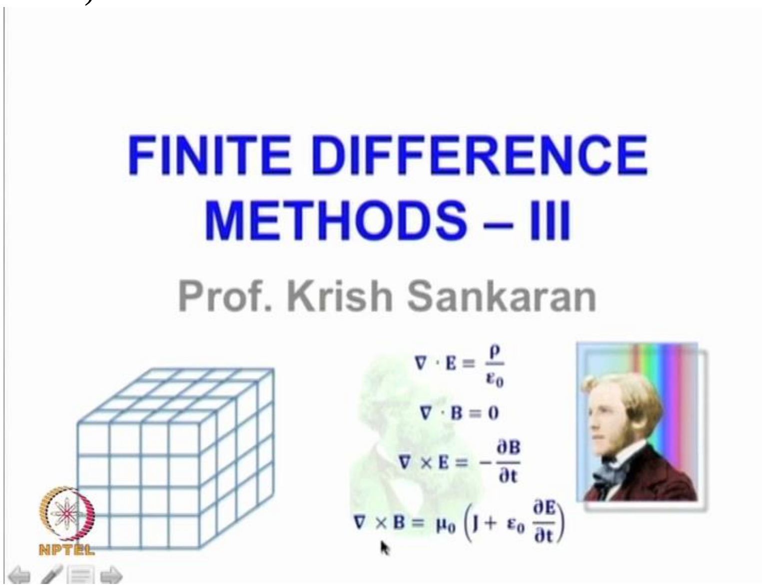
Computational Electromagnetics and Applications
Professor KrishSankaram
Indian Institute of Technology, Bombay
Lecture 15
Finite Difference Methods-II

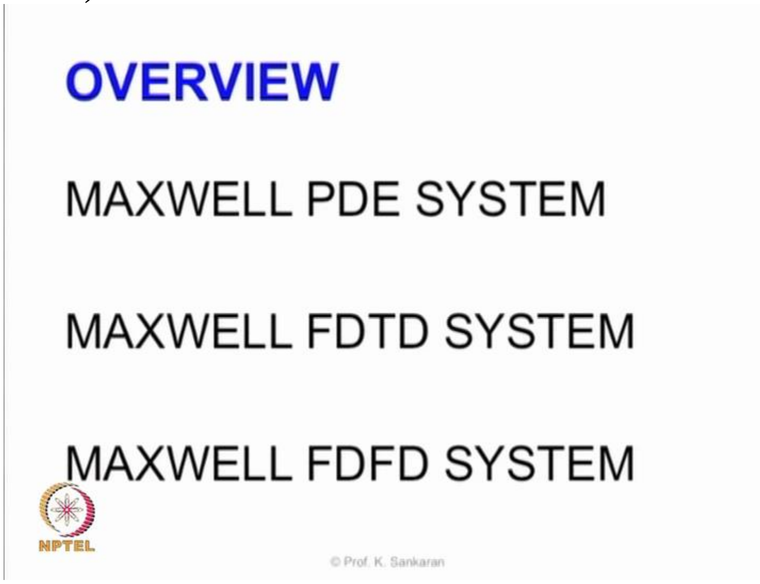
So we have done quite a bit of mathematics and also numerical analysis to get to a point where we are finally ready to look into Maxwell's equation.

(Refer Slide Time: 00:40)



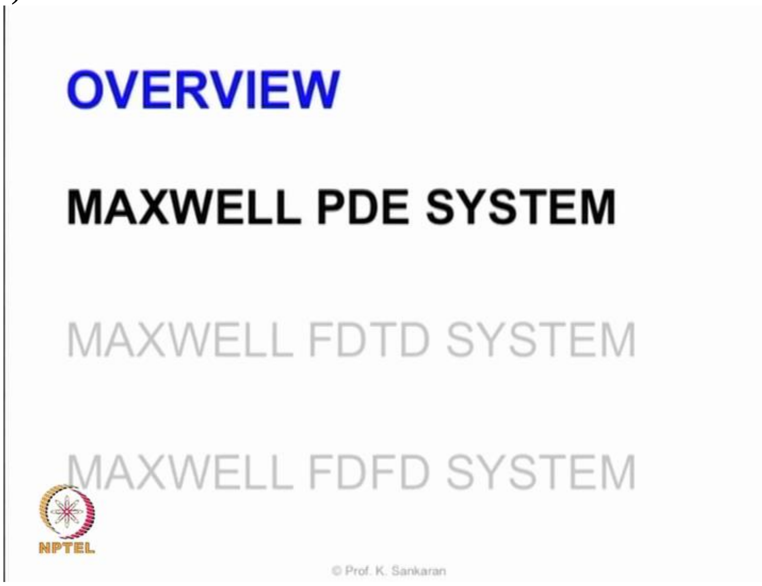
So today is all going to be about Maxwell equation and the slide actually is a tribute to a great Scottish Mathematician who revolutionized by putting together the individual parts of electricity and magnetism into his famous equation and one point of remark here is the kind of equation what we here is not actually due to Maxwell infact when you look at Maxwell's theory it would be more complicated than this. So what we call today as Maxwell equation is actually due Oliver heavy side and in that sense it also a tribute to him.

(Refer Slide Time: 01:17)



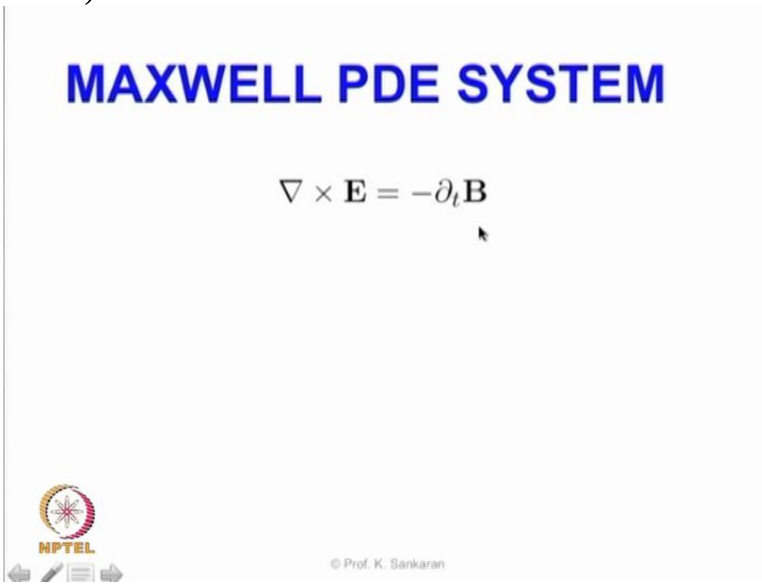
Without further due let us go to todays overview. Todays discussion is going to be on Maxwell system itself. But we will start with a continuous partial differential equation system of Maxwells. And then we will see how this is translating into time domain and frequency domain approach. So when we say FD ED what we mean is Finite difference time domain approach and the other one is Frequency domain approach. So I am quite happy because these earlier modules we have looked into quite a lot of interesting at the same time quite heavy mathematical discussion. Whether it is going to be a CFL condition or finite differencing techniques or Central Differencing or Forward Differencing so on and so forth. All these things converges into a one nice problem space which we call it as Maxwell system.

(Refer Slide Time:)



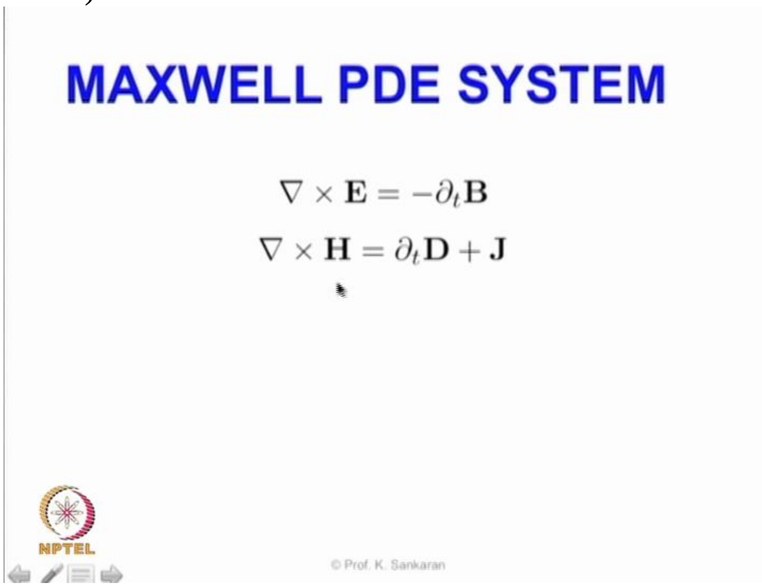
And we are going to make use of those finite differencing algorithms or schemes that we have learnt so far to our benefit of modeling Maxwellian problems. Let us say we are interested in knowing what is inside a Maxwell system.

(Refer Slide Time: 02:28)



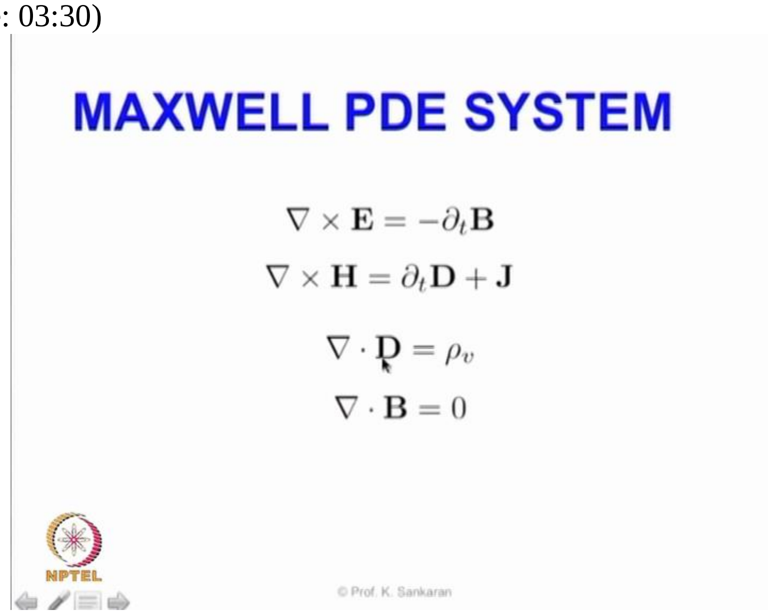
So the first equation is the curl of electric field vector is going to be negative of the time derivative of the magnetic field vector and I am using the word electric field and magnetic field here for E and B. Some engineers might have problem with this but I am trying to be very careful on saying why I am using the word electric field and magnetic field to E and B. And this will become clear in the next slides.

(Refer Slide Time: 03:00)



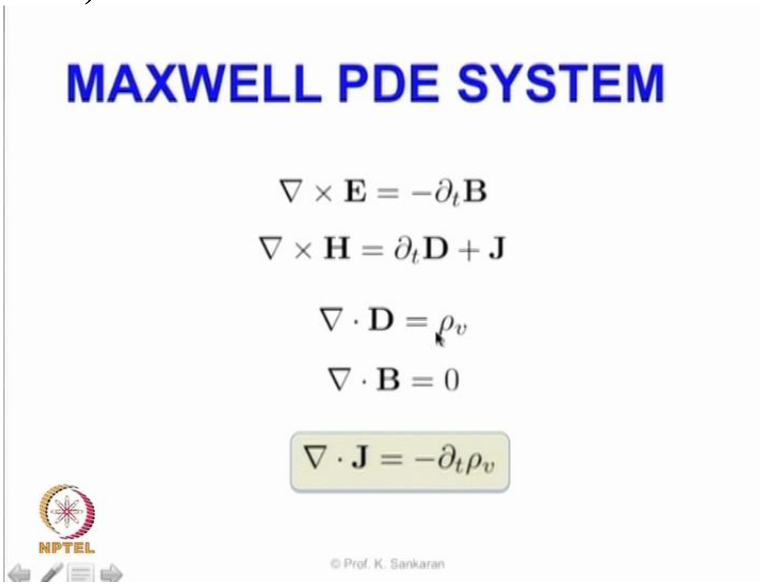
So the next equation is the curl of magnetic excitation is going to be equivalent to time derivative of the electric excitation plus the current density term. Obviously one of the biggest influence of Maxwell is putting this time derivative of the electric excitation and creating the coupling between the electric field magnetic field and the electric excitation and the magnetic excitation.

(Refer Slide Time: 03:30)



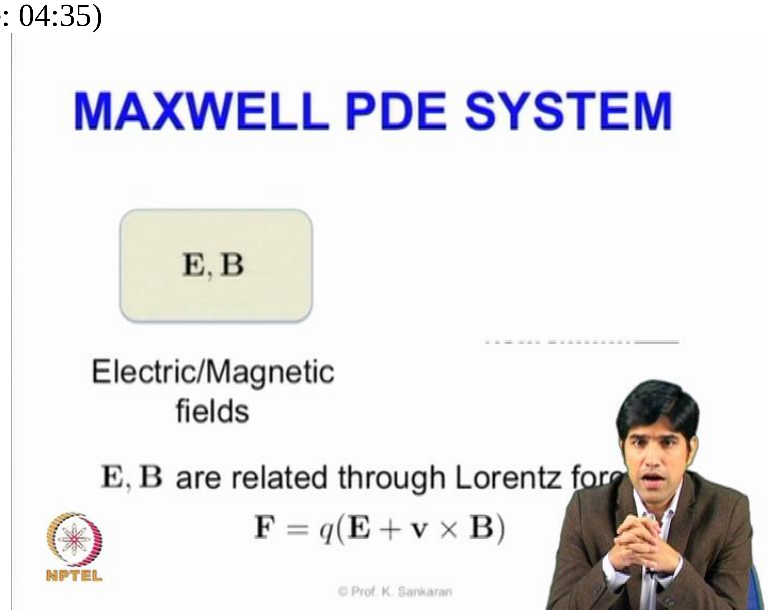
And obviously there are also two other equations which comes quite often. one is the electric Gauss Law which says that diversions of the electric excitation should be equivalent to the charge contained within the volume where Rho is the volume charge density and the diversions of the magnetic field B is equal to 0.

(Refer Slide Time: 03:56)



When you use these equations and manipulate them you essentially get when you take the diversions of this equation and use the values accordingly what you get is the charge continuity equation which says the diversions of the electric current density is equal to the negative of the time derivative of the volume charge density So when you apply certain manipulation on the second equation and substitute the values in the other equations what you will get is let us say if you take the diversions here and what you can apply the values in the other equations you will essentially get the last equation.

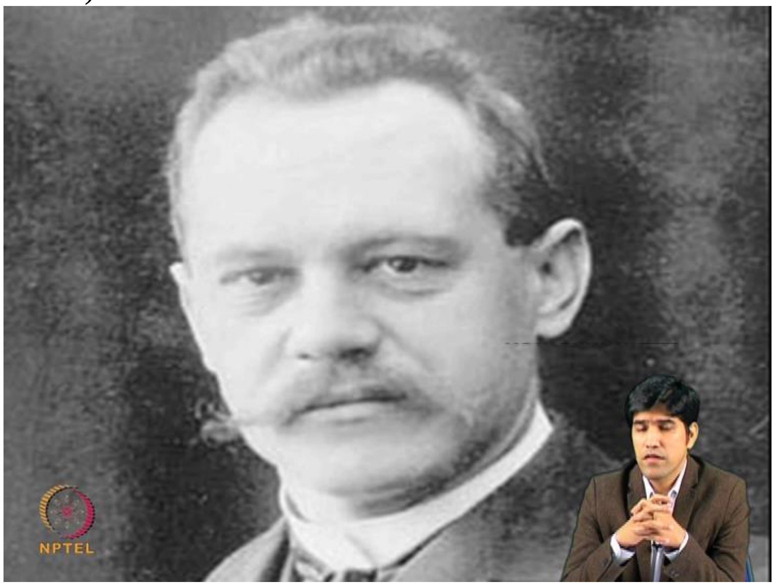
(Refer Slide Time: 04:35)



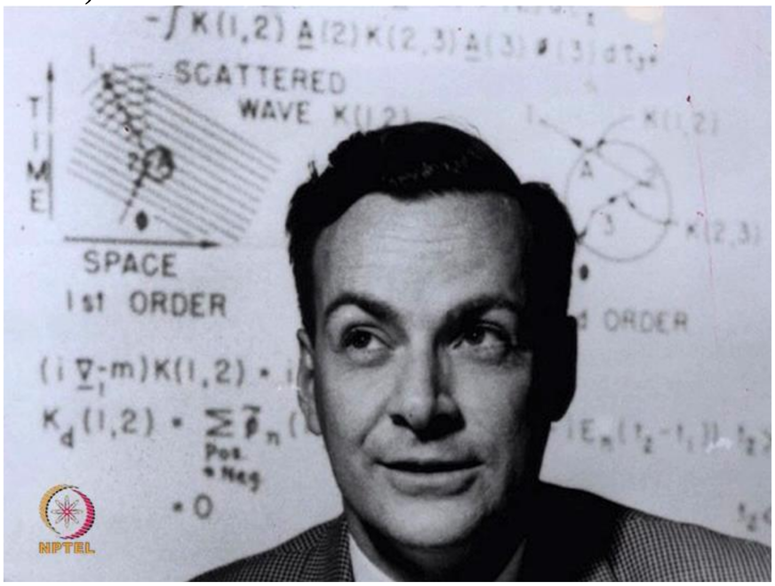
So there was a reason why I mentioned that the E and B fields are called as electric and magnetic field because there is something fundamentally similar about them. And that is something that you can see when you look into the Lorentian force. So Lorentz force is nothing but the force acting on the unit electric charge and also on the charge that is moving at the velocity V. So if you see the value of the Lorentian force is given by the sum of the forces acting on the charge.

One will be due to the stationary charge, the other one will be the charge which is moving at a velocity v. So since E and B but nit E and H that is involved in the force definition E and B are very similar kind of quantities. So that is why it is rightful to call electric field and magnetic field as E and B and not E and H as contrary to many Engineering literature I am using the term here.

(Refer Slide Time: 05:40)

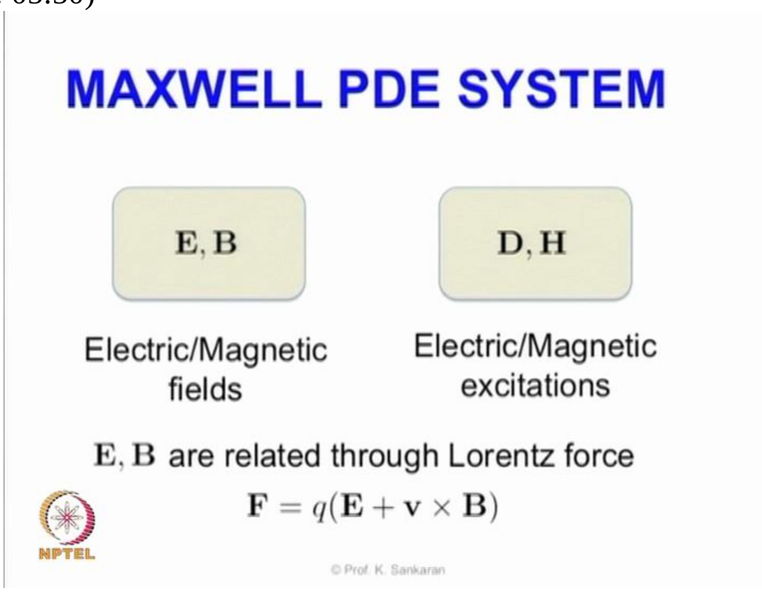


(Refer Slide Time: 05:44)



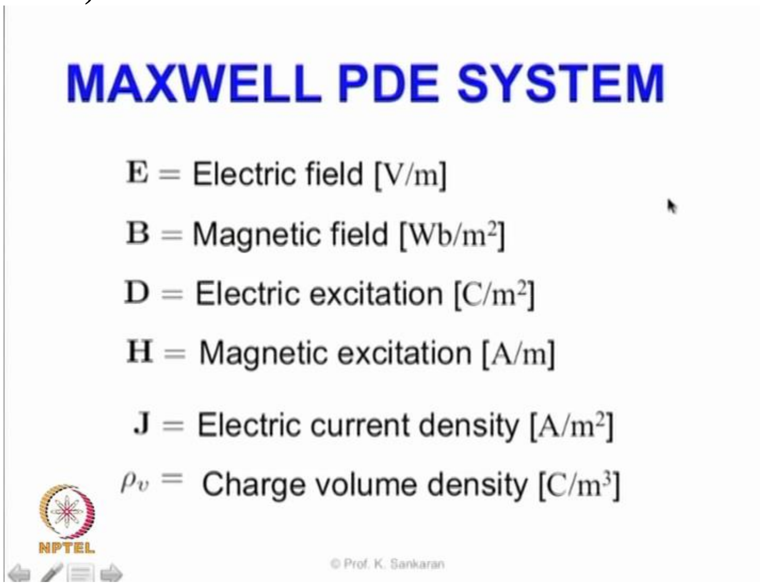
And if you look into some of the very well known physicist Sommerfeld and also Richard Feynman they have always used this form of notation.

(Refer Slide Time: 05:50)



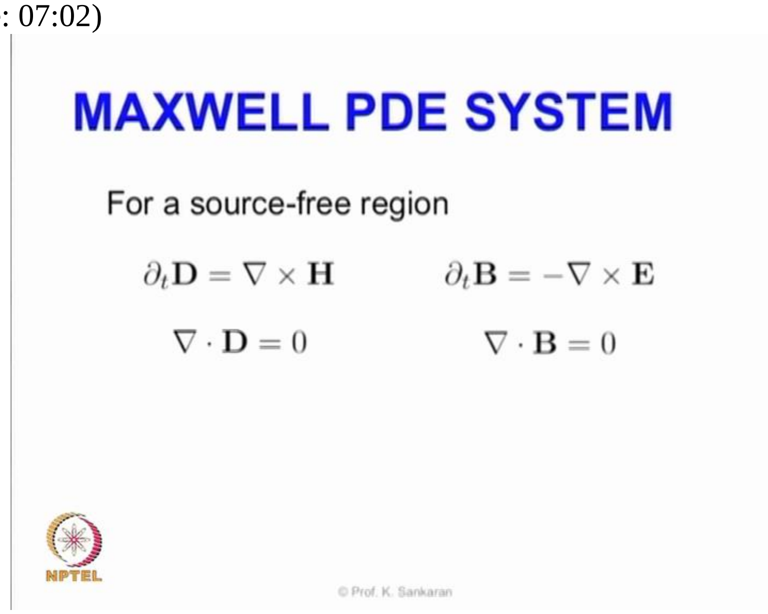
So we will use electric field and magnetic field given by E and B and we will use the word D and H as the electric and magnetic excitation and this is also something will become clear when you look into the Finite differencing algorithm that we will have E and B that are in the same space whereas D and H will be in a different space.

(Refer Slide Time: 06:12)



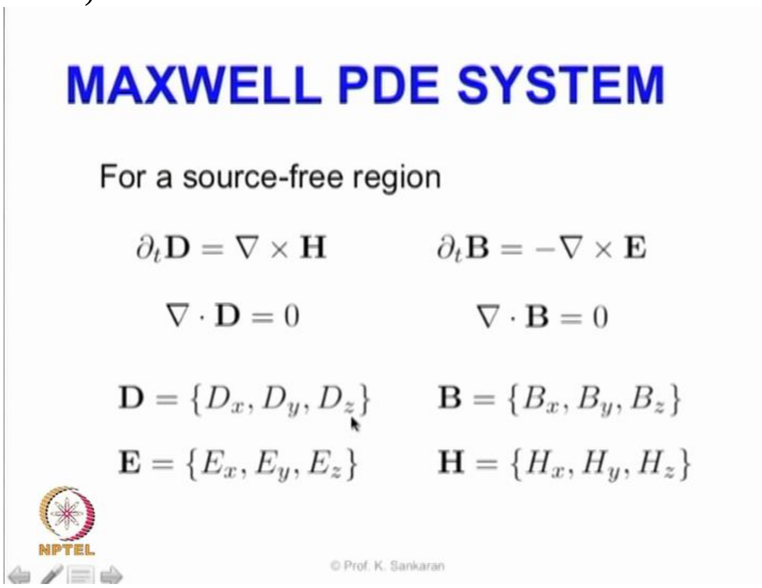
And the D and H are the Electric and Magnetic excitation and the J and Rho v are the Electric current density and Charge volume density respectively and the units are given here. It is also interesting to look at all these quantities as densities of some fundamental quantities. So electric field will be the line density of the voltage. Similarly electric excitation is the surface density of the charge and similarly when you look at magnetic excitation it is the line density of the current and the Electric current density is given by J and the Charge volume density is given by Rho v. So in simple sense what I wanted to say is they are all of them are some kind of densities of some functions.

(Refer Slide Time: 07:02)



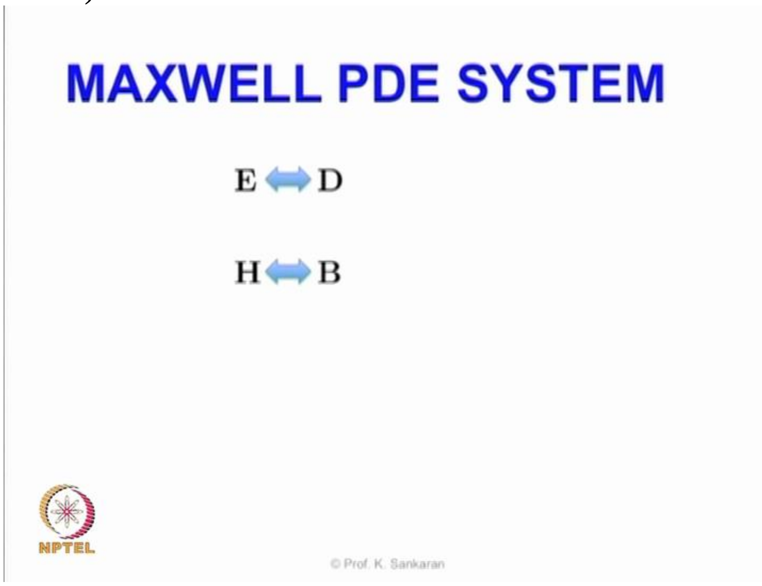
So if we take a source free region what we will have essentially is we wont have the J component in the equation. You will simply have the value given by the time derivative of the D is equal to the curl of H, similarly the time derivative of B is equal to minus curl of E. And the diversions of D will be 0, and the diversions of B will also be 0.

(Refer Slide Time: 07:29)



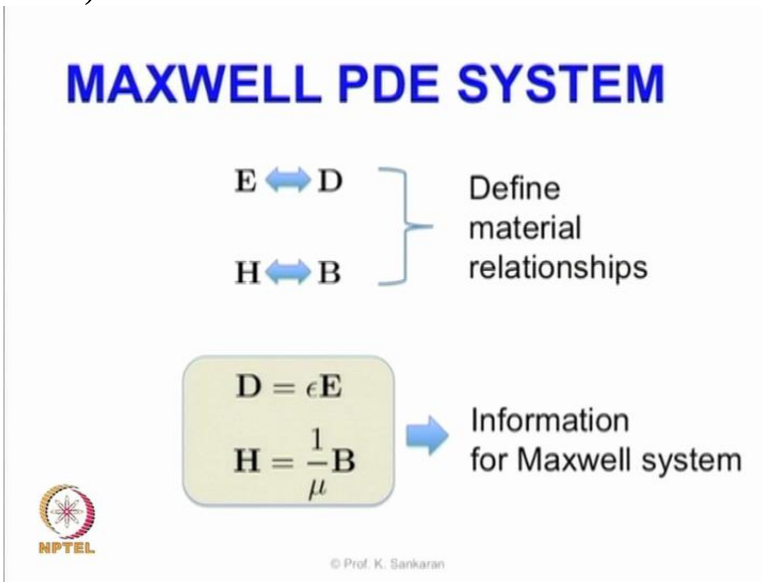
As you know these are all field quantities and three dimensions and they had individually three components where the Dx, Dy, Dz are the scalar components in x, y and z direction for D. Similarly you will have for other field quantities their respective field components. It is important to know so far I have not used any material components, when I am saying material components what I mean is the permittivity and the permeability.

(Refer Slide Time: 08:03)



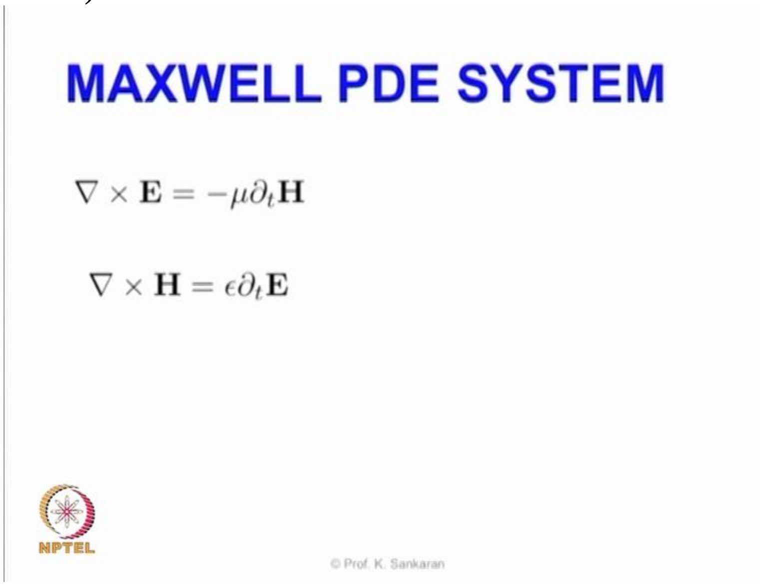
And this will come into play when we are relating B and H or D and E. So that is what we have to define before doing any mathematical modeling so the relationship between D and B H and B will be given by the material relationships. In other words the permittivity and permeability.

(Refer Slide Time: 08:23)



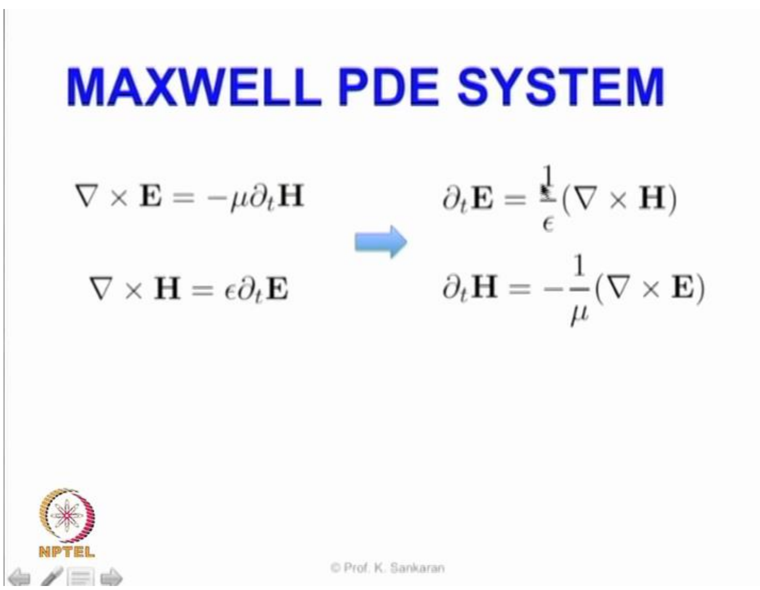
So epsilon is the permittivity and Mu is the permeability. This is the information that we will be using in the Maxwell system.

(Refer Slide Time: 08:35)



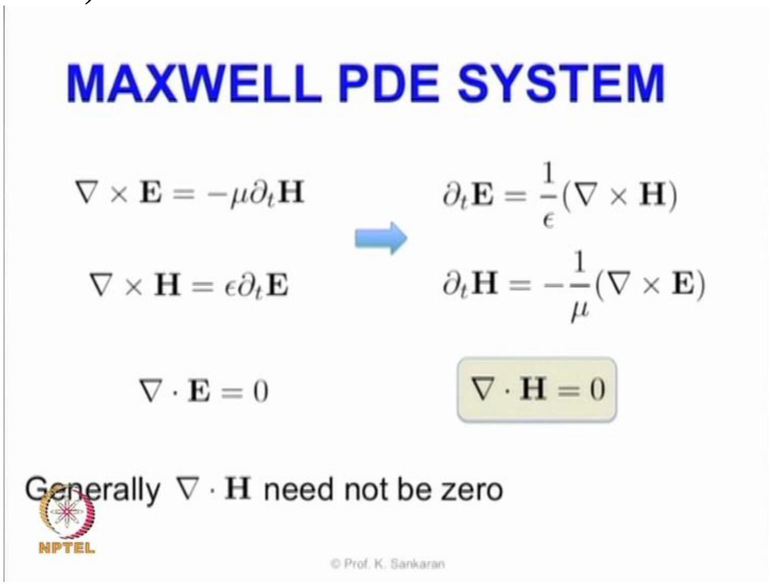
So if we can use this information in the Maxwell system what we essentially transform B on the right hand side will become Mu H and the D on the right hand side of the second equation will become Epsilon E.

(Refer Slide Time: 08:52)



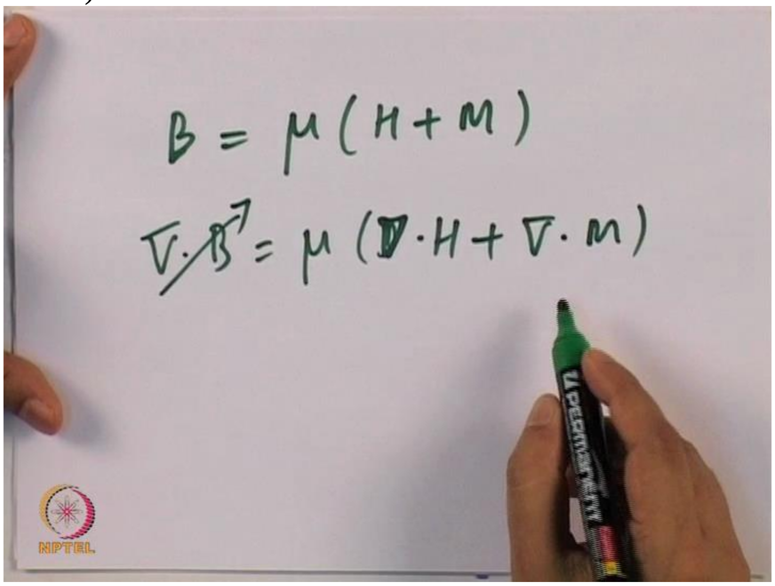
And similarly we will bring the time derivative on the left hand side and keep the spatial derivatives on the right hand side. We will have an equation of this form.

(Refer Slide Time: 08:59)



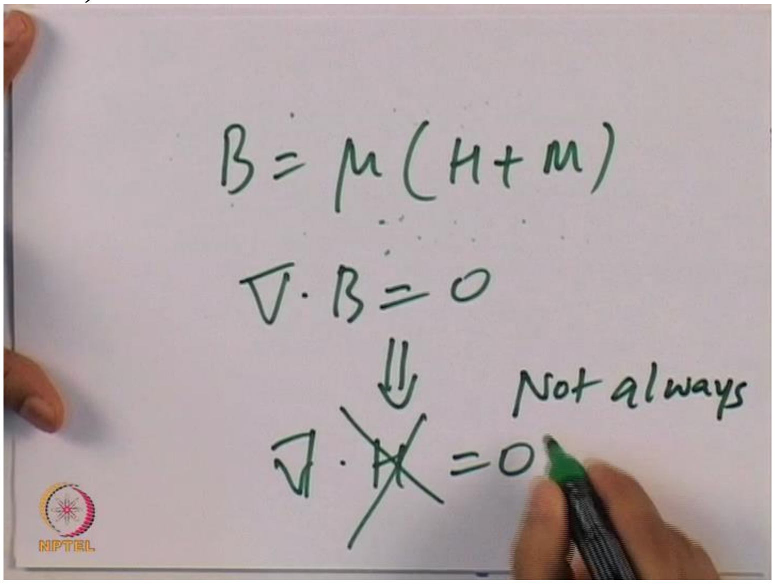
And we can set diversion of E is equal to 0 and diversion of H is equal to 0. It is important to notice that diversions of H need not be 0 all the time. This is something that we need to look into. Because the reason why diversions of v equal to 0 does not naturally give us diversions of H is equal to 0. Will become clear if we understand what is the relationship between B and H in a more general case.

(Refer Slide Time: 09:32)



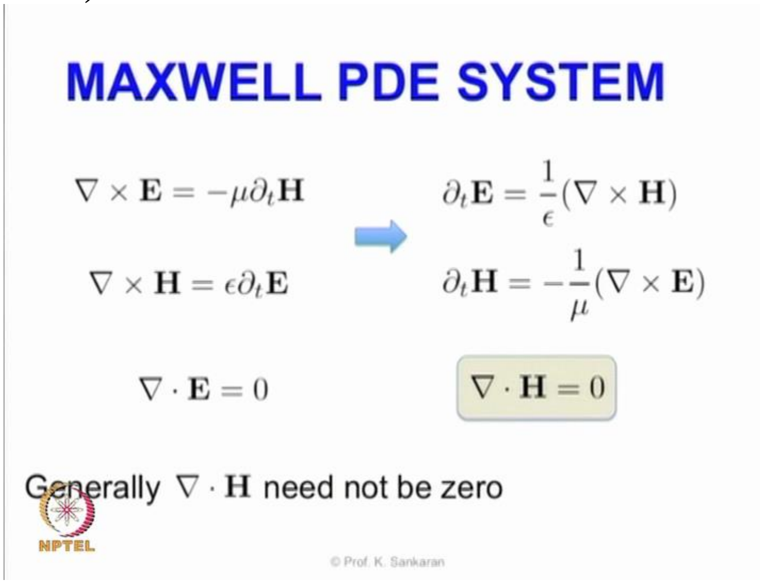
So if you have B which is given by; these are all vector quantities so in the general case the B value will be given by Mu times H plus. So M will be the magnetization parameter. So when we say the diversions of B is equal to then we will have two components here which is basically Mu times the Diversions of H plus the Diversions of M. So when this becomes 0, it does not mean that this becomes 0 because there might be also component that is coming from the diversions of M.

(Refer Slide Time: 10:14)



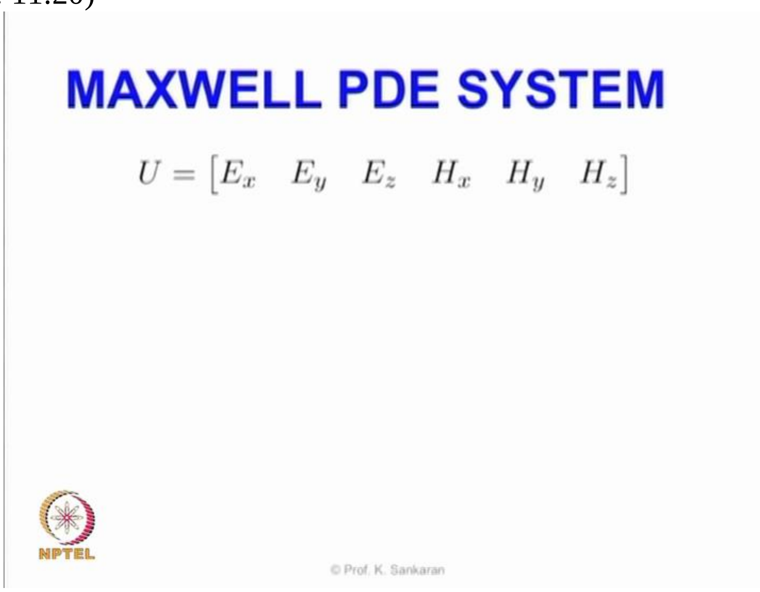
So it is important to know since we have an general equation which is basically saying B is equal to Mu (H plus M), we have to always account for the magnetic component as well. So diversions of B equal to 0 does not mean always the diversions of H is equal to 0. So this value will become 0 only when the magnetization component is also 0. In certain magnetic material you might be knowing that you might compute that diversions of B will be 0. But the there will be still some diversions of the magnetic excitation which is H.

(Refer Slide Time: 11:05)



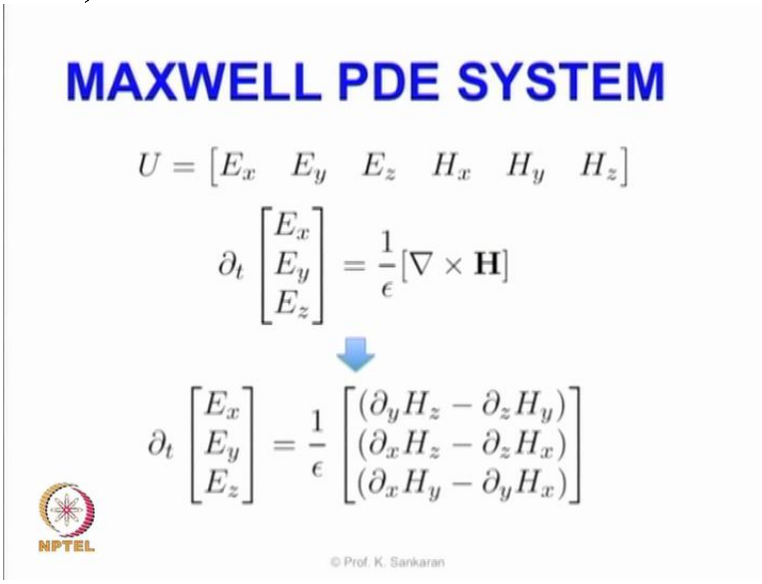
So with that being said it is important to notice that this is not the general case even when B is 0. It is enough to know this, but in this case let us assume that the magnetization component is also 0 hence we will have the diversions of H is equal to 0.

(Refer Slide Time: 11:20)



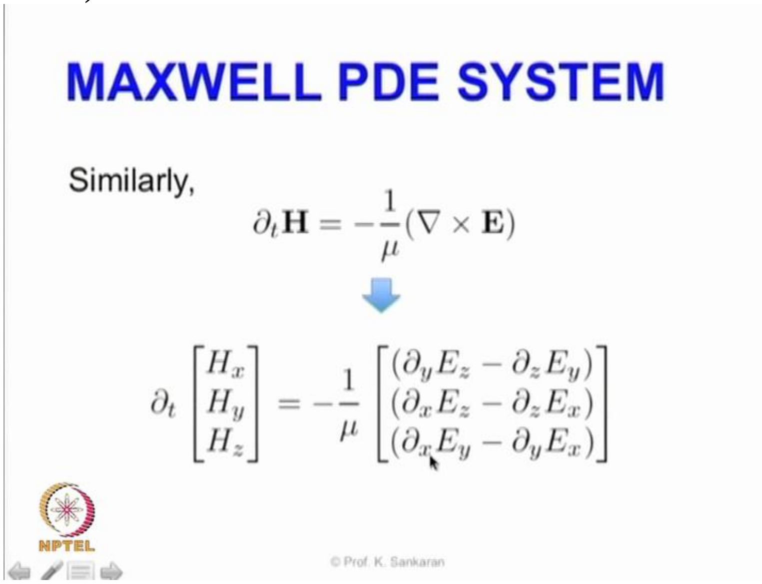
For a general problem as I said we can combine all the components into one vector which we call it as U, and this is vector quantity.

(Refer Slide Time: 11:33)



And we will have the value of the first curl equation written in the matrix form where you will have the individual curl components given here, so if you expand this you will get three components the curl will have three components the X component will given by (DyHz minus DzHy) similarly the Y component will be (DxHz minus DzHx) and the z component will be (DxHy minus DyHx). And remember these are all partial differentiation not normal differentiation, so you have to pay attention to that.

(Refer Slide Time: 12:15)



So similarly what we will get is also a set of three equations for the Maxwell curl equation related to the partial differentiation with respect to time for H is equal to the curl of E. And we will get three components for the x, y and z for Hx, Hy and Hz. And so far we are still been in the continuous case. Continuous because still we are partial differentiation with respect to time which is continuous and the partial differentiation with respect to x, y and z which is also continuous.

So what we will do now is we will find a way to use the finite differencing algorithm that we have learnt in the previous lectures, to derive the finite differencing algorithm and this will give us a very nice starting point to model some of the problems that we have in Engineering applications. So we will get back to you in the next module.

Thank you.