

Power Electronics

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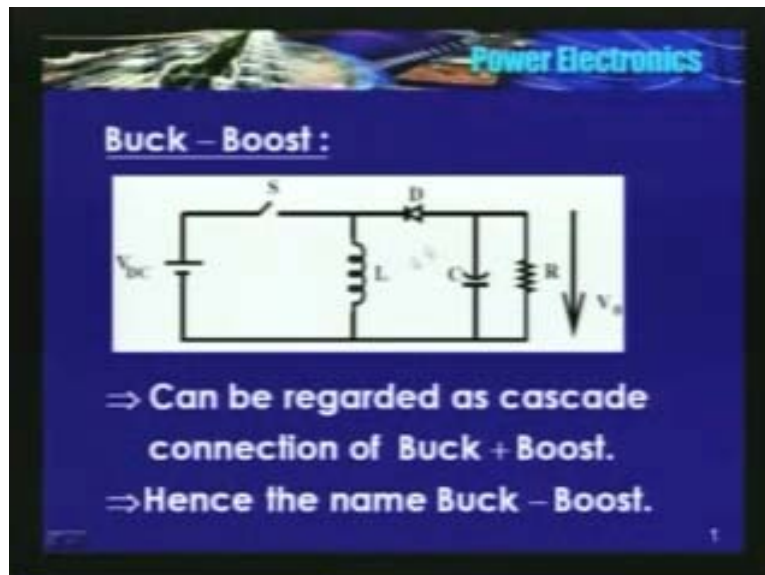
Lecture - 26

In my last class we discussed the operation of boost converter. We found that for a given value of D and the input voltage V_{DC} , output voltage is higher if the current is discontinuous. If the inductor current is discontinuous for a given value of D , output voltage is higher than if the current were to be discontinuous. If it is continuous, output voltage is V_{DC} divided by $1 - D$. Now, if it is discontinuous, now is going to be function of β . We found that this value is the new value. The output voltage when the current is discontinuous is higher than the current rate to be continuous and second point that we discussed was the use of buck as well as a boost converter, the speed control of DC machine.

We found that both speed control, 0 to rated or the operation in the first quadrant and the regenerative braking - feeding energy back to the source is possible by using these 2 converters; buck as well as boost converter. So, 2 quadrant operation of the DC machine is possible just by using a buck and boost converter.

Recall, in the line commutated converter, 2 quadrant, using a using 2 quadrant converter we had to interchange the armature terminal. We interchanged the armature terminals for i_A to reverse, whereas, using a DC to DC converter, buck as well as boost, I do not need to interchange the armature terminals. Having studied buck and boost, there has to be buck and boost converter, buck - boost converter, what is known as a buck - boost converter which is a cascade connection of a buck converter and a boost converter.

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So, here is a **buck**, buck - boost converter; switch S, inductor, diode and this is the output stage. Similar to buck converter, close S. I am storing the energy in the inductor. See, **this sort of**, this feature is similar to a buck converter and this is some sort of a boost converter **...** and stored energy, you pass it through the load, **using** through D. Hence the name, buck - boost converter. How does it operate? I close S for DT duration where D is a duty cycle, voltage across the inductor is the supply voltage itself, diode cannot conduct.

(Refer Slide Time: 4:16)

The slide is titled "Power Electronics" and shows two circuit diagrams of a boost converter. The left diagram is for the interval $0 < t < DT$ and the right diagram is for $DT < t < T$. Below each diagram are the corresponding equations.

Interval	Equations
$0 < t < DT$	$v_i = V_{DC}$ $r i_L + L \frac{di_L}{dt} = V_{DC}$ $C \frac{dv_s}{dt} + \frac{v_s}{R} = 0$
$DT < t < T$	$v_i = -v_s$ $r i_L + L \frac{di_L}{dt} + v_s = 0$ $i_L = C \frac{dv_s}{dt} + \frac{v_s}{R}$

So therefore, at the output stage, capacitor supplies power. See now, **we are all most**, this is a boost converter topology, capacitor supplying power. We never had this sort of situation in buck converter, **buck converter**, whereas, I have a switch in the main path of the flow similar to buck converter. I will open S after some time. So, stored energy in the inductor is transferred to the load and diode starts conducting, this is the circuit.

So, KVL gives for this loop, taking the resistance into account, this is the equation that I will get; $R i_L + L \frac{di_L}{dt} = V_{DC}$ and KCL at this node is i_C , the capacitor current is equal to the load current. So, capacitor is discharging here and similarly here, i_L is equal to i_C plus i_0 . i_L is equal to $C \frac{dV_0}{dt} + \frac{V_0}{R}$ and the circuit equation is $L \frac{di_L}{dt} + R i_L + V_0 = 0$, **plus V_0 is equal to 0**. Voltage across the inductor is minus V_0 now, whereas here, voltage across inductor is V_{DC} .

By the way, what is the polarity of V_0 ? **What is the polarity of V_0 ?** If you see in this circuit, I have shown the direction in this way because when I open the switch current starts flowing in this direction. So, this terminal is positive, this is negative, **this is negative**. But then for the sources, this line is a reference point. So, I cannot have 2 reference points in 1 circuit, remember, you cannot have 2 reference points in 1 circuit. If I am taking the reference as a negative DC bus, it is a reference point **for** to measure the voltages at other point.

So, voltage applied to the load is negative, **negative**. It is minus V_0 that is appearing across the load. If I see this waveform in the oscilloscope with this as the ground, this is plus V_{DC} , whereas, if I touch this, it is minus V_0 , **minus V_0 , is minus V_0** .

So, let us find out transfer function by solving this equation. So, this is the voltage equation; r into i_L plus $L \frac{di}{dt}$ plus V_{DC} and when I open the switch, this is the KVL. So, if I find out average values of this equation, here I need to integrate from 0 to dt and this equation, I need to integrate from dt to T and if I add, see, $r i_L$ is there, $L \frac{di}{dt}$ is there, V_{DC} is present from 0 to DT , whereas, V_0 is present from DT to T .

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Av. values of v_s & i_L are V_s & I_L

$$r I_L + L \left(\frac{di_L}{dt} \right)_{av} + \frac{1}{T} \int_0^T V_s dt = DV_{DC}$$

$$C \left(\frac{dv_c}{dt} \right)_{av} + \frac{V_c}{R} = \frac{1}{T} \int_{DT}^T I_L dt$$

At steady state $\left(\frac{di_L}{dt} \right)_{av}$ & $\left(\frac{dv_c}{dt} \right)_{av} = 0$

$$r I_L + (1 - D)V_s = DV_{DC}$$

$$\frac{V_c}{R} = I_L (1 - D)$$

So, I will get an equation something like this; r into i_L $L \frac{di}{dt}$ average, V_0 is there only from dt to T and V_{DC} is there from 0 to dt . So, **this is the average**, this is the equation giving the average values. Similarly, if I find out the average values here, dV_0 $C \frac{dV_0}{dt}$ is there in both the equations, V_0 by R is there in both the equations and i_L . So, average of this plus average of this should be equal to or the average value of i_L from DT to T . So, here is it, dt to T . At steady state, the $\frac{di}{dt}$ average should be 0. **Current should start** current at T is equal to 0 should be equal to current at T is equal to T **and that is** only then I am saying, circuit has attained a steady state.

Similar is true for the output voltage V_0 , voltage ripple should remain constant over a cycle. So, this term is 0, this term is 0, so therefore, r into i_L plus 1 minus D into V_0 is equal to DV_{DC} and this is the equation giving the load current in terms of the inductor current, V_0 by R is the average load current.

(Refer Slide Time: 9:52)

Power Electronics

$$r \frac{V_o}{R(1-D)} + (1-D)V_o = DV_{dc}$$
$$\therefore V_o = \frac{V_{dc} D(1-D)}{\frac{r}{R} + (1-D)^2} \text{-----} \rightarrow 1$$
$$i_L = \frac{V_{dc} D}{r + R(1-D)^2}$$

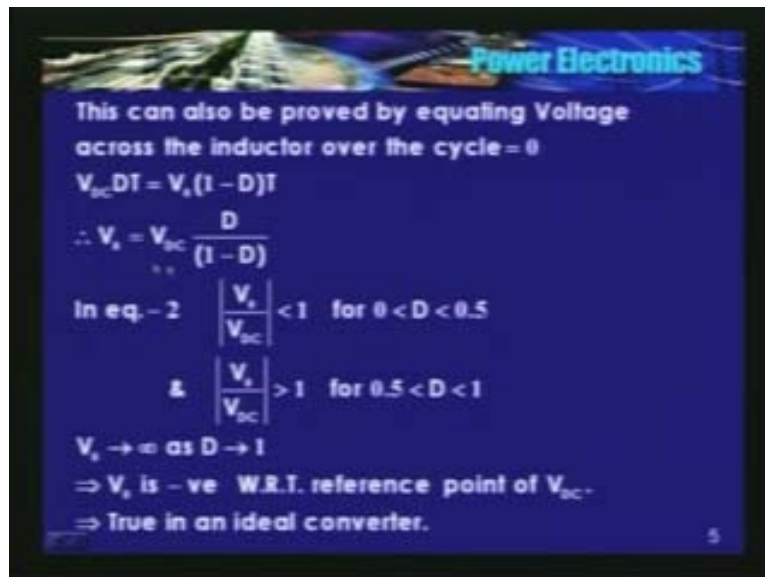
if $r \rightarrow 0$ $V_o = V_{dc} \frac{D}{(1-D)}$ ----- $\rightarrow 2$

$$i_L = V_{dc} \frac{D}{R(1-D)^2}$$

So, if solving this and then substitute it for i_L , substituting for i_L in this equation using this values, I get this equation. So, V_o in terms of V_{DC} and the internal resistance of the inductor is given by this and i_L is this equation. So, if R tends to 0, if I consider inductor to be ideal, I will get a voltage equation which is given by V_o is equal to V_{DC} divided by 1 minus D , 1 minus D and i_L is equal to V_{DC} divided by R into 1 minus D squared into D .

This equation can also be solved by equating the average voltage across the inductor, what we did for buck or the boost converter. So, if I know voltage across the inductor from 0 to DT from DT to T , I will equate it and I will get transfer function. It is very simple. I do not need to write, I do not need to take the value of R into account.

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So, here is the equation - V_{DC} into T , voltage across the inductor during DT is V_{DC} and voltage across the inductor when the switch is open, it is minus V_o . That is for this duration, I will equate it, I will equate it and I will get this transfer function. V_{DC} is the voltage across inductor when the switch is on and when the switch is off, voltage across the inductor is at the output voltage. So, this is for the ideal buck - boost converter. What does it say?

The magnitude of output voltage V_o is less than the input voltage V_{DC} for D varying from 0 to 0.5. So, we have a step down action here or the buck the operation. For 0 to 0.5, magnitude of output voltage is less than the input voltage. This is nothing but buck operation and from 0.5 to D , 0.5 to 1, the magnitude of output voltage is higher than the input voltage. So, this is nothing but the boost operation. But then what happens at D is equal to 1? Similar to boost converter, even in ideal buck - boost converter, magnitude of output voltage tends to infinity as D tends to 1.

I will repeat; similar to boost converter, the magnitude of output voltage tends to infinity as D tends to 1. See, the in the equivalent circuit if you see here, we had assumed that output voltage remains constant, V_o is constant and ripple free and we said that V_o is the voltage that is appearing across the inductor when the switch is opened and we equated it. Equated the voltage across the inductor when switch is on and when the switch is off, we equated and we found that the transfer function is D divided by 1 minus D .

But then what happens at D tends to 1 or as the value of D increases? Most of the time, at input inductor is charging and at the output, capacitor is supplying power. So therefore, capacitor voltage has to fall and it will fall. So, the very basic assumption that I made saying that output

voltage will remain constant is not valid. Similarly, at the input, most of time we are applying a DC voltage to the inductor. Current increases linearly and it will so happen that inductor may saturate.

So, whatever that happened in the boost converter for high values of D, happens in buck - boost also. So, as D tends to 1 or if you see in this circuit here, D tends to 1, output voltage will become 0. Output voltage will become 0 because capacitor is permanently connected across the load and it is discharging and at the input, inductor is permanently connected across the DC supply and it will saturate. So, this we should get from an equation which we have derived taking the resistance into account, inductive resistance into account.

See here, this is the equation we derived taking the non idealities of the inductor r by R. So, if I substitute here, D is equal to 1, output voltage becomes 0. D is equal to 1, output voltage becomes 0, this is 0 and the inductor current is when D is equal to 1 is V_{DC} divided by R, at steady state. So, a very high current and which may damage the source or inductor or the switch. This is what it is.

So, last observation that I have need to make is the output voltage is negative with respect to the reference point of the source voltage. I have taken **negative** DC negative bus as the reference point. So, output voltage is negative with respect to this line, DC line.

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Power Electronics

In non-ideal converter :

$\frac{V_o}{V_{dc}}$ reaches a peak and then reduces.

$V_o = 0$ when $D = 1$

Using eq.1 $\left. \frac{dV_o}{dD} \right|_{D=1}$

$$D_{max} = 1 + \frac{r}{R} - \sqrt{\frac{r(1+\frac{r}{R})}{R}}$$

$$V_{o(max)} = \frac{V_{dc}}{2} \left(\sqrt{1 + \frac{R}{r}} - 1 \right)$$

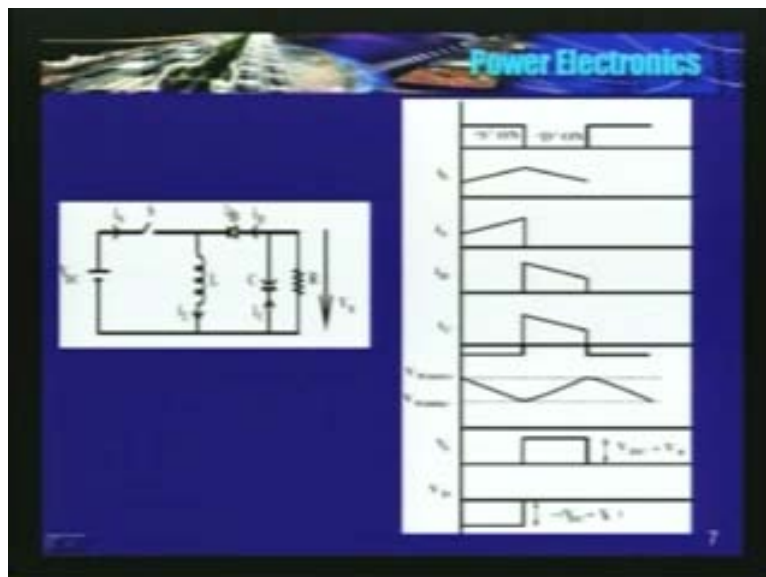
Now, we will ... a non ideal converter, this is the variation; when D is equal to 0, output voltage is 0, whereas, in boost converter, output voltage, the ratio of V_0 / V_{DC} is equal to 1. Buck converter when D is equal to 0, output voltage is also 0, D into V_{DC} . It starts increasing, tends to infinity for ideal buck – boost. At D is equal to 0.5, the ratio is 1.

Similar to boost, the magnitude of V_0 reaches a peak and it comes down and D is equal to 1 which is equal to 0. So what is a value of D for which the magnitude reaches, magnitude of output reaches a peak? We need to differentiate the voltage equation with respect to D. So, D_{max} is found to be this which is relatively bit complicated compared to compared to the boost converter. This is a D_{max} and $V_{0(max)}$ is this value, $V_{0(max)}$ is this value.

Now, let us draw the various wave forms. I have drawn both continuous and discontinuous cases for buck as well as boost. For buck – boost, I will draw only for continuous current. I encourage you people to go back and draw it for a discontinuous case.

So, what happens when I close S? Voltage across the inductor is V_{DC} , current increases linearly, so when I close S, if you see here, the circuit, whatever the current that was flowing through the diode starts flowing through S.

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So, this is the inductor current i_L , this path or during this period, diode is on. So, this is the current that is flowing through the diode. At this instant, we have turned on the switch. So,

whatever the current that is flowing through the diode starts flowing through the switch S. This is also equal to the source current waveform **this is also the source current waveform**.

When switch is on, source supplies power. When switch is off, there is no current from the source, similar to buck **similar to buck**. So, this is a buck converter source current waveform. So, current instantaneously changes. There are abrupt changes in the source current when I turn on as well as when I turn off, whereas, this is not the case in the boost converter. The source current waveform will look like this in the boost converter. Source is always there in the circuit. So, I have a source current waveform which is similar to the buck converter.

So, when I open S, whatever that the current that was flowing through the inductor starts flowing through the load and the diode. **The so diode**, this is a diode current. At this point, there is an abrupt change because we have turned on the switch. How does an output voltage vary or how does the capacitor current waveform look like?

I told you, this is similar to a boost operation **boost operation**, only capacitor supplies power. This set of thing **does** did not takes place or does not takes place in buck converter, **buck converter**. Inductor is always there with the **with the** output capacitor and the load, even when the switch is open as well as when the switch is closed. So, when the switch is closed, diode is reversed biased or diode cannot conduct. Capacitor supplies power. We have assumed that V_0 remains constant. Even if V_0 changes over a very narrow band, V_0 by R remains approximately constant.

So, I am assuming that load current is constant. So, the capacitor discharges. So, this is the capacitor current, constant. KCL gives, here that I_0 is equal to minus i_C . So, this is the capacitor current, discharges at a constant rate. Therefore, capacitor voltage, output voltage varies or decreases linearly here. I have opened the switch, the inductor current starts flowing through capacitor and load. KCL at this point gives i_L is equal to i_C plus i_0 and I am assuming, whatever the assumption that I made while drawing the waveform, in the case of boost converter. I said that in the entire on period of the switch, capacitor is discharging. So I will assume that when the switch is opened or during the off period of the switch, capacitor is charging.

In other words, **in other words**, just prior to closing S, inductor current is higher than i_0 . I will repeat; just prior to closing S, inductor current is higher than i_0 . If inductor current is higher than i_0 , the remaining current has to go through **has to go through** the capacitor, capacitor continuously charging.

So, if you see here, capacitor current is positive at this point because inductor current i_L is higher than i_0 . I am just assuming, it need not be true. **if I** only thing **I have to I need to** I have to apply only KCL at this point. If I know the magnitudes of i_L , i_C , i_0 I assumed to be constant, I can

determine or I can plot i_C waveform. So, at this point I am assuming that i_L , the inductor current is higher than i_0 . So, the remaining current has to flow into the capacitor which is positive. At this point, I am turning on S, D turns off and capacitor supplying the entire i_0 . Mind you, capacitor current can change instantaneously. Only capacitor voltage cannot change instantaneously.

So, if I plot V_0 , so it is minimum at just prior to opening the switch and this voltage is maximum just prior to closing the switch. I am assuming that just prior to closing the switch, capacitor current is positive. Only **this** under this condition it is true, mind it. So, at this point i_L is higher than i_0 . So, i_C is positive, capacitor is charging. But immediately when I close this switch, i_C becomes negative, it starts discharging. But definitely, the peak of the capacitor voltage or output voltage occurs just prior to closing the switch.

Now, what is the voltage rating and voltage rating of the switch and the diode? See the circuit, when I close the switch, what is the cathode potential of the diode? When I close S, this positive point gets connected to the cathode. These 2 are at the same potential. So, in other words, cathode potential with respect to the negative DC bus is V_{DC} . Anode potential is minus V_0 **minus V_0** with the respect to the negative DC bus.

Anode potential is minus V_0 , cathode potential is plus V_{DC} with respect. So voltage across diode is sum of these 2 voltages. Plus minus is the voltage at this point and here is V_{DC} . So, it is a sum of these 2 voltages **that the voltage** that the diode should block. Diode should block the sum of these 2 voltages and this same voltage appears across this switch when diode starts conducting. See, when diode starts conducting, this point is at minus V_0 with respect to the negative DC bus, whereas, this point is at V_{DC} with respect to negative DC bus.

So, the voltage across S is V_{DC} plus V_0 . So, the switch has to block the sum of the output voltage **and the and the** and the source voltage. The voltage rating of S and D is the same and the minimum voltage rating is the sum of the input voltage, V_{DC} plus V_0 . That is about the buck - boost converter. In all 3; buck, boost and buck - boost, when I am saying continuous conduction it implies that **implies that** inductor current is continuous. If it is discontinuous, it implies that inductor current is discontinuous not the output current, remember.

Now, I will compare the buck, buck - boost and the boost, all 3. Just see the source current waveform in all 3 as well as a load current waveform in all 3. What sort of a **current** source current waveform we had in buck and buck - boost? They are same. The source current, **there is the** instantaneously, it has to supply the current that is flowing through the diode. Abrupt change if the current is continuous and it comes abruptly to 0 when I open the switch in both cases; buck as well as buck - boost, there are abrupt changes when I close the switch as well as open the switch in the switch current.

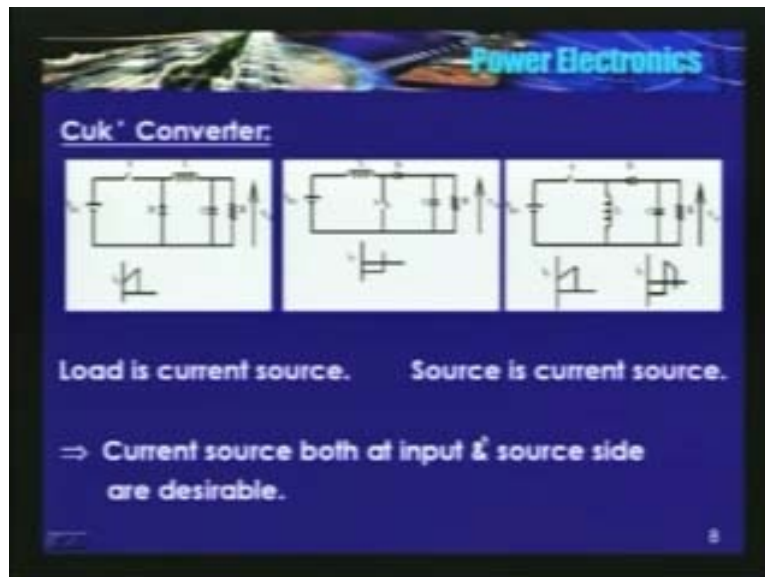
What happens in boost? The source current is approximately a smooth one. There is a large inductor connected in series with the battery. So, I told you yesterday that input stage for a boost converter can be thought of as if it is a current source. The voltage source and an inductor, we can combine it and we can represent it as a current source. So, we have a current source at the input for a boost, whereas, the source current abruptly changes when I open the switch as well as close the switch. So therefore, we require input filters for buck as well as buck boost.

What sort of an output stage I have for a buck converter? The output stage of a buck converter, there is always an inductor comes in series with the capacitor and ... inductor is always there. So, I can represent the output stage of a buck converter as a current source; can represent it as a current source. So, load current or the capacitor current, it is not constantly discharging or capacitor does not discharge at a constant rate when the switch is opened or closed as in the case of boost converter.

In the case of boost converter what happened or in the case of buck boost converter? Both cases; boost as well as buck - boost capacitor was discharging at a constant rate when the switch was closed. Where, that sort of a thing did not happen in the buck converter. Inductor is always present only even when I open the switch or close the switch. So, there is a step change or capacitor current instantaneously changes there.

It is see in this figure, this is source current waveform for a buck converter, same as the buck – boost, whereas, the source current waveform for a boost converter is slowly increasing when the switch is closed and slowly decreasing when the switch is opened. But the output current waveform or the capacitor current waveform in capacitor is discharging at a constant rate. Same thing is true here, capacitor is discharging at a constant rate, discharging.

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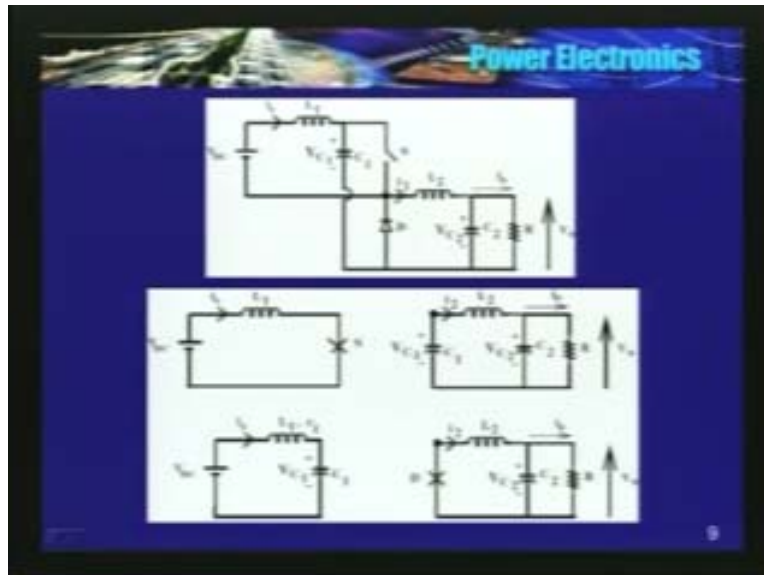
So definitely, if compared to the buck converter, ripple in the output voltage in boost as well as buck - boost is higher. I am saying this because capacitor in both the cases is discharging at a constant rate. I will repeat; the ripple in the output voltage will be higher for a buck - boost and a boost converter because output capacitor is discharging when the switch is closed at a constant rate. The entire load current is being supplied by the output capacitor. Definitely, capacitor has to change, voltage has to change, whereas, this sort of a thing does not happen in a buck converter, inductor is always present there. Some sort of a ... some sort of a current source is there in the output stage of a buck converter.

So, it is always desirable to have a current source at the input as well as at the output. See, **the** since the source current abruptly changing in buck as well as buck – boost, I said we require the input filters and that sort of filter is not required for a buck converter and I told you that output voltage ripple definitely is higher in buck, boost and buck – boost.

I will repeat; output voltage ripple is higher in boost and buck boost because capacitor is discharging at a constant rate, whereas, that sort of a thing does not happen in buck converter. So can I combine the advantages of boost and advantages of buck? **whereas** wherein, wherein, I have a current source at the input in the boost converter and I have a current source at the output in buck converter.

So, both; current source at the input as well as the output, can I have a converter, wherein, have these 2 features. So, in chuk converter both input as well as output has a current source. So, input stage and an output stage of a chuk converter is a current source. How does it look?

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Here is a circuit; source, an inductor and a switch. An inductor and an output stage, **inductor and an output stage** and the diode and am connecting a capacitor between these 2 points. Looks complicated but is very simple and it became very popular, **very popular** and named after the inventor.

How does it work? Close S, what happens when I close S? **L_1 , V_{DC} , S_1 sorry S** , this is the stage. The moment I close S, **diode**, this point gets connected to this point. In other words, V_{C1} appears across D. See, the negative plate is connected to the anode; the positive plate gets connected to the cathode of D because I am shorting this.

In other words, V_{C1} is appearing across the diode in the reverse now. Therefore, diode is reverse biased. So, this point gets connected here and the remaining circuit - L_2 C_2 and R. I am just closing S. There is the short circuit here, V_{C1} appears across the diode, cannot conduct and this V_{C1} , this voltage is a forcing function for this part. I will open S, what happens? The stored energy in the inductor starts flowing through capacitor C_1 , diode D and back to source.

So, open S. Whatever there was the current that was flowing through L_1 starts flowing through the capacitor and the diode, back to source. So, here is the output, when I open the switch, here is an equivalent circuit. What happens to the load side? Current was flowing in this fashion that is i_s , the source current. So, diode is on **diode is on** and this load current i_2 also starts flowing through D_2 .

So, at the load stage or **the** at the output stage, I have a circuit something like this; L_2 C_2 R and D . Current through D is sum of i_2 as well as i_s , the source current. **Sorry, I should have shown here a switch or I need to** I should have shown here D . So, L_1 , C_1 , D , back to source: similarly, L_2 , this combination, D , back to source.

Now, before deriving the transfer function, can I see these equivalent circuits and can I use whatever that I have studied so far; those 3 converters; buck, buck - boost and boost and can I derive or can I write down the relationship between V_{DC} , capacitor voltage V_{C1} and output voltage V_0 ? **We can by** just by inspecting, I can write the equations. What are they? What happens at the input stage? At closed switch S , current started increasing in the inductor. Open the switch S , whatever the stored energy in the inductor is transferred to the capacitor C_1 . Nothing but a **buck** boost converter operation.

Let me repeat; it is a boost converter operation. I closed S , opened S , whatever that the current that has flowing through L_1 starts flowing through the capacitor C_1 . At the input stage I have a voltage source and an inductor. So, I can represent **it** the input stage as a current source, similar to boost. Voltage source and inductor, I said we can represent it, the input stage by a current source. Similarly here, V_{DC} and L_1 , the inductor can be a thought of as a current source. That current source was flowing through, the value of the current was flowing through S when I closed that and when I opened it, it starts flowing through the capacitor C_1 . Same thing what that has happened in the boost converter.

So therefore, the relationship between V_{C1} and V_{DC} should be equal to or the same transfer function that we derived for the boost converter should be valid here between V_{C1} and V_D . The relationship between V_{C1} and V_D is 1 divided by 1 minus D , same as the boost converter. So therefore, V_{C1} is V_{DC} divided by 1 minus D . See the circuit here, **circuit here**, source S , we closed them and opened S , V_{C1} , same thing. **So, relationship between V_{C1} and V_{DC} is same as,** this is nothing but **buck a** boost converter circuit. This is nothing but boost converter circuit. So, V_{C1} is equal to V_{DC} divided by 1 minus V_C .

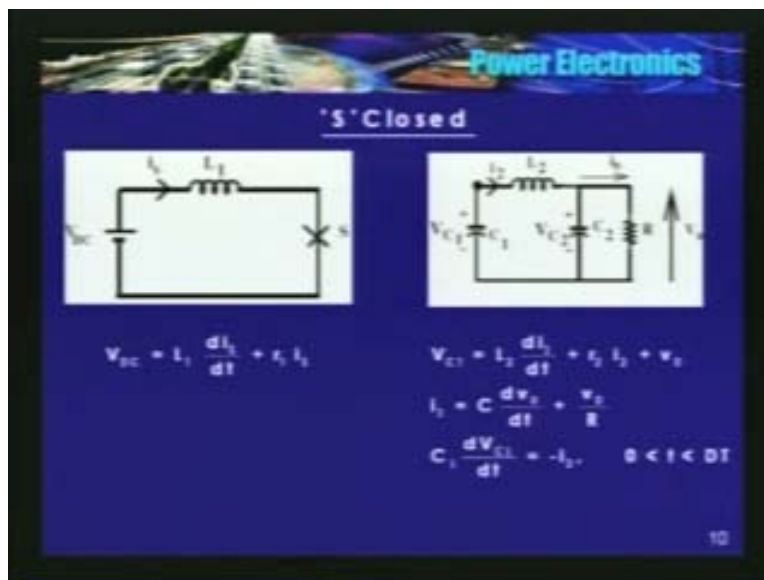
What happens in the secondary side or in the load side? What happens on the load side? Do not you think this is nothing but a buck converter? Buck converter with the input as V_{C1} ? When I close S , input voltage is applied to the inductor as well as the output stage, same equivalent circuit that we got when switch is on in the buck converter. When I opened S , diode starts conducting and this is the equivalent circuit. This is nothing but the buck converter.

So, we have a relationship between V_{C1} and V_0 ? What is that? V_0 is equal to D into V_{C1} , whereas, here we have V_{C1} is equal to V_{DC} divided by 1 minus D . Boost converter, a buck converter, V_0 is equal to D into V_{C1} . So therefore, what is the relationship between V_{DC} and V_0 ? V_0 is equal to D into V_{C1} and V_{C1} is equal to V_{DC} divided by 1 minus D . Therefore, V_0 is equal to D divided by 1 minus D into V_{DC} , nothing but a buck - boost operation, **buck boost operation.**

So, transfer function of a chuk or a chuk converter is the same as that of a buck - boost converter. Again, the output voltage, the polarity is the same as **same as** the buck – boost.

Now, we will analyze this circuit taking the resistances of L_1 and L_2 into account. Now, the process may be bit tedious but then very simple equations. After all, R into I plus L di 1 by dt is equal to V_{DC} , **there is the max is** the term that come into the appearing the equations. **The**, you may or they may appear to be bit tedious, please do not lose patience; solve this equations. Now, we will start writing the circuit equations in terms of resistances.

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S is closed, input is V_{DC} is equal to L_1 di₁ by dt plus r_1 into i_1 . i_1 is the source current, not the switch current and **at the** at the load side, V_{C1} is equal to L_2 di₂ by dt plus r_2 into i_2 plus V_0 and at the output, KCL gives i_2 , the inductor current i_2 is equal to capacitor current C d V_0 dt plus V_0 R **C and** and the entire i_2 is being supplied by the capacitor 1, C_1 V_{C1} . So, C_1 d V_{C1} by dt is equal to minus i_2 . **The entire**, the inductor current i_2 which is the sum of output current and the capacitor 2 current, C_2 current, is flowing through L_2 that is i_2 is equal to the rate of change of the capacitor voltage C_1 , V_{C1} by dt.

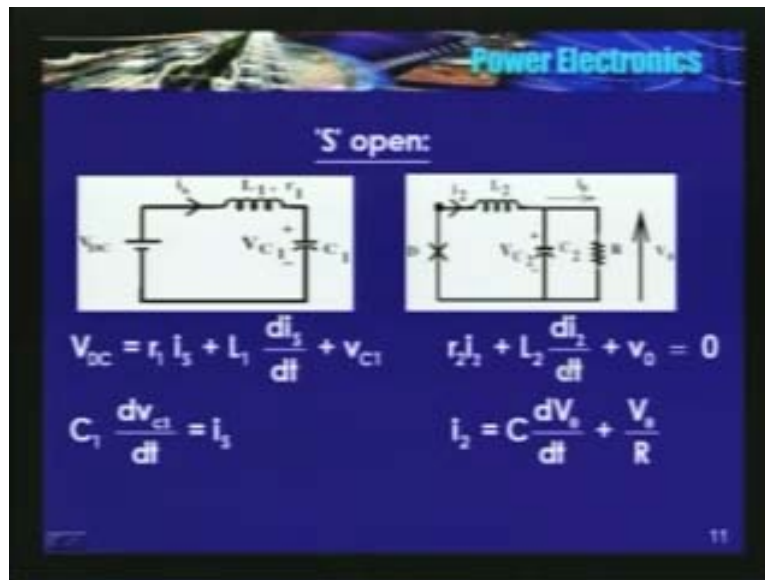
So, if I represent a load side by a current source of i_2 because there is an inductor coming in series with the output stage, I can assume that or I can say that C_1 is discharging at a constant rate of i_2 . See, same thing whatever that happened **in a** in a buck converter. Buck converter, the load side we represented or I said we can represent it by a current source. So, here also if I represent the load side by a current source of i_2 that current is being supplied by the capacitor C_1 .

So, we can say that C_2 is discharging at a constant rate. Current that is flowing is i_2 for 0 to dt when the switch is closed.

What happens when I open the switch? Switch is open, whatever the current that was flowing through L_1 starts flowing through C_1 . I said, the input stage of a cuk converter, the V_{DC} in series with L_1 can be represented by a current source of i_s , similar to the boost converter. Now, that entire current source **current of a current** or the value of the current source is flowing through the capacitor C_1 . In other words, capacitor is charging at a constant rate. Current that is flowing is constant and it is at i_s . I am neglecting the ripple, please. So, V_{C1} increases when the switch is off.

In other words, capacitor C_1 is charging when the switch is off. When I open the switch, capacitor C_1 starts charging. It was supplying i_2 , constant current when the switch was closed, **when the switch was closed**. So therefore, from 0 to DT , V_{C1} is decreasing linearly because I assume that current that is flowing out of **out of** C_1 is i_2 and from DT to T , capacitor voltage V_{C1} is **increasing at a** increasing linearly because I assumed that current that is flowing through it is i_1 or i_s is the current source.

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So, here is the equation; $C \frac{dV_{C1}}{dt}$ is equal to i_s . Capacitor V_{C1} is increasing or C_1 is charging. At the load side, $r_2 i_2$ plus $L_2 \frac{di_2}{dt}$ plus V_o is equal to 0 and i_2 is equal to same, $C \frac{dV_o}{dt}$ plus V_o by R , **V_o by R** .

Now, how would I find out the average values? Same, integrate it over the cycle, **integrate it over the cycle** only when the switch is on as well as switch is off and I will get these 2 equations. See, here are the equations. Please I have just written integration, actually, we have to find the average values. **1 over t is there**, 1 over t is there. I have to find out the average values. So definitely, 1 over t time period has to be there.

(Refer Slide Time: 49:20)

The slide contains the following equations:

$$\int_0^{DT} V_{DC} = \int_0^{DT} r_1 i_1 + \int_0^{DT} L_1 \frac{di_1}{dt} \quad 0 \leq t \leq DT$$

$$\int_{DT}^T V_{DC} = \int_{DT}^T r_1 i_1 + \int_{DT}^T L_1 \frac{di_1}{dt} + \int_{DT}^T V_{C1} \quad DT \leq t \leq T$$

$$\therefore V_{DC} = r_1 i_1 + L_1 \left(\frac{di_1}{dt} \right)_{av} + \frac{1}{T} \int_{DT}^T V_{C1} dt$$

similarly, $r_2 i_2 + L_2 \left(\frac{di_2}{dt} \right)_{av} + V_{C2} = \frac{1}{T} \int_0^{DT} V_{C1} dt$

$$i_2 = C \left(\frac{dv_{C1}}{dt} \right)_{av} + \frac{V_{C1}}{R}$$

$$C \left(\frac{dv_{C1}}{dt} \right)_{av} = \frac{1}{T} \int_0^{DT} (-i_1) dt + \frac{1}{T} \int_{DT}^T i_1 dt$$

This is the equation for 0 to DT and this is the equation, the voltage equation, mind you, from DT to t T **from DT to T**. So, if I add up both, V_{DC} , i_1 to r_1 is there, $L \frac{di}{dt}$ is there average, 1 over T $\int_{DT}^T V_{C1} dt$, this capacitor term appearing only when the switch is opened. This term, average is 0 **0**.

Similarly, in this load side we have this sort of equation. $r_2 i_2$ plus $L_2 \frac{di_2}{dt}$ average plus V_0 is equal to $\frac{1}{T} \int_0^{DT} V_{C1} dt$ and i_2 current is **this** constant. I can represent it by a current source is $C \frac{dV_0}{dt}$ average plus V_0 by R is at load side and at the input side, this is the current. How would I get them? Finding out these 2 values, **this** this is current waveform or this is the equation for the current when the switch is opened and here at the load side, equation for the current when the switch is opened. Similarly, we have here. These are 2 equations, combine them and you get this.

(Refer Slide Time: 51:14)

Power Electronics

In steady state $\left(\frac{di_1}{dt}\right)_{av} = 0$, $\left(\frac{di_2}{dt}\right)_{av} = 0$
 $\left(\frac{dV_{C1}}{dt}\right)_{av} = 0$ & $\left(\frac{dV_C}{dt}\right)_{av} = 0$

Also neglecting ripple in V_{C1} , i_2 & i_1

$$V_{DC} = r_1 i_1 + (1 - D) V_{C1} \text{ ----- (1)}$$

$$r_2 i_2 + V_0 = D V_{C1} \text{ ----- (2)}$$

$$i_2 = \frac{V_0}{R} \text{ ----- (3)}$$

$$0 = -D i_2 + (1 - D) i_1 \text{ ----- (4)}$$

13

So, at steady state, the ripple at the input because we have a V_{DC} and an inductor, we have at the output also, i_2 a current source. At steady state, ripple in both, ripple in these 2 inductors should be the same. Peak to peak ripple should be the same. Similarly in the output voltage, V_{C1} and output voltage V_0 , that is nothing but the voltage across the capacitor C_2 , it should be the same. So, we will get neglecting ripple and all this we will get these 4 equations. V_{DC} is equal to i_1 into r_1 and 1 minus D V_{C1} . So, if I neglect r_1 , see we have V_{C1} is equal to V_{DC} divided by 1 minus D . Nothing but the boost converter transfer function, **boost converter transfer function.**

This is at the secondary side or sorry the load side, the load side. Secondary side, I mean the load side. $r_2 i_2$ plus V_0 is equal to $D V_{C1}$. If r_2 is 0 , V_0 is equal to D into V_{DC} . I told you that output stage is nothing but a buck converter with the forcing function of V_{C1} , **forcing function of V_{C1} .** i_2 is the average load **load** current, I_2 is a average current that is flowing through the inductor that is given by V_0 by R because average value of the current that is flowing through the capacitor C_2 should 0 , mind you. Same, whatever **that I** that we said for a buck converter, there is the inductor, capacitor and R is always present. Even in cuk converter, L_2 C_2 and R is always present. So, average value of the inductor current should be equal to the average current of the load in **in in** buck as well as cuk converter of the load side because average current flowing through the capacitor should be 0 at steady state.

So, i_2 is the average value of the inductor current L_2 should be equal to the average value of the load current and this is the another equation. So, again by solving these equations, these are the 4 equations that we get.

(Refer Slide Time: 53:57)

Slide 14: Power Electronics

$$I_s = \frac{D}{(1-D)} I_L \rightarrow \text{Av. load } I \quad \because \text{av. } I \text{ through } C = 0$$

$$r_1 \frac{V_s}{R} + V_{C1} = D V_{C1}$$

$$\therefore V_{C1} = \frac{V_s}{DR} (r_1 + R)$$

$$V_{DC} = r_1 I_s + (1-D) \frac{L_s R}{DR} (r_1 + R)$$

$$= r_1 I_s + (1-D) I_s \frac{(1-D)}{D^2} (r_1 + R)$$

$$\therefore I_s = \frac{V_{DC}}{r_1 + \left(\frac{1-D}{D}\right)^2 (r_1 + R)} \rightarrow \text{av. source } I \text{ in terms of } V_{DC} \text{ \& } D$$

Substitute for V_{C1} in the equation representing V_{DC} . See, these are the equations that we are getting. I_s in terms of V_{DC} and D . How did I get them? Just by substituting 1 variable in other, in the previous equation, add V_{DC} and substitute here and I will get this. For I_s in terms of V_{DC} is given by this equation; V_{DC} divided by r_1 plus whole square into r_2 plus R . Average source current is the source current in terms of the input voltage and the duty cycle, and the duty cycle.

(Refer Slide Time: 55:00)

Slide 15: Power Electronics

$$V_s = I_s R = \left(\frac{1-D}{D}\right) R I_s = \left(\frac{1-D}{D}\right) R \frac{V_{DC}}{r_1 + \left(\frac{1-D}{D}\right)^2 (r_1 + R)}$$

$$\therefore V_s = \frac{V_{DC}}{\frac{r_1 + D}{R(1-D)} + \left(\frac{1-D}{D}\right) \left(\frac{R+r_1}{R}\right)} \rightarrow \text{av. } V_s \text{ in terms of } V_{DC} \text{ \& } D$$

$$V_{C1} = \frac{V_s}{DR} (r_1 + R) = \frac{V_{DC}}{(1-D) + \frac{r_1 + D}{(r_1 + R)} \cdot \frac{D^2}{(1-D)}}$$

Similarly, output voltage V_0 in terms of V_{DC} and the duty cycle D , V_0 in terms of V_{DC} into D . Same, V_0 is equal to I_2 into R . I will substitute now, I know the relationship between I_2 and I_S , I will substitute here. This what I will get? **This is what I will get**. These are the equations and here is the equation, capacitor V_{C1} in terms of V_{DC} and this is ...

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Power Electronics

If $r_1 \& r_2 = 0$

$$I_1 = \frac{D}{(1-D)} I_S = \left(\frac{D}{1-D} \right) \frac{V_{DC}}{R}$$

$$V_{C1} = \frac{V_{DC}}{(1-D)} \quad V_s = V_{DC} \frac{D}{(1-D)}$$

OR $V_s = DV_{C1}$

$$V_{DC}DT = (V_{C1} - V_{DC})(1-D)T$$

$$\therefore V_{C1} = \frac{V_{DC}}{(1-D)}$$

$$\therefore V_s = DV_{C1} \text{ (is buck converter with } V_{C1} \text{ as the input voltage)}$$

16

So, if I make I_1 and r_1 , if I neglect the resistances, **neglect the resistances**, I will get this equation. Substitute for r_1 and r_2 and make them, make r_1 and r_2 is equal to 0. Substitute, simplify, we will get this equation. See here, V_{C1} is equal to V_{DC} divided by 1 minus D , boost converter and we have a buck, this is V_0 and V_{DC} , a buck - boost converter, **buck - boost converter, buck - boost converter**. That is about the cuk converter. We will draw the various wave forms tomorrow.

Please, compared to buck, buck - boost and boost, they may look bit, the cuk converter may look bit complicated. But it is very simple and **and** it is very interesting circuit, **very interesting circuit**. You can derive the transfer of function just by drawing the equivalent circuit when S is equal to on and S is equal to off, buck and the boost operation. This is more about it, we will see tomorrow.

Thank you.