Computational Electromagnetics and Applications
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Lecture 01
Finite Difference Methods - 1

Good Morning! So welcome to the new lecture series on Computational Electromagnetics and its Applications. So, this particular topic is quite interesting for people working in industries or in Academia; mostly focusing on modeling and also modeling related to experiments. Computational Electromagnetics has been a topic that is gaining advantage and attention since the last 50 years owing to various changes in computer architecture, availability of fast computers, and also numerical solvers that are able to moral quite accurately quite complex problems.

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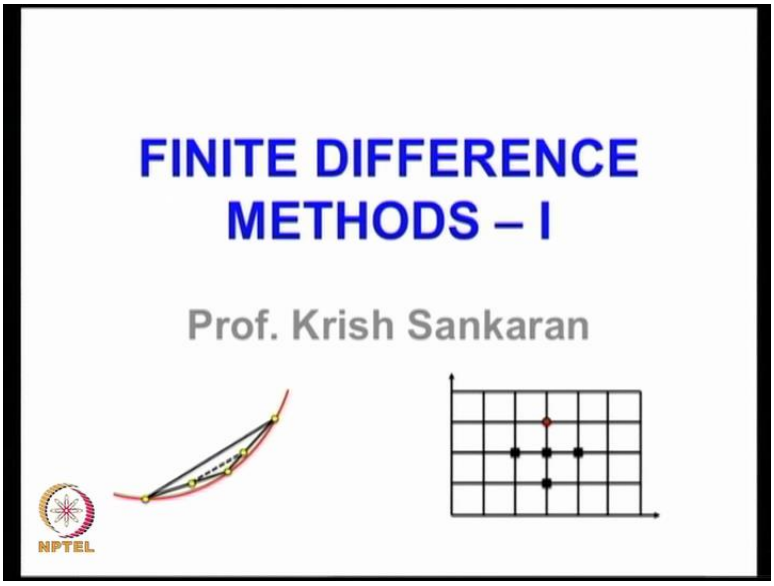

So in this particular module series we will focus on finite difference methods.

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So we will see what is going to be our motivation for this particular lecture. And we will follow the order as it is given here. Following the motivation we will look into the background and background of particularly the finite differencing method. And we will introduce certain finite differencing schemes. So let us go directly to the motivation;

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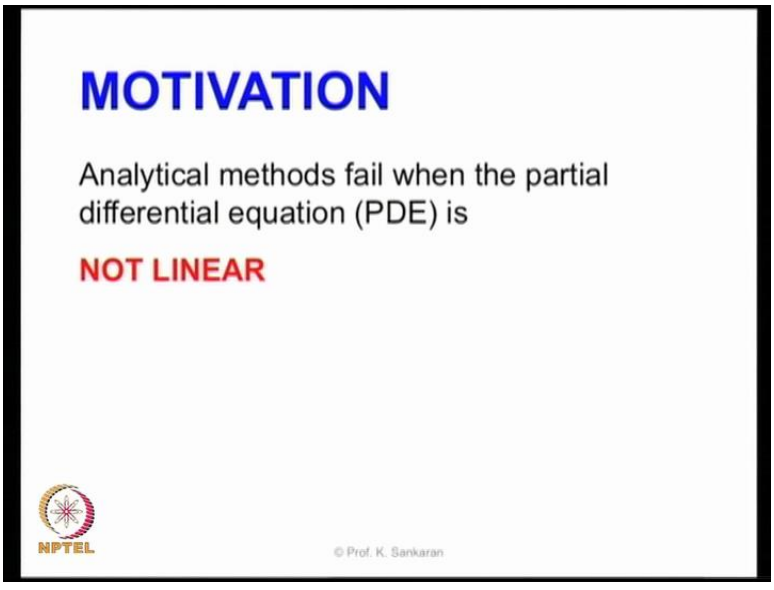

The motivation for Numerical Methods is we cannot use analytical methods quite straight forwardly for most of the practical problems, because practical problems has always certain sense of non linearity. So, analytical methods fail when the partial differential equations are not linear.

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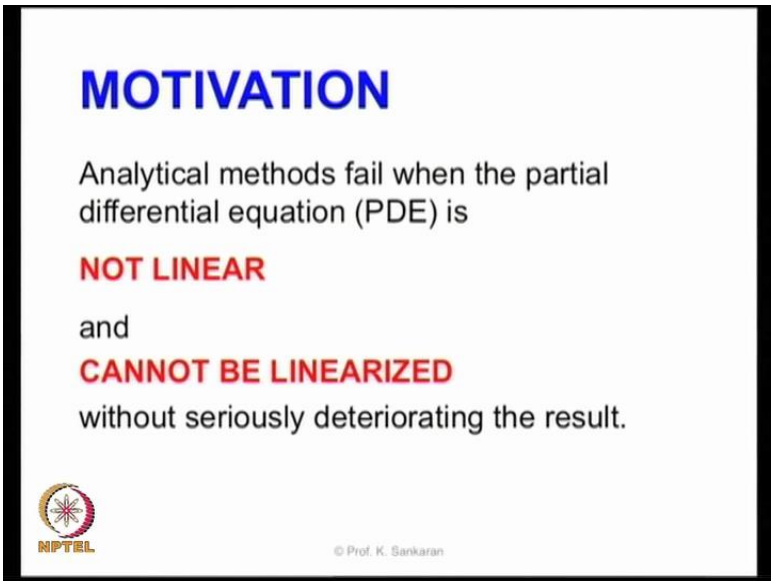

We see that linearizing create serious errors. In other words, inaccuracy in the solution space. So, that is the first motivation for going into the non analytical or any kind of numerical methods.

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The second thing is what happens when computational domain is complex, what I mean by that is for example here, let us say we are talking about a spiral antenna, which has various spirals several arms of spirals metallic with certain properties let us say conductivity, permittivity and permeability. And then backed by certain other materials which could be dielectric medium or whatsoever. And then has also metallic surfaces on the side. So this is quite a complex structure for us to analytically model. So, Analytical method also fails when the domain becomes complex.

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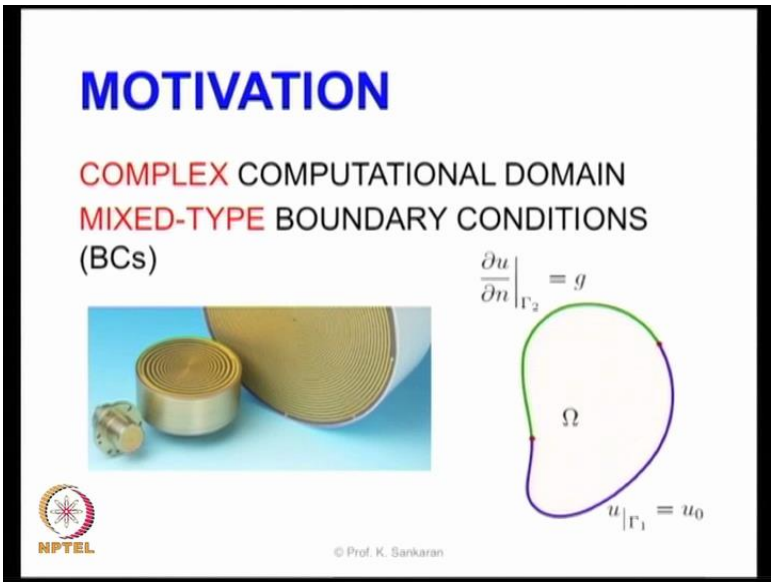

And also when we have boundary conditions let us say, we have a set of boundary conditions defined on the gamma 1 which is equal to u of 0 let us say this is a very hard boundary condition. And then other part of the boundary of the domain let us say gamma 2 has certain flux normal component of the flux define, so this is called as the mixed boundary condition. So when we have two three different types of boundaries, also in those conditions it is very difficult to use if not impossible to use analytical methods. Likewise for example when we have boundary conditions which are also time dependent; let us say this is in the previous case we had straight forward as a constant.

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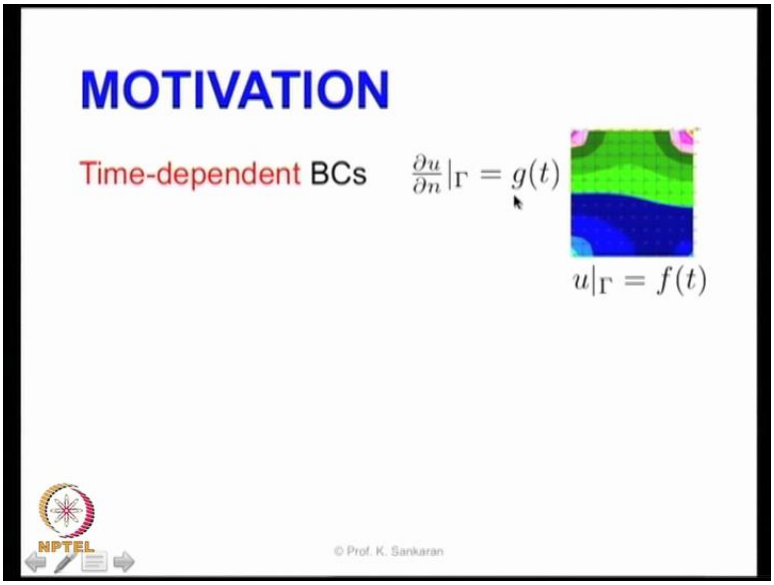

Whereas in the other case when we have instead of constant if we have a time variation also on the boundaries. So then also the analytical methods become quite difficult. For example here, we have an aluminium plate (())(04:08) into you know various units and then we see that the four corners are having different boundary conditions which are dependent on time.

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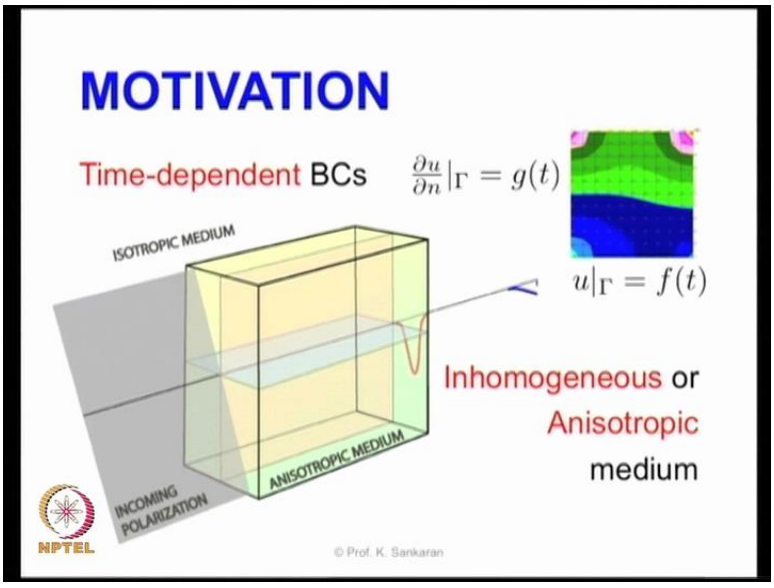

The last but not the least motivation is when you are talking about inhomogeneous and also anisotropic medium. For example in this case when you are trying to model a medium which is

anisotropic in a particular direction and then isotropic elsewhere, so what do you see is it's very difficult to model such mediums accurately using analytical methods.

So with this we see that what we have in the case of finite difference method or any numerical method is it gives us quite a bit of flexibility and also face out the disadvantages or the lack of flexibility of the analytical methods for modeling more accurately complex problems. It had been said I am going to go into one of the most basic methods which is called as the Finite difference method. Without specifying whether frequency domain or a time domain, we will just look at the special discrimination for the time being.

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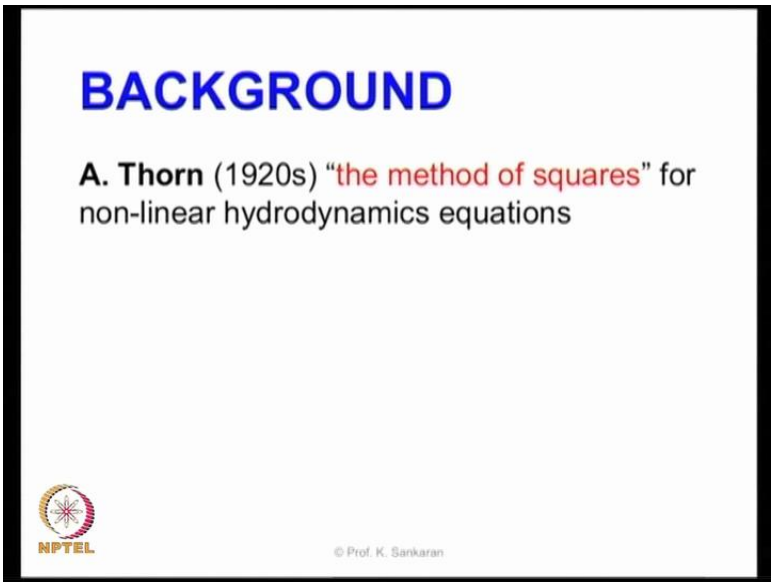

So the method itself was introduced in 1920s by Thorn. And he actually named the method, the method of squares. And it was mainly used for nonlinear hydrodynamics equation. Because they found out that it's very difficult to use classical methods like we talked before. So he invented a new method which he called the method of squares to model nonlinear hydrodynamic problem, that is the background.

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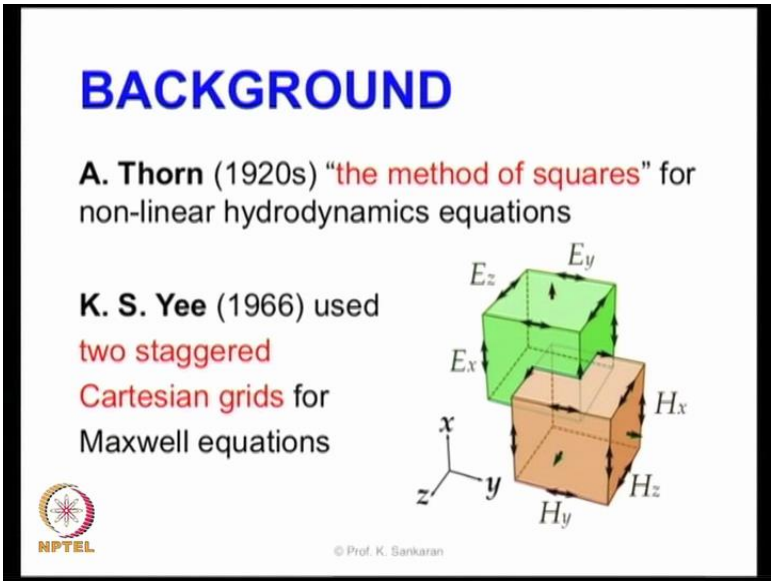

But in Electromagnetics the scheme itself was broaden by Ken Yee. So Ken Yee introduced the method in Electrodynamics as you can see for Maxwell equation using two staggered partition grids. So what I mean by staggered partition grid will become more clear later on, but the pictorial representation here gives us a little bit understanding. There are two grids one is green grid the other one is the brown grid and they are staggered in space and in fact they are also staggered in time. This is something we will see later on, but this is the most important point. So what I want to do also give a little bit background on this methods. I mentioned that algorithm itself was introduced in 1966 by E but the method itself did not gain attention for almost a decade. Nobody really bothered about using the method for almost a decade.

So that being said what was the other methods or what was keeping numerical scientist or computational scientist busy. They were more busy with a well established method called as method of moments. Which we will see later on. It was much more evolved it had much more numerical and mathematical tools involved in it so people were basically using method of moments. They did not pay attention to find a different time domain or find a different method in general for almost a decade. There were other problems also related to finite difference method. The method itself works fine but somehow if you wanted to do any practical problems, you need to define the method along with the boundary conditions, if not you are going to recharge-

assimilate a very large problem although what you are interested is very small area of computational domain.

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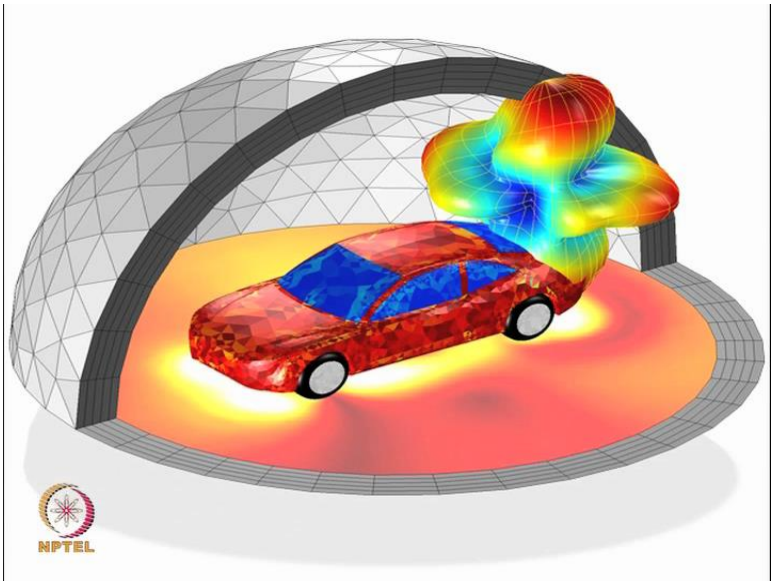

So, if for example if I wanted to simulate scattering problem, let us say I want to understand what is a scattering of a particular object, let us say I have a car and I am talking about electromagnetic waves scattered by the car. So what I would do is I would model the car surrounded by certain atmosphere, maybe I am using a standard air, free space but since I do not have a proper termination so I have to simulate the car surrounded by a very very big volume, and although I am only interested is what is going on around the car, I need to simulate a very big problem because I did not have very accurate and stable boundary conditions. So that was one of the biggest problem of this method for a long long time, that changed over a period. So what happened was in 1975 it was both Tafloveand Broadwyn, they brought certain improvement to the stability of this method they basically computed the stability criteria and they also improved the methods functioning for a steady state solution for a sinusoidal input and also in 1977 Holand, kuns and Lee applied this method for a broadband application. They basically send in a pulse and simulated it for a broadband. But all these things are only still working on the method itself. And later on someone called as Murr came and did something called as absorbing boundary condition.

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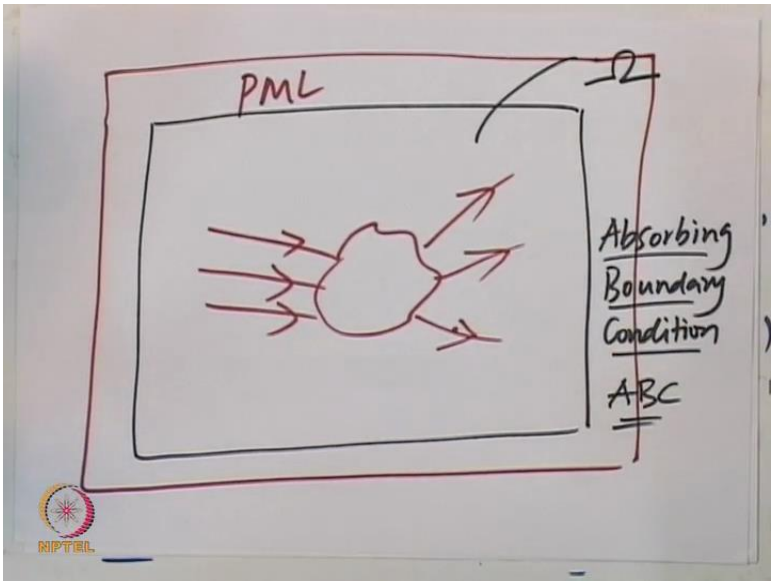

Let me explain this in a slide. Let us say this is a scatterer and then the scattering is happening, so there was a possibility to truncate the entire problem using a certain absorbing boundary conditions. So we will not focus too much on the absorbing boundary condition right now , we will talk about it later on but its important to know that the computational domain let us say we call it as omega, it is being truncated using certain conditions here at the boundary and this is what we call it as ABC : Absorbing Boundary Conditions. And some called as Beroje a French Engineer working for the electricity corporation of France. He broadens an idea which revolutionized and even popularized this method further called as perfectly matched layer.

We will talk about this also during this course what is the meaning of perfectly matched layer and how does it work. But right now for the motivation it is enough to know instead of putting one single boundary what Beroje did was quite revolutionary. We will see why it is revolutionary later on. So he put a truncation layer and this is the perfectly matched layer.

So we will discuss all these terms while we go forward. But, I wanted to give you a little bit on the historic front of the development of this method. So it is quite clear there were quite a lot of other things that were required for a method to be popularized or widely applied. This is not just the method itself but also the tools that are required around the method. It could be on the stability conditions, it could be on the requirement of certain truncation techniques, or perfectly match layer so on and so forth.

So with this we will start looking at basically what is the meaning of this finite difference method. We will look at it, what are the structure of it in the next slides.

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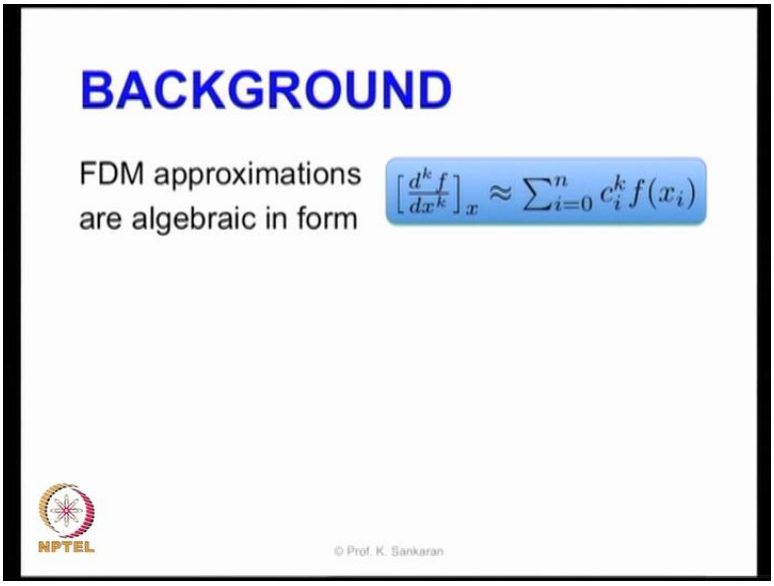

So the finite difference method itself is an algebraic form. Let us explain this a little bit further, let us say I am interested in finding out the value of certain differential on a particular grid space.

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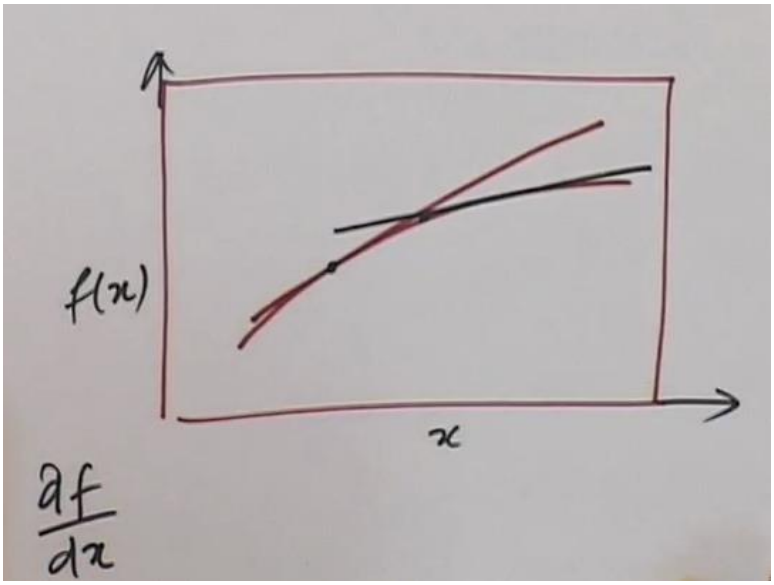

Let us say I have a grid space, and I have to compute the value of my function on the grid space. So, let me say I am interested in finding out what will be the functions first derivative on each of these points. At Point number 1 this is the derivative, Point number 2 this will be the derivative and so on and so on and so forth. So what is happening here is, if this is x and this is f of x what I am doing is I am computing the value of f of x and each of these points and then I am differentiating.

What I am interested is, I am interested in the different by dx, this is what I am interested in. So, if these are the two axis and umm the X axis is the independent variable and the Y axis is the dependent variable, I am computing the df by dx in each point. Algebraically this is equivalent to say I am writing down this as a function of certain weights.

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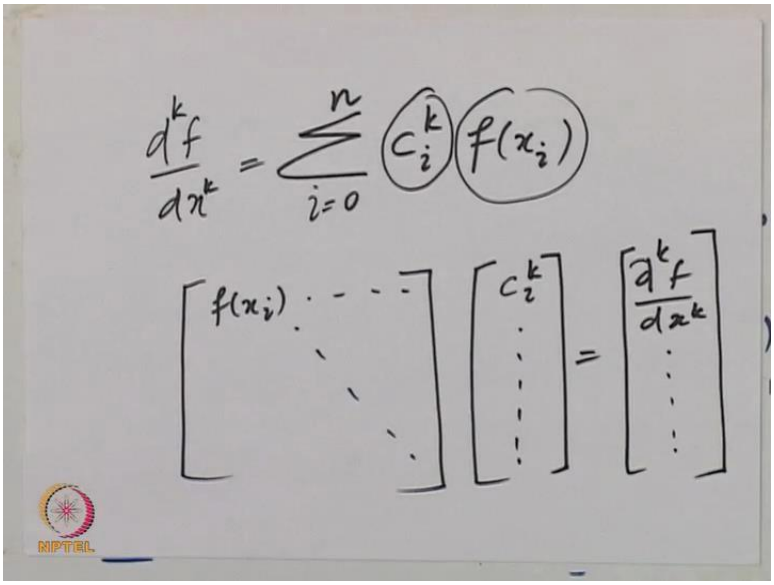

This will become clear later on, but what will essentially happen is if I have a function df by dx, let us say I am interested in first differential, second differential so on and so forth. And this is basically given as a sum of certain weights, which I call it here C, I, F, K times the function itself define in those points. So I goes from 0 to n, if I equal to 0, we are talking about only one thing and so on and so forth.

So in this case, what is happening is there are certain weights, I am multiplying on this so you can basically write this as value which is containing f of x i, and then there is certain weight

functions c of k, will be the value of d to the power of k of f by dx to the power of k. So this essentially algebraic equation, where the weight functions are multiplied for those value so that is what we mean by saying the finite difference approximations are algebraic in nature.

Right now if you are not able to understand this, it is totally fine we will explain this step by step later on. Right now what you need to know is the finite difference approximations are basically algebraic. They are in the form an algebraic equation.

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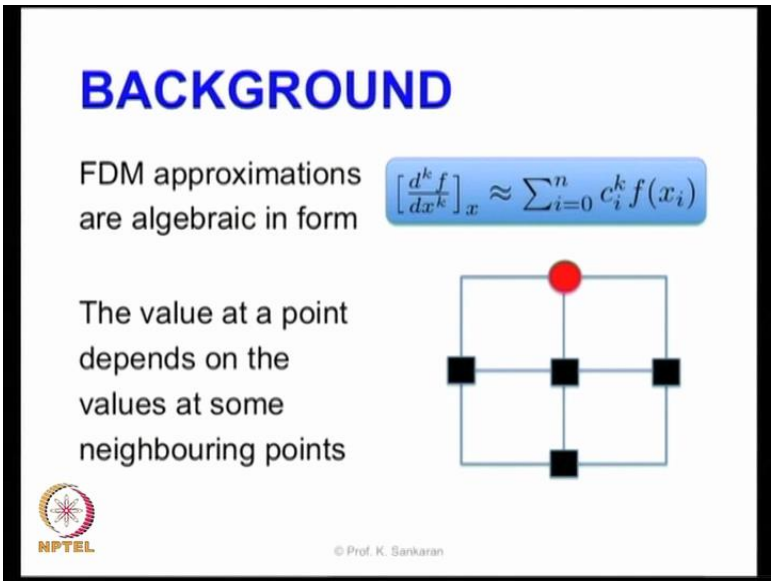

The second thing is as you can see here the value at a point depends on values at some neighboring points. What I mean that by this is, I know the value of these points given in black. I am interested in knowing a value let us say at this point, so the value at this point can be found out the using the values at these neighboring points. So we are talking only in terms of special derivative here ,special aspect here but if you also take the time axis into play so you can compute the value of certain points in space and time using the values of certain neighboring points in space and time as well. So the entire process of finite difference goes through 3 simple steps. So when I say three simple steps what are those steps? That what we are going to say. Let us say we take simple one dimensional example to illustrate the logic and we can expand it to multi dimensions.

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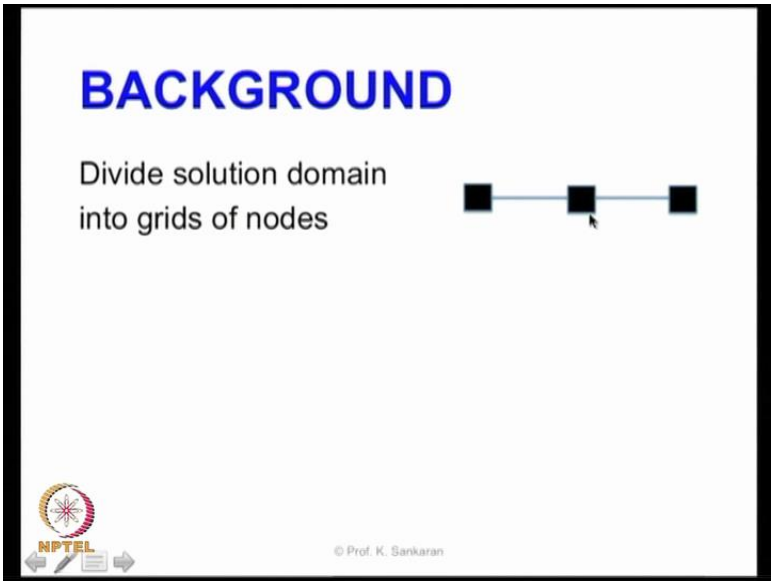

Let us say we have a domain which is a one dimensional domain given by this line, what we are essentially doing here is we need to divide this solution domain into grids of nodes. Say for example 3 nodal points. Of course when you do numerical methods your domain will be much larger and your number of nodes will be also much larger, here we are illustrating it only for the sake of simplicity with three points.

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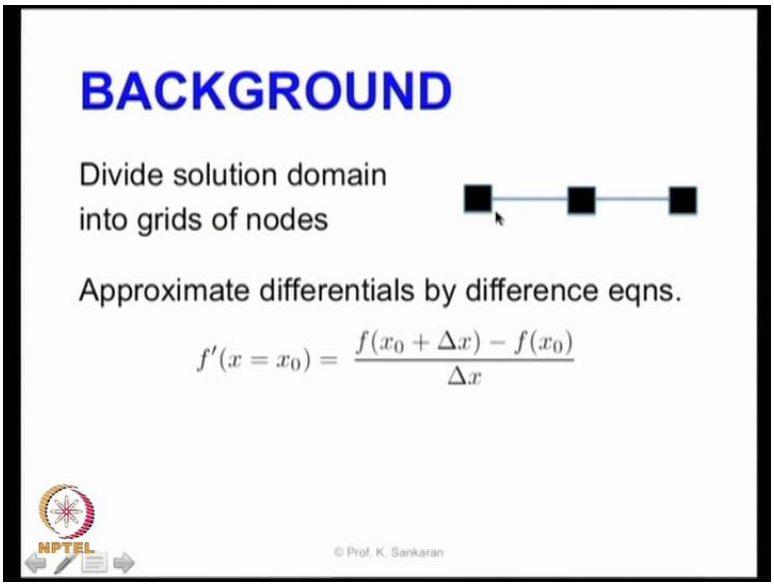

And then the second step is, you are trying to approximate the differentials, what I mean by differentials is the differential equations by certain difference equation. For example in this case this f prime at x equal to x not is nothing but first derivative of f with respect to x at x not is equal to certain value that we are computing for f at x not plus delta x minus f x not divided by the step size in x. So the step size in x is the value here. In other words we can write this in a much more simple way as follows:

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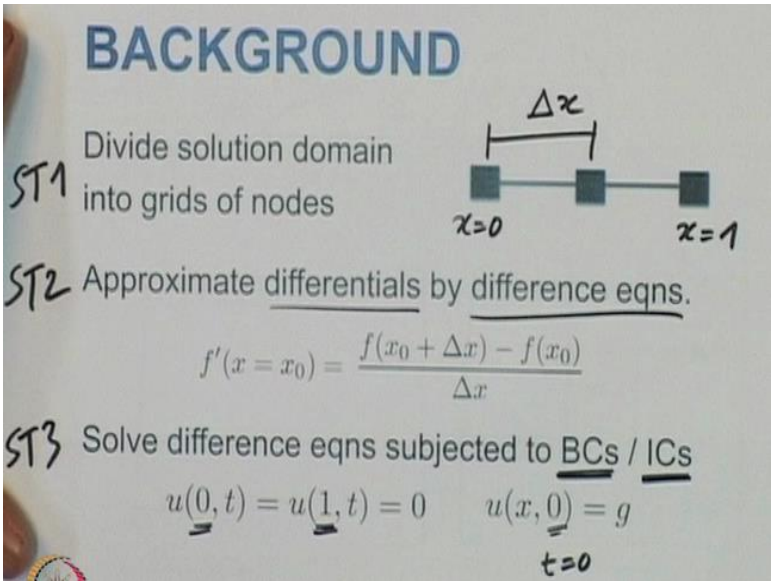

So let us say this is the grid and what we are having here is this is the size of delta x. So what we are doing next is we are trying to go forward with certain difference equation in order to approximate the value of the differential at certain point. Likewise as you can see we have to have certain boundary conditions and initial conditions BCs are the boundary conditions and ICs are initial conditions.

So, when we say boundary conditions we are interested in the x coordinate of the boundary. Since it is a one time (())(18:19) problem what we are talking about is; let us say x equal to 0 here and x equal to 1. So we are skipping certain values of u at x equal to 0 here and x equal to 1 as 0 for all time variables. Similarly, we are talking about certain initial conditions if it is a static problem then we are not interested in the time variables, but in the case of the time varying problem we set the initial conditions as time equal to 0 certain value. So this value is a t equal to 0.

As you can see the number of initial condition also changes depending on the order of the problem itself that we will see in the next slides so what you need to know is when we have a problem, the solution process for finite difference method is you create a step 1 and then you do the following thing you divide the domain into certain nodal points and then in the step 2 you approximate the differentials using certain difference equations and then in the step 3 you use certain given boundary conditions and initial conditions so that you can compute the value of the problem space or the solution space serially in time.

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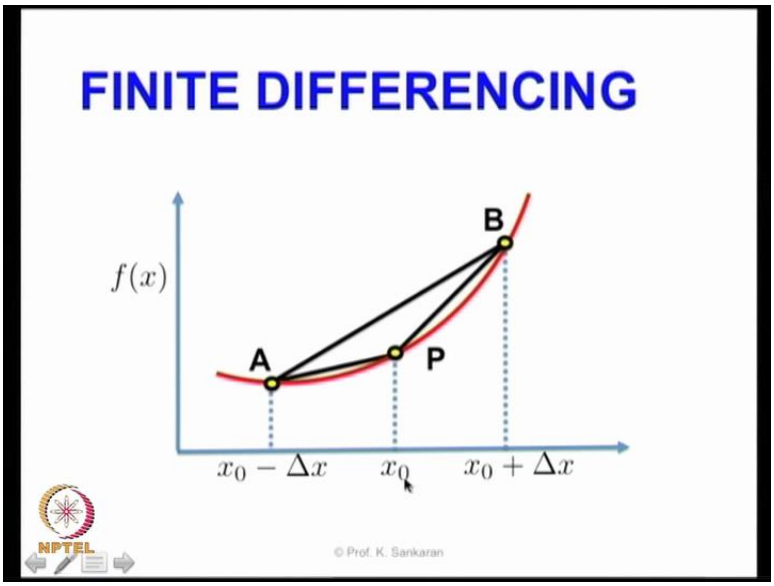

So let us look now at certain fundamental form of finite differencing. So let us say we have a problem where the solution is f of x and the value of the x coordinate is given we have a step size of delta x, this is x not a step before is x not minus delta x and the step one forward is x not plus delta x.

So let us say we are interested in knowing what is the value of certain differentials based on this f of x. So let us say we are interested in knowing the value of the first differential of f of x at the point x not. So we can do this in different ways.

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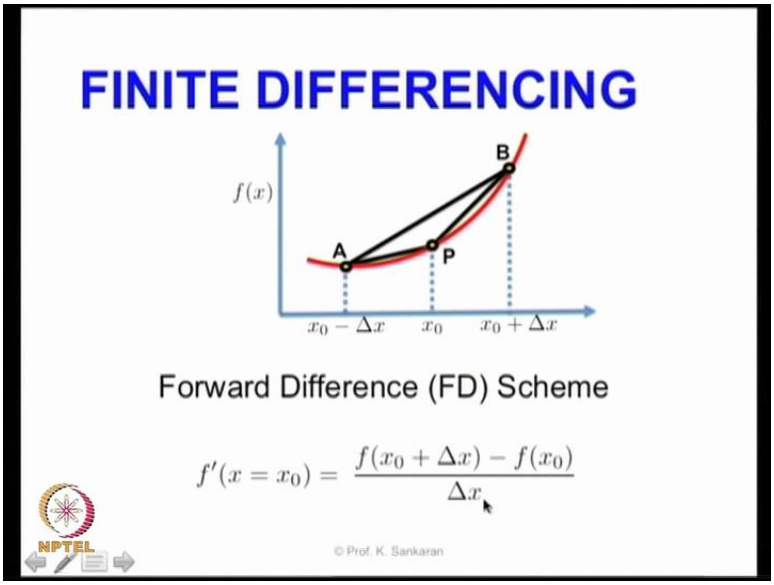

The first way to do that is we take the forward differencing. What we mean by forward differencing is we say at x not the value of f dash that is the first differential at x equal to x not is going to only depend on the value that is one step in the forward direction from x not and x not itself.

So it does not really matter what the value is here. And the value is given by this equation where we are taking the value at d minus the value at p divided by the step size itself. So this is a very simple forward differencing scheme.

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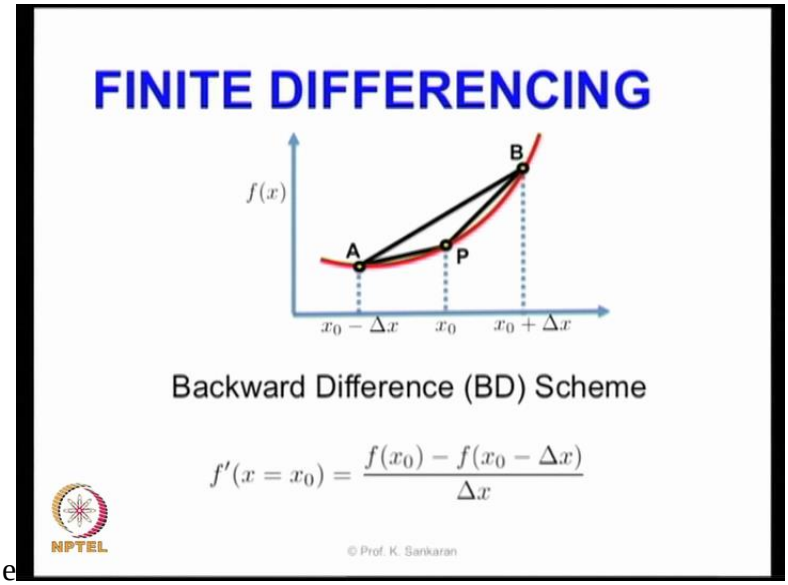

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Likewise we can do the same thing with backward differencing scheme. So here we say that the value at x not the first differential of f at x not with respect to x is going to depend only on the value at x not minus delta x and x not itself. It does not matter what the value is here it is going to depend only on this. So as you can see both these methods whether you are doing a forward differencing or backward differencing, there is a kind of a bias. The bias is it is saying what ever going to be the value of differential at one point is going to be only dependent on what is in the forward direction or in the backward direction.

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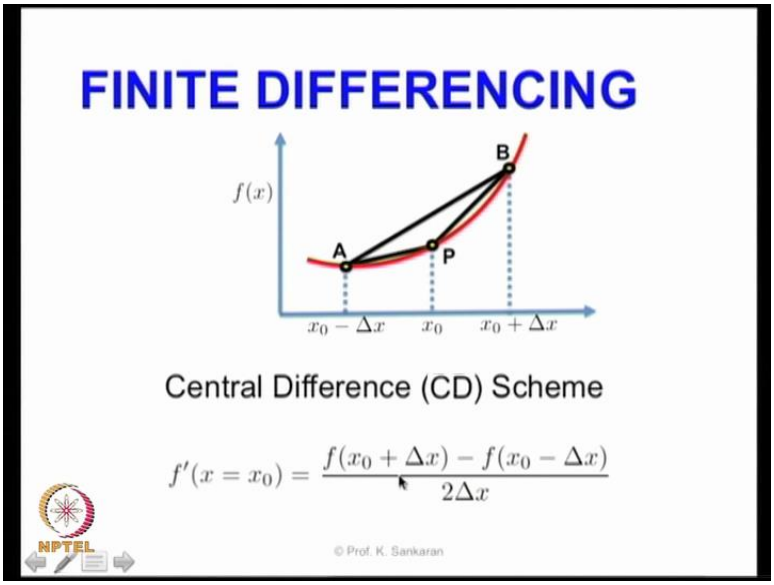

So a safer bet would be is to take the value at both x not minus delta x and x not plus delta x; and that is exactly what we do in the scheme of central differencing, as you can see here.So what we are saying is instead of going and doing the differencing either only focusing on x not plus delta x or only focusing on x not minus delta x let us take both of them. That is what we are exactly doing here. We are saying the value of the first differential at x not is going to depend on both these values at x not minus delta x and x not plus delta x and of course here would be twice the delta x. delta x here plus delta x here. And this is the central differencing method.

So let us summarize all of them in one slide; so we have the forward differencing scheme which is focusing only on forward value the backward differencing scheme which is focusing mainly on the backward value, and the central differencing which is taking both to delta x.

So with that being said we can stop here, and we can summarize what we have seen. So we have seen now the motivation for going into the finite differencing method, why we are approaching finite difference scheme as compared to analytical method and we also gave you the historical background on the method itself. And we have introduced some very basic notions of differencing. Of course we have to develop them further if we wanted to model any meaningful problems for applications in electromagnetics. So with that being said we will come back again and we will focus on the further techniques in finite differencing. Thank you