

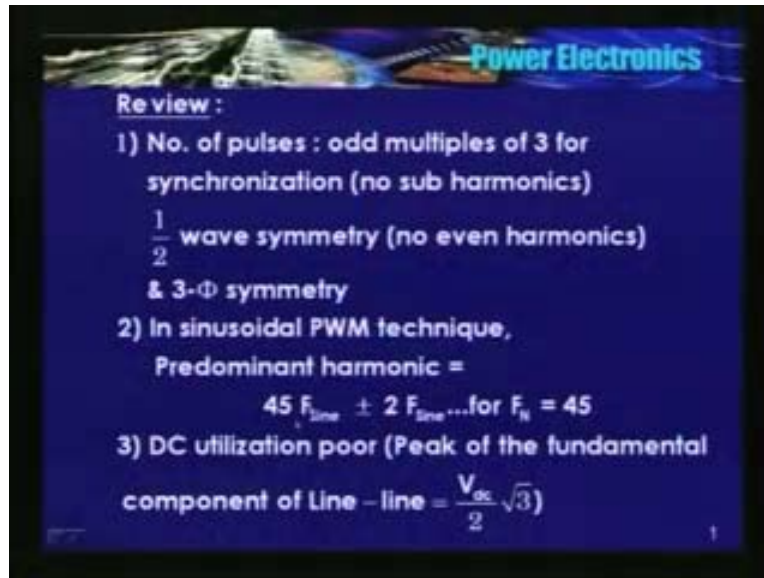
Power Electronics
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Lecture - 38

In my last class we discussed the use of pulse width modulation in inverter voltage control. See, I will repeat; pulse width modulation in inverter output voltage control, not inverter harmonics control, no. Pulse width modulation is used for **for** inverter output voltage control and if you switch the device in a particular fashion, harmonics can be eliminated. So remember, if you switch that device in a particular fashion, you can eliminate certain harmonics. So, if there are or if the number of pulses is an odd multiple of 3, then you have no sub harmonics, no even harmonics and there will be 3 phase symmetry.

We also discussed about the sinusoidal PWM technique, wherein, a sine wave of desired magnitude and a desired output frequency are compared with a high frequency triangular. The frequency of the sine wave or the modulating wave decides the frequency of the fundamental component of the output voltage and magnitude of the sine wave decides the magnitude of the output voltage and we found that the harmonic spectrum has a 2 side bands near the switching frequency or near the carrier frequency or near the frequency of the triangular wave.

See, in the last class **I did** I did show you the expression and we have taken the ratio to be 45. The ratio of the triangular wave to the ratio of sine wave, we took it as 45 and then the frequency of the predominant harmonic is 45 into the frequency of the sine wave plus or minus twice the frequency of the sine wave.

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We found that as the modulation index increases or as m increases above 1, the inverter from pulse width modulation **to a transfer** transfers to the 6 step operation. I will repeat it; till modulation index is equal to 1, inverter is under pulse width modulation control: above 1, it goes into a 6 step operation or a square wave mode.

The moment it goes to square wave mode, harmonics reappears. The frequency of the predominant harmonic in square wave is again the fifth and the seventh and so on, $6N$ plus or minus 1. So, using sinusoidal PWM technique, we could push the predominant harmonic to higher frequency till the rated **rated** frequency. But then we found that the utilization of the DC link or the input voltage source reduces or the utilization becomes poor because the peak of the fundamental component of the line to line voltage, we found that it is V_{dc} by 2 into root 3.

See, this is the peak of the fundamental component of the line to line voltage in a PWM inverter which is less compared to a square wave operation. **That is why we** that is the reason, in the sense, using PWM technique, we eliminated the harmonics. In the process the utilization comes down.

I told you that instead of using a sine wave, we could use any of the modulating waves having other waveform. But then the criterion is it should not affect the harmonic spectrum or it should not deteriorate the harmonic spectrum and you should try to improve the DC link utilization. So, that is the reason we added a third harmonic component. See here, that is the fundamental component.

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4) Can be improved by adding 3rd harmonic component to the modulating wave.

Expression for modulating wave = $1.15 \sin \omega t + 0.19 \sin 3\omega t$

∴ Peak fundamental component of $V_{L-L} = (1.15) \sqrt{3} \frac{V_{dc}}{2} = V_{dc}$

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It was just $1 \sin \omega t$, $V_m \sin \omega t$ or is equal to $1 \sin \omega t$, whereas, in third harmonic injected PWM technique, see, we have increased the fundamental or magnitude of sinusoidal modulating wave by 15%. See, it is $1.15 \sin \omega t$ plus $0.19 \sin 3 \omega t$. We added these 2 waveforms and we got a new modulating wave.

See, in this new modulating wave, there is no peak which appears in a sine wave. The area is improved here but then if you see the harmonic spectrum of the line to line voltage, there is no third harmonic **there is no third harmonic**. What did we achieve by adding a third harmonic component to the modulating wave?

We could improve the peak of the fundamental component that increased by 15%. How did it improved by 15%? Because, the peak of the sinusoidal modulating wave itself is being improved by 15%. So, **so** peak of the fundamental component is now 1.15 times root 3 V_{dc} by 2. That is V_{dc} itself. So, this is about sinusoidal PWM technique and a third harmonic injected PWM technique.

There is yet another PWM technique what is known as the harmonic elimination technique. It says, undesirable harmonics can be eliminated and the fundamental component can be controlled by creating notches at pre determined angles. I will repeat; see here, undesirable harmonics can be eliminated and fundamental component can be controlled by creating notches at pre determined angles.

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Harmonic elimination Techniques

Undesirable harmonics can be eliminated and fundamental can be controlled by creating notches at pre-determined angles

\Rightarrow If 'n' switchings / $\frac{1}{4}$ cycle

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Now, we will see how to determine these angles. Before that I will explain to you this statement. What it says is if there are n switchings per quarter cycle, remember, n switchings per quarter cycle, n minus 1 harmonics can be eliminated and you can control the fundamental. I will repeat; if there are n numbers of switchings per quarter cycle, n minus **n minus** 1 harmonics can be eliminated and the magnitude of the fundamental can be controlled.

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\Rightarrow (n-1) harmonics are eliminated & magnitude of fundamental can be controlled

\Rightarrow 4 switchings / $(\frac{1}{4})$ cycle

(a_1, a_2, a_3, a_4)

$a_1 < a_2 < a_3 < a_4 < \frac{\pi}{2}$

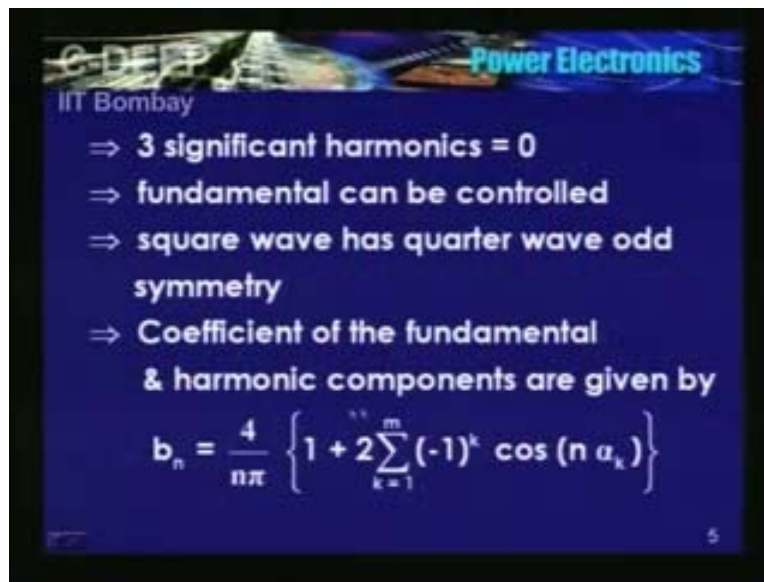
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See, I will explain to you. See, this is the pole voltage waveform of a square wave inverter. See, I am creating 4 notches **I am** or I am switching at 4 different places, at α_1 α_2 α_3 and α_4 . $\pi/2$ is here of quarter cycle; similarly, at $\pi - \alpha_4$, $\pi - \alpha_3$, $\pi - \alpha_2$, $\pi - \alpha_1$. Condition here is α_1 is less than α_2 less than α_3 which is less than α_4 which is less than $\pi/2$. So, there are 4 switchings per quarter cycles. So, you can eliminate 3 harmonics and you can control the fundamental.

Now, which 3 harmonics that you want to eliminate? See, V_{a0} has a quarter wave symmetry and an odd function. **What is the**, what is the Fourier series of this square waveform of V_{a0} ? It is an odd function, so average value is 0, all $\cos n \omega t$ terms are 0, like Fourier series is given by a_0 plus $\sum a_n \cos n \omega t$ plus $b_n \sin n \omega t$.

So, it is an odd function. So, all $a_n \cos n \omega t$ terms are 0, so I need to know or I need to calculate what are the various odd or odd harmonic components that are present in this waveform. See, the expression for the coefficient of the fundamental and the harmonic component is given by this equation.

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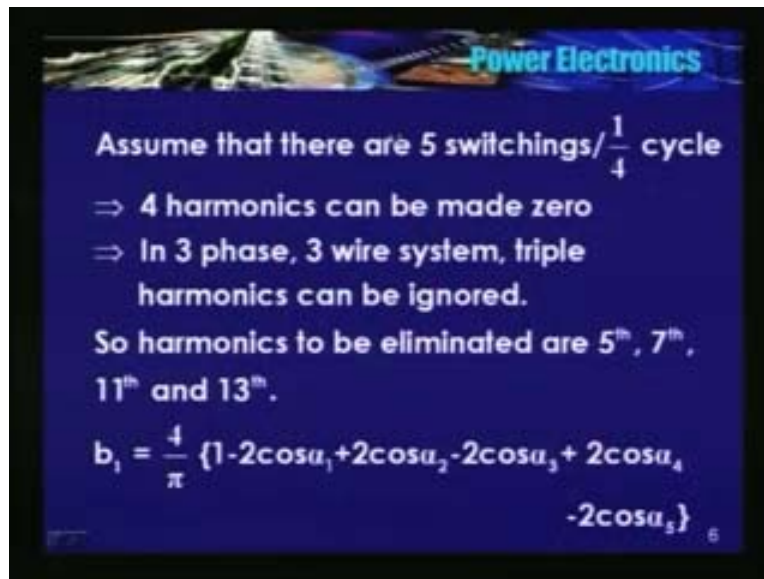


I have just written this equation, you need to determine. This is obtained, this obtained by determining the harmonic or writing the Fourier series of the waveform and determining various components is as simple as that. So, it is given by b_n is equal to this equation. Now, **I will**, using this equation, I will show you how you can eliminate the harmonics.

Now, assume that there are 5 switchings per quarter cycle, 4 harmonics can be eliminated. Now, what are the 4 harmonics? System is a 3 phase 3 wire system. So, you can conveniently neglect triple N harmonics. No even harmonics are present. So therefore, the harmonics that are to be

eliminated are fifth, seventh, eleventh, thirteenth. See now, using that equation which I showed you just now, I will write this equation.

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See here, b_1 is equal to 4 by pi to 1 minus 2 cos α_1 plus 2 cos α_2 minus 2 cos α_3 plus 2 cos α_4 minus 2 cos α_5 ; 1 to 5, because there are 5 switchings per quarter cycle, see here, 1 to m. There are so many switchings per quarter cycle. This is for fundamental; b_1 4 by pi n cos, whatever, α_1 , α_2 , α_3 , α_4 , α_5 .

Now which is the harmonics to be eliminated? 5, they are the fifth one. So, b_5 is equal to 4 by 5 pi and wherever alpha comes, you put 5 alpha. See, in this equation you substitute α_1 substitution 5 α_1 to α_1 and same equation and here b_5 4 by 5 pi.

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$$b_5 = \frac{4}{5\pi} \{1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 - 2\cos 5\alpha_3 + 2\cos 5\alpha_4 - 2\cos 5\alpha_5\} = 0$$

$$b_7 = \frac{4}{7\pi} \{1 - 2\cos 7\alpha_1 + 2\cos 7\alpha_2 - 2\cos 7\alpha_3 + 2\cos 7\alpha_4 - 2\cos 7\alpha_5\} = 0$$

$$b_{11} = \frac{4}{11\pi} \{1 - 2\cos 11\alpha_1 + 2\cos 11\alpha_2 - 2\cos 11\alpha_3\} = 0$$

$$b_{13} = \frac{4}{13\pi} \{1 - 2\cos 13\alpha_1 + 2\cos 13\alpha_2 - 2\cos 13\alpha_3\} = 0$$

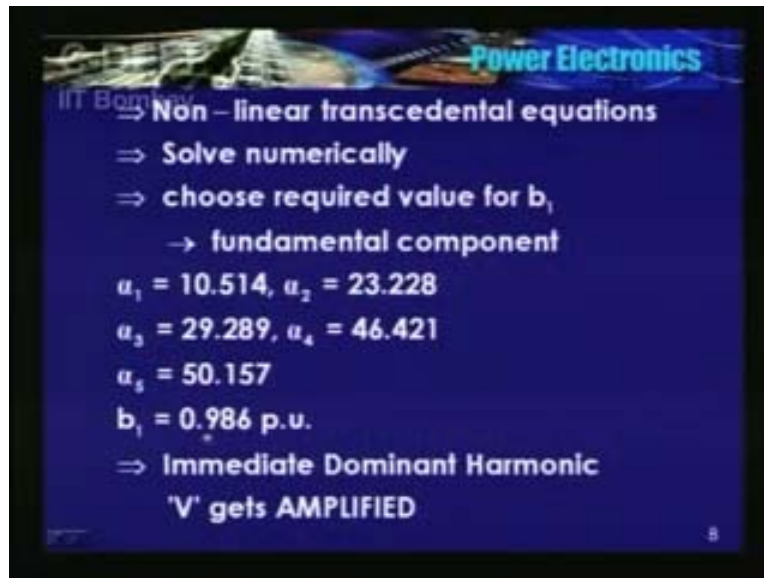
See, here is the equation; b_5 is equal to 4 by 5 pi and same thing, same equation. So, this has to be eliminated. I will equate it to 0. What is the next harmonic? It is the seventh harmonic. See here, I will write the same equation. I will substitute 7 for n, 4 by 7 pi and wherever there is 5, I will write 7. Again, I have to eliminate it, I have to or I will equate it to 0.

The next one being the eleventh one, see here, n is equal to 11 now. 4 by 11 pi again, cos 11 alpha₁ 11 alpha₂ so on. Say, **is equal to** again equal to 0 and the last one is the 13th, 4 by 13th and same thing. So, I have 5 non linear, transcendental equations that had to be solved. How do I solve this? There are 5 equations, 5 variables. So definitely, I need to use some numerical technique to solve them.

Now, how do you choose the value of the fundamental? Because that all 4 equations are been equated to 0, because those harmonics should be eliminated and fundamental should be controlled. Now, how do you know or what should be the value of the fundamental? The fundamental component is a system requirement. It depends on a V by F curve, is a V by F curve.

Suppose, the frequency of the fundamental is say if I know the V by F curve and if I know the frequency of operations, **I can** I can get the magnitude of the fundamental that is required. See, I will repeat; if the V by F curve is known, depending upon the frequency of the fundamental, I can get the frequency of the fundamental. That is nothing but b_1 . So, you need to know the value of the fundamental that you want at a particular frequency. So, I equated from a V by F curve to **be** b_1 to b equal to 0.986 per unit and I got these values for alpha.

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See here; α_1 is 10.514 degrees, see here, I mean 23.228 degrees, again, 29.289 degrees, α_4 is equal to 46.421 degrees and α_5 is equal to 50.157 degrees. So, if you switch at these angles, these particular angles; you can eliminate fifth, seventh, eleventh, thirteenth and you can control the fundamental and its magnitude is found to be 0.986 per unit. So that is about the harmonic elimination technique.

What are the disadvantages of harmonic elimination technique? Disadvantages are very obvious. See, as the number of switchings increases, you will have a larger number of equations that you need to solve them. See, if there are say, 15 switchings per quarter cycle, so you will have 15 equations or non linear translated equation that you need to solve them. Now, of course, this could not be a disadvantage because powerful numerical techniques are available, powerful computers are also available.

The second disadvantage is; see the angles that you are getting, α 10.514 degrees, 23.228 degrees. See, I will request you to do this exercise. You will you approximate this these angles, say, 10.5, here 23.2, 29.33 and 46.4 and 50.2; you approximate these angles and substitute in those equations. You will find that there is a significant increase in the harmonics. See, only when you switch at these angles, you can make fifth, seventh, eleventh, thirteenth, 0. If I approximate it, there is going to be a significant rise in the harmonics which I have already eliminated. In fact, they will not get eliminated.

See, after all we need to implement it in real time. In the sense, we have to implement it in a for an inverter. So, you need to use a powerful processor or an intelligent controller. So, while implementing these angles, there is a for any processor, there is a maximum value of accuracy.

Now, below or above that you cannot achieve it. So, resolution is always there, in the sense, at what value of θ or what is the lowest value of θ that you can implement it.

I do not think you can go in or you can implement it to a value to a third decimal value, may be, it is going to be extremely difficult **extremely difficult**. So, **so** if I am approximating it to first decimal, there is going to be a significant increase in the harmonics that I have already eliminated. And, in this harmonic elimination technique what happens is **you might have eliminated forth sorry you might have eliminated 4 harmonics** or till the thirteenth, the next immediate dominant harmonic that is seventeenth, it really gets amplified **really gets amplified**.

So, if you determine the magnitude of the seventeenth harmonic because till eleven you have eliminated, next immediate is seventeenth, you may get a value which is equal to the value of the fundamental itself. I will repeat; see, value of b_1 we found to be 0.986 per unit. Now, using these angles without approximating it, if you try to find or try to calculate the seventeenth harmonic, you will find that the magnitude of the seventeenth harmonics **which is** may be equal to or may be slightly higher than 0.986 per unit itself. It really gets amplified.

But then again, the frequency of the seventeenth harmonic may be higher and it gets filtered out by the machine inductance that whatever that I have told in my last lecture holds good. But this is the disadvantage of harmonic elimination technique. This is quite attractive because you can try to optimize certain performance index of a machine.

So, what **what** the harmonic elimination says? You need to switch at pre determined values or in other words, if there are n number of switching per quarter cycle, $n - 1$ harmonics can be eliminated and the fundamental can be controlled. So, that is about sinusoidal PWM technique and the harmonics elimination technique.

So, if you make certain observations on whatever that you have studied so far, I can **sum them** sum up them to the first one. **The** there should not be or **there should not** no pulses of opposite polarity in the half cycle of line to line voltage waveform.

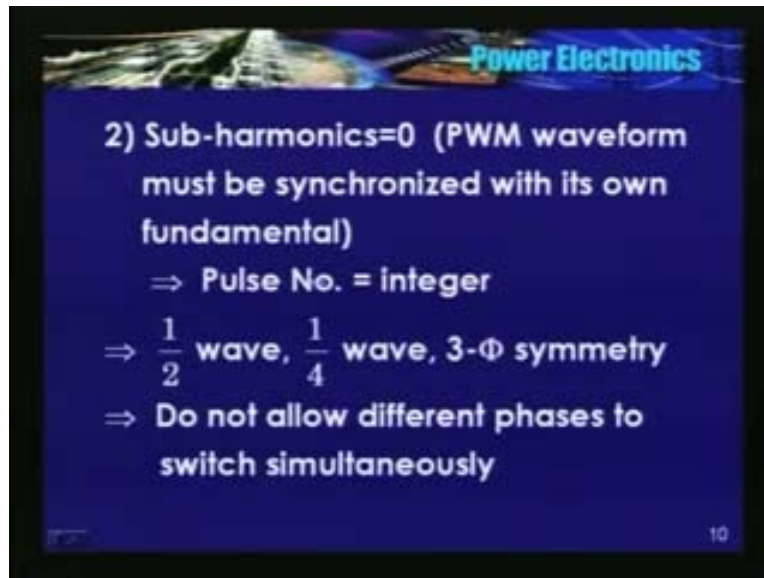
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The slide is titled "Power Electronics" in a blue font at the top right. Below the title, the text reads: "A well designed PWM strategy should have the following:" followed by a list item: "1) No pulse of opposite polarity in $\frac{1}{2}$ cycle of line-line voltage waveform \Rightarrow large ripple current". To the right of this text is a diagram of a PWM waveform. The diagram shows a horizontal axis with a vertical arrow pointing upwards. The waveform consists of several pulses. The first pulse is positive, followed by a zero-voltage interval, then a negative pulse, and then another zero-voltage interval. This sequence of positive, zero, negative, zero pulses within a half-cycle illustrates the concept of opposite polarity pulses.

A well designed PWM strategy should have a well designed PWM strategy should have these features; one is there should not be a pulse of opposite polarity. See here, suppose to be only positive to 0 and positive transition. This is the first half cycle of line to line voltage. Same thing should repeat in the negative half. But then here you may have a pulse of negative polarity. You may find in sinusoidal PWM technique. Now, what happens if this sort of pulse is there in the voltage waveform? You will have a large ripple current. Ripple current increases.

Second one is there should not be sub harmonics. Sub harmonics should be 0. Sub harmonics are those harmonics whose frequency is not an integral multiple of the fundamental component. The frequency of the sub harmonics is not an integral multiple of the frequency of the fundamental. So, sub harmonic should be 0. They will be 0 only when the PWM waveform is in synchronism with its own fundamental. See here, PWM waveform must be synchronized with its own fundamental or in other words, pulse number should be an integer. This, I have already told you in my last lecture.

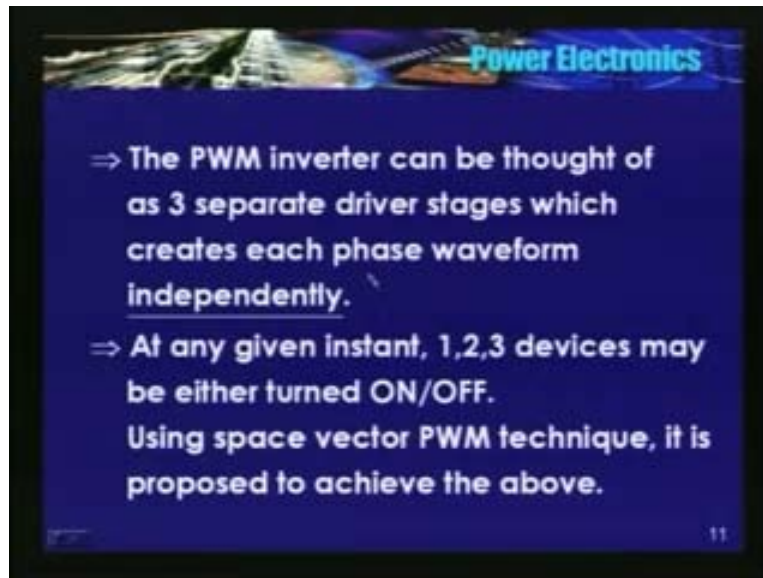
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In addition, there should be a half wave, a quarter wave and 3 phase symmetry. These are all desirable. Also, one should not allow different phases to switch simultaneously. I mean, see, when S_1 is switching, you should not allow S_3 or a complement of S_3 in b phase and in c phase, S_5 and complementary of S_5 , you should not switch or at a time **at a time** only 1 switch should change its conducting state. I will repeat; it is always desirable to have 1 switch changing its conducting state.

So, you can determine the frequency of the ripple of the current very accurately. But then so far we have observed that you can independently control one leg of the VSI. In other words, a 3 phase PWM inverter can be thought of as 3 separate driver stages, 3 separate legs of a driver; each creates phase waveform independently.

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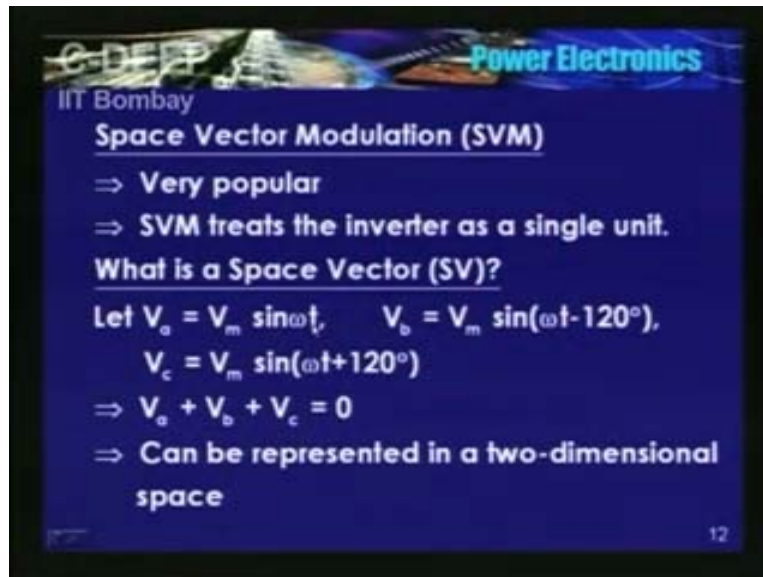
See, I will repeat, read this; a PWM inverter can be thought of as 3 separate driver stages. In the sense, there are 3 independent legs. I can control them independently and they create the phase waveform independently. While controlling phase A, we do not bother about what happens in phase B and what happens in phase C. That is what we did in sinusoidal PWM technique. We took a sine wave compared with the triangle. It may so happen that **phase A upper** phase A device may be switching along with phase B and along with say, phase C.

So, at any given instant, either 1 or 2 or all 3 switches may be either turned on or off. So, one of the feature that I told you here is do not allow phases to switch simultaneously, whereas, in the sinusoidal PWM technique, you cannot ensure this. There is a PWM technique what is known as the space vector PWM technique. I will repeat; space vector PWM technique which does or which achieves all the above features, the space vector PWM technique. That is the reason it is very popular **very popular** and is very simple to implement also. I will explain to you how to implement a space vector PWM.

So, if you see the modern digital signal processors, DSPs, there are dedicated pins in the DSP which give a pulse width modulated waveform using **space vector** space vector modulation. See, **if from here** from this you can conclude as to how popular this space vector PWM technique is. There are DSPs, digital signal processors; they have dedicated pins, wherein you can get a pulse width modulation waveform using space vector PWM technique. It is very popular and very simple to implement also.

So, before explaining what is space vector modulation, I will explain to you what is space vector. So, let see, I will take 3 phase waveform. That is V_a equal to $V_m \sin \omega t$, V_b is equal to $V_m \sin(\omega t - 120^\circ)$ and V_c is equal to $V_m \sin(\omega t + 120^\circ)$.

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Space Vector Modulation (SVM)

- ⇒ Very popular
- ⇒ SVM treats the inverter as a single unit.

What is a Space Vector (SV)?

Let $V_a = V_m \sin \omega t$, $V_b = V_m \sin(\omega t - 120^\circ)$,
 $V_c = V_m \sin(\omega t + 120^\circ)$

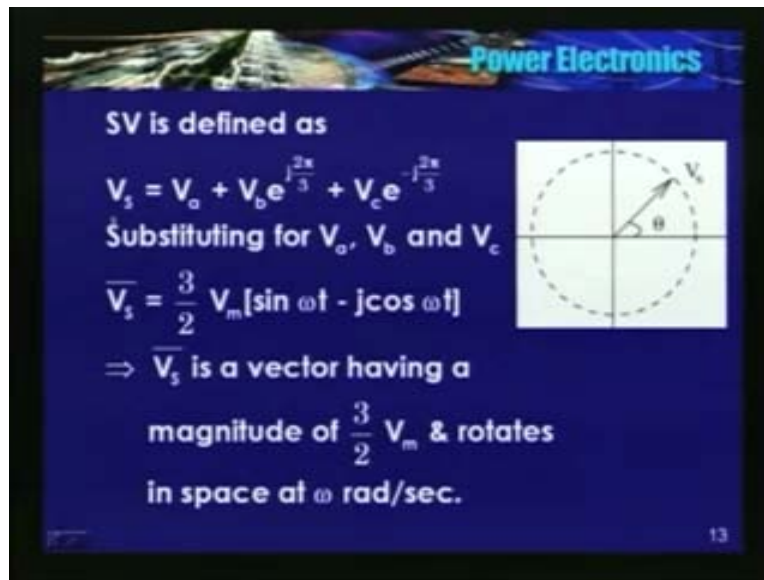
- ⇒ $V_a + V_b + V_c = 0$
- ⇒ Can be represented in a two-dimensional space

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I have 3 vectors which are displaced by 120 degrees. So at any given time, the instantaneous value; V_a plus V_b plus V_c is equal to 0. So, I can represent these 3 vectors by 1 vector what is known as the space vector. This vector, I can represent it in 2 dimensional spaces. So, it has an X - axis component and it has a Y - axis component.

So, I will repeat; a 3 phase vectors V_a , V_b , V_c or I can have currents also, 3 phase current not a problem, so you can represent it by what is known as one vector. So, it has an X - axis component and a Y - axis component.

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SV is defined as

$$V_s = V_a + V_b e^{j\frac{2\pi}{3}} + V_c e^{-j\frac{2\pi}{3}}$$

Substituting for V_a , V_b and V_c

$$\bar{V}_s = \frac{3}{2} V_m [\sin \omega t - j \cos \omega t]$$

$\Rightarrow \bar{V}_s$ is a vector having a magnitude of $\frac{3}{2} V_m$ & rotates in space at ω rad/sec.

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So here, the space vector is defined as V_s is a space vector is V_a plus V_b into e to the power $j 2\pi$ by 3 plus V_c into e to the power j minus 2π by 3. This is the definition of the space vector. Now, I will substitute for V_a , V_b and V_c and I will simplify, I will get this equation. So, what is this equation? It says that space vector is a vector having constant magnitude of $\frac{3}{2}$ times V_m , it has a constant vector of $\frac{3}{2}$ times V_m and **and** it rotates in space at ω radians per second. Has a magnitude of $\frac{3}{2}$ V_m and rotate in space **at ω is equal to** at ω radians per second. What is this ω ? ω is a frequency of the sine waves itself, these sine waves, $\sin \omega t$.

See, in space I can represent it by this. The locus is the circular because I have assumed all 3 to be sinusoids **all 3 to be sinusoids**. So, V_s rotates in a circular locus, magnitude is $\frac{3}{2}$ times V_m , **ω is same as the ω** this ω is same as the ω of those V_a , V_b and V_c . This is something similar to may be, rotating magnetic field in an induction machine. That also rotates at $\frac{3}{2}$ times the magnitude of this one and at ω . So, this is space vector.

Now, I said space vector can be represented or you can represent it in 2 dimensional spaces. See here, I have a, b and c. Angle between them is 120 degrees. At any given time, they are in this fashion. May be, a, b, c may be continuously rotating, no problem.

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SV can also be written as

$$V_s = V_x + jV_y$$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_x = V_a - \frac{1}{2}[V_b + V_c] = \frac{3}{2} V_a$$

$$V_y = \frac{\sqrt{3}}{2} [V_b - V_c]$$

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So, I will represent it in a 2 dimensional space; X - axis component and a Y - axis component. Sometimes it is known as a d - axis component and a q - axis component, direct axis and a quadrature axis. You can also say that X - axis and Y - axis, does not matter and this is the space vector V_s .

How do you determine the X - axis component and a Y - axis component or how do I represent these 3 vectors in 2 dimensional spaces? It is very easy. I need to just resolve them along X - axis and Y - axis. It is as simple as that. Please, do not get confused. It is I have to just resolve them along X - axis and Y - axis.

What is sigma V_x ? What is sigma V_x or what is the sum of all the X - axis components of these 3 vectors? V_a itself, X - axis component of A phase is V_a itself, B phase is V_b into $\cos 120$ **cos 120**, this is 120. $\cos 120$ is minus half **and V_c is** X - axis component of V_c is V_c into $\cos 240$, that is again minus half. So, V_x is V_a minus half of V_b minus half of V_c . That is sigma V_x .

So, if they are balanced, **so** I have V_x is equal to 3 by 2 times V_a itself because this is V_b plus V_c nothing but minus V_a and what is V_y ? The y axis component; y axis component of A phase is 0, B phase is sine 120 that is root 3 by 2 and C phase is sine 240 that is minus root 3 by 2. So, I will take V_y is equal to root 3 by 2 V_b minus V_c . So, if I know the value of V_a , V_b , V_c ; I can determine V_x and V_y and therefore V_s .

So, to determine the space vector, I need to know V_{an} , V_{bn} and V_{cn} . I have a 3 phase inverter; so, assume that it is feeding a star connected load, how do I determine V_{an} , V_{bn} , V_{cn} ? See, **we** I have drawn the phase voltage waveform for a star connected load in 180 degree conduction, inverter with 180 degree conduction; we found that the magnitude is 1 third and 2 third, a 6 step wave

form that we got. I had used a equivalent circuit approach there, in the sense, so I took 6 cases or 6 possible conduction states of an inverter and for each case for each case I determined what is V_{an} , what is V_{bn} and what is V_{cn} , please recall.

I said, if upper switch is on that point gets connected to A phase when the lower switch is on that phase is connected to sorry, when the upper switch is on, it gets connected to the positive DC link, when the lower switch is on, it gets connected to negative the DC link and the potential of point a, point b, point c with respect to the centre point to the DC link is known if the conducting state of the switches are known.

Here, I will derive an a very generalized expression and you will find that this is bit simpler. I will I will we can determine or will write it in a matrix form, a very simple matrix.

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V_{a0}, V_{b0}, V_{c0} can be either $+\frac{V_{dc}}{2}$ or $-\frac{V_{dc}}{2}$

$V_{a0} = V_{an} + V_{n0}$;

$V_{b0} = V_{bn} + V_{n0}$;

$V_{c0} = V_{cn} + V_{n0}$

But $V_{an} + V_{bn} + V_{cn} = 0$,

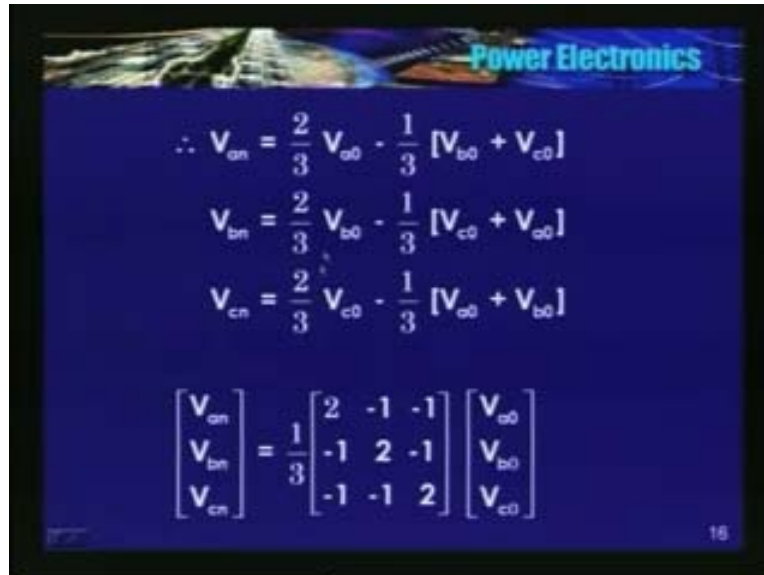
$V_{n0} = \frac{1}{3} [V_{a0} + V_{b0} + V_{c0}]$

See, consider an inverter feeding a star connector load; again, I am taking centre point to the DC link as a reference point. So, when the device upper device is on, potential of a, b, c could be or is V_{dc} by 2 and if the lower switch is on, it is minus V_{dc} by 2. So now, I can write V_{a0} or V_{a0} , V_{a0} is equal to V_{an} plus V_{n0} . Similarly, V_{b0} is equal to V_{bn} plus V_{n0} , V_{n0} ; similarly for V_{c0} , equations.

So, I have 3 equations here. I will add these 3 equations. When I add them, I will find that V_{an} plus V_{bn} plus V_{cn} become 0. So therefore, $3 V_{n0}$ $3 V_{n0}$ is equal to V_{a0} plus V_{b0} plus V_{c0} . So therefore, V_{n0} is equal to 1 third of this equation. Now, I will substitute for V_{n0} . So therefore, I can get V_{an} in terms of V_{a0} , V_{b0} and V_{c0} . See, when the inverter is switching, I know or potential of a, b, c with the respect to 0 are known. So, now I found an expression for V_{n0} in terms of V_{a0} , V_{b0} , V_{c0} .

So, I will substitute this value in these equations and I can get expression for V_{an} , V_{bn} and V_{cn} in terms of V_{a0} , V_{b0} and V_{c0} . See, if you substitute this equation in first, I will get this - 2 by 3 V_{a0} minus 1 third of V_{b0} plus V_{c0} .

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The slide displays the following equations and matrix form:

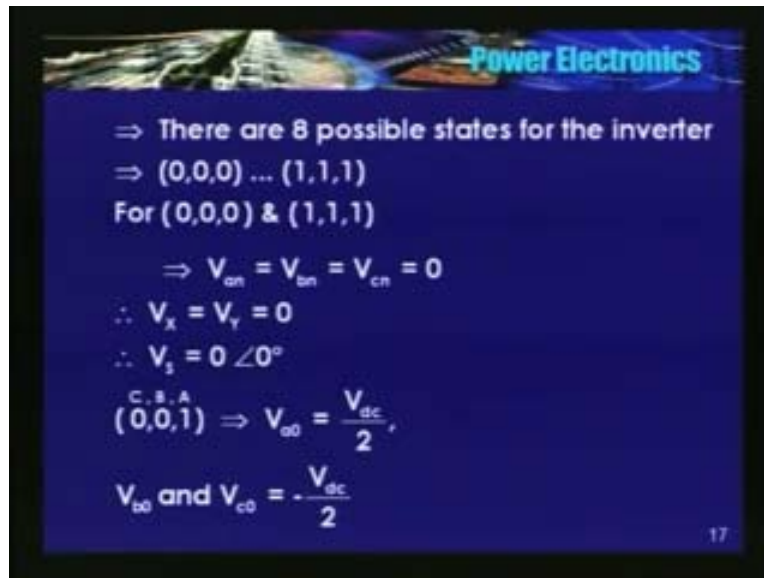
$$\begin{aligned} \therefore V_{an} &= \frac{2}{3} V_{a0} - \frac{1}{3} [V_{b0} + V_{c0}] \\ V_{bn} &= \frac{2}{3} V_{b0} - \frac{1}{3} [V_{c0} + V_{a0}] \\ V_{cn} &= \frac{2}{3} V_{c0} - \frac{1}{3} [V_{a0} + V_{b0}] \end{aligned}$$

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix}$$

Similarly, V_{bn} is equal to, same thing; b is here, so 2 by 3 V_{b0} minus, **now** since b is here, b should not be here. See here, a is here, there is no a here. Similarly cn, 2 by 3 V_{c0} and no c here. Coefficients are same. So, if I write in matrix form; see, V_{an} V_{bn} V_{cn} , so 1 third I will take it out, I have a very simple matrix, very easy to remember - 2, minus 1, minus 1, minus 1, 2 minus 1, minus 1, minus 1, 2. V_{a0} , V_{b0} , V_{c0} .

See, all the diagonal elements are 2 and remaining elements are minus 1, multiplied by 1 over 3. So, I can determine V_{an} , V_{bn} , V_{cn} by knowing V_{a0} , V_{b0} and V_{c0} . So, V_{a0} , V_{b0} , V_{c0} are known if the conducting state of the switches are known. So now, you do not need to determine or you do not need to use the equivalent circuit approach or **which** the approach which we used while drawing the phase voltage wave form in a 180 degree conduction inverter. But then if you **if you** use this approach, you will get the same result.

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I have told you this; there are we are we have 3 switches in the in the upper 3 arms. So, they could be either on or off. So, there are 8 possible states of the inverter. What are the 8 possible states? They are 000. In other words, all 3 switches are off to all all 3 switches are on. I am just monitoring the upper switches. That is 1, 3, 5 or $S_A S_B S_C$. Now, what happens when all 3 are on or all 3 are off?

When all 3 are off or when upper 3 switches are off, lower 3 switches are on. Therefore, point a, point b and point c, they are connected to negative DC bus. So, if you substitute these values here, you will find that V_{an}, V_{bn}, V_{cn} , they are equal to 0: all three 0, which is obvious. Also, all 3 points are connected at the same potential sorry all 3 points are connected to to the same point.

So, V_{an} is 0, V_{bn} is equal to 0, V_{cn} is 0. Therefore, X - axis component or sigma V X - axis component of all 3 phases; that is nothing but V_x , that is also 0, also, the V_y , the sigma Y - axis component of these vectors is also 0. So therefore, this space vector; magnitude is 0, angle tan inverse Y by X is also 0. So, we are at the origin. So, when all upper 3 switches are on, that is 111 and when all the upper 3 switches are off, that is 000; the space vector magnitude of space vector is 0 and angle is also 0. So, in the XY plane, you are at the origin.

Now, what happens when A phase, upper switch of the phase A is on and remaining 2 switches are off? That is S_1 is on, S_3 and S_5 are off, the remaining that is S_6 and S_2 are on? Since, A phase is on, it is V_{a0} is V_{dc} by 2, V_{b0} and V_{c0} are minus V_{dc} by 2. Now, I will substitute these values in that matrix and will get this value. So, V_{an} is equal to 2 by 3 into V_{a0} that is V_{dc} by 2 plus this. So, it is equal to 2 by 3 V_{dc} .

See in this matrix, I will repeat; you just substitute for V_{b0} and V_{c0} . V_{b0} and V_{c0} , both are equal, equal to minus V_{dc} by 2. So, this becomes plus. So, V_{an} is 2 by 3 V_{dc} . V_{bn} and V_{cn} are minus 1 third V_{dc} . See, same result we got in that or the procedure that we used to plot that 6 step wave form, phase to neutral. When one of the switches is on, one of the upper switches is on, only 1 switch is on in the upper half; at that time, the phase voltage of that phase is 2 by 3 V_{dc} and the phase voltage of remaining 2 phases is minus 1 third V_{dc} , remember. Here **here** also we are getting the same thing.

So, what is V_x or the sigma X - axis component of V_{an} , V_{bn} , V_{cn} ? A V_x is nothing but 3 by 2 times V_{an} . This is expression that we have derived. That is equal to V_{dc} itself.

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SV can also be written as

$$V_s = V_x + jV_y$$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_x = V_a - \frac{1}{2}[V_b + V_c] = \frac{3}{2} V_a$$

$$V_y = \frac{\sqrt{3}}{2} [V_b - V_c]$$

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See here, V_x is 3 by 2 times V_a and V_y is square root of 3 by 2 times V_b minus V_c . So, what is the Y - axis component? V_{bn} and V_{cn} are same, minus 1 third V_{dc} . Therefore, V_y you will get it as 0. So, what is the space vector? Magnitude of space vector is V_{dc} itself because y is 0, square root of V_x square plus V_y square and angle is 0, tan inverse Y by X, y is 0; so, V_{dc} by 2. So therefore, for 001, I have a space vector which is equal to V_{dc} at an angle 0.

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Similarly, $V_s = V_{dc} \angle \pi$ for $(1, 1, 0)$

Now $(0, 1, 1) \Rightarrow V_{a0} = V_{b0} = \frac{V_{dc}}{2}, V_{c0} = -\frac{V_{dc}}{2}$

$V_{an} = V_{bn} = \frac{1}{3} V_{dc}$ and $V_{cn} = -\frac{2}{3} V_{dc}$

$\therefore V_x = \frac{1}{2} V_{dc}$ and

$V_y = \frac{\sqrt{3}}{2} [V_{bn} - V_{cn}] = \frac{\sqrt{3}}{2} V_{dc}$

$\therefore V_s = \frac{1}{2} V_{dc} + j \frac{\sqrt{3}}{2} V_{dc} = V_{dc} \angle 60^\circ$

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So, I can straight away write for 110, the magnitude of the space vector is V_{dc} at an angle π radians. For 001, it is V_{dc} at angle 0. So therefore, for 110 which is the complementary of this, my vector is also at 180 degree to the previous position; so, V_{dc} at an angle π . If you are not convinced; you just workout, you will get this answer. More about the space vector PWM technique, I will cover in the next class.

Thank you.