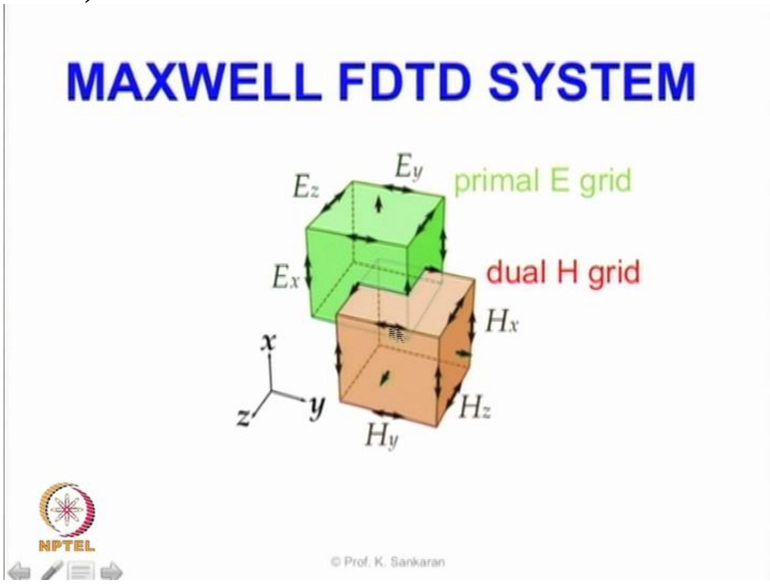

And that is what you see in this grid here, and when you do that the same way we have defined electric field, we will also define the magnetic field which are on the edges of those the staggered grid. And when you see only from the primal grid point of view what you will see as if they are coming from the center. So here if you see the magnetic excitation what you see is it is seen as it is coming from the center. And that is what we saw in the previous slide that the H field is coming from the center.

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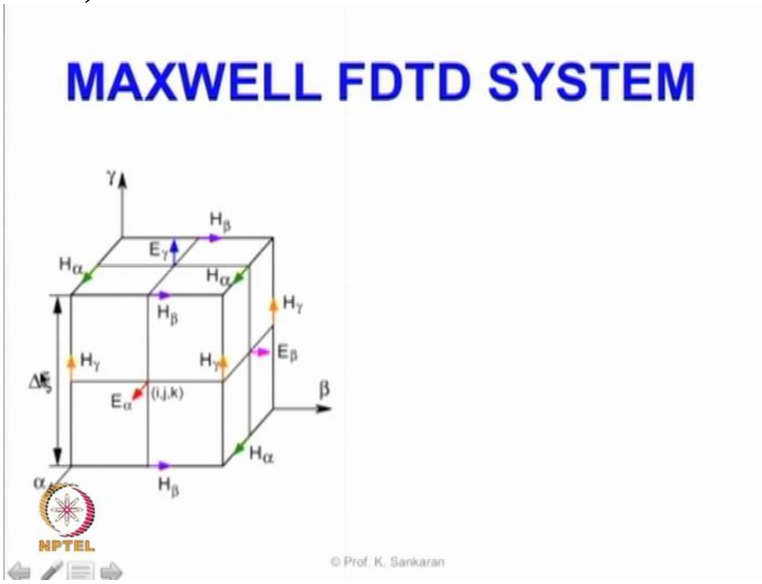

So a more clear understanding of this is in this slide where you see the values are given for the different discretization and you see that the purple color refers to the magnetic excitation and the electric field is defined as coming from the center. In the previous case we had the magnetic excitation that is coming from the center, whereas in the second case where we see that the electric field is coming from the center depending on what is your main cube. \

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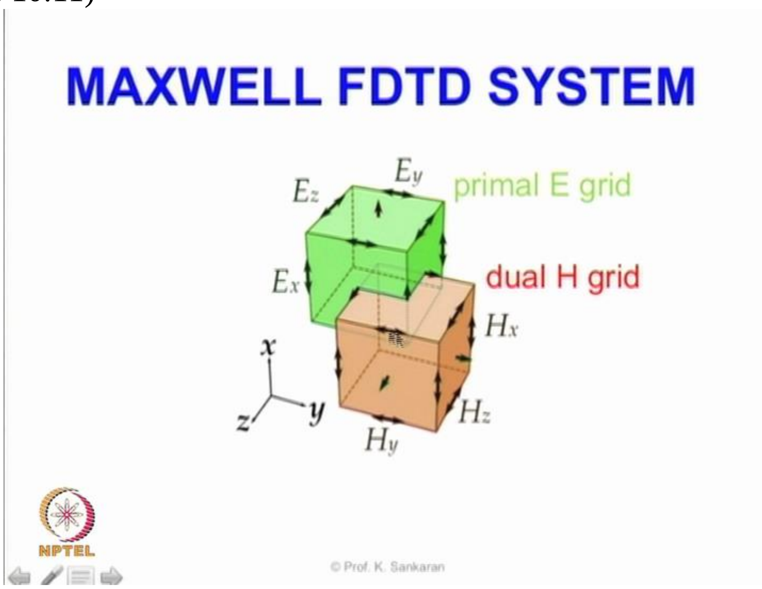

In this case the main cube is the dual grid. And in this case we see both the dual grid and the primal grid

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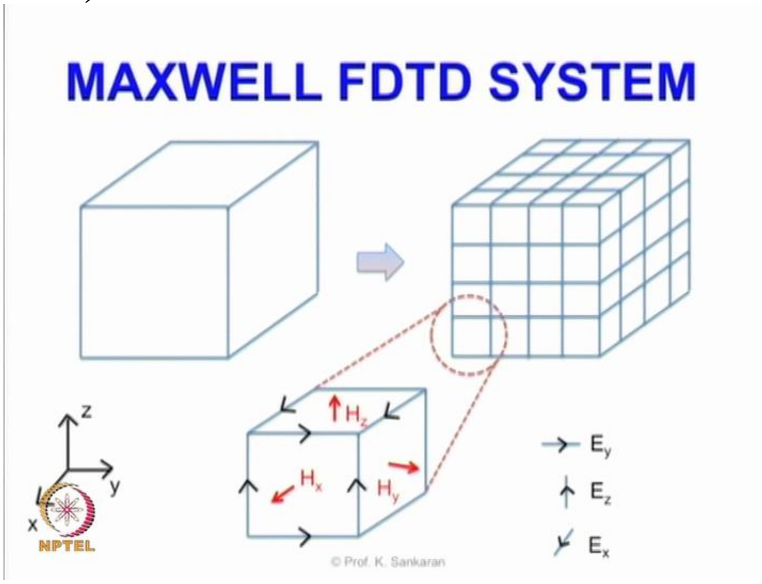

Whereas in the previous case here we saw the primal grid and we saw the h field coming out of the primal grid center.

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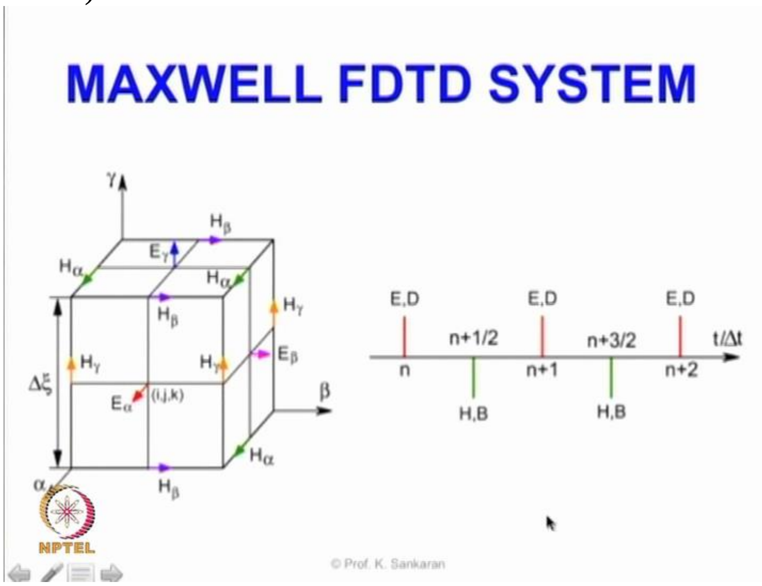

So with that being said so here we see the dual grid and we see electric field components with respect to the dual grid coming from the face centers of those cubes. And this is the spatial discretization. The question is what are we doing for the time discretization. So remember we have both time and space in case of the Maxwell equation. We have somehow found a way to model electric field and magnetic field in the two spatial coordinates which are the primal coordinates and the dual coordinates which are the staggered half the space length in all the three directions.

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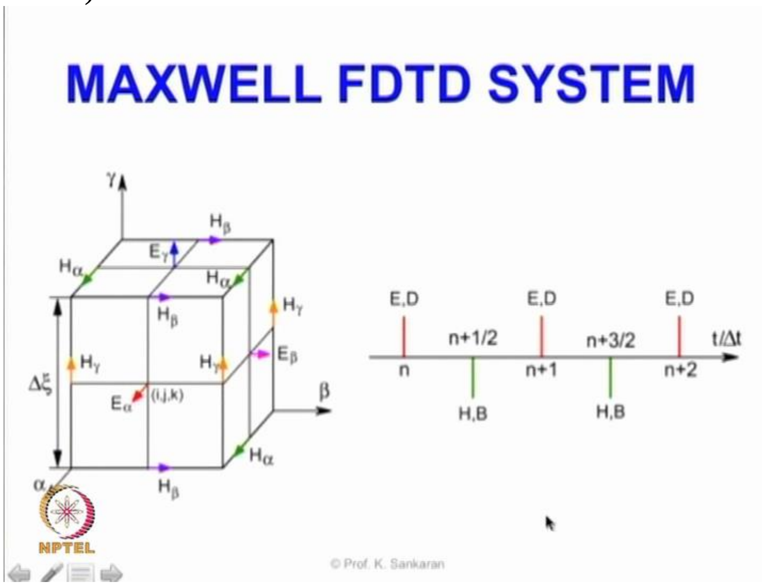

So we will go and take the simple example of scalar equation of the Maxwell system and do the modeling step by step so that you know how the spatial and temporal discretizations are done using the finite differencing method that we have learnt so far. So in this case as I said the electric field and the magnetic field are going to be staggered in space and also in time.

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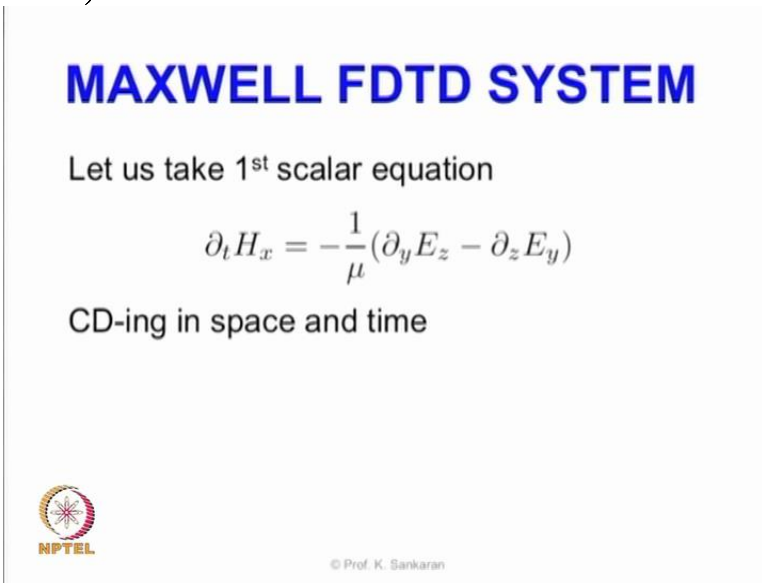

So let us take this scalar equation which is curl of E equal to will be here. What you see is you are going to do central differencing in space and time. So what we have is basically you take the central differencing in space and time.

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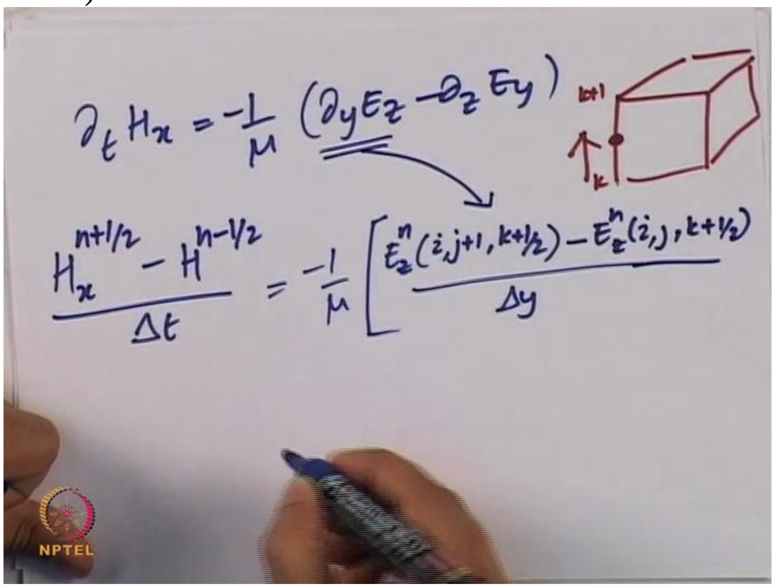

So let us derive this the partial differentiation with respect to x this is equal to the minus of 1 by Mu (dy minus dz) so this part which is here I know the component are going to be at half steps so I take the value at half n plus 1 and n minus 1I am going to divide it by the distance in time so it will be delta t and I will have 1 by Mu this particular partial differentiation with respect to y is going to be replicated in the finite differencing as delta y and component at time step n and the location of it is going to be i,jplus 1,k plus half).

So since it is a z component, the z component is going to be let us say this is our cube and let us say this is our z component direction. So what I am interested is exactly the value at the center of the edge so that is why the z component is having the half here k plus 1 by 2. So this is going to be k this is going to be k plus 1. So k plus half is going to be exactly at the center of the face. So we assume that the value of the electric field is going to be taken as a value that we are going to approximate at the center. So that is the reason you have half here. minus E n value of the z component (i, j,k plus half). So that is going to be the first component of this one.

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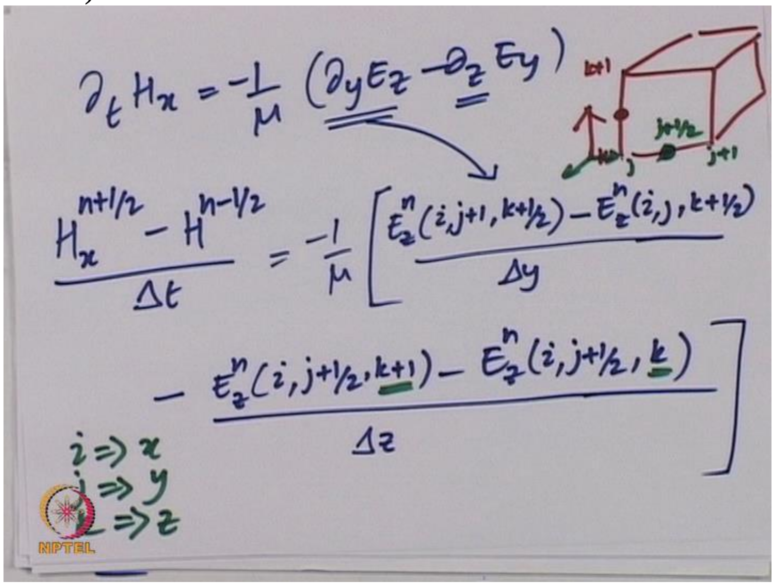

Similarly the second component is going to be minus since it is a partial differentiation with respect to z it will be delta z and the first component is going to be at time step E n z and it will be a z component. And we know that we are going to find out for y, so the y component will be let us say, so this is our x direction this is our y direction. So this is going to be j, this is going to be j plus. So the exact middle point is going to be j plus half. So that is what we are going to have here so we are going to have i,j plus1 by 2, k plus 1 minus E z n (i,j plus 1 by 2,k). So we are seeing here k so the i component refers to x so the j component refers to y and the k component is going to refer to z. So what we have is k plus 1 and k is the finite differencing stencil and we are dividing it by delta z.

Similarly here it is j plus 1 and j it is going to be the stencil for y and differentiating with respect to y. So this is the first curl equation component and similarly you can think of deriving it for the other component. So we have shown how to do the finite differencing for individual scalar equations. Likewise you can do it for the other 5 equations that we will get from the Maxwell curl equations.

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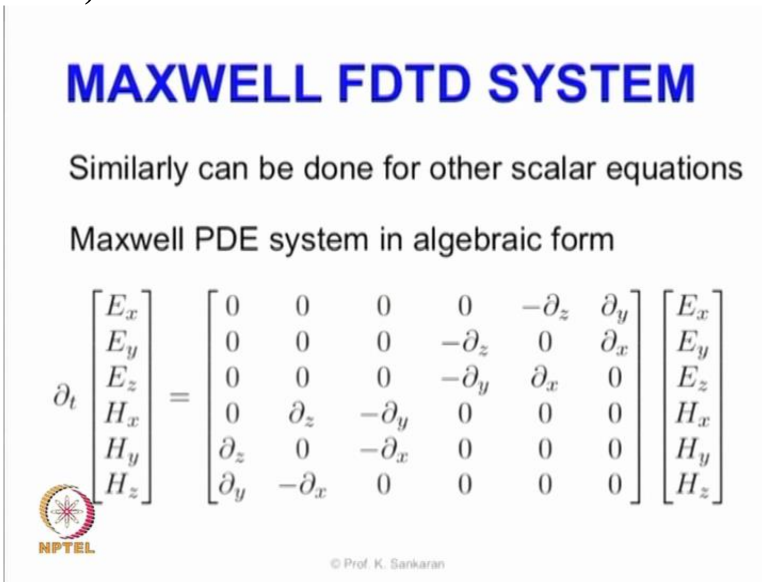

In essence what we will have is a set of equations represented by the Maxwell PDE system as we can see on the slide and they will have a algebraic form where the left hand side will be the time derivative component and the right hand side is going to be the spatial derivative components. The spatial derivative components are given by the curls formulation itself. And there are going to be kind of symmetry here and since we have a minus sign on one of the curl equations the signs will change and the rest of the things are the same.

So pretty much what you need to pay attention to is the electric field components are going to be in the full time step and the magnetic field components are going to be in the half time step. Likewise the electric field components are going to be in the primal grid and the magnetic field components are going to be in the staggered dual grid.

So you have to look into the numbering accordingly and once you have that you can go forward to do multiple problems. This is the more general case for a three dimensional Maxwellian system. The question comes is what happens when we do a two dimensional modeling.

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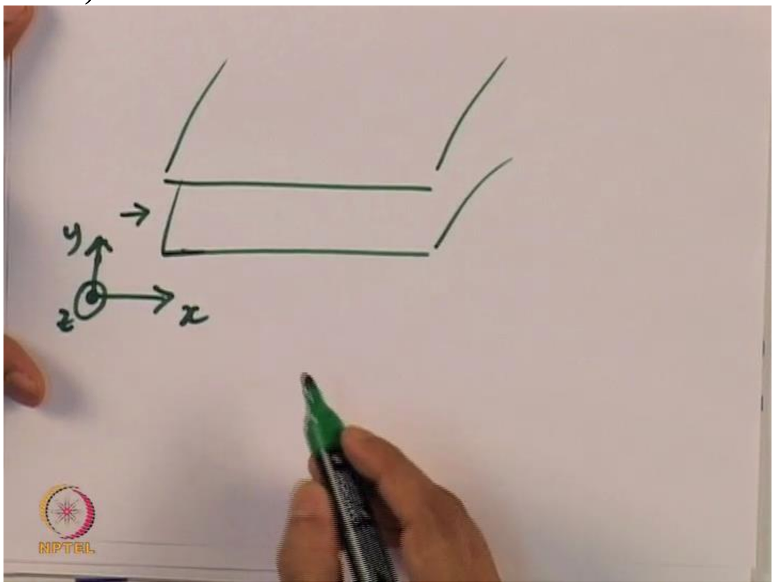

So let us say you are modeling a particular problem, let us say wave guide problem and the wave guide this is the propagation, so let us say we are talking about a two dimensional wave guide and so this particular direction is the y direction and this particular direction is the x direction. And let us say the z direction is coming out, so these are parallel plates and what we are interested in knowing is if I have a source coming from this direction how is the going to be the propagation.

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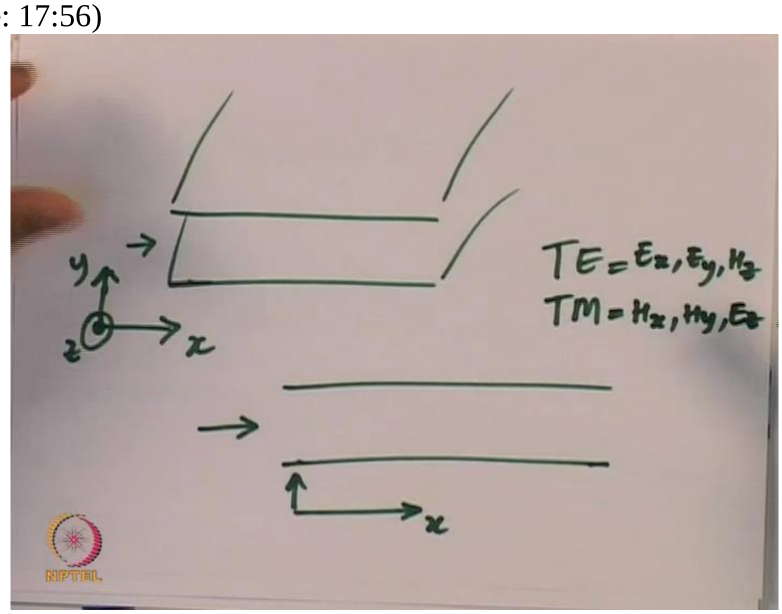

So for this kind of problems what we can do is we can assume that we are interested in modeling the problem as a two dimensional problem. So the z component is something that we will not take into count in the spatial derivative. So basically it will have the wave propagation direction which is given by the x direction. And the y direction is the direction which is going to define the distance between the parallelplates. And when you have that you can look at the problem as either a transverse electric or a transverse magnetic case, So when you have a Transverse electric case, the electric field components are going to be in the xy plane and the magnetic field components are going to be in the z direction. Similarly the counterpart will be the transverse magnetic case where the fields are going to be in the x and y direction and the electric field is going to be in the direction given by the component.

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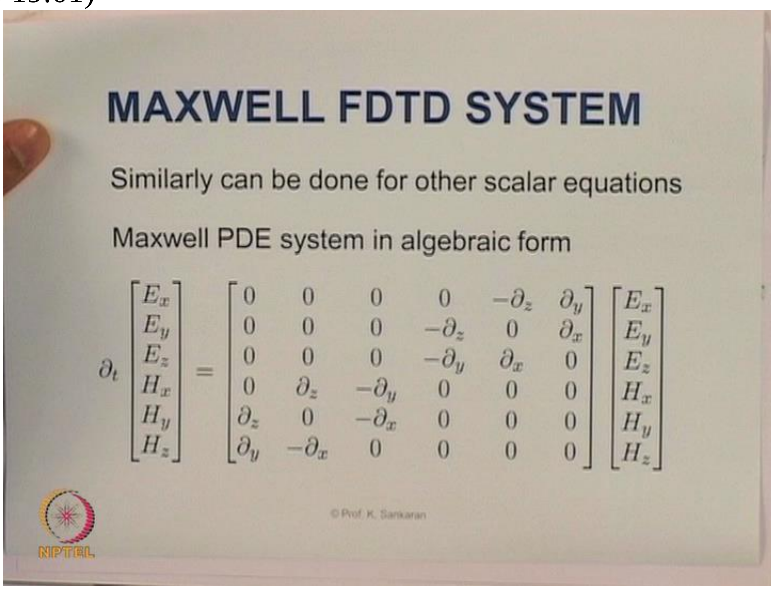

So what will happen is when you try to model such a problem you can essentially take the partial differentiation with respect to z as 0. And this algebraic form what we have here will have only the dx and dy components, the dz components will go away. So if you try to simulate this as a two dimensional problem essentially your PDE will look like this.

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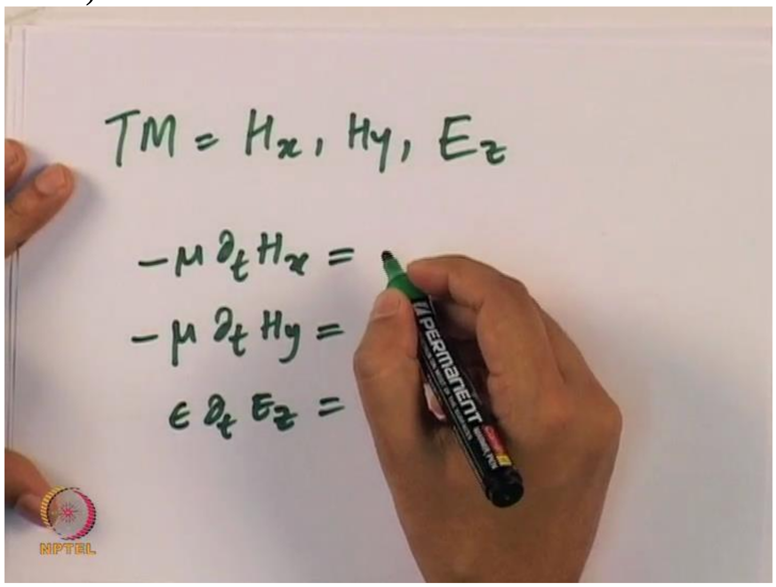

Let us say I am taking the case of a TM where my magnetic components are going to be in the x and y. And I will have electric field in the z direction. So what I will have is I will have the partial differentiation with respect to time for the. And I will have the partial differentiation with respect to time for the Hy. And I will have the electric field counterpart which will be given by Ez. So I am having instead of 6 I will have 3 equations and obviously the right hand side are to be taken directly from the Maxwell case where we will have only the dy and dz component.

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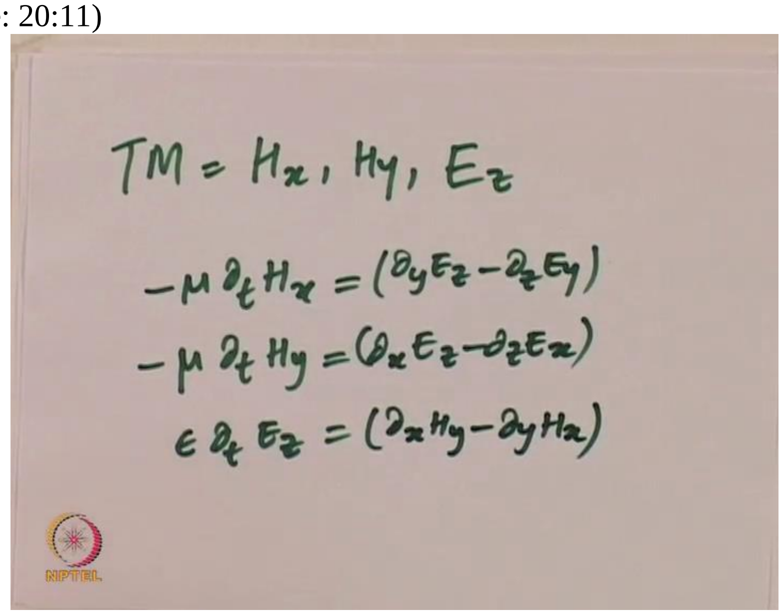

Remember in the case of a three dimensional problem you will have for H x the counterpart will be (d y E z minus d z E y). And similarly the y component will be (d x E z minus d z E x), similarly you have for the E z component (d x H y minus d y H x).

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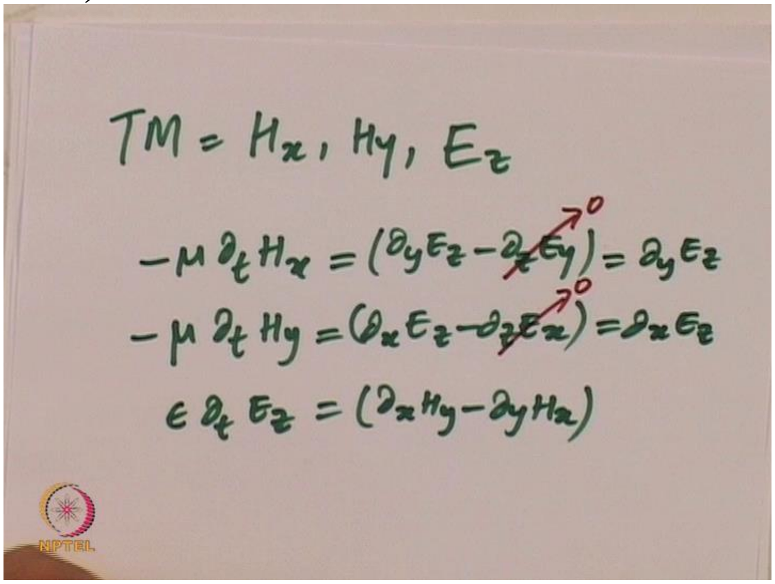

So I said when you are doing for a 2D problem you can basically put these two terms with respect to z as 0. So what you will have is for the first two equations only d y E z is equal to d x E z and the third equation will be the same. So basically with this you can pretty much model two dimensional problem where the H x, H y and E z time differentiation is exactly done the way we did it before and the partial differentiation will have only one component. So if I write down let us say for one particular case.

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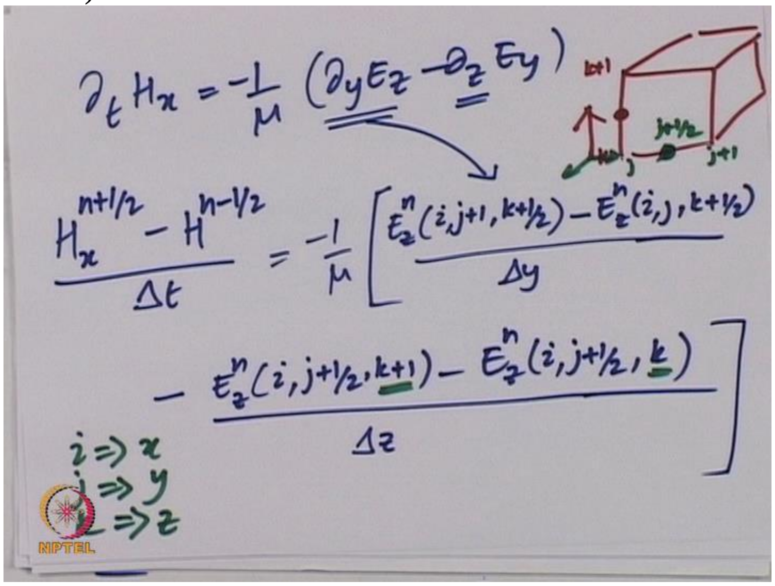

Let us say I am writing it for the first equation what I will have is in the earlier case we had an equation like this. So this particular term will not be there only this particular term will be there.

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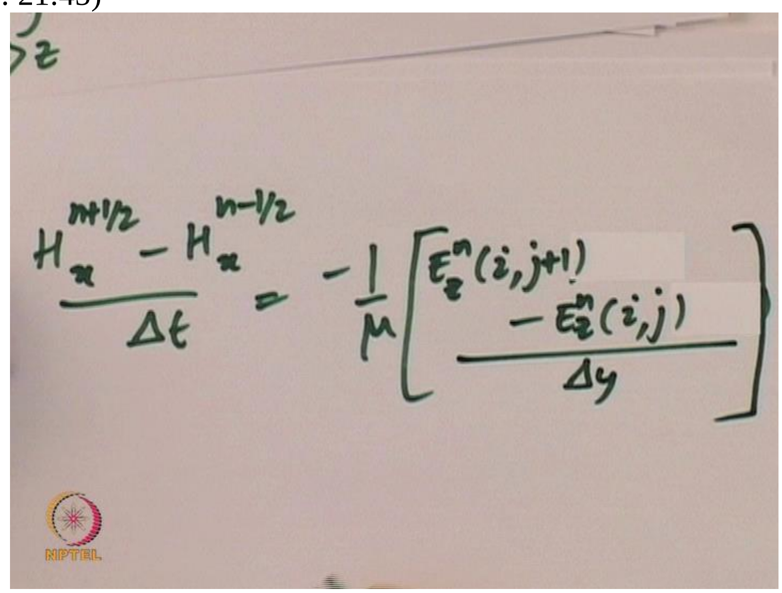

So in other words what you will have is H n plus 1by2 x minus H n minus 1by2 x divided by delta t is equal to minus 1 by Mu [E z n (i,j plus 1) minus E z n (i,j divided by delta y)]. Likewise you can do the other two equations and that will give you the case of transverse magnetic problem. So pretty much you can do that also the same way for the transverse electric case and this will be useful for modeling any two dimensional problem where the z axis has certain axis of symmetry or infinity in case of a parallel plate wave guide which we saw before in our case.

And in the next module we will be looking into the frequency domain approach of doing finite difference problems and with that being said come back in the next module.

Thank you!