

Control Engineering
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Lecture - 09
Steady-State: Derivatives Zero

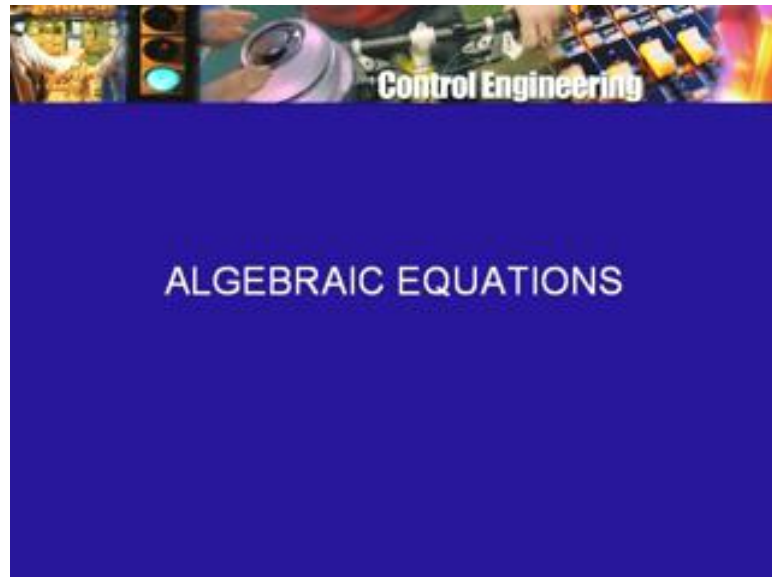
Let us look at the equations in the steady state that is assuming that the motor is running at a constant speed, the load torque is constant the power supply voltage is also constant as a result, the derivative terms become 0 there is no change of speed so $\frac{d\omega}{dt}$ is 0 there is no change of armature current, so $\frac{di_a}{dt}$ is equal to 0 and because of that the equations become simply algebraic equations and therefore they will be the applied voltage E_a will be equal to the back EMF, the back EMF will be $K_b \times \omega_m$ the motor speed plus only the armature drop term $R_a I_a$ the $L \frac{di_a}{dt}$ term is 0 and the second equation is the mechanical system torque equation, the torque is k_T times the armature current and that equals the constant load torque T_l plus the frictional torque $K_f \omega_m$. The moment of inertia term $J \frac{d\omega}{dt}$ is equal to 0.

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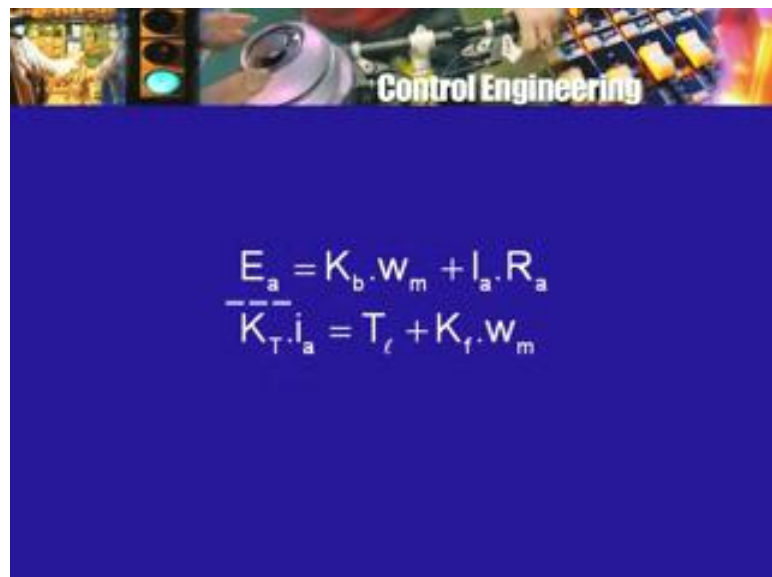


So these are the steady state equations what information can we get from here or how these equations can be made use of for the purpose of designing our control system or the motor. Now this is where I will take some very simple numbers, so that we do not get bogged down in detailed calculations and numbers with 3 or 4 places of decimals and what not. But in practice the numbers will not be such nice rounded of numbers and one will have to work with the calculator and sometimes even the computer to make detail calculations

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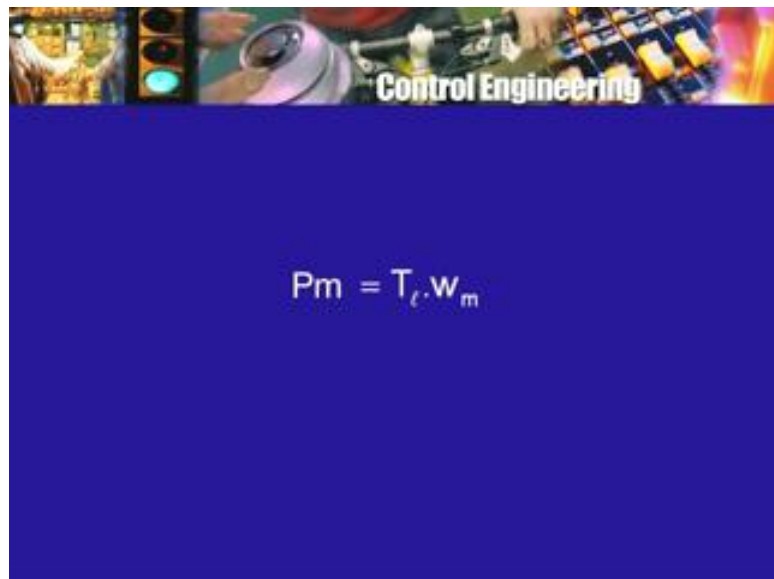
So keep this in mind that I am only using some very simple round figures. So, that we can understand the point without getting lost in to all the computation. Now what was the objective of our control system or the drive the objective is to drive the load at a particular speed. So the load there are two aspects of it which are there in this equation about the torque namely the load torque and the desired speed ω_m . As we have seen the product of these two the load torque in to the desired speed or the speed at which the load will run gives you the power the mechanical power that is to be spent on the load and that much

power and in fact a little more has to be provided to the motor that is the electric power that will be taken from the supply.

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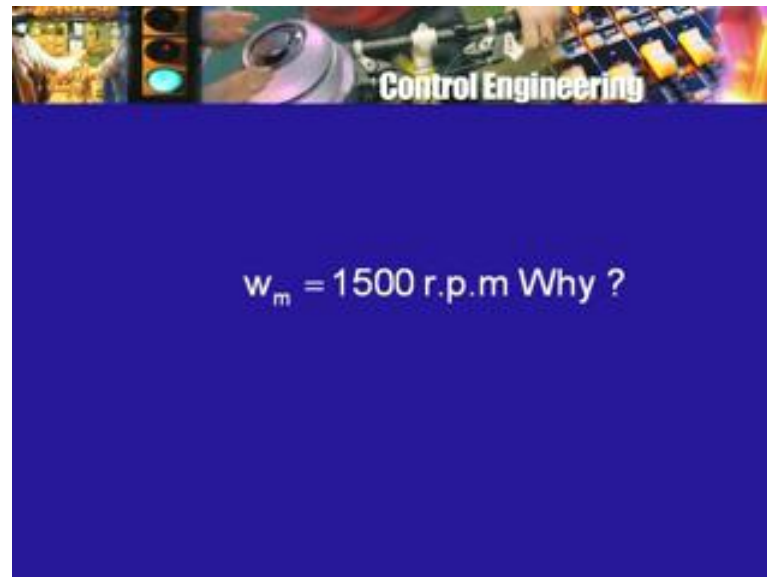
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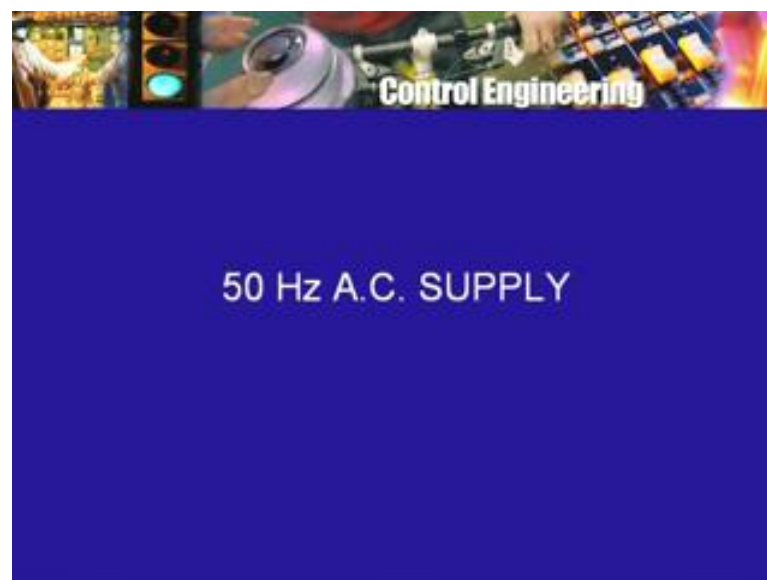
So the mechanical power P_m is equal to the load torque T_l in to ω_m . Now depending on the particular load application of these 3 numbers P_m , T_l and ω_m of them will be specified in the grinding wheel example perhaps based on some previous experience or measurement one will be able to figure out what is the load torque, when you are doing a

particular grinding operation, what is the resistance of the surface being ground and therefore what is the torque that is required and of course the speed at which the grinding wheel should operate, a practical grinding wheel will have to operate perhaps at a much higher speed than whatever numbers we will select. Similarly, the torque number that we with choose will also be just for our convenience of calculation and does not reflect any actual figure.

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So let us choose some rounded numbers ω_m for ω_m we will choose a speed roughly corresponding to 1500 revolutions per minute 1500 rpm and there is reason for this particular figure 1500 rpm. The 1500 rpm figure arises in the following way. Remember that the power supply that we normally get in our house or in an industrial establishment is an alternating current power supply and in India our frequency is 50 hertz or 50 cycles per second. You can think of that roughly as 50 revolutions per second of some generator somewhere and therefore that will correspond to 3000 revolutions per minute and as you know it turns out that depending on the number of poles on the generator that you have or the number of pole pairs because the poles come in north south pairs. The generator will have to be run at an appropriate speed this is what actually happens in a generating station and therefore there is a speed control problem at the generating station itself.

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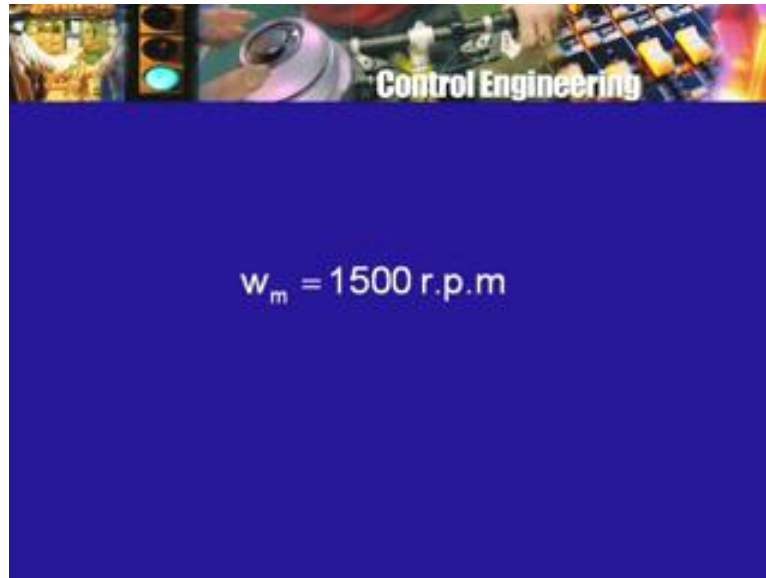


The generator of course is not driven by a motor the generator is driven by a turbine, a hydraulic turbine in the case of hydroelectric station or a steam turbine in the case of a thermoelectric generating station or a thermal generating station. In order to produce an AC voltage of 50 hertz frequency therefore it will be necessary to maintain the turbine speed at a constant value and turbine speed control and in addition to the frequency of course the generator voltage has to be maintained constant. So the voltage control these are two major control problems in a generating station, voltage control and frequency control, frequency of the AC produced but that is dependent on the turbine speed.

So essentially it amounts to controlling the speed of the turbine maintaining it reasonably constant and maintaining the generator output voltage reasonably constant. So there are two major control problems in a electric generating station anyway for the movement the figure therefore 3000 rpm can be understood as 50 hertz or 50 cycles and if you think of a cycle as a revolution then 50 cycles or revolutions per second amounts to 3000 revolutions per

minute and since you have at least one pole pair therefore the frequency at which the generator should run will be 1500 rpm. It can run of course at a lower speed if you have more poles then the more pole pairs, you can run the turbine at a lower speed typically a steam turbine can be run at a high speed whereas a hydraulic turbine will run at a lower speed.

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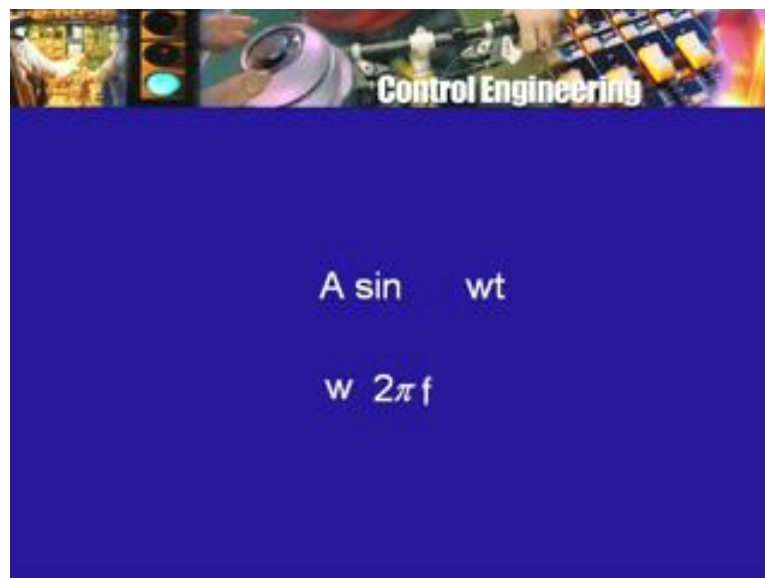


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Therefore a steam turbine driving a generator or an alternator as it is called will require a smaller number of pole pairs for the alternator and for the hydraulic turbine it will be a larger number of pole pairs. So that is the reason for choosing this number 1500 rpm an actually grinding wheel may run much faster than that and in fact I will make some further rounding of as we go along otherwise as I said I will have to write down several places of decimals.

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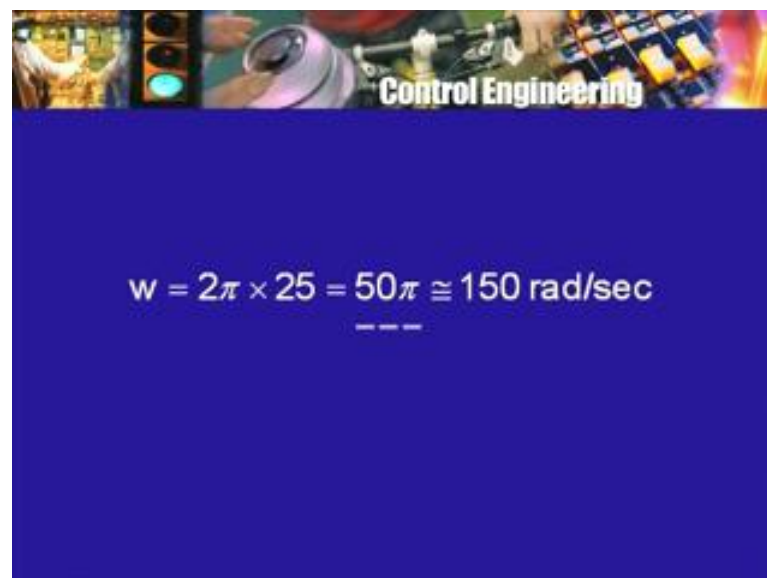


Now a 1500 rpm and roughly how much is it as in terms of radians per second. Now these are some of the calculations which as a engineers one should be able to do mentally without putting down or using the calculator rough calculations I am saying not precise figures. So once again 1500 rpm just as 3000 rpm corresponded to 50 hertz, 1500 rpm will correspond to 25 hertz, 25 hertz.

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So 25 into 60 is 1500 that is how 25 hertz. Remember the expression for the sinusoidal voltage that we wrote down earlier it had something like $A \sin \omega t$ where this ω is called the frequency, angular frequency for the power supply voltage and it of course as we can see corresponds to some turbine rotating somewhere. So this ω although it is the frequency almost not quite ω is $2\pi f$, where f is the frequency in hertz. This ω will be what 1500 rpm corresponds to 50, 25 revolutions per second, so with that ω will be 2π times that and therefore ω will be nearly 50π radians per second. The angular speed in radians per second is 50π , a better way of thinking of how it is this $2\pi f$ business is more electrical than mechanical will be one revolution is 360 degrees and how many radians is that. Now everybody knows that it is 2π radians.

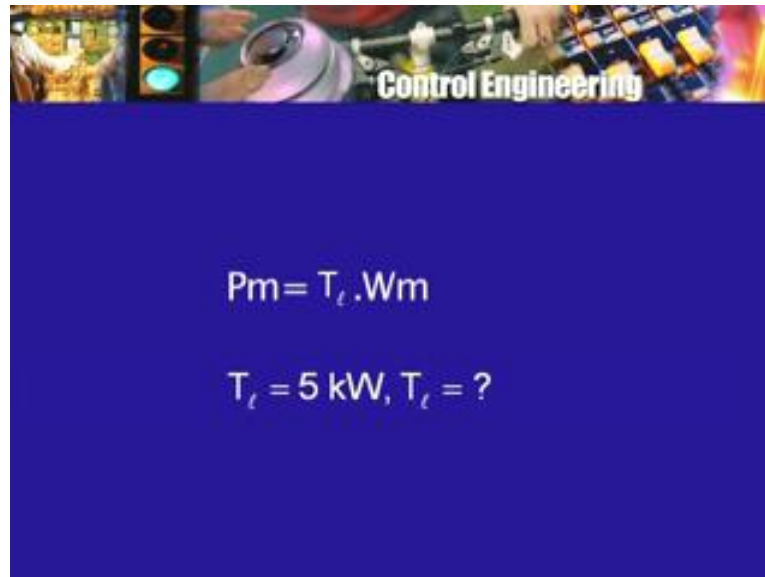
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So in fact we talk about 360 degrees as an angle of 2π radians right. So one revolution is 2π radians so 25 revolutions will be 2π into 50, 50π radians in a second so ω will be roughly 50π and may be as we proceed we will take it approximately as 150 radians per second, just a rough figure precisely of course it will be 50π and you can take π into whatever number of decimal places it will be a little more than 150 radians per second. So that is ω , the motor speed.

Now the torque we have to take it Newton meters then the product of the torque in Newton meters and the angular speed in radians per second will give us the mechanical power in watts. Now as I said one may be used to thinking of power in the horsepower units though nowadays because we are using the international system of units one would use the watt or the kilowatt as the unit of power.

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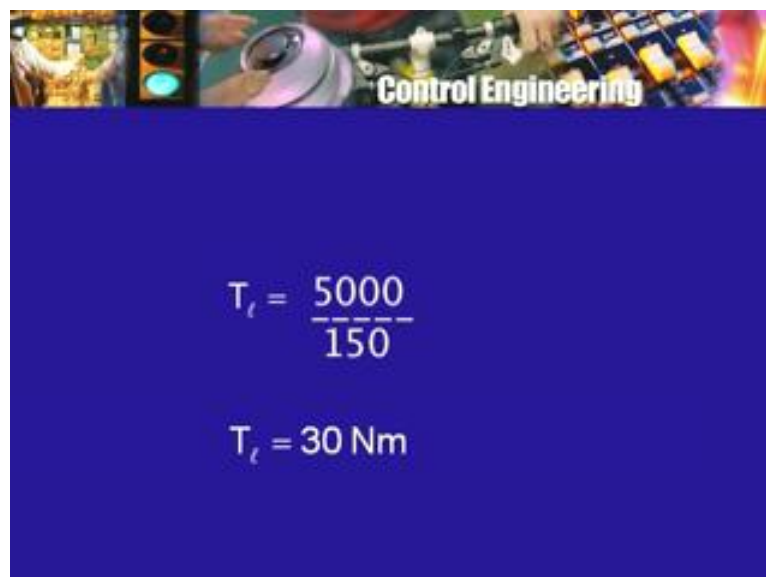


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$$P_m = T_l \cdot \omega_m$$
$$T_l = 5 \text{ kW}, \omega_m = ?$$

So thinking of that then P_m the mechanical power is load torque into ω_m ω_m is about 150 rad per second. So if I think of a motor power of the order of let us say we are thinking of a motor which is about 5 kilowatt or the load which requires a power of about 5 kilowatt so what will be the magnitude of the torque it will be 5 kilo watt, so that is 5000 watts divided by 150 that is a little more than 30, so let us say 30 Newton meters.

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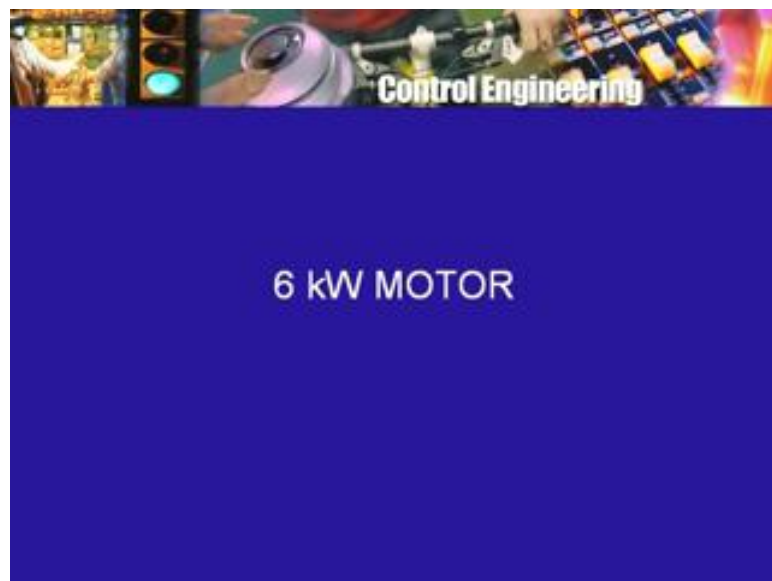


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$$T_l = \frac{5000}{150}$$
$$T_l = 30 \text{ Nm}$$

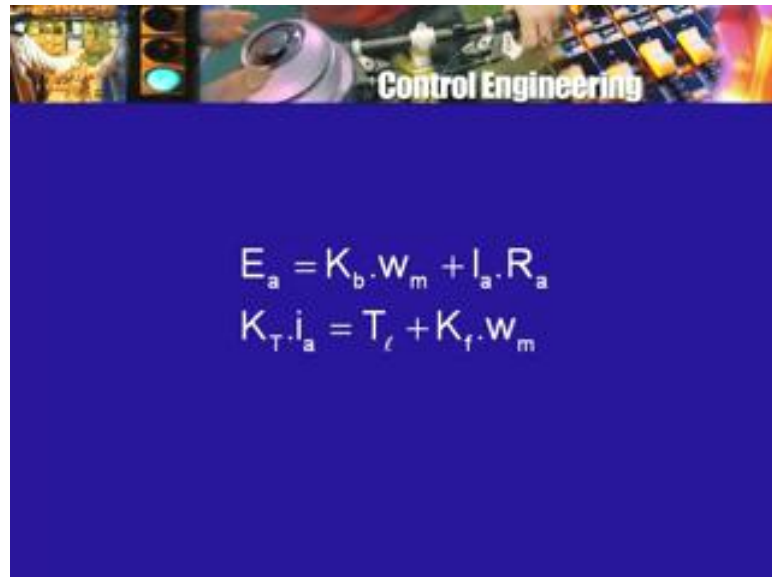
As you can see what I am trying to do is I am trying to cook up an example. So that we can work up with some simple figures and get some idea of what is really important and what is it that is need to be done so now I can pose the problem like this a load requiring 13 Newton meters maximum torque is to be driven at a speed of 1500 rpm that is where the problem will start here is a particular mechanical load which requires at a speed of 1500 rpm a torque of so many Newton meters what kind of motor should I buy. So you can see how the calculations go omega m is known T l if it is known one calculates the mechanical power in this case 13 Newton meters into 150 rad per second will be about 4500 watts and so on, we will say well I will choose a motor whose capacity is more than that but not much more than that.

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So may be a 5 kilowatt motor or a 6 kilowatt depending on what is available off the shelf one will go for okay. So the load torque that is to be provided at a specified speed are now known the us, so we have two figures and we have the mechanical power. Now of course the motor is being chosen so that it can deliver that much power or even more. So we will have no problem in finding such a motor what next can we calculate, so if you look at the equations then you have the motor torque generated by the motor equal to the load torque which we have found out or which we have assumed is 13 Newton meters plus the coefficient of friction into the angular speed at which the motor is going to run. So the motor is going to produce more than what the load itself is going to absorb because some power will be lost in friction it will appear as heat elsewhere in the bearings may be the fan will be turning and so on.

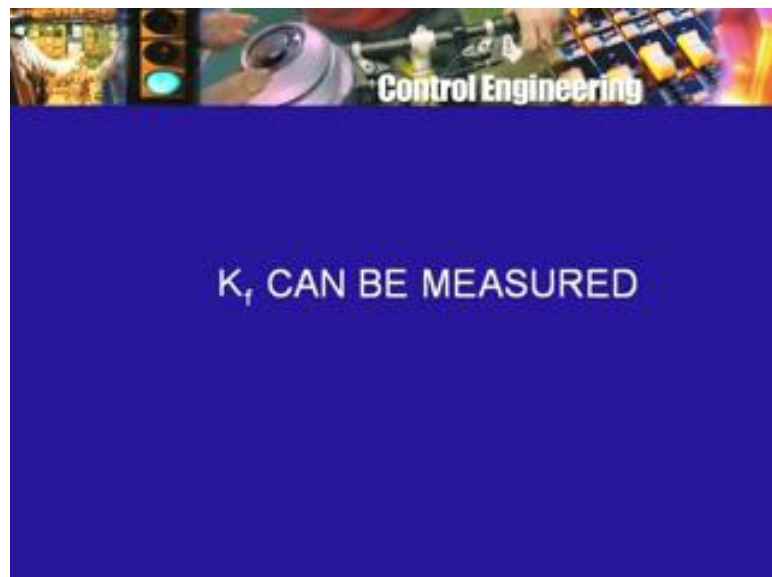
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$$E_a = K_b \cdot \omega_m + I_a \cdot R_a$$
$$K_T \cdot i_a = T_l + K_f \cdot \omega_m$$

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K_f CAN BE MEASURED

So the motor power therefore has to be more than the mechanical power which is to be delivered to the load and knowing K_f , now how do you know K_f . Well, once you know the load if you have decided about what kind of motor you are going to use may be the motor comes with a fan, you will have to decide what kind of bearings you are going to provide and then one can thought of set up the drive preliminary in a preliminary manner and if necessary even measure this friction coefficient K_f .

So the design is not simply a matter of sitting at the table and just doing some calculations it will involve going back and forth looking at the load what finding out what is the requirement then looking in to what is commercially available, cheap, easily, reliably what kind of motor quality of course we have decided that we will use a separately excited DC motor. So one is only looking for a DC motor what power, so rough calculations are shown if you know K_f of course K_f you will not know completely unless you have chosen the motor to start with so a little back and forth will enable you to decide as to what should be the motor.

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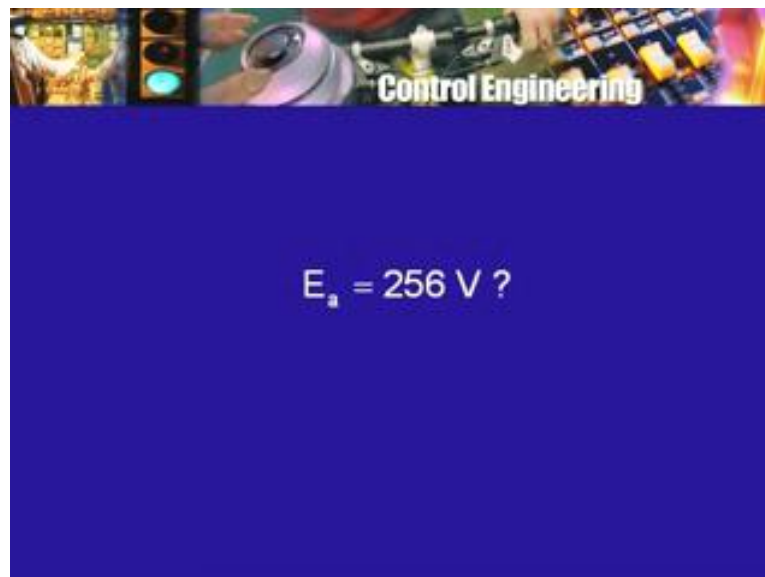
So probably if 4500 the product 13 Newton meters into 50, 150 rad per second was about 4500 watts. So with 6 kilowatt as the maximum power output of the motor probably we are on the safe side, so having chosen such a motor then one will find out what is this K_f and those therefore complete the calculation. Now what is the result of all this calculation T_L is known the load torque K_f is now known the coefficient of friction of the whole drive the angular speed is known, so we know the torque that the motor has to produce. The motor will produce torque $k_T I_a$ equal to this sum knowing this then and knowing the motor torque constant I have chosen the motor already.

So I will know the motor torque constant I can calculate the armature current I_a , so that is the next thing that we will then be able to find out armature current I_a , what next the motor has been chosen. So we know the armature resistance R_a and therefore we know the armature drop $R_a I_a$ when the armature will carry this current to produce the required torque. Now this plus the back EMF what about the back EMF, K_b we know we have already decided what motor to use so we know K_b in fact approximately it is going to be equal K_T .

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So K_b into ω_m will give us the back EMF and so all of this will add up to give us the applied voltage E_a okay. Now suppose I do all this calculation and I found find out that this E_a turns out to be say something like 256 volts. Now obviously I am not going to be able to find a readymade DC power supply of this value 256 volts. In the old age when even the houses use to have DC power supply like the AC power supply voltage today there was a standard DC power supply voltage and once again it was 230 although you know that 230 volts rms is not the same thing as 230 DC as for as electric shock is concerned.

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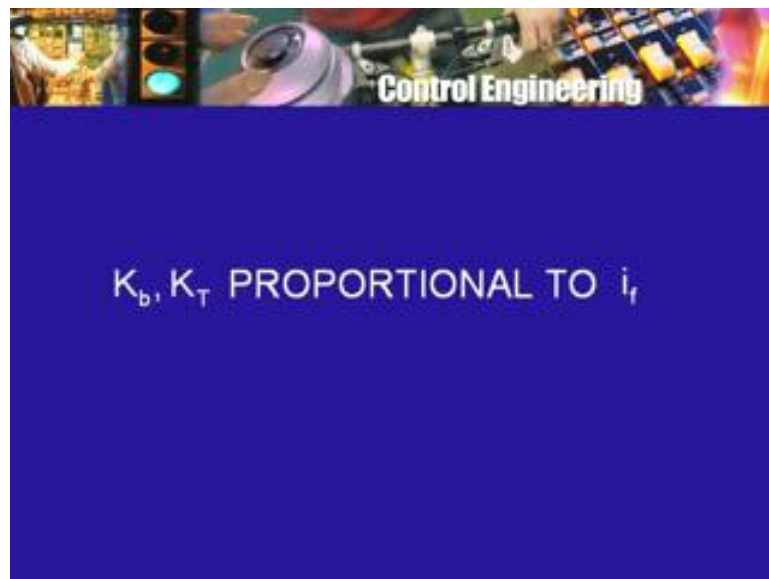


So now suppose the power supply voltage available to me unless I want to design it specially is 230 volts but my calculations has thrown a figure of 246 volts. So what do I do I go back to choose another motor and things like that, no because there was one assumption that I was making. So that is something that will have to be decided the assumption that I was making is that I know k_T or I know K_b for the motor but if you remember I mentioned earlier that these constants the EMF, the back EMF constants and the torque constant both of them depend on the field current I_f . So in fact the these two

constants are determined by the field current, so when I say I know k_T and K_b what I really can find out is for this speed current what should be k_T and K_b .

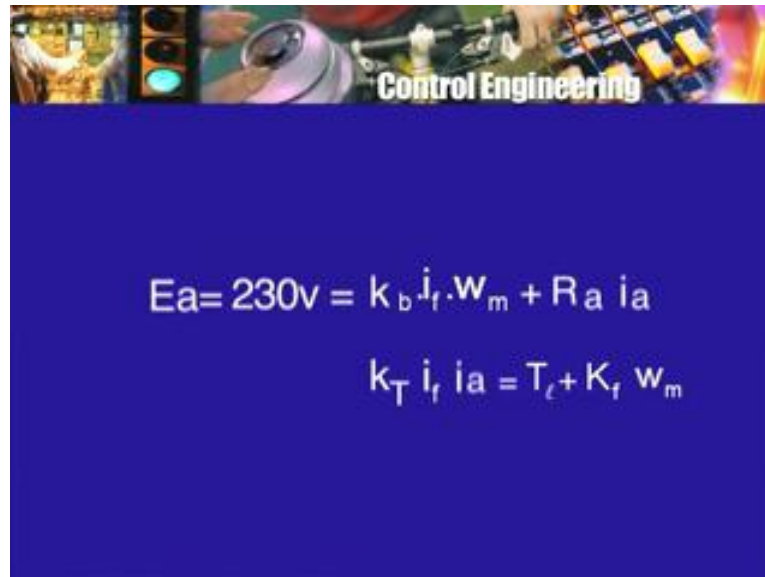
So now we will have to do our calculations in a little different manner, we will say that okay the supply voltage is known to us. So let us say it is 230 volts, so 230 volts is equal to now it is a good approximation that the torque constant and the back EMF constant are proportional to the field current. Of course this approximation holds provided the field current is not very large if you apply a very large field current then what is known as saturation can take place in the magnetic circuit and therefore the flux will not be proportional to the field current but if you do not reach that level which normally one will not reach then the torque constant and the EMF constant will be roughly proportional to the field current.

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So instead of K_b into ω_m I can write it as say another coefficient into field current into ω_m so I will write it as let us say k_b into ω_m that is the back EMF term plus R_a into I_a that is the armature drop now of this we know k_b it is the back EMF coefficient of per ampere of field current per rpm of speed. So it is not just volts per rpm that was the case with capital K_b into ω_m rpm multiplied by volts now it will be or radians per second. Now it will be volts per rpm per ampere of field current, so that is this term the second term is the torque and therefore there also you will have a k_T small k torque Newton meters per ampere of armature current, per ampere of field current. So I will have k_T into I_a equal to the total torque which is used up by the load and the friction so T_l plus K_f into ω_m . Now in these two equations of course I have assumed already that I use a power supply voltage of 230.

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The slide features a header with the text "Control Engineering" and a background image of various electronic components and a motor. Below the header, on a dark blue background, are two equations:

$$E_a = 230\text{v} = k_b \cdot i_f \cdot \omega_m + R_a i_a$$
$$k_T i_f i_a = T_l + K_f \omega_m$$

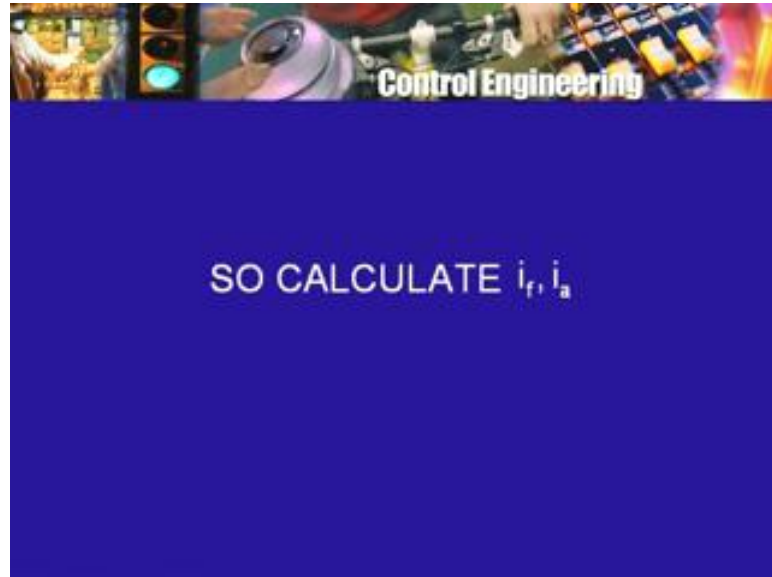
So what else is what is known and what is not known ω_m is the speed which we want so we know 1500 rpm or roughly whatever number of radians per second. Load torque is known, armature resistance is known coefficient of friction is known and this constant or these constants little k_b and small k_T in fact they will be nearly equal which are the torque and the back EMF constant giving back EMF or torque per ampere of field current and per rpm or per ampere of armature current these are known.

So then from these 2 equations what are the unknowns here, the unknowns are essentially the field current and the armature current ω_m is known everything else is known I have assumed that E_a , I am choosing because of their supply availability as 230 volts. So everything else is known except the 2 unknowns I_f and I_a . Now these two equations are fairly simple although you can see that in one equation I have what is called a linear combination of I_a and I_f , 230 equal to some number in to I_a plus some other number in to I_f whereas the other equation is some number into the product of I_f and I_a equal to some other known number. So this is an example of what is called a non-linear equation but the non-linearity is really very simple just the product of these two currents and so certainly it is not beyond your capability to solve these 2 equations for the 2 unknowns here field current and armature current.

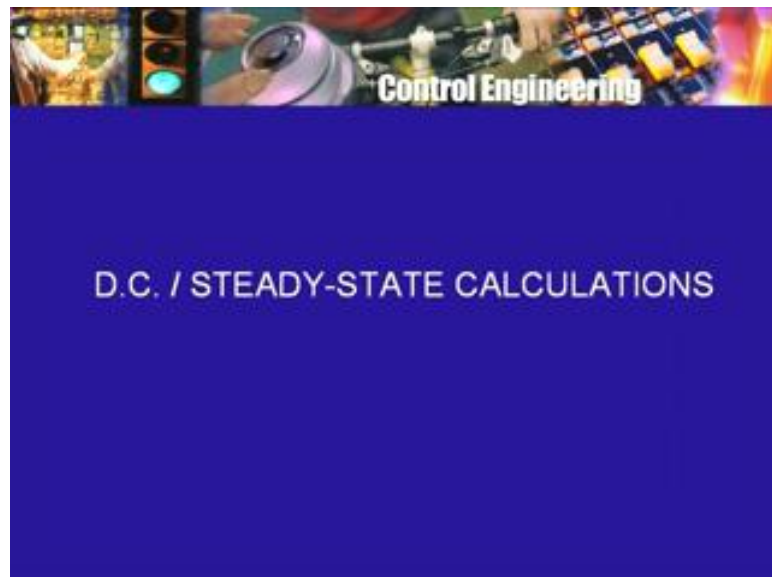
So that at the end of all of these then you would have arrived at the value of the field current that should be maintained and the value of the armature current that should also be maintained. So that given the constant load torque specified constant load torque given the applied voltage 230 volts assuming that the parameter values are known to 100 percent accuracy, armature resistance etcetera, etcetera. The motor will run at the desired speed, so this is the DC or steady state calculation which one can go through which is necessary to proceed further. Of course we can stop here we can say all right let us buy the motor, let us

get this power supply voltage 230 volts arrange for the field winding may be resistance in series with the field winding whatever arrangement.

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So that the calculated field current will result we can measure it actually and then set it once and for all and now you are ready to turn the motor on with the motor shaft, the load mounted on the motor shaft and hopefully things will go all right. Of course there is a

problem here which is a practical problem and as an engineer you cannot overlook it, it is also very important but it is a different problem of control namely starting of the motor.

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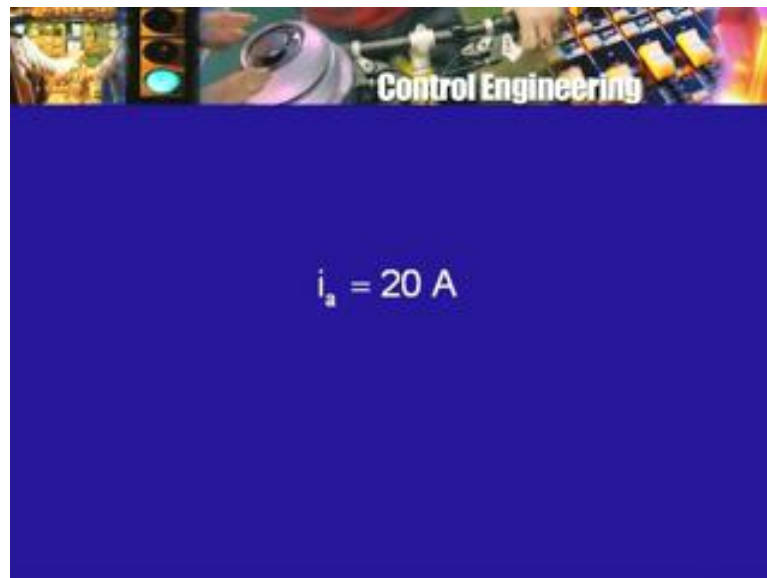


A DC separately excited motor is normally not directly started by applying the full voltage or rather applying the voltage and doing nothing else and we should be able to see the reason for this and in fact I mentioned earlier that when the motor starts from rest, the back EMF is

0 and so the electrical equation tells us that the applied voltage E_a is equal to R_a into I_a that is all so if E_a is 230 volts the armature resistance.

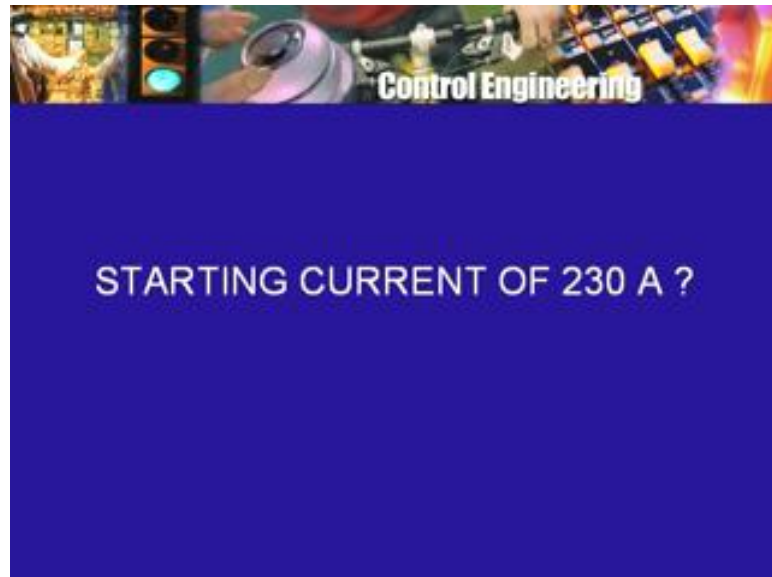
Let us say is of the order of ohm or less or a little more the armature current will be of the order of 100 amperes. Now is the motor expected the motor winding armature winding expected to carry this kind of a current 100 amperes certainly not for a longtime, how do we know that well what was the motor power the motor power was let us say about 6 kilowatt maximum. So, that is 6000 watts that is the maximum motor power multiplied that is equal to the motor applied voltage 230 into the armature current I_a . So 6000 watts equal to 230 volts in to how may amps.

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So that will give a answer of may be about 20 amperes, so what we expect is perhaps and this is what the previous calculation would I have shown as approximately that sorry the previous calculation whereas I shown as a much a smaller current but in the steady state an armature current of about 20 amperes will be flowing in the motor whereas if we do not take any precautions and just connect the motor armature directly to the supply or close the switch 230 volts one ohm of resistance would result in a current of 230 amperes 10 times the current which the armature is expected to carry in the steady state. Of course the motor will pick up speed but then that depends on the moment of inertia and we are not looking right now at how the motor starts, how the speed builds up, how the speed changes etcetera because we are only looking at the steady state when everything is constant.

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So in any case then one does not start the motor that way what is done is in series with the armature a starting resistance is put which of course will have to be chosen sufficiently high. So that the armature current when the motor is just starting will not be such a high value for example if the steady armature current value is 20 amperes then you can figure out how much resistance we should connect 230 volts divided by 20 amperes will be about say 10 or 15 ohms or whatever it is and therefore we will need a starting resistance of that value this may have to be a rheostat as because it has to carry a lot of current, it will have to be made

of thick wire and what not and in the laboratory perhaps we have already done an experiment on a DC shunt or a separately excited motor and you have already come across this problem associated with starting of the motor.

So this is a different problem altogether and it is important as a engineer we have to face it and we should know what is to be done but in the steady state when the motor has reached the desired speed, when the load torque is there then the armature current we have calculated the field current we have calculated certainly the motor if it is to deliver mechanical power of the order of 6 kilowatt drawing from the power supply voltage of 230 volts the current would be of the order of say 20 or 25 or whatever it is and therefore the motor designer would have taken care of this when I go to the market and say well I want to buy a 230 volts DC, 6 kilowatt motor, certainly the motor designer has thought about the armature current that the motor armature will have to carry so there is no difficulty on that account okay.

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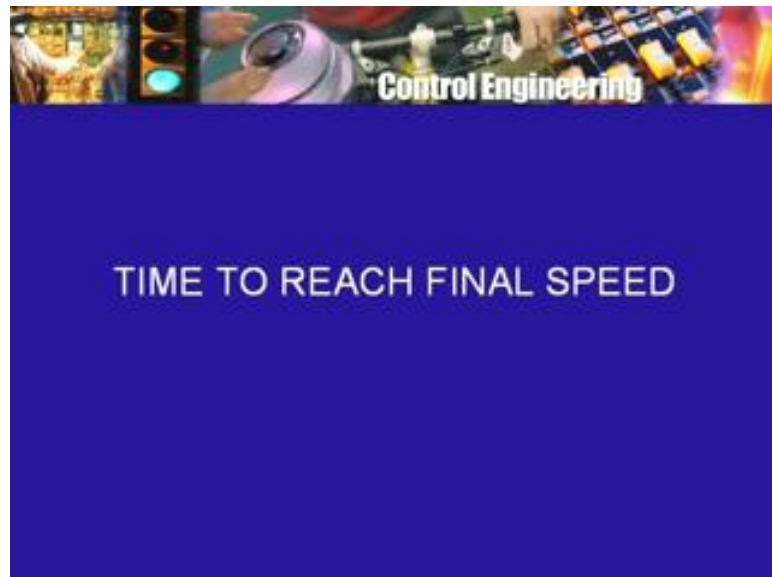


So we are ready with the drive except for the starting where one will have to take some precautions and maintain the field current constant somehow that will have to be arranged we are through. So why not say that the calculations are over and now we can operate the drive or if you are going to sell it to somebody you can say that all right I have done the design take this package. Now of course it is not as simple as that and what is the reason or what are the reasons. One thing which we will look into a little later is not the steady state but transient that is the motor will run at the desired speed etcetera. For example, the starting current must be low of course the series resistance has ensured that the starting current was sufficiently low but one would like to know what will be the behavior during what is called the transient period or what will the transient behavior something that does not stay on for a longtime but is there any way when you make a transition from 0 condition motor at rest to motor running at the desired speed.

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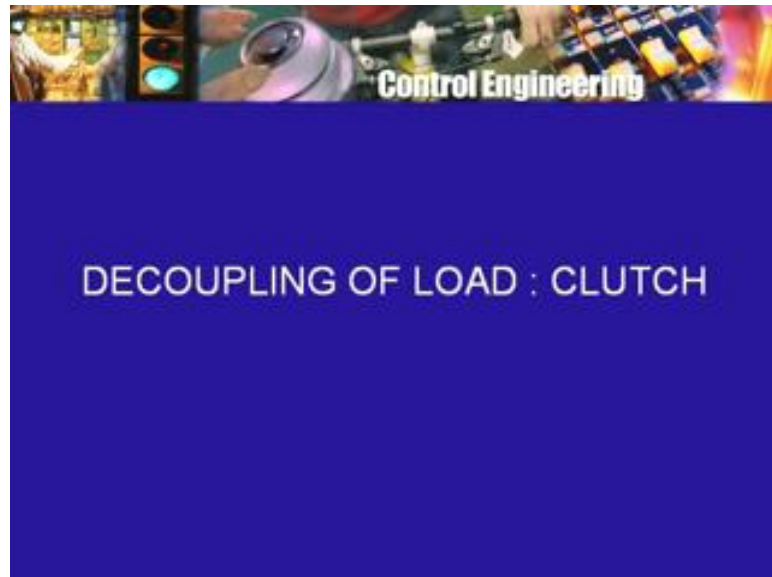


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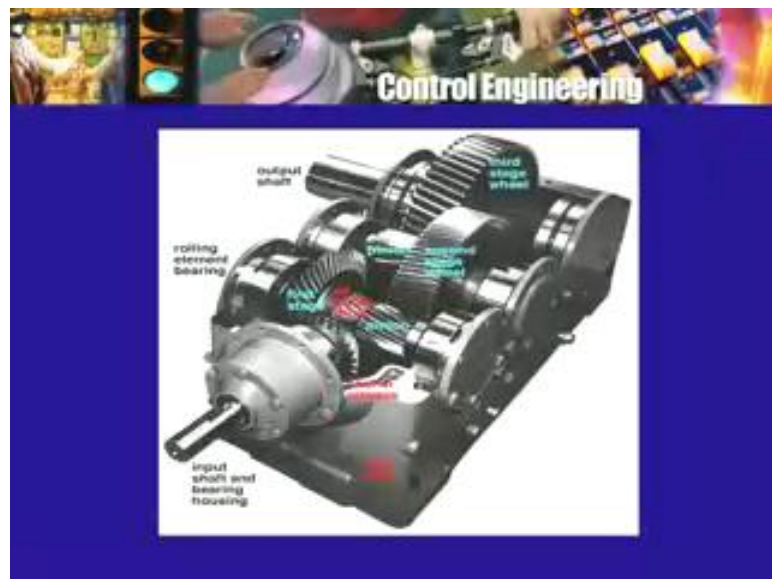


Another important consideration will be well with the load on how long does the motor take to reach the final speed because one would like the motor to speed up to the final speed as early as possible. In fact again in practice the situation will not be that simple the motor will be started without any load that is the load will be decoupled, the as you know in an automobile or in a scooter there is this arrangement of decoupling the engine from the wheels because there is a clutch of course in those cars or scooters where there is a gear box and therefore there is also a clutch.

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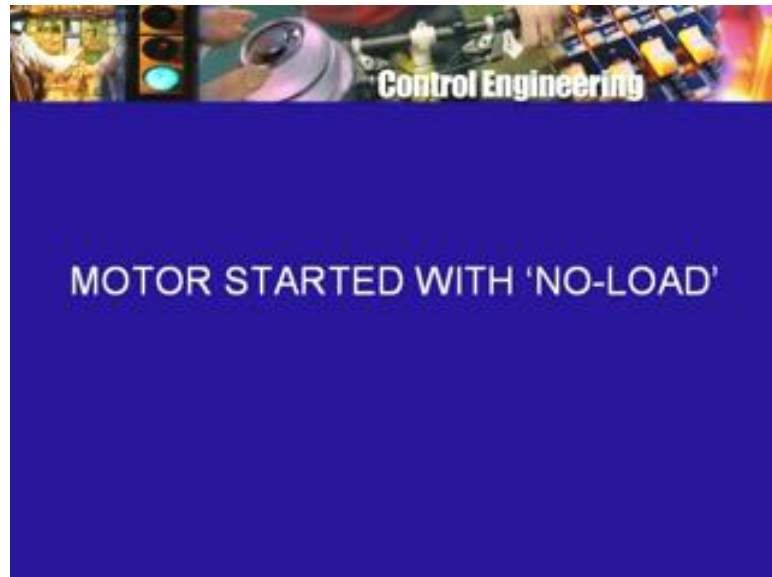


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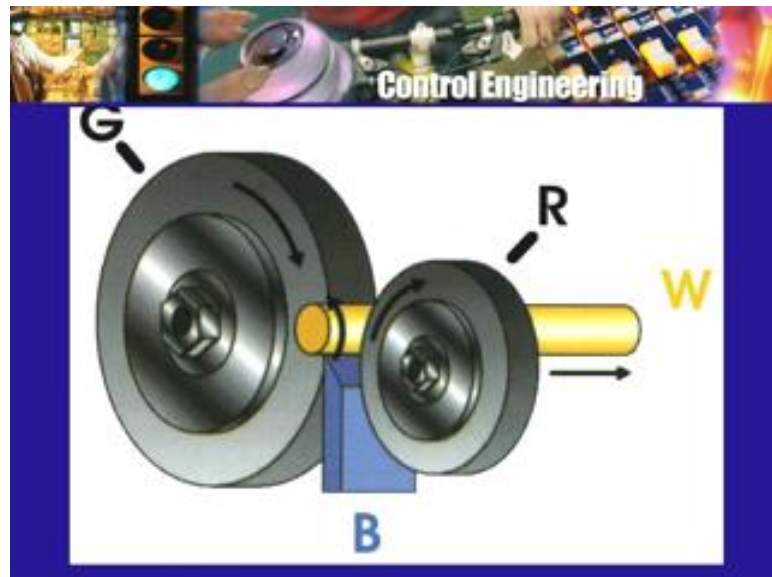


So this arrangement is usually there the mechanical load is not just left connected on the shaft all the time but due there is an arrangement for detaching it whenever required. So what one will do is one will start the motor with no load as it is called that is this load torque is absent of course the friction will still be there perhaps the inertia may be little lower now because the load is disconnected and then the motor is brought up to a certain speed and when it is close to the desired speed or after waiting for some time the load is thrown as it is called that is the load is connected.

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So how long will it take for the motor to build up its speed so that then I can connect the load and my grinding will start. Remember, the grinding job usually will not start with the grinding wheel at rest I will have to put the grinding wheel into motion and then only bring it close to the job and start my grinding operation. So there are these additional considerations which are also important and therefore one will have to look at these aspects of the drive how much time will the motor take to come up to the no load speed because you have to wait till that time before you can connect the load, one would like this time to be as

short as possible of course but it cannot be done instantaneously because there is a inertia, the motor armature has inertia. So it just just not speed up instantaneously.

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So transient studies will have to made what happens between the starting moment and the moment the final conditions are reached and the load can then be connected and it to run at the desired speed etcetera. So this is one side of considerations and we will look at what is known as transient analysis or analysis of transient behavior of control systems or of

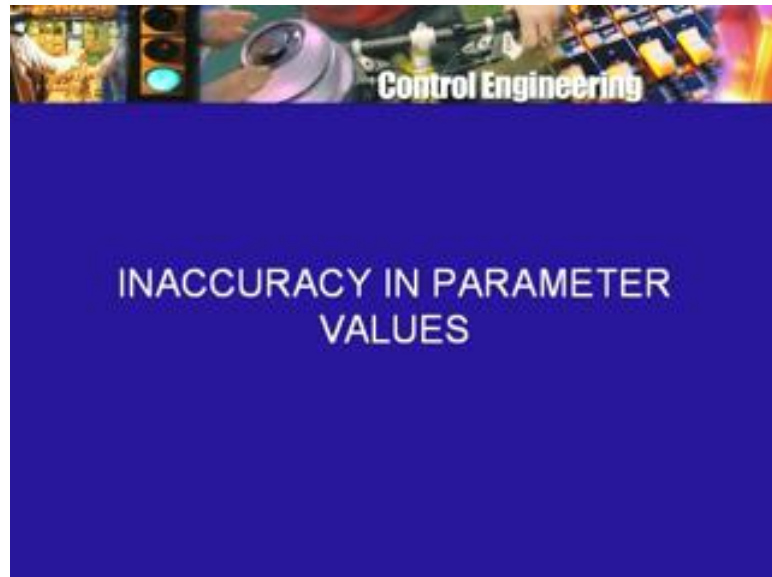
systems which are described by differential equations and so on and for that we will make use of tools such as the Laplace transformation and others the second thing about which I mentioned earlier was the possibility that things will not be as simple as they look. The load torque may not remain absolutely constant at the value we choose for our design purpose.

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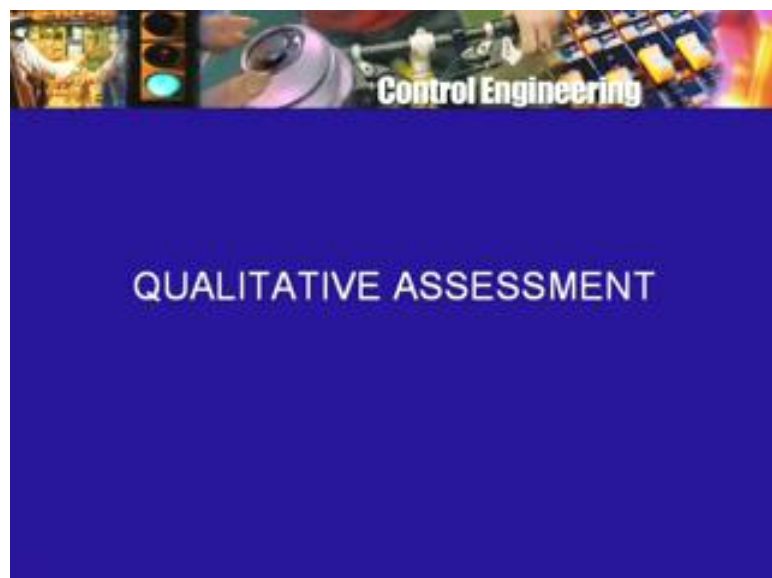
The grinding job will vary perhaps and therefore the load torque will not necessarily be all the same all the time, one may still like the speed to remain at a desired value grinding operation will have to be still carried out at a sufficiently high speed. The supply voltage for some reason or the other may not remain constant at the value that we choose 230 volts. The field current may not remain constant for some reason the field is separately excited therefore there is a separate voltage source for the field perhaps and that may not remain constant because of some disturbances and finally as always we have assumed that we know all the parameters k_T , k_b , R_a etcetera completely confidently up to 100 percent accuracy to the last 10s place of decimals but that we do not because we make some measurements and the measurements are always inaccurate to some extent and also imprecise measuring instruments that we use can only take us up to millivolts or milli amperes or a fraction of a revolution per minute or what have you therefore we cannot assume that we know these parameter values exactly.

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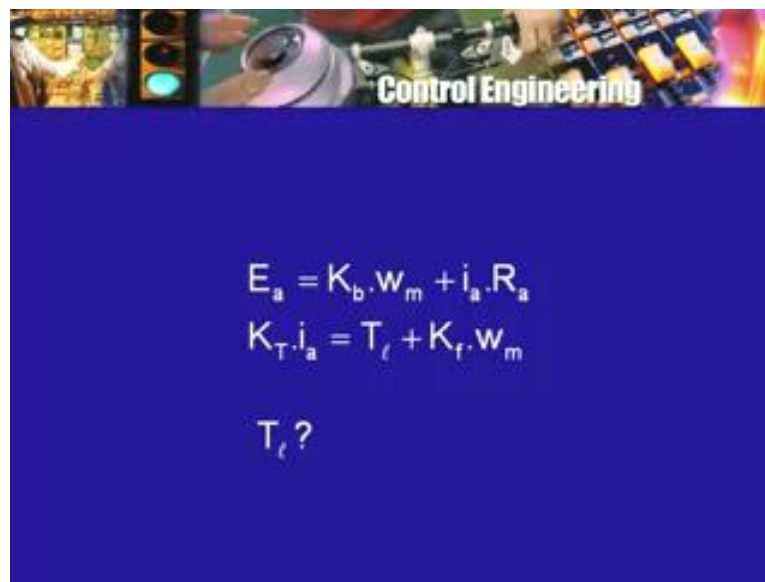
So one will have to consider the effect of what happens because of these factors. Now some of these can be seen qualitatively but then one will have to also make actual computations. So what I will do is I will indicate in a qualitative way what we can expect and then you should go ahead and work out these things at home working with more exact figures that is with actual numbers and not just making qualitative assessment.

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So for that let us write down the 2 equations once again, the applied voltage E_a is equal to $K_b \omega_m + I_a R_a$ and the torque equation is $k_T I_a = T_l + K_f \omega_m$. These are the two basic equations and what we did was knowing the parameter values knowing E_a and of course K_b and K_T 's where known as in terms of the small k_b and small k_T . We calculated the field current and the armature current so having done all those calculations. Now let us look at these equations once again and see what is the effect of these things happening for example and now when we make this analysis we have to start with simpler situations but look at more complex or difficult situations.

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So perhaps one can start of like this of all these things E_a the supply voltage that is I am reasonably sure that it is going to remain constant at 230 volt. But I am not so sure about the load torque we made some design calculations assuming a load torque of say 30 Newton meters but suppose the load torque is less than that or is more than that for example at no load the load torque is very close to 0 there is of course the frictional term still with K_f may be only from the motor and not from the motor and load combined but T_l is 0 that is the no load condition normal load will be 30 Newton meters but occasionally the piece of material that one is polishing or whatever may be a little harder and so it may increase the requirement of the torque.

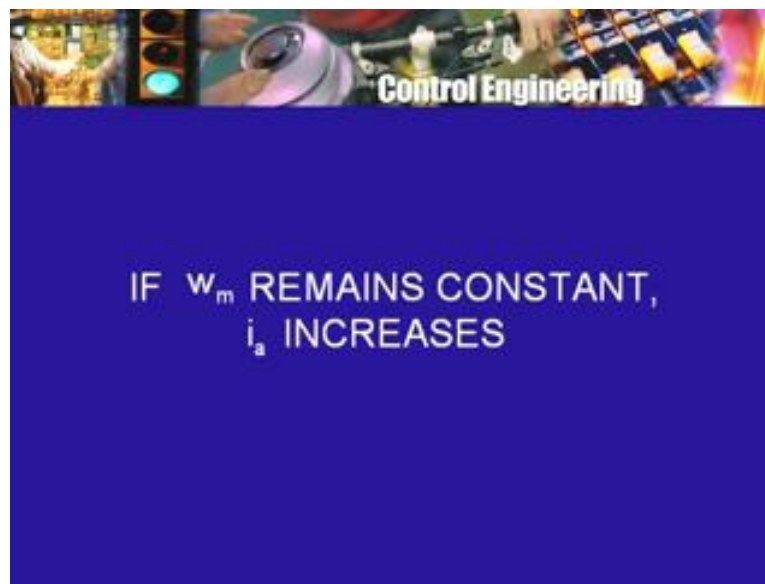
So the torque may not be 30 Newton meters but may be say 35 Newton meters how much will it be we really do not know unless one knows more about the practical drive considerations. So what one does is one makes a provision under the assumption that the load torque may exceed what is called the rated value, rated in the sense we have made these calculations assuming certain value it is like price of a commodity, it is supposed to be sold at such and such price of course you may buy it for more or less.

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So the rated low torque was 30 Newton meter so we will allow for any an increase in it of how much. Now this is where you have to choose what figures and then accordingly you will say all right my drive will take care of such and such variation. So one may say rated torque plus may be 10 percent in excess or may be one wants to be a little more not conservative but a little more pessimistic. So we will provide for 20 percent excess in fact we **will can** then advertise that well our drive will provide for up to even 20 percent excess power.

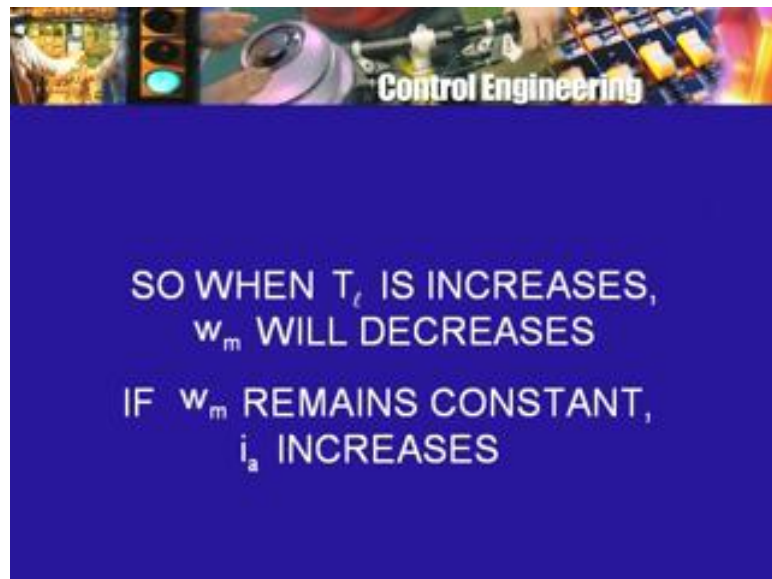
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So 30 Newton meters normally but it could be increased by 10 percent or by 20 percent. So suppose that happens E_a in our very simple drive we are just getting 230 volts DC and we are not doing anything about it. So E_a is 230 the parameters values let us assume are known exactly so if you look at these equations then what is going to be the effect of this change of T_l from 30 Newton meters to a higher value, a little thinking and this is where one should not just go and solve equations blindly or in a hurry but we will in to argue a little qualitatively as to what is likely to happen and the torque equation tells you what is likely to happen and one argues this way that T_l is not going to be 30 but it is going to be say 10 percent more so 33.

Now if ω_m remains the same of course it may not but if it does remain the same then the motor torque will have increase, the motor torque comes from the armature current. So the armature current will have increase now the if you look at the electrical equation the armature current increase increases the armature drop and the armature drop is increased the supply voltage is the same then the back EMF will reduce a little bit but the back EMF is proportional to ω_m so what does all of these then tell you this qualitative analysis then tells you that when the load torque is increased my assumption that the angular speed will remain constant will no longer be justified of course the armature current will not be at the old value what will happen is the speed will decrease a little bit or decrease and the armature current will have increase from the rated value.

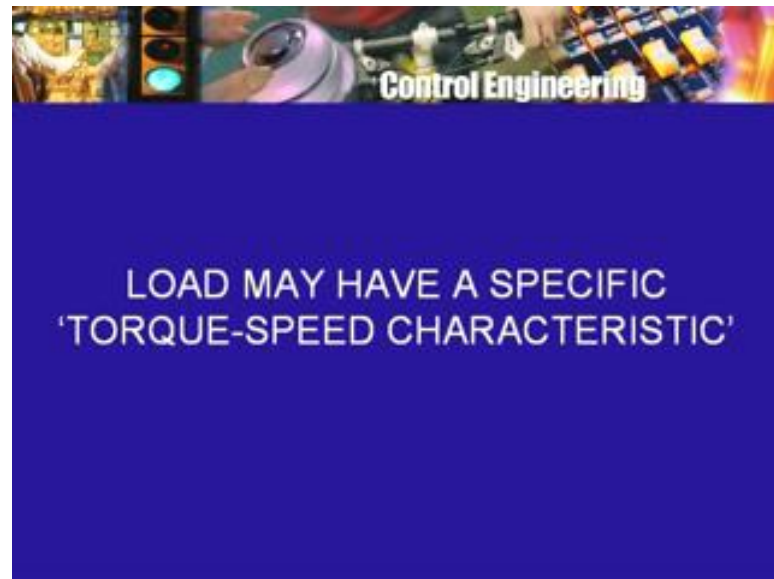
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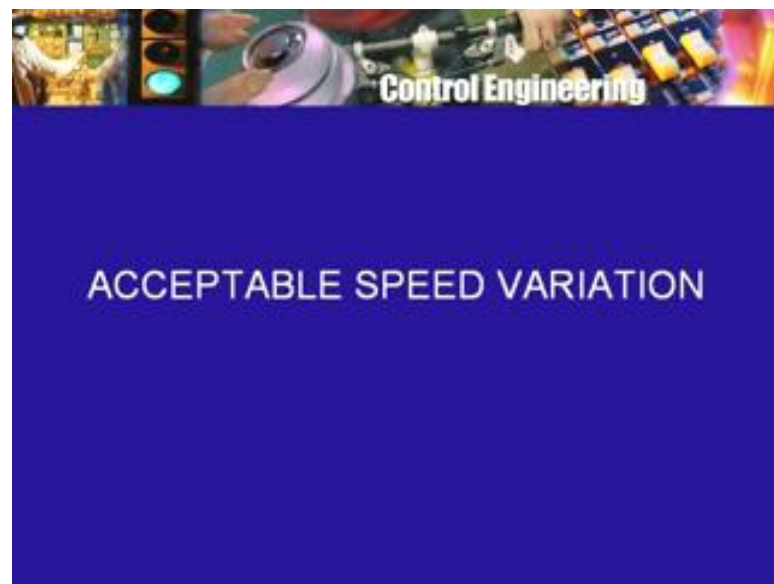
So the drive will no longer run at 1500 rpm it was designed to run at 1500 rpm under rated conditions assuming supply voltage was this torque was this etcetera, etcetera. When the torque is increased then the speed will be less than 1500 rpm the armature current would have also increased a little bit.

Now of course there is there are some additional complications here which we will overlook but it is necessary to be aware of it, now it is not as if these 2 are completely independent of each other there are many mechanical loads for which the torque and the speed do not just behave in any independent manner and therefore there is what is called for many devices, what is called a torque speed relationship or a torque speed characteristic of the mechanical load.

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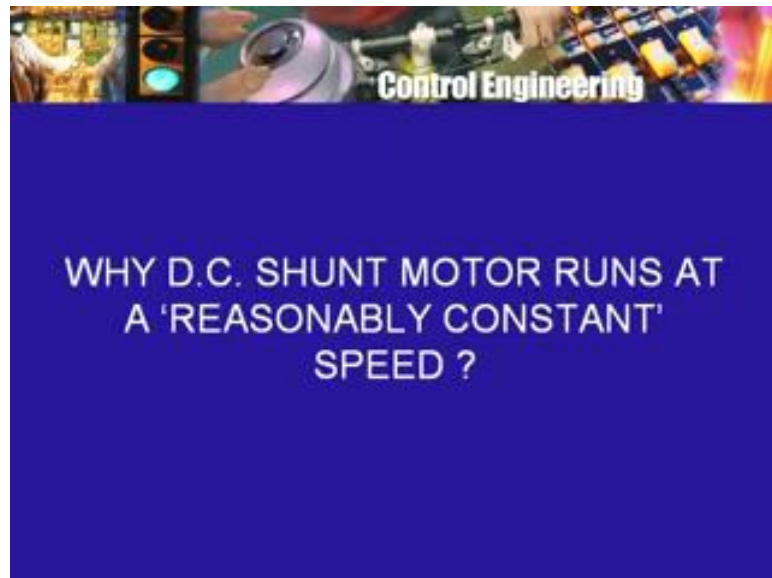
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I am assuming for these preliminary calculations or simple calculations that the torque and speed are independent of one another that is the grinding wheel can require a higher torque even though the speed drops a little. So what one has to do now is assuming a 10 percent increase in the load torque calculate or recalculate or calculate the new values of the angular speed or speed of rotation and the armature current, speed of rotation is of course our main concern. Now if 10 percent increase in load results in 10 percent decrease in speed there may be the situation is not expectable but if it results in only one percent decrease in speed may be it is acceptable but if the user wants the speed to not to change by even that amount but may be as little as .1 percent then one will have to rethink about the whole thing.

Similarly, the armature current the armature current may go up by 10 percent and we have to make sure that the armature current does not increase too much because otherwise the armature winding will get overheated. Of course as is the case with any practical situation there are protective mechanisms that are to put into the circuit for example there could be a fuse which will blow out if the armature current increases too much that is if you are trying to get from the motor much higher torque which requires a much higher armature current the fuse will simply blow and therefore the motor will come to a stand still hopefully of course such things will not happen too frequently.

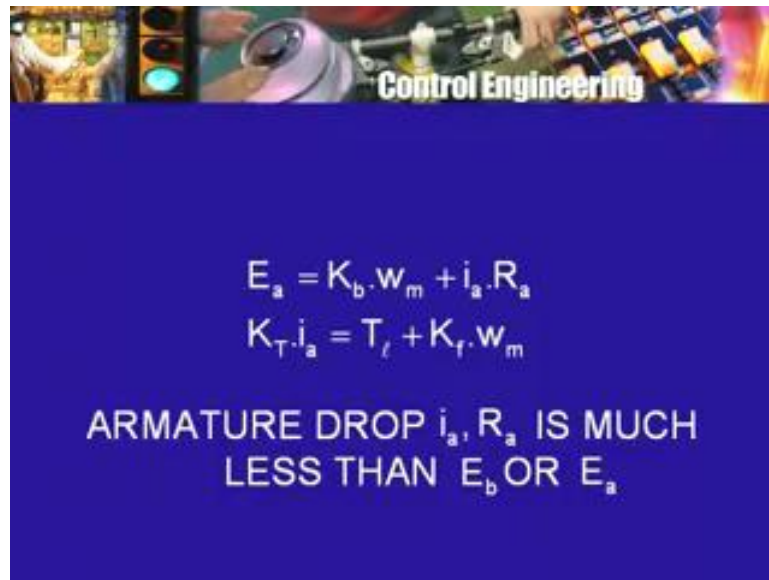
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Now I said earlier that a DC shunt motor by its nature is something which runs at a nearly constant speed. Although if you do these calculations and I would like you to do these calculations now for these calculations you can look at your textbook and take the figures from there or do it the way I have done it that is assumed 13 Newton meters at 1500 rpm the values of R_a , you can take some simple number like even 1 ohm the frictional torque K_f into ω_m .

We can assume that it is much less than the load torque, load torque is 13 Newton meters how much friction may be one Newton meter or write choose K_f into ω_m equal to 1 Newton meter, so that gives you the coefficient K_f . So in this way sought of view of you can cook up an example where you can figure out the values of these parameter although in practice when I have chosen the motor I get R_a , I cannot chose it.

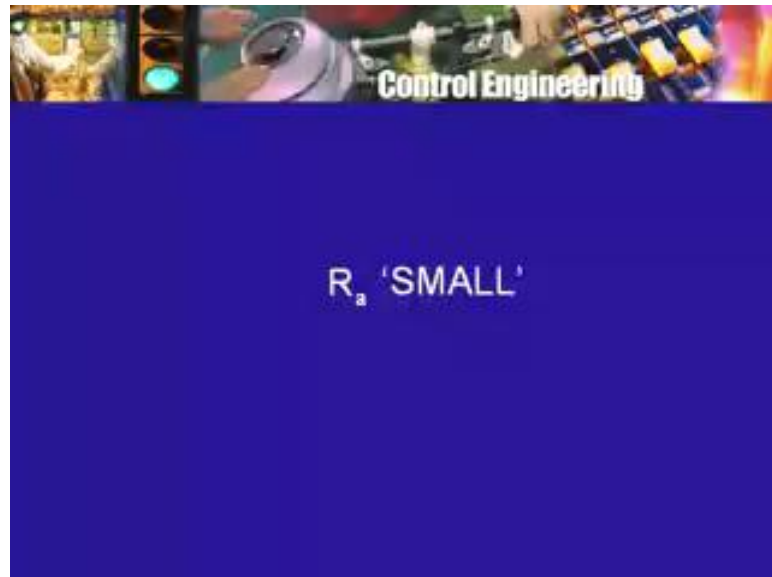
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Now why is it that the DC motor will run at a reasonably constant speed and that is the following if you look at the electrical equation, their supply applied voltage E_a is a sum of 2 terms. Now typically the armature drop is a much smaller thing than the back EMF the 230 volts applied does not get used up in the armature drop in fact very little of, it will get used up in the armature drop. In our example, let us say an armature current is of the order of 20 amperes it is a high side perhaps armature resistance may be as much as say 1 ohm that is also little high the armature drop is just about 20 volts as compared with 230 volts applied voltage and therefore the rest of it 210 volts is the back EMF voltage.

Now if the armature current increases say even by a factor of 2, the armature drop will increase from 20 to 40 but the back EMF does not become one half say it was originally 20 armature drop 230 was the applied voltage, so the back EMF was 210 if the armature current has doubled so the armature drop is 40, 230 still being applied the back EMF is 190. So from 210 it would have reduced to 190 and that is not a reduction of 50 percent. So although the armature current has gone up by 100 percent the back EMF would not have gone down by 50 percent armature current doubled back EMF is not becoming half and therefore the speed will not become half.

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So there is this kind of a built in mechanism, so to speak it is not built in or it is not really designed but that is the way the shunt motor equations are that of course this means that the armature resistance has to be small. So I have to use big wire for the winding I cannot economize by using very fine wire for the armature winding because then the armature resistance will be high and that goes contributes to wastage of power because that is just what is called the $I^2 R$ loss or the heating the armature gets heated up because of flow of current in it. So if R_a is large then $I^2 R$ is also large and that is the problem also.

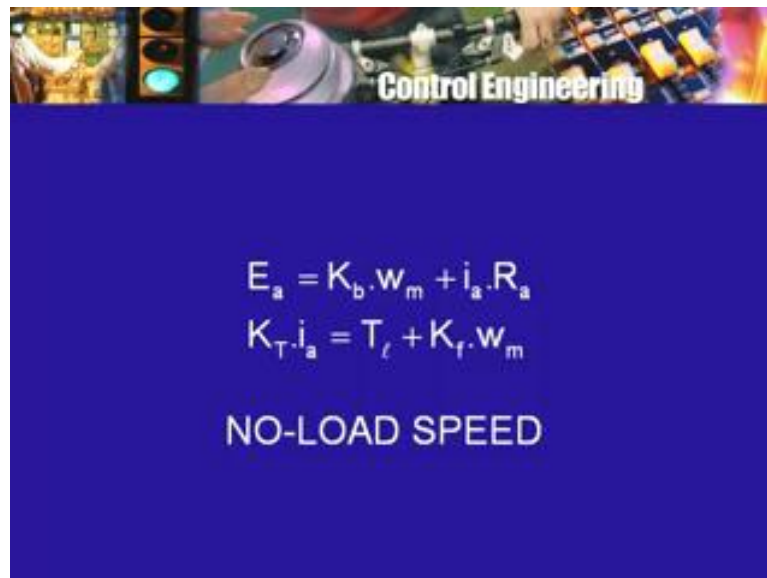
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So usually the decrease in speed and the increase in armature current when the load torque has increased by 10 percent will not be by 10 percent but will be by much less than that but by what amount one will have to actually calculate. But this qualitative analysis tells us that if E_a remains constant the coefficients are known exactly therefore they do not change we assume they do not change field current does not change then if T_l increases then ω_m will decrease it cannot remain absolutely the same and the armature current will increase it cannot also remain the same.

So with change of load torque the drive cannot operate at constant speed and therefore one can in this sense talk about an error, if the drive was expected to run at the constant speed it will not do so, if the load torque changes, if the load torque changes then there will be an error and so either we have to live with it or else do some redesigning or modify the design in some way or the other which will of course what we will look in to. So do these calculations yourself for say 10 percent increase of load torque what is the decrease in speed and typically one thinks in percentages rather than absolute figures then you can re do it with 20 percent increase in load torque, what is the decrease in speed like, so what is the speed regulation of this drive this very very simple crude drive. Most probably this will not be acceptable but this is a first step towards a better design at the other extreme of course we can go to the case when the load has been removed, so T_l is equal to 0, when T_l is equal to 0 if the frictional part is very small the torque requirement is very small then the armature current required also will be much smaller.

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Control Engineering

$$E_a = K_b \cdot \omega_m + i_a \cdot R_a$$

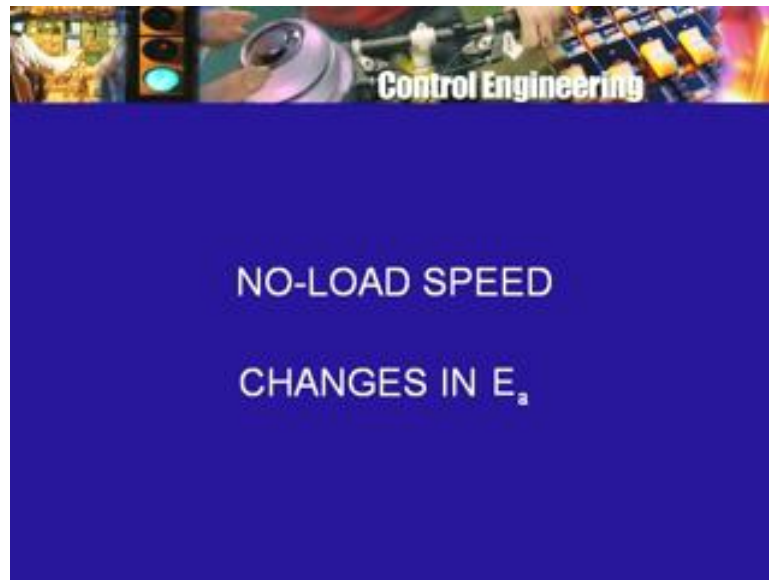
$$K_T \cdot i_a = T_l + K_f \cdot \omega_m$$

NO-LOAD SPEED

So instead of 20 amperes under no load conditions you may require an armature current of only one ampere let us say if that is the case I_a is one ampere R_a is one ohm then the armature drop is just 1 volt. So the back EMF will be now 229 volts whereas under rated conditions the back EMF was 210 volts. Now it is 229 which means what which means that

the no load speed will be greater than the rated speed of 1500 rpm how much greater again one can figure that out and this may be acceptable because there is no load connected the motor may well run at a slightly higher speeds and what is desired but the moment you connect the load of the specified value everything else is as assumed the motor speed will return or will eventually become steady at the value that is require.

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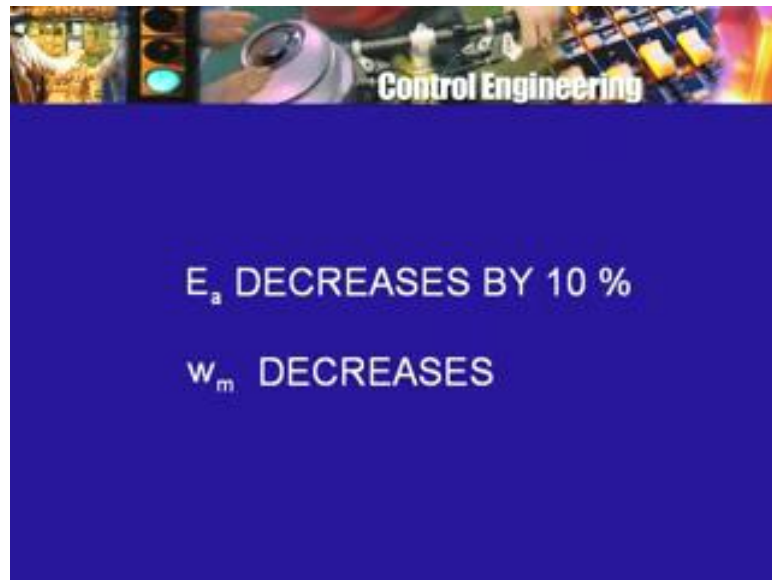
So no load conditions also have to be looked in to besides that changes in the load torque especially on the higher side this is again something one has to look in to. So, so far we looked at what happens when one of the things which we assumed in the design T_l is no longer at that value, no longer 13 Newton meters but 10 percent greater may be or 20 percent greater or no load case so 0. Now we can look at something else we can look at the supply voltage 230 volts who knows the supply voltage may go down there may be other loads that are connected on the power supply and therefore the supply voltage may go down typically it will go down and not go up. So again then we will have to make calculation.

So suppose instead of 230 if the supply voltage goes down by 10 percent then what will happen to the speed to the armature current assuming the load torque remains at the same value. Now again qualitatively one can see what is going to happen E_a is less if the torque is the same assuming ω_m is the same, the armature current is the same, the armature drop remains the same. So if E_a has decreased this means the back EMF must have decrease but the back EMF has decreased that means the motor speed must have got down.

So our assumption that the motor speed will remain constant when the supply voltage decreases is not justified but the qualitative analysis indicate that when the supply voltage decreases the motor speed will decrease by how much that is again the critical question. So with 10 percent reduction in supply voltage how much will the speed go down as we can see

looking at the magnitudes 230 volts, 210 volts back EMF 20 volts armature drop if the supply voltage goes down by 10 percent the armature term which is small anyway the back EMF will go down by nearly 10 percent and therefore the speed will go down by nearly 10 percent not nearly I mean not exactly 10 percent could be a little less than 10 percent but it could go down to that kind of amount.

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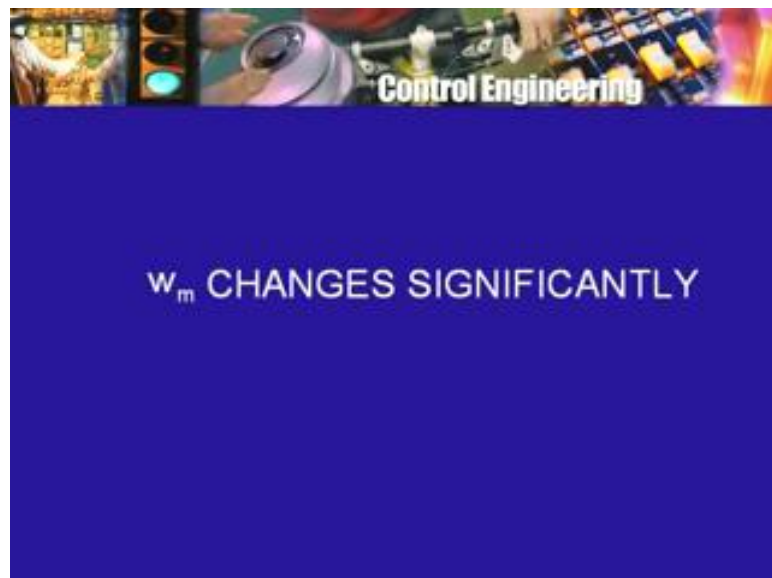


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E_a DECREASES BY 10 %

w_m DECREASES

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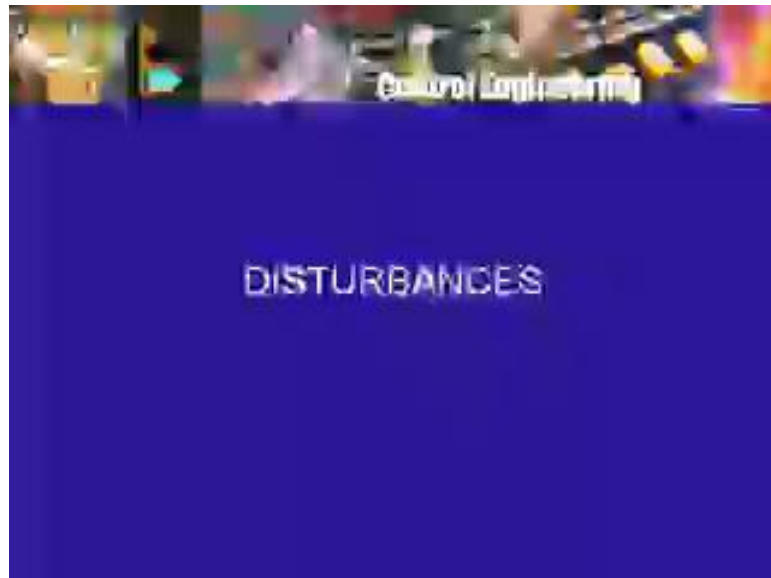
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w_m CHANGES SIGNIFICANTLY

So this drive has this bad feature that if the supply voltage decreases the speed of the drive is going to decrease and this is again a property of the DC separately excited DC motor that for torque changes the speed changes are negligible but for supply voltage changes the speed change can be significant. So if we are operating under a situation where the supply voltage is not guaranteed to remain constant but could go down by as much as 10 percent or even more then our drive will be unsatisfactory if 10 percent of speed change cannot be tolerated and so we will have to do something about it. Now this analysis is still very simple in one case I assume that everything else is okay except T_l is different, now I am saying everything else is okay except E_a is decreased.

Now you can see what is going to happen if these two can be combined that is unfortunately for us as in luck could have it the supply voltage has gone down and at the same time the load torque demand has gone up. So T_l is greater E_a is less, now what is going to happen now when the situation is going to be worse. The motor speed is going to decrease still further then it would have if only one of these things had happen and so one does try to think of such situation, so we may say okay 10 percent increase in load torque, 10 percent decrease in power supply voltage, so what will be the change in speed in fact it will not be really acceptable because already with 10 percent decrease in supply voltage the change in speed may be by as much as 10 percent.

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So the design is not really satisfactory, these are the two things that one normally worries about, one is the input voltage, the other is the output requirement, the torque and I used earlier the term disturbance although for T_l it is not really disturbance, it is the main reason for the drive but E_a changes in E_a can certainly be thought of as disturbance power supply voltage was expected to remain constant at 230 volts but it has not remained so because of some disturbance, some load whatever.

So effect of disturbance is to change the performance of the system it will no longer operate in a satisfactory manner the change or the error may not be acceptable, the other things that cause some worry is the assumptions that we have made about that parameter value R_a , K_b , K_f in this case. We have perhaps made the calculations on the assumption that the armature resistance is one ohm but may be somebody made some mistake and it is really 1.5 ohms or the armature for some reason the motor has been running for a longtime the armature is getting heated may be outside temperature is also high, we have not provided adequate cooling as a result the armature resistance has increased by may be 10 percent or 50 percent.

So either the value that we choose was not correct or was not known accurately enough or has changed now because of operating conditions what will be the effect. Now as you can expect if R_a is greater than what it was calculated or assumed to be then the back EMF term will decrease a little bit so the speed will decrease a little bit. But you can see that the effect of armature resistance on the speed or the change in armature resistance on the change in speed is going to be much less because there is 20 volts in comparison with 210 volts but what about the back EMF and the torque coefficient and why should it change it may change because it is dependent on the field current and the field current I have assumed somebody has made arrangements for it to remain constant but suppose that arrangement is not full proof some power supply voltage which is connected to which the field winding is connected that voltage has gone done.

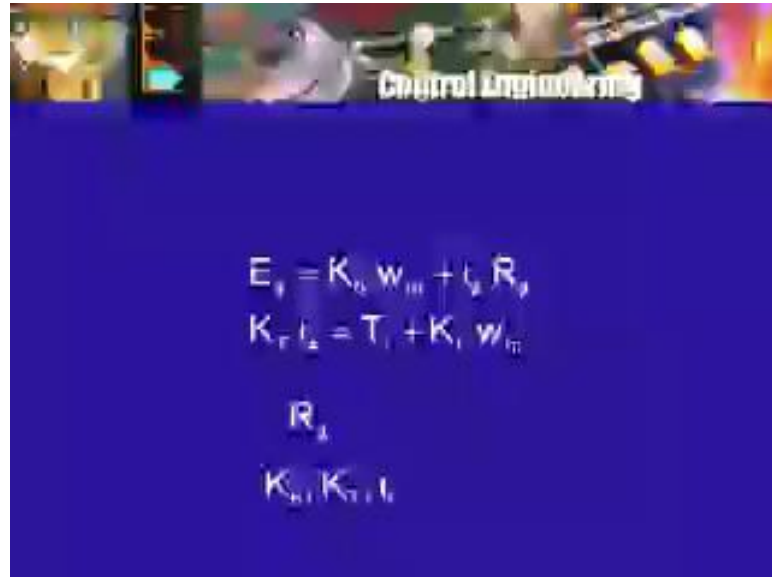
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So the field current has gone down by ten percent which means K_b and k_T will be down by 10 percent then what is going to happen to the speed assuming the load torque is constant at the old value and the supply voltage E_a is constant at the old value. So change in parameter value or alternately uncertainty K_b was assumed to be this much but there was a mistake

done in the measurement. So it is not really that but it is something else what would be the values of speed and armature current that would result in the steady state.

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$$E_f = K_b \omega_m + I_a R_a$$
$$K_T I_a = T_L + K_T \omega_m$$

R_a
 K_b, K_T, I_a

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Control Engineering

BODE, NYQUIST, BLACK

SENSITIVITY STUDIES

Now here again you can see roughly what is going to happen if K_b and k_T are less than what it was assumed to be because let us say the field current has decreased the load torque if it remains the same then the torque equation, the mechanical equation tells you T_L is the same for the moment assuming ω_m is the same $k_T I_a$ has to remain the same but k

T has decreased, so I_a will increase the armature current will increase, so the armature drop will increase, so the back EMF available will decrease in addition K_b has decreased. So ω_m what is going to happen well there are these 2 forces working? So one will have to actually calculate and see what happens to the speed on the other hand if the field current is increased then what will happen.

So all these calculations have to be done and this is the kind of thing that people like Bode and Nyquist in the early days or Black in the early days in the 1930 and 40's were worried about and these can be roughly said to be sensitivity studies and in fact Bode introduced a particular measure of sensitivity, how sensitive are some of the things for example the steady value of the speed to variations in load torque variations in applied voltage error made regarding the value of K_b or k_T or change in the field current things of this kind, sensitivity coefficients of various kinds can be calculated and I will leave it as an exercise to you to look at these 2 equations and write them in several ways. So that one way will show you the equation will show you how ω_m depends on the load torque.

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So ω_m equal to some coefficient in to load torque plus a another term which will then tell you that if the load torque changes by this much percent then the speed will change by this much amount and which may amount to so much percent. Similarly, ω_m can be expressed in terms of the supply voltage E_a rather than the load torque. So that will tell you the dependence of the of the speed on the supply voltage and one can do it for each one of the other things but what we can see right now is that the drive is not necessarily going to be adequate for many practical purposes. So we will have to look into how to get over this problem.