

## Power Electronics

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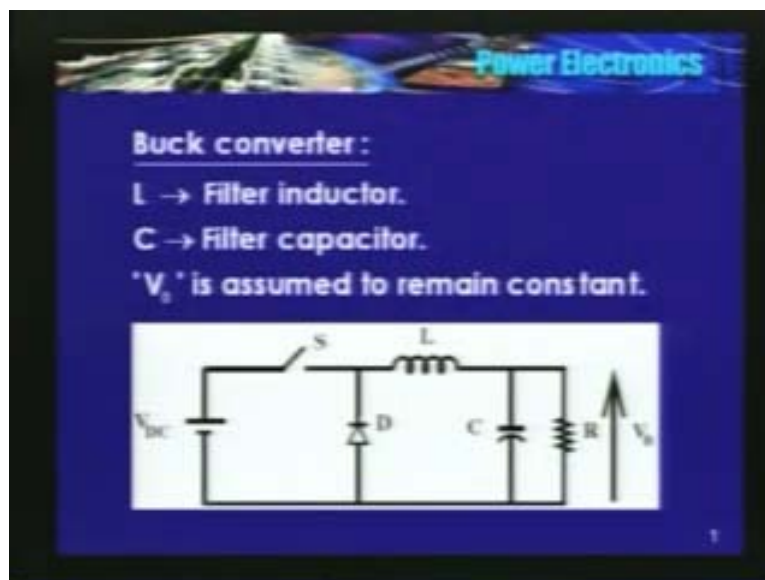
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### Lecture - 24

In my last lecture we discussed the limitations of a linear regulated power supply. I told you that linear regulated power supplies are bit heavy and bulky because they use a 50 hertz step down transformer. Second is source current is peaky, has a predominant third harmonic component because source supplies power only for a very short duration. When the output voltage or unregulated voltage **is** becomes equal to the input voltage, diodes gets forward biased. So, **very**, for a very short duration source supplies power and at that time when the instant diode turns on, a large current flows. Basically, source current is peaky, power factor is poor.

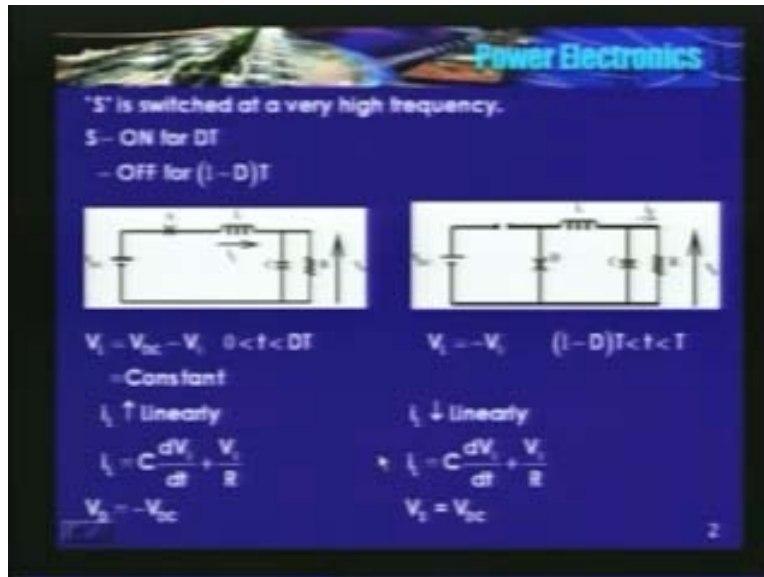
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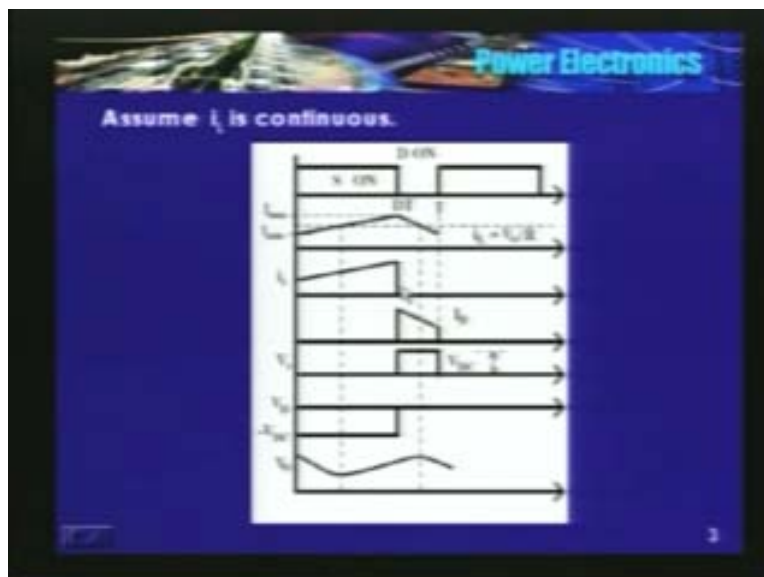
Third is efficiency is low because the series element or the regulating element comes in, **is** connected in series, series with the load or it is in the main path of the power flow and it **is** always acts in or is operated in linear or active mode. The difference between unregulated and regulated is dropped across the series element. So, it is invariably, it is operated in linear region, losses are high, **temperature**, temperature rise **then**, therefore, you require a larger heat sink, whereas, a switched mode power supply, they are operated in very high frequency. The magnetics which are used in SMPS or switched mode power supplies, they are operated in very high frequency. Then I told you that as the frequency of operation increases, size of a transformer or an inductor or for that matter the capacitor, filter requirement comes down.

So, first one in the series is the buck convertor. I had explained you the principle operation. When I close S, inductor current builds up, diode is reverse biased and when I open S, the inductor current freewheels through the diode. So, while deriving the transfer function, we made assumptions. The assumption that we made are  $V_0$  is constant and ripple free, all the circuit elements are ideal, switches are also ideal and these are the equivalent circuits. Inductor current builds up here, inductor currents freewheels through diode and it falls.

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And, I did discuss the various waveforms here. Switch is on, inductor current increases: diode is on, inductor current falls. At steady state,  $I_{\min}$  at T is equal to 0 is equal to T is equal to T or beginning of the next cycle. See the switch current, the moment I turn on the switch, the current that is flowing through the diode at this point, **at this point** the current starts flowing through the switch or the source.

So, source current jumps here, increases and when the switch is off, source current becomes instantaneously 0. So, source supplies power only for a short duration here or only for the duration when the switch is on. Same thing, when the diode current of the instantaneously rises to  $I_{\text{peak}}$ , reduces and at this instant it falls abruptly. When the switch is on, voltage across the switch is 0 and when it is off **when it is off** see here, voltage across this switch is  $V_{\text{DC}}$  itself because diode is conducting. This point gets connected to this point. So, voltage across the switch is  $V_{\text{DC}}$  itself and when switch is on,  $V_{\text{DC}}$  appears across the diode. So, in both the cases: or the voltage rating of S as well as D is  $V_{\text{DC}}$  itself and this is the variation in the capacitor voltage.

Over here, we assume that or in analysis we assume that **voltage** output voltage remains constant but then this does vary but over a very narrow range. Generally, output voltage ripple is specified, it is very small. So, when **that is** when does it starts charging and when does it start discharging? At any given point, KCL has to hold good.  $i_L$  is equal to  $i_C$  plus  $I_0$ , ripple in  $V_0$  is very small. So therefore,  $V_0$  by R is a ripple in  $I_0$ . That can be really neglected. So, I am assuming that load current remains constant at  $I_0$  which is equal to  $V_0$  by R.

This is the positive direction of current. If the current flows in this direction, capacitor charges and if the current reverses, capacitor discharges. So, out here, current starts from a minimum value, increases, reaches a peak and this is the average value of the load current,  $I_0$ . This is  $I_0$ , is equal to  $V_0$  by R. From T is equal to 0 to till this point, you find that inductor current is less than **inductor current is less than** the load current,  $I_0$ . So therefore, the remaining current has to come from the capacitor, **remaining current has to come from the capacitor.** So, capacitor is discharging, capacitor  $V_0$  is changing.

From this point to till here, we will find that inductor current is higher than the load current, mind you. See the load, switch is opened here, S is open, diode starts conducting. Since, the current is higher than  $I_0$  capacitor continues to charge till here. So, from here capacitor starts charging till here and beyond this point again it starts discharging. So, do not think that the moment you turn on S, output capacitor starts charging, no; it all depends on the inductor current and the average load current.

We have assumed that inductor is ideal, switch is also ideal, therefore the losses that are taking place in the convertor is approximately 0. So, under that condition, input power should be equal to the output power. What is the input power? Input power is the battery voltage, source voltage into source current,  **$V_{\text{DC}}$  into  $i_S$** ,  $V_{\text{DC}}$  into  $i_S$  should be equal to  $V_0$  into  $I_0$ . This is the output power,  $i_S$  is the source current, average.

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Neglect losses  
Input power = Output power.  
 $V_{dc} I_s = V_o I_o$   
 $= D V_{dc} I_o$   
 $\therefore I_s = D I_o$

Avg. source current < Avg. load current.  
→ Similar to step-down transformer.  
Source current waveform jumps from peak to zero.  
→ peak value of  $i_s > I_o$   
→ L-C filter at the input side.

We have derived an expression, output voltage  $V_o$  is equal to  $D$  into  $V_{DC}$ . So therefore,  $i_s$  source current is equal to  $D$  into  $I_o$ . So therefore, average value of the source current is less than the average value of the load current. See here, if you see **in the** in the waveform also, **source current supply** source supplies power for only till  $DT$  or when the switch is on, source supplies power **and** but then the moment the switch is turned on, instantaneous value of current is high. This is the,  $I$  minimum is **the current value** the current that starts flowing through the source and is abruptly starts falling.

What is desirable? Desirable is source also supplies the constant value of current. The moment, see, it has to supply abrupt value of current. There are stresses, stress **on the** on the source increases. Take for example, you are just sitting, all of a sudden  $I$  will come and put 10 kgs on your head, immediately, instantaneously. So, there is going to be a heavy load coming on you. Instead, if  $I$  gradually increase or gradually increase the load, it is better or **I can**  $I$  will not get exhausted. All of a sudden, **I am** instantaneously,  $I$  will come and put a heavy load on you, definitely,  $I$  do not think you can sustain, **you may** you may collapse.

Though at steady state you may be able to carry that much of load. Same thing happens to in the any any physical device, wherein, if you instantaneously load it, it may not be able to supply that much of current or the stress **on the** on the equipment increases. So it is always is better to have a constant value of current. But in this case it is just not possible. So definitely, in order to reduce the stress **on the** on the source, we need to have some sort of a LC filter at the input. **You need to have some sort of a LC filter at the input.** A source, a small LC filter and a switch, just to reduce the current stresses that on the source.

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Expression for current ripple in  $i_L$  :

Assume that  $V_o$  is constant (neglect the voltage ripple)

$$\frac{d i_L}{d t} = \frac{V_{DC} - V_o}{L}$$
$$= \frac{V_{DC} - D V_{DC}}{L}$$
$$= I_{min} + \frac{V_{DC}}{L} (1 - D) t$$
$$I_{max} = I_{min} + \frac{V_{DC}}{L} (1 - D) DT$$

Now, what is the expression for the ripple, that is the ripple **ah ripple** in the current that is flowing through the inductor or current in  $i_L$ ? After all, how do I choose the value of inductor and how do I choose the value of C, output capacitor? Definitely, I need to specify the ripple in the current as well as the ripple in the output voltage. So, we will make 1 assumption. That is while deriving an expression for the ripple in the current that is flowing through  $i_L$ , I will neglect the ripple in the voltage, output voltage  $V_o$  or can be vice versa.

If that is the case, rate of change of current  $d i_L$  by **dt**  $dt$  is given by  $V_{DC}$  minus  $V_o$  divided by  $L$ , when the switch is on. So,  $V_o$  is nothing but  $D$  into  $V_{DC}$ . So, the equation to this line is  $I_{min}$  plus the slope of the line into  $1 - D$  into  $t$ . So,  $I$  is equal to  $I_{max}$  at this point, at that time  $T$  is equal to  $DT$ . So, this is the expression for  $I_{max}$ .

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For  $DT < t < T$

$$\frac{di_L}{dt} = \frac{-V_L}{L} = \frac{-DV_{DC}}{L}$$
$$\therefore i_L = I_{max} - \frac{DV_{DC}}{L}(t - DT)$$

$i_L = I_{min}$  at  $t = T$

$$\therefore \Delta i_L = I_{max} - I_{min} = \frac{V_{DC}}{L}(1 - D)DT$$
$$\Delta i_L|_{max} \text{ when } D = 0.5 = \frac{V_{DC}T}{4L}$$

For during this duration from when the switch is open, current falls. Current falls from  $I_{max}$  to  $I_{min}$ . The slope of this line is now minus  $V_0$  by  $L$ , remember, because voltage across the inductor is minus  $V_0$ . So, substitute for  $V_0$ , we will get  $D$  into  $V_{DC}$  divided by  $L$ . So, equation for  $i_L$  is given by this,  $i_L$  is equal to  $I_{max}$  into this. That is nothing but very simple, if I know the the coordinates, coordinates at this 2 point, I can write the equation of a line.

Again,  $i$  is equal to  $I_{min}$  at  $T$  is equal to  $T$ . Substitute at this point, so you will get  $I_{min}$  is equal to  $t$ , time period  $t$  minus  $DT$ . What is the ripple in the current? This is the ripple in the current,  $I_{max}$  minus  $I_{min}$  is given by this term, this is the ripple in the current. So, when it is maximum? Of course, you differentiate it with respect to  $D$  and equate it to  $D$ , equate it to 0, you will find that ripple is maximum when  $D$  is equal to 0.5 and it is given by  $V_{DC}$  by  $T$  into 4 by  $L$ .

So,  $L$  can be selected or  $L$  is selected in such a way that having specified the maximum ripple, the input voltage and  $T$ ,  $L$  can be determined. Switching frequency is decided, input is the source voltage, current ripple is specified, that is the desired value, so you choose the value of  $L$ . Now, how do I determine the ripple in the output voltage,  $V_0$ ?

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Capacitor voltage ripple :

for  $0 < t < DT$

$$i_c = \frac{-\Delta I_L}{2} + \Delta I_L \frac{t}{DT}$$

for  $DT < t < T$

$$i_c = \frac{-\Delta I_L}{2} - \Delta I_L \frac{t - DT}{T - DT}$$

$$\Delta V_o = V_{max} - V_{min} = \frac{1}{C} \int_{DT/2}^{(1+DT)/2} i_c dt$$

$$= \frac{1}{C} \Delta I_L \frac{T}{8}$$

This is the expression for or this is the variation in the inductor current,  $I_{max}$   $I_{min}$  to  $I_{max}$ . When the inductor current is less than  $I_0$  less than  $I_0$  capacitor is supplying that current, capacitor discharges and at this point, both are same. Inductor current as well as the load current is the same and beyond this point, inductor current increases, capacitor starts charging till here. So, in this region capacitor is charging. Capacitor discharging, charging, discharging so this is the change in or ripple in the output voltage. Ripple in the output voltage.

From 0 to T, what is the equation for the capacitor current  $i_c$ ? It is given by this. This is  $\Delta I_L$  by 2, varies linearly, linearly with the time. So, this is the expression for  $i_c$ , this is the expression for  $i_c$  and again from DT to T variation is linear and expression for current is given by this. So, variation in the output voltage that is  $\Delta V_o$  is equal to 1 by C, integral of  $i_c dt$  in this region. In this region, there is a change in the voltage. So that is a capacitor current,  $i_c dt$ . So, integrate it, you will find that expression for  $V_o$  is given by this. In both cases, I have written the expression there.



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$$\therefore \Delta V_o = \frac{V_{DC}}{RC} (1-D)DT^2$$

for constant L, C & T,  $\frac{\Delta V_o}{V_{DC}}$  is maximum

for  $D = \underline{0.5}$

Now, substitute for  $i_L$  from the previous value. You will get  $\Delta V_o$  is given by this equation. So, **if I** again,  $\Delta V_o$  is maximum for D is equal to 0.5 both cases; **delta**, ripple in the current as well as ripple in the output voltage is maximum for D is equal to 0.5. So **if I mention**, if I specify value of  $V_o$ , I can choose the value of C because all other values are fixed. T is fixed,  $V_{DC}$  is fixed, L is fixed from the previous value. So, that is about the buck converter operating in continuous zone.

In other words, inductor current varies from  $I_{min}$  which is non 0 to  $I_{max}$ . It may so happen that the inductor current may become 0 and remain 0 for a finite time. So, what are that conditions or in what way the inductor current is related to  $I_o$  or what is the relationship or what is the critical value of R above which the **the** inductor current is going to be a discontinuous?



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**Discontinuous Conduction:**  
Inductor current  $i_L$  and NOT  $I_L$ .

$$I_L = \frac{V_o}{R}$$

Now  $i_L$  is continuous if

$$\frac{V_o}{R} > \frac{\Delta I_L}{2}$$
$$\frac{D V_{DC}}{R} > \frac{V_{DC}}{2L} (1-D) DT$$
$$\therefore R < \frac{2L}{(1-D)T} = R_{cr}$$

This is the variation in  $I_L$ ; increases, decreases. This is the constant value of  $I_0$ . Now, inductor current is continuous or it is positive if  $I_0$  is higher than  $\frac{\Delta i}{2}$  ripple, this ripple. Definitely, if this value is higher than  $\frac{\Delta i}{2}$ , inductor current is always positive. If  $I_0$  is less than  $\frac{\Delta i}{2}$  then the inductor current is going to be discontinuous, discontinuous.

Now, in AC to DC conversion, especially in a line commutated converters, when it is discontinuous, it implied that what does it imply? The load current will going to be discontinuous, whereas, in DC to DC conversion or in power supplies, the discontinuous current implies that current in the inductor is discontinuous. Please, do not say that load current is discontinuous. How can you have a load current discontinuous?

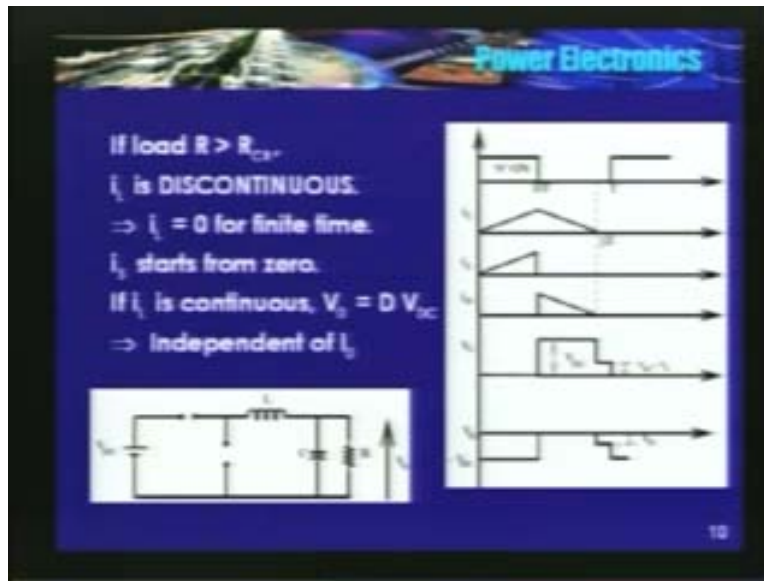
We have assumed that output voltage remains approximately constant,  $V_o$  supposed to be constant and ripple free. So therefore, if I connect a load and the circuit is complete, current has to flow. Therefore, invariably we assume that  $I_0$ ,  $I_0$  the load current is constant and ripple free. The question of discontinuity does not arise unless until load circuit is open. So, in buck converter or a few more power supplies that we will be seeing, when I am saying discontinuous current it implies that the inductor current.

So, this is the relationship,  $V_o$  by  $R$  is average value of current, should be greater than or equal to  $\frac{\Delta i_L}{2}$ , this value. So, if I substitute for  $i_L$  from the previous value, you will get this equation. Critical value of critical value of  $R$  should be this. If the load resistance is less than this value, you have always continuous conduction. So, if the load resistance is higher than this value, you will go in for a discontinuous. What is the concept of this, what will happen?

We have derived a transfer function -  $V_o$  is equal to  $D$  into  $V_{DC}$ . We derived it assuming that inductor current is continuous. When the diode is continuously conducting, when diode conducts from  $DT$  to  $T$ , only then voltage across it becomes minus  $V_o$  and we equated this voltage is ...

when the switch is on, voltage across inductor is positive; when diode is conducting, voltage across inductor is negative. When we equated it, since average voltage across inductor should be 0 and we got  $V_{DC}$  is equal to or  $V_0$  is equal  $D$  into  $V_{DC}$ . And, it is independent of the inductor current  $i_L$ .  $V_0$  is equal to  $D$  into  $V_{DC}$  and  $i_L$  does not appear there. Only, **only** condition is  $i_L$  should be continuous. What happens if  $i_L$  is discontinuous?

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So, when the diode has stopped conducting, what happens to the voltage across the inductor now? So, current has become 0 much before the beginning of the next cycle. So, **when you** when I turn on the switch  $S$ , current starts from 0 and starts increasing or varies linearly, may be, for the source is better now, it starts from 0. And, when the current has become 0 or when the diode has turned off, what is the voltage that is appearing across the switch? No current is flowing through the inductor. So, potential at this point, **potential at this point** is same as the potential at this point. So, this potential is  $V_0$ , **this potential is  $V_0$  this potential is  $V_0$** , plus  $V_0$ . This potential is  $V_{DC}$ . So, voltage across the switch is  $V_{DC}$  minus  $V_0$ ,  $V_{DC}$  minus  $V_0$ .

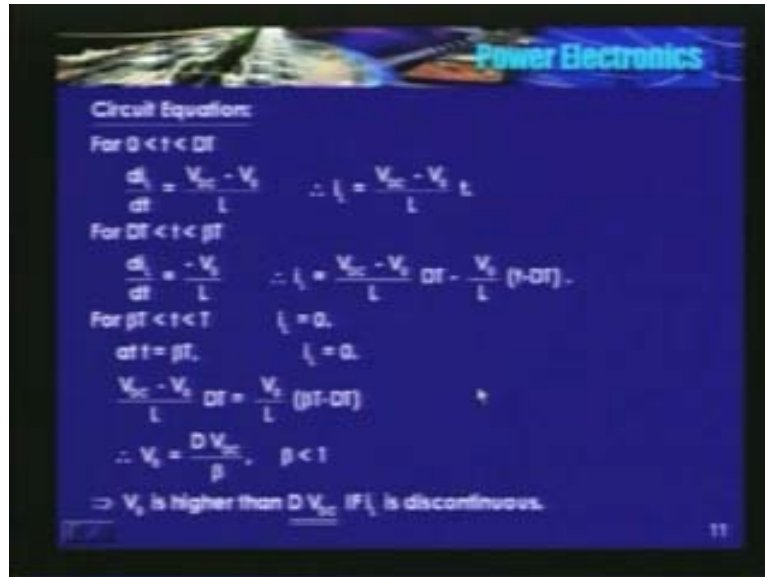
If the current is continuous, voltage across the switch is  $V_{DC}$  itself. For the entire region we would have add  $V_{DC}$ . Now, current has become 0, voltage at this point is  $V_0$ , no current is flowing through the inductor. So definitely, voltage at this point is equal to voltage at this point. Both points are at the same potential. Therefore, no current, so this potential is  $V_0$ , this is  $V_{DC}$ . Difference is appearing across this switch, so,  $V_{DC}$  minus  $V_0$ . What happens?

The voltage across the diode, **voltage across the diode** is same as  $V_0$  because what is the potential that is appearing at this point, it is  $V_0$ . So therefore, when the diode is on, voltage across it is 0. When it turns off, voltage that is appearing across the diode is  $V_0$  and when I turn on the switch again, current starts increasing now.

So, voltage across the switch is  $V_{DC}$  itself because this point closes. So, entire source voltage appears across the diode. So, it is  $V_0$  from  $\beta T$  to  $T$ . So, **this** during this period, voltage across

the diode is  $V_0$ , voltage across the switch is  $V_{DC}$  minus  $V_0$  and beyond  $T$  when it turn on  $S$ , it jumps to minus  $V_0$ .

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So, what are the circuit equations? Circuit equations are same;  $d i_L$  by  $dt$  is equal to  $V_{DC}$  minus  $V_0$  divided by  $T$ . But only thing is if the current is continuous, circuit equation when the switch is open is  $d i_L$  by  $dt$  is equal to minus  $V_0$  by  $L$  and it is true for, **true from**  $DT$  to  $T$ , whereas, now this equation is true from  $dt$  to  $\beta T$ . When  $\beta$  is inserted the current becomes 0. So, this is the expression for current,  $V_{DC}$  minus  $V_0$  into  $T$  and  $i_L$  was the current.

This is the peak value of current or the current that is attained just prior to opening the switch that  $V_{DC}$  minus  $V_0$  by  $L$  into  $D$  into  $t$  when  $D$  into  $t$ . This is  $I_{max}$ , this is  $I_{max}$  minus the reduction in the current from  $\beta T$  to  $t$ .  $i_L$  is 0 and current become 0 at  $t$  is equal to  $\beta T$ . So,  $i_L$  is equal to 0. You equate it, so you will find that  $V_{DC}$  minus  $V_0$  divided by  $L$  into  $D$  into  $T$ , this term should be equal to this term. So, expression for  $V_0$  is now  $D$  into  $V_{DC}$  divided by  $\beta$  and  $\beta$  is less than 1,  $\beta$  is less than 1,  $\beta$  is less than 1.

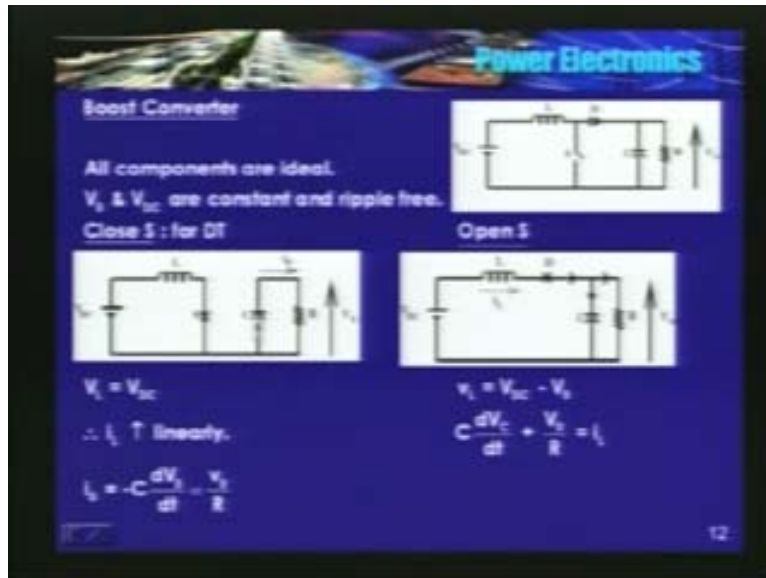
So, what is the result if the current is discontinuous? Output voltage  $V_0$  is  $D$  into  $V_{DC}$ , independent of the inductor current. If the current is discontinuous for a given value of  $D$ , output voltage is higher than  $D$  into  $V_{DC}$ . Output voltage in the discontinuous case is higher than  $D$  into  $V_{DC}$ . It is given by  $d V_{DC}$  divided by  $\beta$  where  $\beta$  is the instant where the current becomes 0 and is always less than 1. That is about the buck converter, wherein, the output voltage  $V_0$  is always less than or equal to  $V_{DC}$ , depends on  $D$ .

The duty cycle is almost the same as a step down transformer in AC. Output voltage is less, depends on number of turns  $n$ , whereas, the source current or **the** in primary, current again is higher than. That is about the buck converter, wherein, the output voltage is always less than the input voltage. It is almost the same as a step down transformer in AC. Whatever that happens in

a step down transformer, it happens in a buck converter. Therefore, we can call it as a step down DC transformer. We can call buck converter as a step down DC transformer.

In AC, we had a step up transformer. So definitely, we will have or we should have a step of DC transformer also. So that is nothing but a boost converter. We have already studied this and switch mode rectification. I will repeat it here and we will derive the other expressions.

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Shown here; switch, inductor, diode and the output stage. Switch is close for D into T, diode is off, capacitor supplies power. Open S, stored energy in the inductor is transferred to the output. Out here, I am making an assumption saying that remember, again I will emphasize V<sub>0</sub> is constant and ripple free, **constant and ripple free**.

So, what is the circuit equation here? V<sub>L</sub> is equal to V<sub>DC</sub>. Therefore, i<sub>L</sub> increases linearly, capacitor is supplying power, i<sub>0</sub> is equal to minus C d V<sub>0</sub> dt that is V<sub>0</sub> by R. Here, V<sub>L</sub>, a voltage across the inductor is V<sub>DC</sub> minus V<sub>0</sub> and this should be negative only then current decreases. Here V<sub>DC</sub>, voltage across the inductor is positive, i<sub>L</sub> increases. So, at steady state, current should increase here and current should decrease in this case. So, i<sub>L</sub> should fall when I open the switch or when the switch is opened, i<sub>L</sub> should decrease. So, V<sub>DC</sub> minus V<sub>0</sub> is negative. In other words, V<sub>0</sub> should be higher than V<sub>DC</sub>. Hence the name, boost converter and KCL at this point is capacitor current, C d V<sub>0</sub> by dt or d V<sub>0</sub> by dt, **d V<sub>0</sub> by dt**, capacitor voltage is same as V<sub>0</sub> here plus V<sub>0</sub> by R should be equal to i<sub>L</sub>.

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So, when the switch is closed, voltage across it is 0, voltage across the diode is minus  $V_0$ , voltage across the diode is minus  $V_0$  here. Entire  $V_0$  appears across the diode, this point gets connected at this point and when the diode is conducting, the switch is on. Again,  $V_0$  appears across it. So, in both the cases, voltage rating of switch as well as the diode is  $V_0$ . I will assume I had assumed that  $V_0$  and  $V_{DC}$  are constant and ripple free. I will equate it and I will find that  $V_0$  is  $V_{DC}$  divided by 1 minus D, this is the transfer function.

So, if I plot, I will get this sort of a variation,  $V_0$  by  $V_{DC}$  is equal to 1 when D is equal to 0. That means switch is opened, switch is opened, average voltage at inductor is 0. So therefore, average value of the input is same as the average value of the output and increases and goes towards infinity for D is equal to 1, provided, system is ideal. I have assumed that all the circuit elements are ideal, all the circuit elements are ideal, inductor is lossless, both voltage sources are constant and ripple free. So, there is a flow in our assumptions, will we will find out what are their flows.

Now, I assumed the system to be lossless. So, input power should be equal to output power. So,  $V_{DC}$  into  $i_s$  where  $i_s$  is the source current, should be equal to  $V_0$  into  $I_o$ . We substitute for  $V_0$ , we will get the relationship between the source current and the output current. So therefore,  $i_s$  is equal to  $I_o$  divided by 1 minus D. So,  $i_s$  is given by  $i_s$  is given by  $I_o$  divided by 1 minus D, simple.

Now, we will derive our transfer function taking into account the non idealities or taking into account the internal resistance of the inductor. I have not derived it for the buck converter. Now, what may happen in buck converter when D is equal to 1? When D is equal to 1, source continuously supplies power to the load. So, D is equal to 1. So, average value of  $V_0$  is equal to average value of the input voltage,  $V_0$  is D into  $V_{DC}$ , whereas, if I approach or if make D is equal to 1 there, output voltage, you know ideal converter, that is what the expression says that output voltage stands to infinity, looks like there is a problem. Looks like there is a problem, if I take into account or if I take, consider the converter to be an ideal one. So therefore, we will take the

non idealities into account and we will find that what are the flows or what in our assumptions that we made?

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Effect of  $\gamma$ :

$$V_{DC} = r i_L + L \frac{di}{dt} \quad \left| \quad 0 < t < DT \right.$$

$$C \frac{dv_C}{dt} + \frac{v_C}{R} = 0$$

$$V_{DC} = r i_L + L \frac{di}{dt} + v_0 \quad \left| \quad DT < t < T \right.$$

$$C \frac{dv_C}{dt} + \frac{v_C}{R} = i_L$$

So, we will consider a small  $r$ , resistance of the inductor. Now, KVL is  $r$  into  $i_L$  plus  $L$   $di$  by  $dt$ , the secondary correction is the same and when the switch is opened, it is  $V_{DC}$  is equal to  $r$  into  $i_L$  plus  $L$   $di$  by  $dt$  plus  $V_0$ . There is a KVL here, KVL here.

Now, I will find out the average values, this is true for  $0$  to  $DT$  and this is from  $DT$  to  $T$ . I will take the average values and I will add them. So,  $V_{DC}$  is here,  $V_{DC}$  is here,  $r$  into  $i_L$ ,  $r$  into  $i_L$ ,  $L$   $di$  by  $dt$ ,  $L$   $di$  by  $dt$  and  $V_0$  is here only from  $DT$  to  $T$ .



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Let the avg. values of  $v_s$  &  $i_L$  be  $V_s$  &  $I_L$  respectively.

$$V_{DC} = r I_L + L \left( \frac{di_L}{dt} \right)_{avg} + \frac{1}{T} \int_{DT}^T v_s dt$$

$$C \left( \frac{dv_c}{dt} \right)_{avg} + \frac{V_c}{R} = \frac{1}{T} \int_{DT}^T i_L dt$$

Average values of  $\frac{dv_c}{dt}$  &  $\frac{di_L}{dt}$  are zero at steady state.

Also, variation of  $v_s$  and  $i_L$  is assumed to be linear.

$\Rightarrow$  avg. values of  $v_s$  &  $i_L$  during  $(DT, T)$  are equal to avg. values of them varying on the whole cycle.

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So, what do I get?  $V_{DC}$ , average value is the same,  $r$  into  $i_L$ .  $L \frac{di}{dt}$  is the average value plus  $\frac{1}{T} \int_{DT}^T v_s dt$  because this  $V_0$  is there only from  $DT$  to  $T$ . Similarly, the current equation if you see,  $C \frac{dV_0}{dt} + \frac{V_0}{R} = \frac{1}{T} \int_{DT}^T i_L dt$  here from  $DT$  to  $T$  is 0 here.

So, if I have to find out the average value or integrate and you do it and you will get this value,  $C \frac{dV_0}{dt}$  average,  $V_0$  by  $R$ . So,  $\frac{1}{T} \int_{DT}^T i_L dt$ . So, at steady state, the variation in inductor current, average value should be 0 at steady state. Current increases and decreases, it has attained the same value at  $T$  is equal to  $DT$  is equal to beginning of the next period. Similarly, the output voltage. Of course, capacitor would have does changed but then average value should **be** remain constant or in other words, average current flowing through the capacitor should be 0 at steady state. Average value of the current that is flowing through capacitor should be 0 at steady state. So, I will get 2 equations because this is 0, this is 0.



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$$V_{DC} = r i_L + (1-D)V_0 \rightarrow (1)$$
$$\frac{V_0}{R} = (1-D) i_L \rightarrow (2)$$

multiply (1) by (1-D)

$$V_{DC} (1-D) = r i_L (1-D) + (1-D)^2 V_0$$

using (2)

$$V_{DC} (1-D) = r \frac{V_0}{R} + (1-D)^2 V_0$$
$$\therefore V_0 = \frac{V_{DC} (1-D)}{\frac{r}{R} + (1-D)^2} \rightarrow (3)$$

So, the 2 resulting equations are RDC is equal to  $r$  into  $i_L$  plus  $1$  minus  $D$  into  $V_0$ . See, if  $r$  tends to  $0$ , you have the same equation;  $V_{DC}$  is equal to  $1$  minus  $D$  into  $V_0$ . The difference between an ideal and non ideal is only this,  $r$  into  $i_L$  but looks like it effects significantly. We will see some time later. Similarly,  $I_0$  is equal to  $1$  minus  $D$  into  $i_L$ . Now, I will do sum jugglery, I will multiply this equation 1 by  $1$  minus  $D$  and I will get this equation. Multiply this equation by  $1$  minus  $D$ , I will get this. Now, I have here,  $1$  minus  $D$  into  $i_L$ , I have  $1$  minus  $D$  into  $i_L$ . So, I will substitute, so I will get this equation, very simple maths.

So, now I will write an expression for  $V_0$  in terms of  $V_{DC}$ . So, I will get this. Again, if you substitute here  $r$  is equal to  $0$ ,  $r$  is equal to  $0$ , so you will get same as  $V_0$  is equal to  $V_{DC}$  divided by  $1$  minus  $D$ . So, this term remains  $0$ , this gets cancelled on this, so same equation. But then what will happen for finite values of  $r$ , small  $r$ ?

If  $D$  is equal to  $0$ , if  $D$  is equal to  $0$ , what is the average value of the output voltage? It is no longer is equal to  $V_{DC}$ . Now, I have a potential divider, small  $r$  here. See here, a small  $r$  here and a  $R$  here. Input is  $V_{DC}$ , so output voltage  $V_0$  is  $V_{DC}$  divided by  $r$  plus  $R$  that is the current that is flowing multiplied by this  $R$  is the output voltage. It is not equal to  $V_{DC}$  itself.

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$$D = 0, V_o = \frac{V_{DC} \cdot r}{r + R}$$

$\Rightarrow$  As  $D \uparrow$ ,  $V_o \uparrow$ .

From (3),  $V_o = 0!$  when  $D = 1$   
( $V_o \rightarrow \infty$  at  $D = 1$  when  $r = 0$ )

Let  $D = D_{max}$  at  $V_o = V_{max}$

$$\frac{dV_o}{dD} = 0, D = 1 - \sqrt{\frac{r}{R}}$$
$$\therefore V_{o(max)} = \frac{V_{DC}}{2} \sqrt{\frac{R}{r}}$$

So, as  $D$  increases,  $V_o$  also increases. If you see in this equation, see in this equation, when  $D$  is equals to 1,  $D$  is equals to 1, output voltage is 0, output voltage is 0. See, we have a very interesting equation here, if I neglect  $r$ , if I neglect  $r$  here and  $D$  is equal to 1,  $D$  is equal to 1,  $V_o$  becomes infinity and if I consider  $r$  at  $D$  is equal to 1,  $V_o$  becomes 0,  $V_o$  becomes 0. If I neglect  $r$ ,  $D$  is equal to 0, output voltage is same as input voltage. If I take  $r$  into account, output voltage is  $V_o$  divided by  $r$  plus  $r$  into  $R$ , small  $r$  plus  $r$  multiplied by the load resistance, capital  $R$  which is less than  $V_{DC}$  itself. Fine, there is no much difference because the winding internal resistance is very small compared to the load resistance.

What happens when  $D$  is equal to 1? If I neglect the winding resistance  $R$  or the internal resistance of the inductor,  $D$  is equal to 1,  $V_o$  is equal to infinity. Now, if I take  $r$  into account,  $V_o$  becomes 0,  $V_o$  becomes 0. Why such a large difference, ideal case infinity, non ideal case 0? Such a large difference; ideal transformer, no load current is very small sorry ideal transformer no load current is 0, non ideal transformer, no load current is of the order of 5, 2 to 5%, a good transformer.

Here, ideal boost converter,  $V_o$  is infinity; non ideal boost converter,  $V_o$  is 0. Poles apart, why so? I will address, I will come to that point. But then, as  $D$  increases, in both cases,  $V_o$  starts increasing initially and  $D$  is equal to 1 and in the second case, it becomes 0. So definitely, for 1 value of  $D$ ,  $V_o$  approach  $V_o$  becomes maximum and becomes and from there onwards, it starts decreasing.

How do I find? I will differentiate that equation and equate it to 0, differentiate it with respect to  $D$  and equate to 0, you will find that you find that this value of  $D$  is equal to 1 minus  $r$  divided by  $R$ , square root of this equation. You need to differentiate this equation;  $V_o$  is equal to  $V_{DC}$  1 minus  $D$  into this. Differentiate it with respect to  $D$ , equate it to  $D$ , you will find the value of  $D$ . At that instant,  $V_o$  is maximum. So, that value of  $D$  is found to be this.

So, if you substitute  $r$  is equal to 0,  $D$  is equal to 1. Same, it is ideal case, infinity and if you put this value, you substitute this value in this equation; substitute for  $D$ , you will get  $V$  is equal to  $V_{\max}$  and that is equal to  $V_{DC}$  divided by  $R$  divided by  $r$ ,  $R$  is the load resistance, this small  $r$ . Therefore,  $V_0$ , maximum voltage that you can get at the output is the strong function of the internal resistance of the inductor.

It depends on the ratio of load resistance to the internal resistance of the boost converter. As this ratio increases, as the internal resistance becomes 0, in other words, I am going towards the ideal case voltage, I can get a higher, maximum voltage  $V_0$  in  $V_0$  max increases.

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$$D=0, V_0 = \frac{V_{DC} \cdot r}{r+R}$$

$\Rightarrow$  As  $D \uparrow$ ,  $V_0 \uparrow$ .

From (3),  $V_0 = 0!$  when  $D = 1$

( $V_0 \rightarrow \infty$  at  $D = 1$  when  $r = 0$ )

Let  $D = D_{\max}$  at  $V_0 = V_{\max}$

$$\frac{dV_0}{dD} = 0, D = 1 - \sqrt{\frac{r}{R}}$$

$$\therefore V_{0(\max)} = \frac{V_{DC}}{2} \sqrt{\frac{R}{r}}$$

See here,  $R$  is equal to sorry it does not decrease, it increase, goes on increasing, please at 1 and goes on increasing, does not decrease. It attains the peak and comes down, it attains the peak and comes down. Please, it starts from 1, increases, attains the peak and comes down. If  $R$  is equal to 0, it becomes infinity. So,  $R$  by  $r$  for this curve is higher compared to this.

Now, what is the difference, 0 and infinity? Before answering this question, I want to ask; what are the assumptions that we made? We said that  $V_{DC}$  is  $V_{DC}$  and  $V_0$  are constant and ripple free,  $V_{DC}$  and  $V_0$  are constant and ripple free.  $D$  is equal to 1 or  $D$  approaches 1, what does it imply? Most of the time switch is closed and what is the equivalent circuit when the switch is closed? See, here is equivalent of the circuit if the switch is closed.

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Power Electronics

$$D = 0, V_o = \frac{V_{DC} \cdot r}{r + R}$$

$\Rightarrow$  As  $D \uparrow$ ,  $V_o \uparrow$ .

From (3),  $V_o = 0!$  when  $D = 1$   
( $V_o \rightarrow \infty$  at  $D = 1$  when  $r = 0$ )

Let  $D = D_{max}$  at  $V_o = V_{max}$

$$\frac{dV_o}{dD} = 0, D = 1 - \sqrt{\frac{r}{R}}$$
$$\therefore V_{o(max)} = \frac{V_{DC}}{2} \sqrt{\frac{R}{r}}$$

Capacitor continuously supplying power to the load and **switch is** inductor current increases linearly and when S is equal to 1, what does it imply? S is equal to 1 implies that switch is permanently closed, **switch is permanently closed**, capacitor is permanently supplying power to the load. What is this situation? This situation results into a case where output voltage becomes 0, capacitor has to discharge and capacitor will discharge, it is continuously supplying power, it does not receive power at all when D is equal to 1. So, capacitor voltage gradually decreases and becomes 0.

What happens at the input stage? A  $V_{DC}$  is permanently applied across an inductor. A constant voltage is permanently applied across the inductor. So,  $L \frac{di}{dt}$  is  $V_{DC}$  that is positive. In other words,  $\frac{di}{dt}$  goes on increasing. The device, the inductor or the source have their own current rating capacity. Beyond a point,  $V_{DC}$  will fail or inductor will fail or switch will fail.

At steady state, if the switch is permanently closed, **current is** steady state current is  $V_{DC}$  divided by small  $r$ . Inductor is saturated, **inductor is saturated**,  $V_{DC}$  divided by small  $r$ ,  $r$  is very small, so  $L$ , large current in flow and it will definitely damage all 3 or one of them and at the output,  $V_o$  becomes 0,  $V_o$  becomes ...

So, there is a flow in our assumption or in other words, assumptions are valid only if the value of  $D$  is low. As  $D$  approaches 1, our assumptions are not valid. What are they?  $V_o$  is constant and ripple free, no,  $V_o$  will decrease,  **$V_o$  will decrease**. Here, current increases and it may saturate the inductor and may damage the source or inductor or the switch. So, our assumptions are not valid for high values of  $D$ . They are valid only for low values of  $D$ . For low values of  $D$  implies I close the switch, dumping the energy, closing the switch, dumping an energy. So, I can safely assume that output voltage will remain approximately constant and here inductor does not saturate, that is.

So, remember, our analysis is everything correct, only the problem with our assumptions. So, what is the average value of the current? See,  $V_{DC}$  divided by  $R$  by  $r$ . It is true, voltage input voltage is  $V_{DC}$  for  $D$  is equal to 0. In other words, switch is permanently open. The current that is flowing is the total resistance of the circuit,  $V_{DC}$  divided by total resistance is  $R$  divided by  $r$ . Average value of this current is goes on increasing with the  $D$  and at  $D$  is equal to 1, it is  $V_{DC}$  divided by  $R$ .  $D$  is equal to 1, switch is permanently closed. Current that is flowing is  $V_{DC}$  divided by small  $r$ . It will be very high. I do not think you can achieve that value.