Control Engineering
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Lecture - 08

After this short exposure on mathematical modeling of physical systems, let us get back to our motor control problem what is that we are going to model there. Our problem consists of driving a mechanical load by the separately excited DC motor, what are the various parts of the motor and of the load for which we can write down equations and then perhaps we can combine them into one or more equation. I mentioned the armature winding of the electric motor and the same holds for the armature winding of a DC generator also, in fact as you perhaps know already the same machine can be used either as a motor, DC motor or as a DC generator in a way there is no basic difference between the two in both cases magnetic field or flux is involved, current is involved rotation takes place torque is produced. The only difference is that in an electric motor the power is drawn from a electric supply and mechanical power is given to a load where as in the case of an electric generator the power is drawn from a mechanical supply such as a turbine and is delivered to an electrical load such as in air conditioner or heater or what have you.

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So if one looks at the armature winding of the motor or generator, one can use a crude but fairly good model for the behavior of the armature winding. The armature winding is of course made up of series connection of a large number of conductors which are located in slots on the armature. So we can think of it as one single long piece of wire wound around a magnetic core material. Now when this armature that the winding on it rotates in the magnet field produced by the field winding, electromotive force is produced that is the winding looks as if it is generating electromotive force and there is a simple relationship between the EMF or the electromotive force that is produced in the winding, as a result of the motion of the conductors in the magnetic field the magnetic field itself and the speed of rotation of the armature.

We are assuming that the magnetic field current is kept constant and therefore the magnetic field to a good approximation will remain constant, constant in the sense at a given point in space, in the air gap the magnetic flux is not going to change with time although at different points of the air gap, the magnetic flux density will not be the same under this assumption that the field current and therefore the flux or the field density is kept constant, the back EMF as it is called or the EMF produced in the armature will be simply proportional to the angular velocity.

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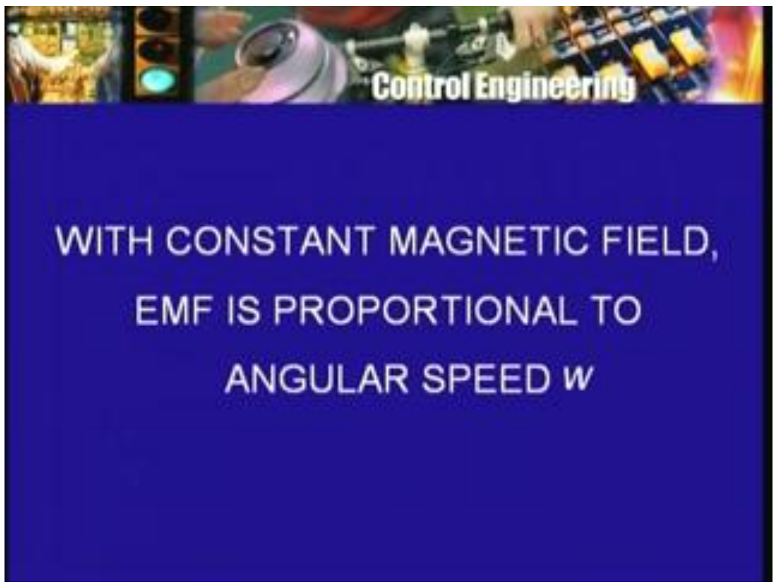

In a way the armature behaves like an inductor and you will recall Lenz's law or Faraday's law and Lenz's law together that wherever there is a change of current in an inductor, the inductor tries to oppose it so to speak that is in the inductor an EMF is produced proportional to the rate of change of current which opposes the EMF which drives the current through it in the first place. In exactly the same the armature winding rotating in a negative field produces this EMF and this EMF as a particular direction and for the machine to run as to motor the applied EMF or electromotive force or voltage will oppose this generated EMF and therefore this EMF is called the back EMF, E b and therefore we will start writing down our first equation for the armature namely the back EMF, Eb produced in the armature winding is proportional to the speed of the motor omega m, m for motor and there will be constant of proportionality relating the two and since we are taking about the back EMF this constant will denote it with the subscript b.

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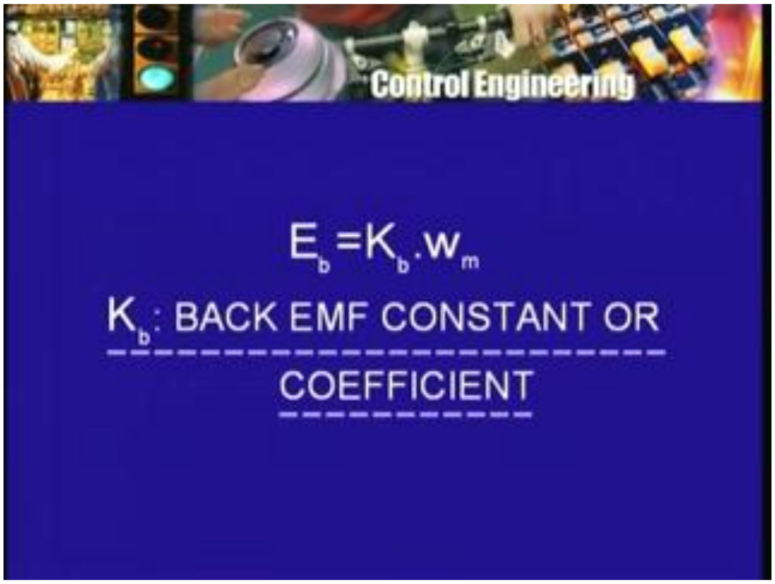

So we have the simple relationship the back EMF, E b is equal to K b into omega m where omega m is the motor speed. This K b is therefore called the back EMF constant or coefficient of the motor since the back EMF will be in volts or if it is expressed in volts and omega, the angular speed may be expressed in rpm or radians per second. So the back EMF coefficient or constant will have the dimensions of volts per rpm or volts per radians per second. For a given motor then with a give field current or the field current is also referred to some times as the excitation and that is why the motor was called or is called separately excited motor that is it gets it speed current from a separate source of supply this K b will depend of the field current of course.

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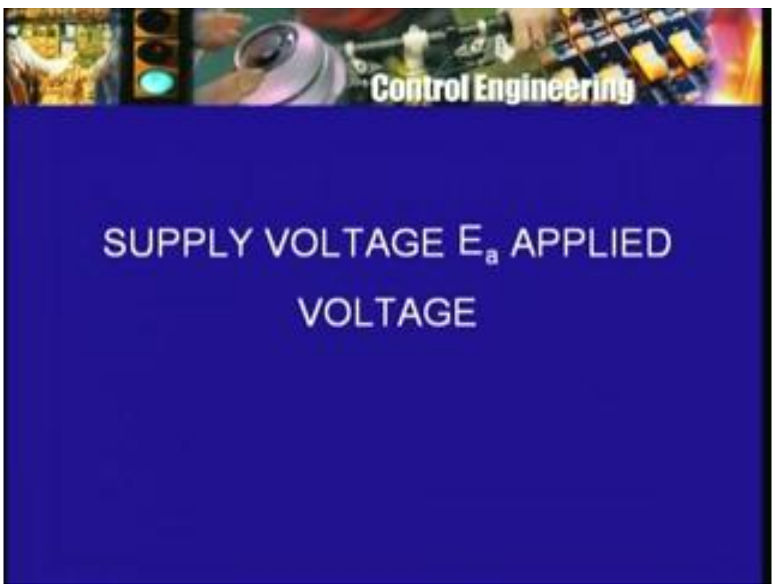

So for a given motor for a given field current there will some particular number K b or value of this back EMF constant or back EMF coefficient K b and the back EMF produced in the armature as a result of motion will be given by K b equal to K b into omega. There is one more equation that electrical equation that we can write for the armature and this is because of the fact that the armature is connected to a source of DC voltage or DC electromotive force or the supply voltage. This supply voltage we will denote by capital E sometimes with a subscript a, a to remind you of the applied voltage that the machine is to be run as a motor, so you have to apply a voltage to it rather then get a voltage and power out of it.

So E a is the applied voltage there is the supplied voltage and of course assuming that the motor is somehow rotating there is the back EMF produced in the armature and these two would be equal and opposite in their directions. But for the fact that in general there will be a current flowing in the armature and the armature winding is a long electrical wire and therefore has resistance and therefore as a first approximation which is fairly good we can write down the following equation the applied voltage is equal to the back EMF produced in the armature winding plus a term that corresponds to the fact that the armature winding has a resistance and therefore this term as given by Ohm's law will be R a into I a, I a as usual will denote the armature current R a will be the armature resistance.

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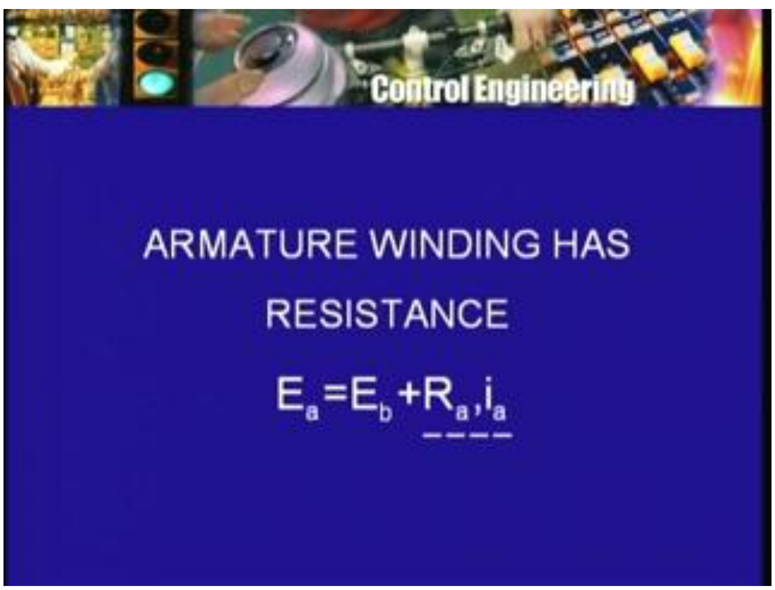

We can take the machine and instead of using it as a motor we could drive it by some mechanical means that is rotate it is armature at angular speed omega. Now connect anything electrically to the armature winding then with the voltmeter we can measure the voltage E b the back EMF and that time no current will be flowing in the armature therefore the armature drop as it is called the term R a, I a is called the armature drop, this will be 0 because there is no armature current flowing. But the back EMF is produced and the back EMF can actually be measured this is referred to sometimes as the open circuit test of the machine and your machine's course you might have carried out an experiment called the open circuit characteristic of a DC machine.

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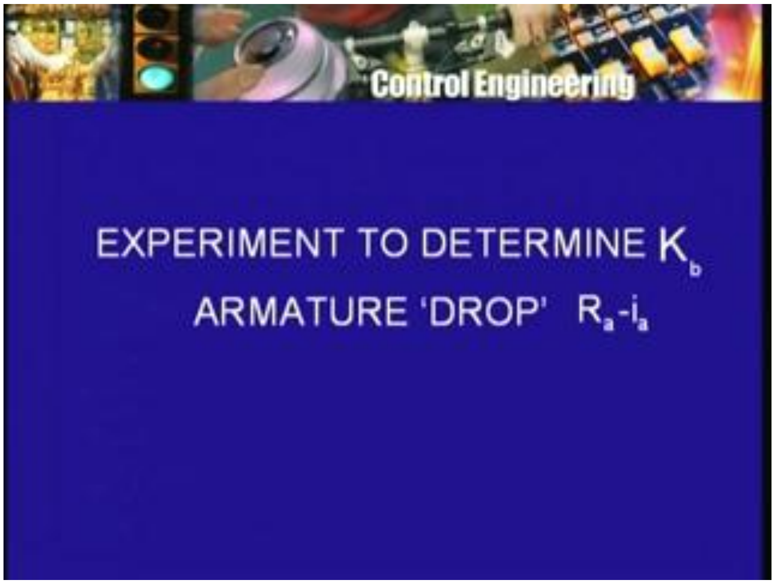

You turn the machine somehow and for various values of the field winding current and various speeds of rotation, you find out the open circuit voltage that is produced and the open circuit voltage is reasonably proportional to the speed so therefore this constant K b, so the constant K b can be actually find out by this open circuit test.

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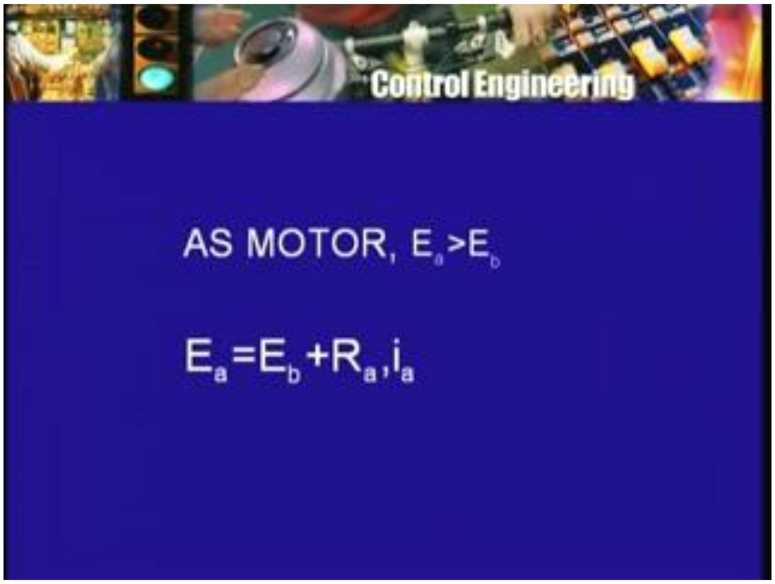

However when the armature is connected to a DC power supply or a voltage source, you have the back EMF E b and you have the applied voltage E a and if the armature is not being driven externally it will function like a motor in the sense the applied voltage E a will be greater than E b and therefore there will be an armature current flowing in the armature and therefore we will have the equation E a equal to E b plus R a, I a. There are some other phenomena that take place in the machine there is something know as armature reaction because the magnetic field that is produced by the field winding current continuously exists but the armature current itself produces a magnetic field which cannot be ignored and therefore this phenomenon called armature reaction takes place.

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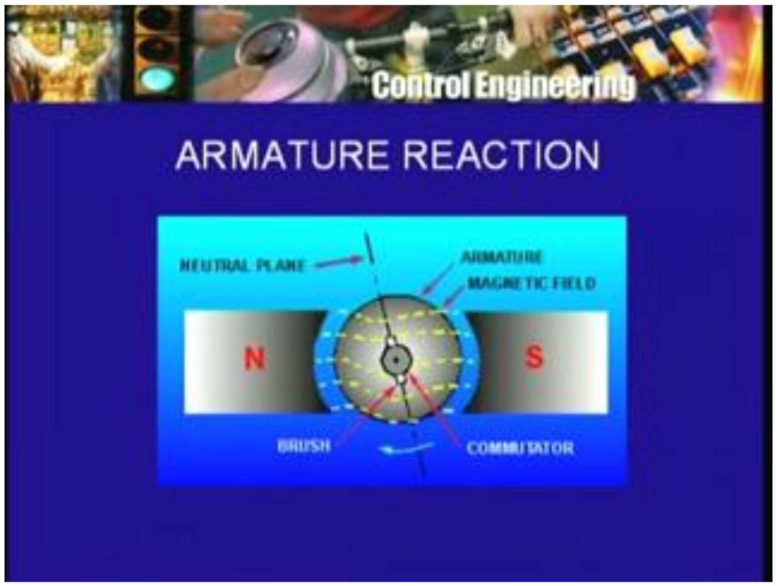

We will ignore it, we will simply assume that the armature resistance can be taken care of by this armature drop R a, I a. Now I say E a is greater than E b because you can see what is happening you can think of what is going to happen when the armature is stationary and you apply a source E_a at that time there is no rotation therefore there is no back EMF produced, the term E b is equal to 0 and therefore if there is no additional resistance connected in series with the armature a very large current will flow given by E a divided by R a. The applied voltage may be on the order of 100s of volts, let us say 230 volts the armature resistance would be of the order of at most a few ohms or even less than a ohm and so you can see that if to a motor at stand still you apply the full supply voltage, a large current will start flowing because there is no back EMF, there is no rotation.

So, E a is greater than E b because E b is 0 and therefore current starts flowing. Now this current because it is in the presence of the magnetic field therefore results in the production of a force on the armature conductor and because the armature is mounted on a shaft and capable of rotation the armature will start rotating. Now as soon as the armature starts rotating of course a back EMF is produced and if one looks at the appropriate rules that are involved that give you the direction of the back EMF and the direction of the torque or the force that is produced, it will turn out that E b will start increasing and will be in a direction opposite to E a as a result the armature current will reduce from the original very large value and as the motor speeds up.

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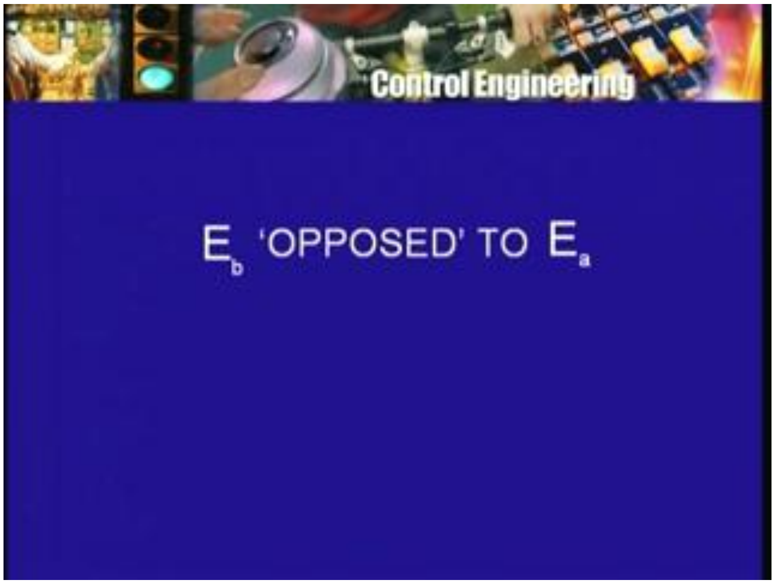

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We are assuming that a torque is produced and the torque is greater than the load torque in fact the starting torque of the motor if the motor is started without any resistance can be very large and therefore the motor will begin to accelerate and as the motor picks up speed the back EMF increases the difference between the applied voltage and the back EMF which can be thought of as causing the current, armature current will go on decreasing because of the decrease in the current of course the torque that is produced will also be reduce till finally the situation is reached when this speed and the armature current are such that the back EMF produced with the armature drop is exactly equal to the applied voltage and the torque that is produced exactly balances the load torque and therefore the load and the motor does not accelerate any more and runs at a constant speed. We have written down therefore 2 equations for the electrical part the back EMF equation E b equal to K b omega m and the electric circuit equation for the series circuit as it were of armature resistance, the back EMF and the applied voltage. However, as we have seen the armature current may not remain constant because the load may change because the supply voltage may change among other things and therefore armature current may vary with time.

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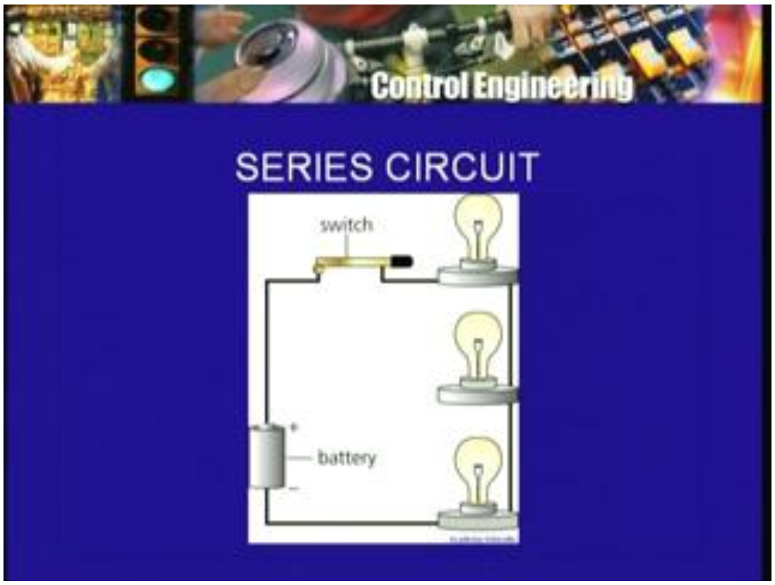

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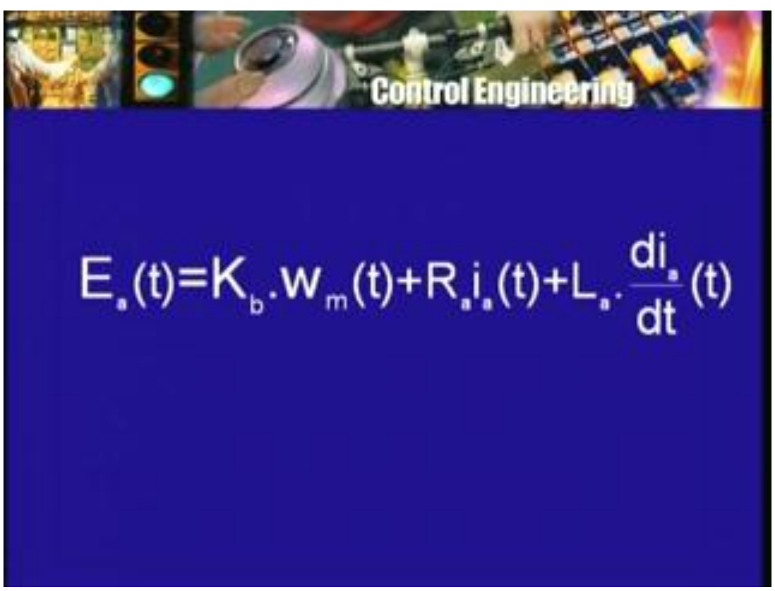

Now when it varies with time then there is an additional term or factor that we must consider and I told you that the armature can be thought of as some kind an inductor because ultimately it consists of conductors in the presence producing a magnetic field because there are in the presence of magnetic material, of course these conductors are in a presence of a strong magnetic field produced by the field winding and therefore the equation will get modified as follows and I will write it in such a way that every term perhaps could change with time, the applied voltage E a at time t equal to the back EMF and the back EMF we will write therefore as K b into omega m at time t plus R a into I a at time t, the armature drop plus one more term which can be thought of a some kind of an inductive self-inductive effect of the armature and therefore L a di a dt, this will be the total electrical equation for the armature applied voltage consisting of 3 terms the back EMF, the armature voltage armature drop and the inductive effect because of changing armature current.

As I said earlier this is all an approximation for the actual behavior but it is a reasonably good approximation or at least one can make this approximation make some calculations and see what happens actually by going to the laboratory and doing tests. Incidentally, all of you should if you have not already seen an electric motor should go the laboratory and have a look at the DC motor and see it in operation. This is for the electrical part of the armature; remember I am assuming that the field current is somehow maintained constant.

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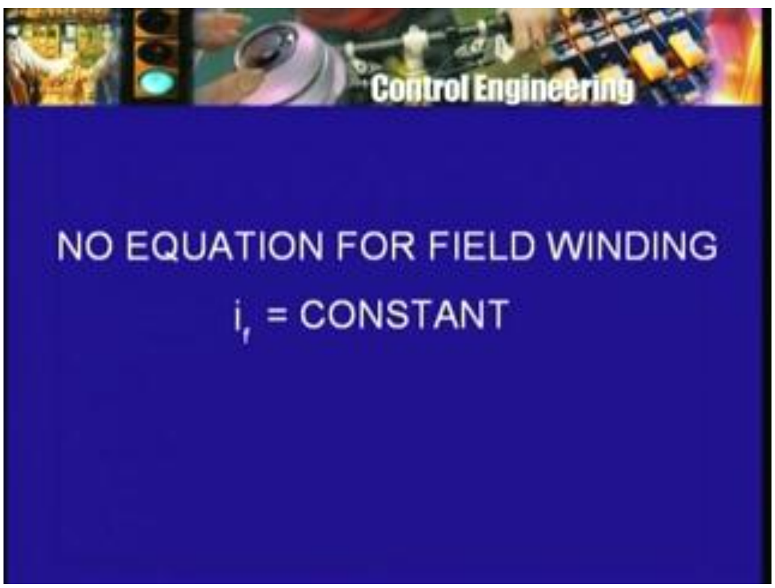

So there is no equation for the field current or for the field winding we will simply say that the field current I f has a function of time t is just some constant value it is not changing with time. Later on we will consider the possibility of changing the magnetic field on the field winding current. So right now the kind of control action that we are thinking of does not involve changing the field current but could involve changing the applied voltage E a and therefore this is referred to as armature controlled motor or armature controlled situation for a DC motor, separately excited DC motor. If the field current can also be changed through some arrangement instead of the armature applied voltage being changed then one refers to if as a field controlled motor or field control of the speed of the motor or of the drive.

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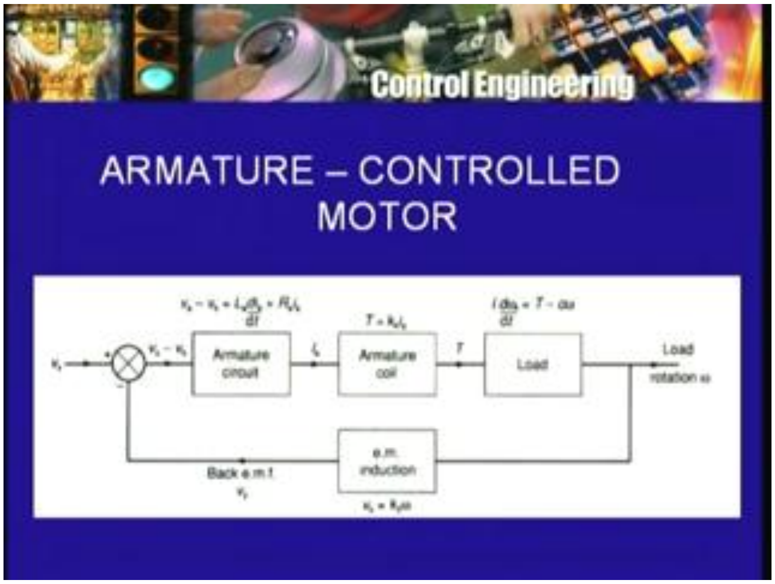

So at the moment we are looking at armature controlled drive, what about the mechanical part of the armature and the load. Here again, we will make some simple assumption which may not be valid in some particular cases the armature is wound on a core quite heavy it is mounted on a shaft which is going to be rotating and at the other end of the shaft I am assuming that you have a load such as a grinding wheel or may be a pulley which through some belt drives some mechanical machines and so on. You could think of even a flywheel at the one end of the shaft.

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So what I am implying is that there is a rotational motion of not only the shaft but also the armature the load and therefore one has to consider the moment of inertia of this rotating part. If there is no gear train the load is directly driven then we can combine the moments of inertia of the armature that is the motor, rotating part of motor and the load itself under a single moment of inertia and the symbol were you very often used is J, capital J. If one has to talk about the motor moment of inertia separately from the load moment of inertia. Then one will use two symbols J m and J l, m for motor, l for load as before. But for the moment we have to assume that there is no gear drive this shaft is directly connected and therefore we are talking of a single moment of inertia and therefore I will denote it by J.

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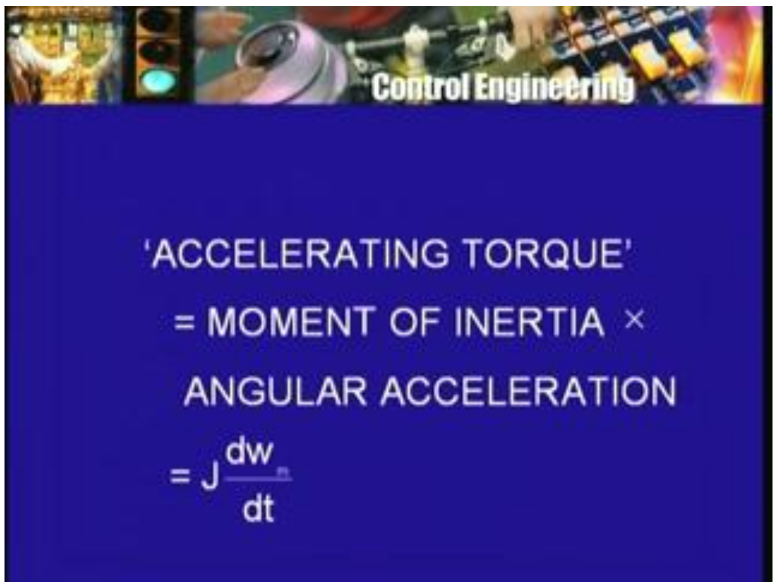

Now the dynamical equation for the rotational motion as you know is similar to the Newton's equation for motion of a particle namely mass into acceleration is the total applied force. So here then we will have the moment of inertia J multiplied by the angular acceleration and since we have denoted the angular speed by omega m therefore we will have it as d omega m dt this will be the accelerating torque that this will be the torque which is related to the acceleration of the motor, of course the acceleration may be positive or negative one uses the word in a neutral sense omega m may be increasing or omega m may be decreasing.

So this is the torque which is required to accelerate or change the speed rotational speed of the shaft and the armature and the load all of it having a moment of inertia J. Now this where is it coming from, what is accelerating or what is turning the shaft in the first place? The force that is produced by the interaction between the armature current and the magnetic field or that is a torque which is produced in the motor and as I indicated earlier, we will use the letter capital T for torque and m for motor and therefore we can talk about the motor torque T m.

Now once again it is a fairly good assumption to make that the motor torque T m is proportional to the armature current of course the motor torque is the result of the interaction of the armature current and the field or the flux or the flux density and therefore it is also going to depend on the field but we have assumed that the field winding current is kept constant and therefore the motor torque produced T m will be proportional only to the armature current and therefore we will write the equation T m is equal to K subscript T, remind us of torque multiplied by armature current I a. This K T for an obvious reason is called the torque constant of the electric motor, it relates the armature current to the torque and so its units will be torque is Newton meters current normal unit is Ampere, so Newton's meter per ampere.

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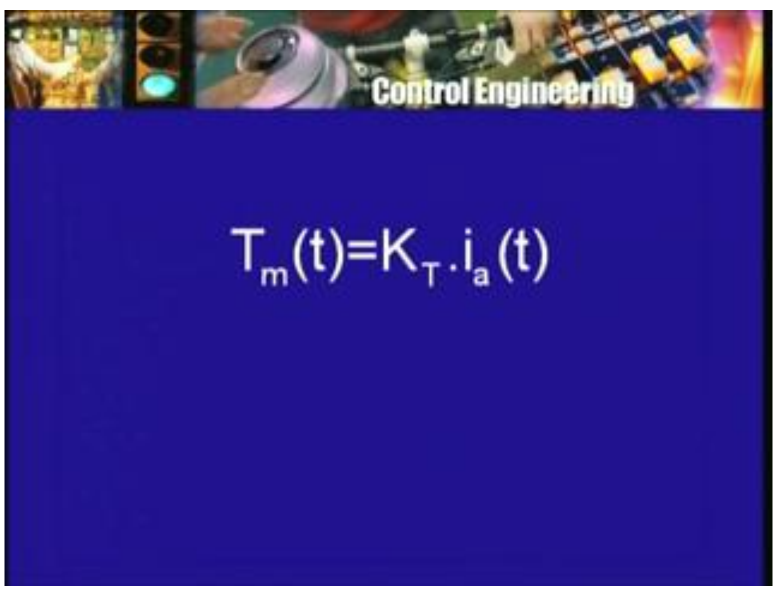

So this is the equation for the production of torque in the motor winding, T m equal to K T, I a because I said that we will have to consider a situation when the armature current is not going to remain constant and therefore I a will be a function of time then the motor torque also will vary accordingly and therefore I will write T m of t equal to K T the constant torque constant into I a of t. This is an equation which is not purely electrically equation its an electromechanical equation and this is one of the two equations that reflects the fact that the electric motor or the electric generator or the induction motor DC, AC what have you, they are all electromechanical systems they involve both electrical as well as mechanical variables. Torque mechanical variable rotation is related to armature current and electrical variable and the other equation of course was the very first one that we wrote namely the back EMF equation we have EMF or voltage that is produced an electrical quantity and the quantity that is related to the motion namely angular velocity omega m.

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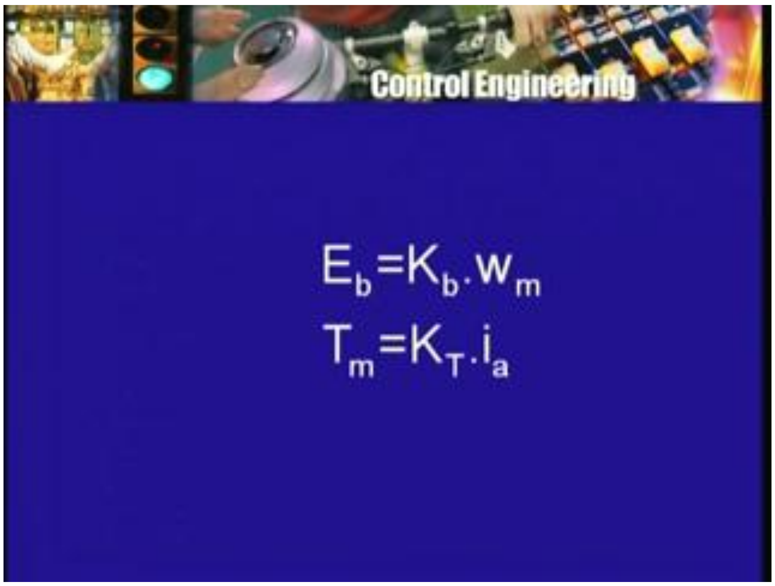

So these are the two equations of electromechanical interaction and right now we are, we are looking at the torque equation torque is equal to K T into I a, t this torque constant can also be measured experimentally because it is a little harder to measure than the back EMF constant. Electrical measurement of voltage is much easier than measuring torque on something which in fact if it is rotating it is even more difficult. So what is done is the motor is forced to remain at rest because the motor winding or the armature has to shaft as to be clamped and then some force will be produced on it that will be counter balanced by some other force which is measured and so on.

So these are referred to as dynamometer tests and some of you in your machines course may have done such tests. So from such tests the value K T can be obtained. Now again it is a fairly good practical approximation and one can show that in a ideal theory it must be true namely that these 2 constants, the back EMF constant which relates EMF to speed and the torque constant which relates the torque produced through the armature current these two are equal. Their units look different volts per rpm or volts per radians per second and Newton meters Ampere but if you look at the dimensions that are involved you will see that the dimensionally there are the same. You should do this yourself, this will also give you some practice of looking at dimensions of physical quantities.

So look at the dimensions of torque and electric current, electric current can of course be taken as one of the fundamental dimensions or you could take charge as a fundamental dimension and similarly voltage then they may be derived dimension and angular speed of course has a certain dimension and you can check dimensionally that K b and K T will have both the same dimension. There is also an interesting effect or a corollary of this relationship K b equal to K T because if I take the equation E b equal to K b omega m and take the equation K T, I a equal to T m and take the product of the two equations then I will get K T into E b into I a equal to K b into T m into omega m.

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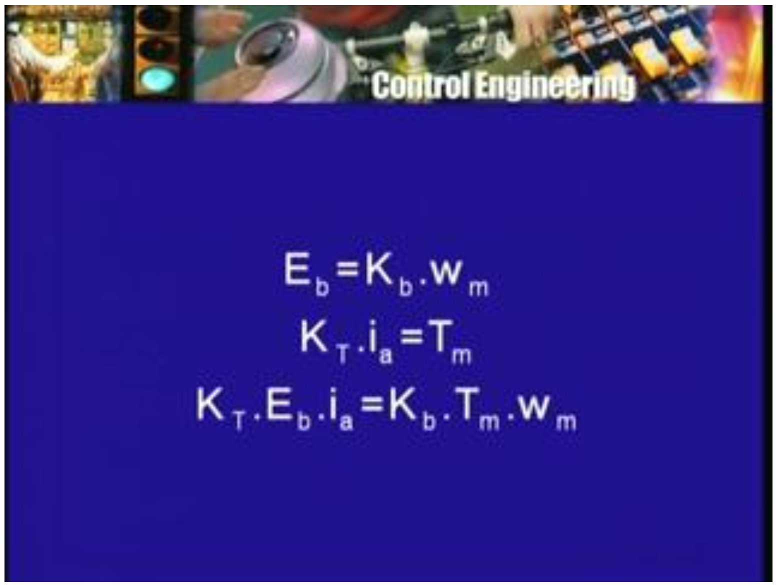

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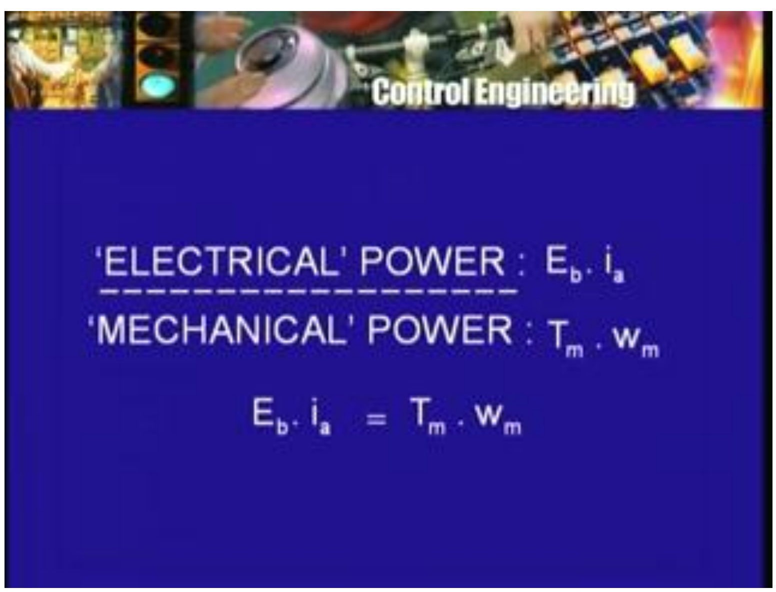

Now what do the 2 terms from the 2 sides of this equation mean the variable terms K T and K b are coefficients E b is EMF or voltage I a is current. So the product can be thought of as electric power Volt Amperes or one volt and one ampere will give rise to an electric power in this case DC power of one Watt on the other side you have power Newton meters and angular speed and their product is also power that is the power in mechanical engineering or in a mechanical system and so you have electric power on one side and you have mechanical power on the other side.

Now if K T is equal to K b this means that these two powers are equal because if this equation is true K T, E b, I a equal to K b, T m, omega m and of course it is true then if K T is equal to K b we can cancel of that factor and therefore we will get E b, I a equal to T m, omega m that is the electric power that is if one looks at the motor from the electric aspect that is the back EMF of E b which is very nearly equal to E a multiplied by the armature current the electric power is equal to look, look at the mechanical side there is a torque T m and there is rotation at omega m speed and therefore there is a mechanical power T m into omega m.

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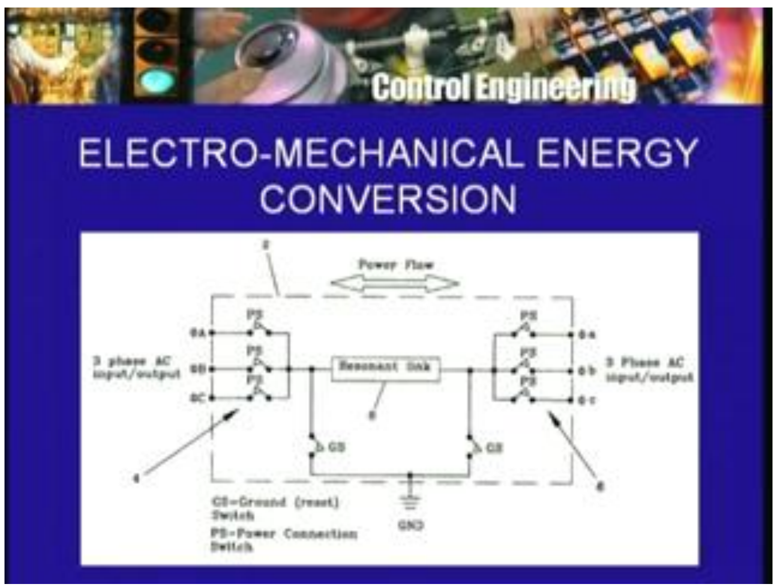

So this is some kind of conservation of power or better it is a transformation of power as one would say from electrical to the mechanical and therefore the motor is electromechanical energy conversion device, converting electrical power into mechanical power where as an electric generator will act in the opposite direction it will convert mechanical power into electrical power. This suggest that K b and K T may be equal and as I said if you go into the theory of it then under certain ideal conditions one can actually prove by calculating the forces and the EMF’s using the appropriate laws and equations that these two coefficients must be equal. But we will continue to use the two symbols although it should turn out to be the case that these two are nearly equal if not absolutely equal and of course both of them are dependent on the field winding current. It is for the same field current or field winding current that I will talk of K b and K T and then they will be equal. If we change the field current the torque production will change and the back EMF production will also change in the same by the same factor or in the same proportion.

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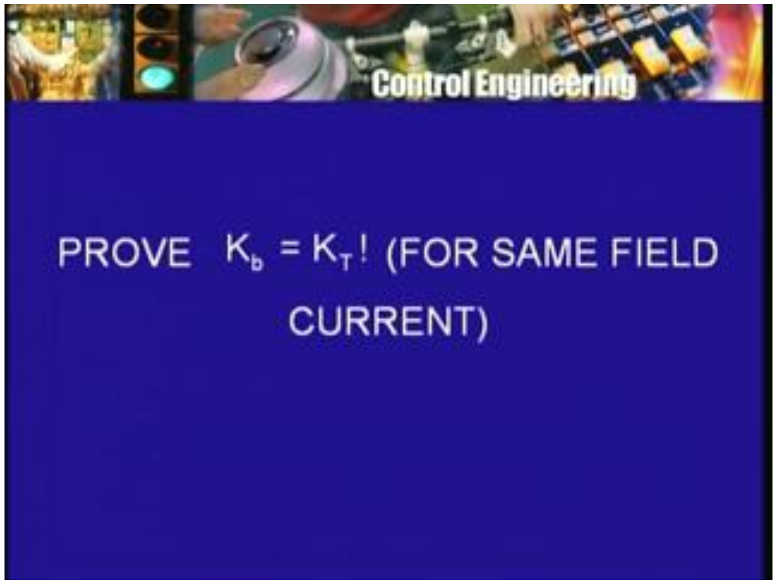

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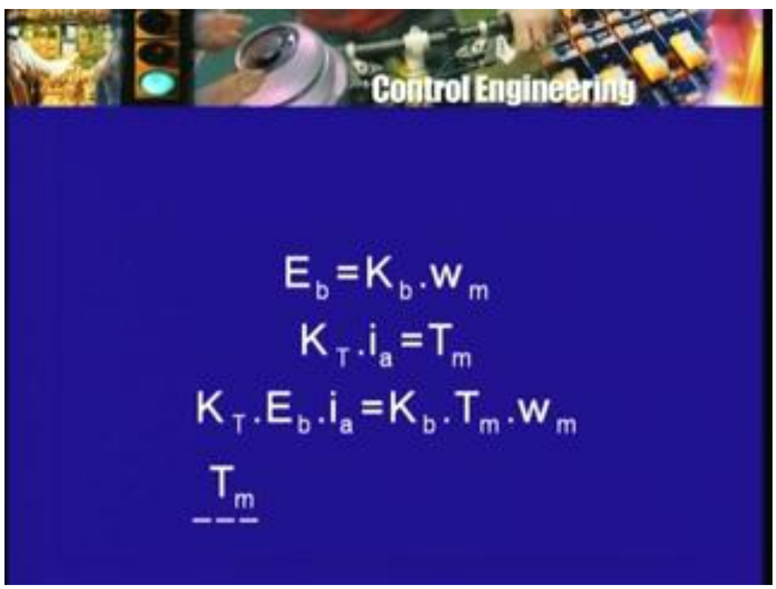

So for the same field winding current K b will still remain equal to K T okay. So this is the torque production equation, what about the load now, going back to the load we have talked about the moment of inertia J of the whole system there is this motor produced generated torque T m now the load itself we could think of it as some thing which opposes the motion of the shaft actually we want the load to rotate but there is some opposition to its rotation somewhere think of the grinding wheel example, the grinding wheel is mounted on the motor shaft and the grinding wheel is pressed against some surface which is through machine and there is resistance to that motion.

So there is a counter torque or rather the grinding wheel must be applied certain torque for it to rotate, the surface, the machine is opposing this motion therefore in this sense we can talk about the load torque and therefore T m will be equal to T l the load torque. For example, the action of the grinding wheel requires a certain amount of torque it will depend on how what the material is and how far you have pressed the grinding wheel against the surface and so on.

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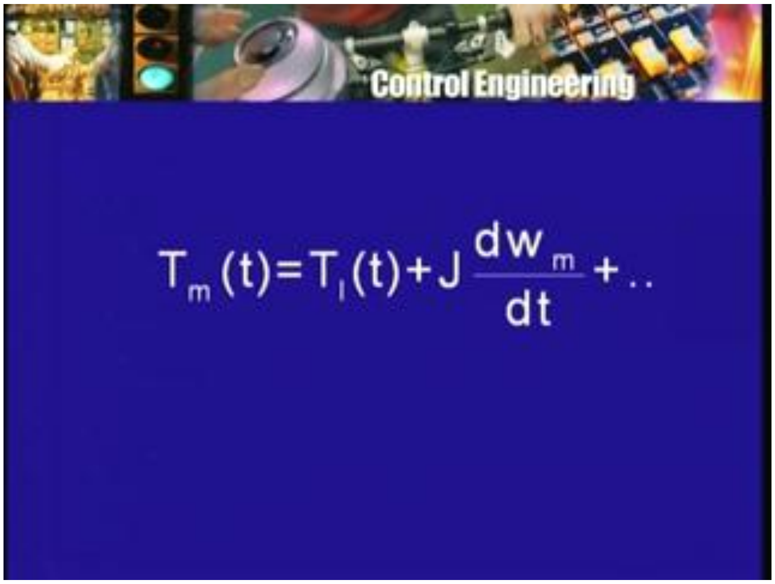

So there will be a load torque T l and of course if the speed is not remaining constant there will the moment of inertia term t, J into d omega m dt and in addition the shaft is mounted in some bearing there may be a cooling fan on the motor shaft as the fan rotates there is a resistance of the air. So all of this we can think of a some kind of friction and therefore an opposing force to the rotational motion of the shaft and the armature if it is a fan then we refer to as to the wind age, it is the air resistance which requires some electrical power to be supplied to this ceiling fan otherwise the ceiling fan would rotate without any power required but it has to move air and therefore there is this resistance to movement which is called wind age.

So this wind age plus friction in the bearings again its not a bad approximation to assume that all of this is a torque which is proportional to the angular speed and therefore I will use one more coefficient K f, f for friction into omega, omega is the angular velocity of the shaft. So this is the mechanical equation for the drive the generated torque, the torque generated by the force or acting on the conductors equal to the load torque, the opposing torque somewhere grinding machine lathe what have you plus whenever there is a speed change there is this accelerating or decelerating term J d omega m dt and there is the inevitable friction and therefore we have an opposing torque due to friction which we are modeling by K f into omega.

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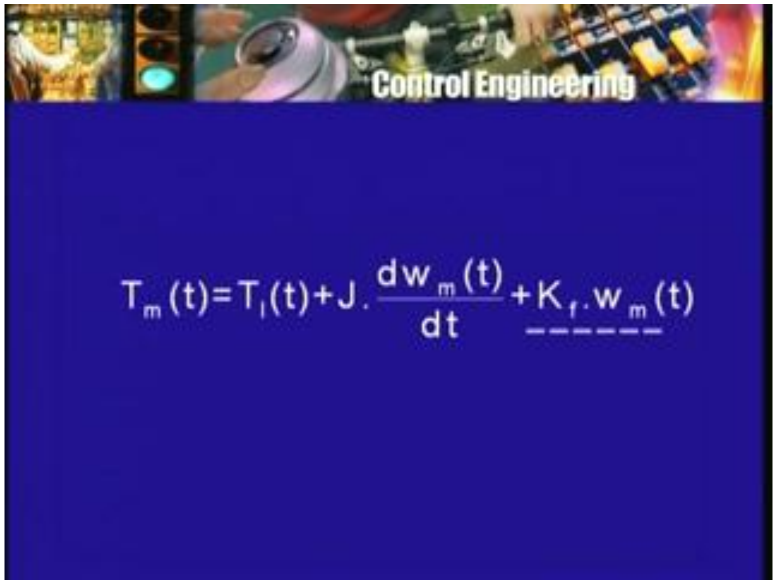

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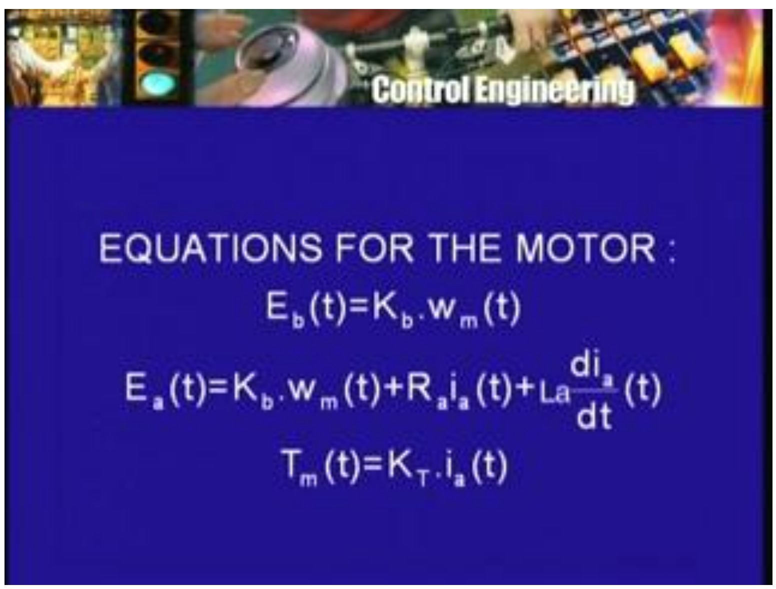

So with this we would have completed writing down the equations for the motor and let us look at them all together once again. The back EMF equation for the motor is E b equal to K b into omega m both of them may be varying with time. So E b of t equal to K b into omega m t, the electrical equation for the winding once again because the current may be varying with time will be E a (t) equal to K b into omega m t the back EMF term plus R a, I a of t, the armature drop term plus L a into the rate of change of armature current. The inductive effect of the armature winding that is the effect of the fact that the armature itself produces a magnetic field. Then the electromechanical interaction equation like the back EMF equation, the other equation torque motor torque produced is equal to K T into I a and the mechanical equation for motion of the system namely the motor torque equals the load torque T l plus the accelerating torque J d omega m dt plus the frictional opposition torque K f into omega.

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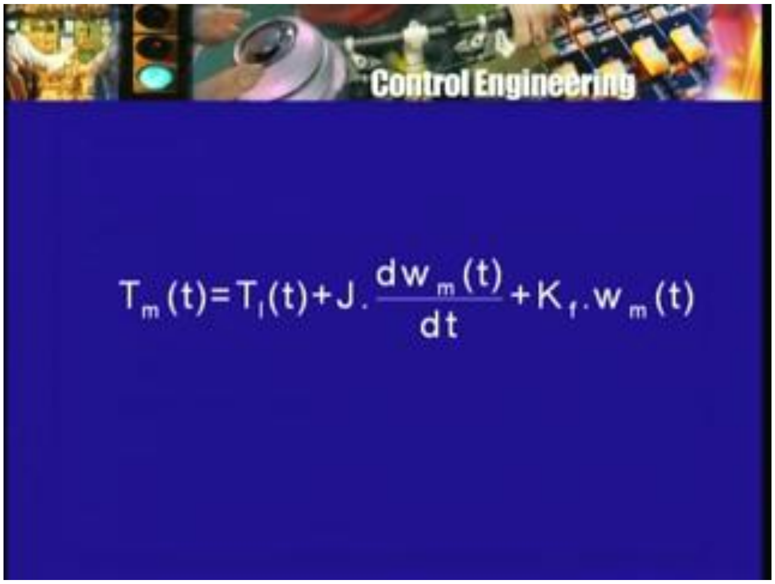

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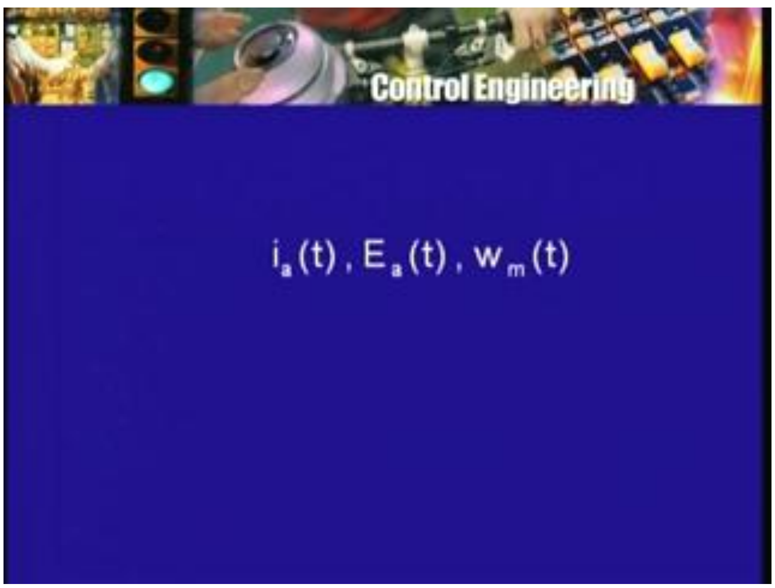

So these are equations with which one will then have to start thinking about the control of the speed of the motor or whatever one wants to achieve, what are the various physical variables that are involved in our model armature current of course I a of t the applied voltage E a of t, the external voltage DC power supply voltage that is applied to the armature winding E a of t. The speed of rotation of the shaft omega m of t if we have a gear train then we will have two separate speeds, the motor speed, the load speed we are assuming that it is a directly driven load for simplicity.

So there is only one speed to talk about omega m or omega L the shaft speed and of course there are other variables these variables I a (t) of course can be measured simply connect an ammeter in series with the armature you can measure the armature current. The applied voltage can be measured with the help of a voltmeter, no problem about that, angular speed, how does one measure the angular speed of a rotating shaft. One could use what may look like more mechanical devices for measuring speed but since we are already dealing with an electromechanical system namely a motor. One might use the system in the opposite direction in other words have a generator mounted on the shaft which produces a voltage which can then be measured and in fact this is what is usually done. You have a very small generator because you are not interested in producing power you only want to get an indication of the speed of the motor that is you want to measure the speed of the motor.

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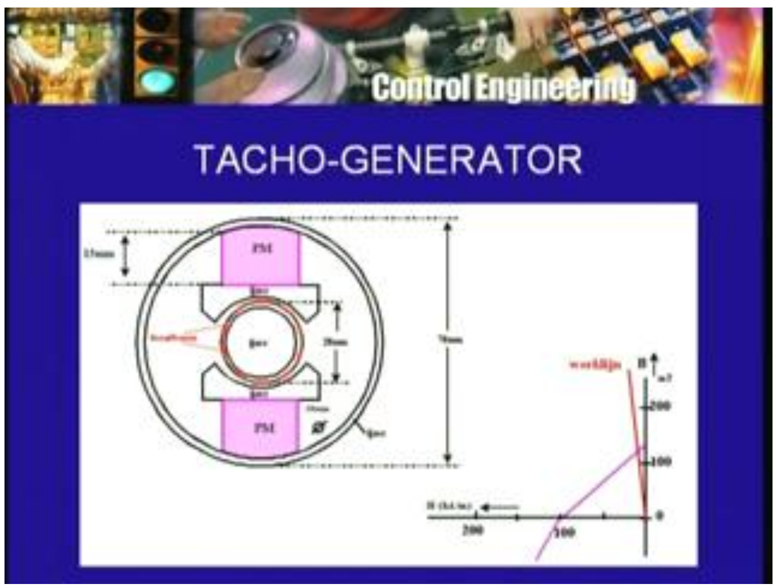

So a small DC generator is mounted on the same shaft and because it measures speed the Greek root for the word speed tachos is involved, this generator is called a tacho generator. It is essentially a small generator which is mounted on the shaft to the motor whose voltage, DC voltage will be proportional to the speed and therefore it gives you a simple, simple in a way simple means of measuring the speed of the electric of the motor or the drive and is useful because its voltage and things can be handled more easily with electricity than with mechanical things. As you know voltages can be added or subtracted, amplified, reduced or attenuated much more easily than what can be done with mechanical quantities.

So these are the three main variables then the armature current I a of t, the applied voltage E a of t and the angular speed omega m of t. There are of course other variables there is the load torque T l and I should put a t there also because that can be a culprit. The load torque may be change and therefore I am should think of it as a function of time and in fact therefore it is one more variable that is important the grinding wheel, in action the surface is changed the pressure with which you are press against the surface could be changed, the quality of the grinding may be changed as a result there will be a change in load torque.

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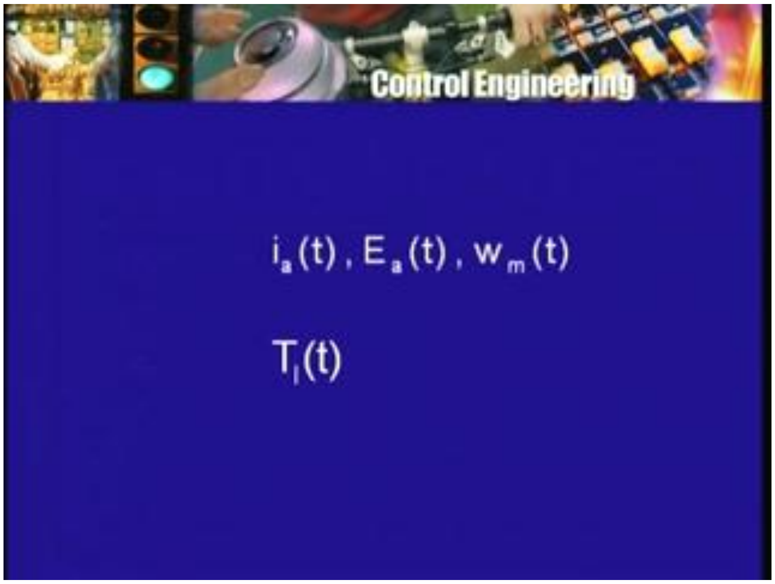

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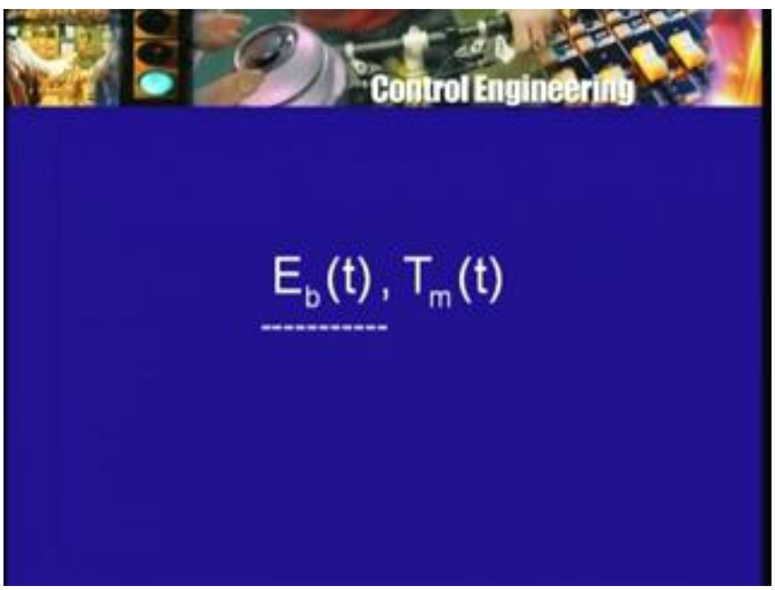

So this is an important variable T l of t and there are the other variables which it is not really possible to measure that easily while the motor is operating one is the back EMF E b, I told you that you could measured the back EMF when the machine is not run as a motor actually but is driven somehow and the armature is not connected to anything so you measure the open circuit voltage. But we are now operating the machine as a motor, so there is no longer any possibility of measuring E b if you connect a volt meter across the armature what you will measure is the applied voltage E of t and not E b of t.

Similarly, the generated torque Tm that's going to be an not so easy to measure it is not going to be easy to measure the force on the conductor or the torque that the armature or the shaft transmits from the armature where it is produced to the load where it is used up so to speak. Now of course torque can be measured and some of you may have been exposed to little bit of mechanical engineering to realize that when a shaft like this one transmits a torque that is one end of it is the motor and some about something the electromagnetic effect is pushing the thing round and at the other end is the load which is opposing this motion and a torque therefore is being transmitted in a shaft. The shaft undergoes a twist may be very hard to believe, you may think of a shaft as something very rigid but it is both a matter fact as well as theory that the shaft undergoes a twist just as in our earlier example of a mass spring dash pot system if you exert a force on the spring, if you pull it, it will stretch, if you push it, it will compress.

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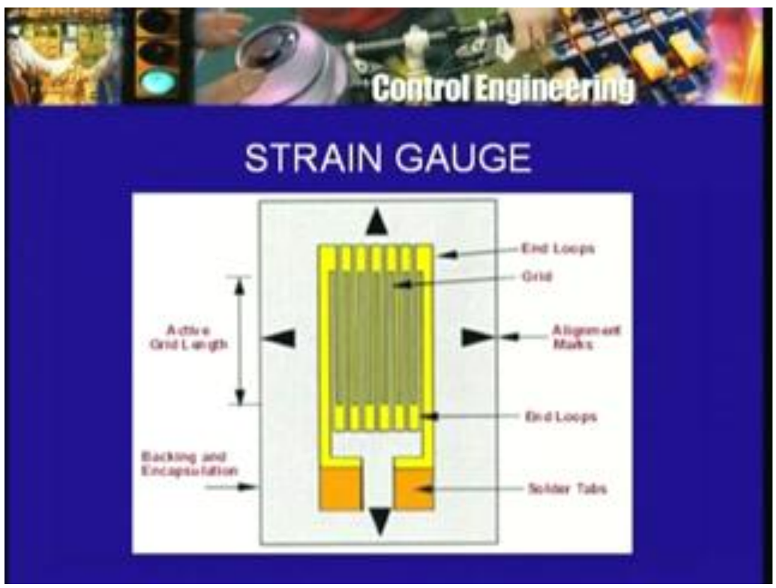

So similarly when a shaft is transmitting torque one end of it is turned and as a result the other end also turns there is a twist in the shaft. Now this twist can actually be measured it is not, so very difficult easier to measure the twist than to measure the torque being transmitted and now something like the rotational version of Hooke's law will tell you that the twist is proportional to the torque. Just as for the spring the force was proportional to the extension there is a rotational Hooke's law which can be used and therefore the torque generated by the motor can actually be measured, of course what is really measured is torque that is transmitted at any one particular cross section of the shaft by using things like strain gauge and so forth this twist can be measured, can be converted into a voltage finally as usual and therefore that motor torque or the torque and that section of the shaft can be measured. But it is not so easy, it is not as easy as measuring the speed of the shaft simply by having a tacho generator and looking at its output okay.

So we have these four variables of importance besides the back EMF and the torque that is produced at every moment of time and we have to know the values of the various parameters that are associated with the drive there are motor parameters there are the two constants K b and K T which will be nearly equal for a given field current of course there is the armature resistance R a there is the armature inductance L a which we must know these are the electrical parameters of course K b and K T are the electromechanical parameters of the motor and then for the load and the armature thought of as a single rotating entity, we need to know the moment of inertia J and the coefficient of friction K f relating the torque the opposing torque caused by friction to the angular speed omega omega.

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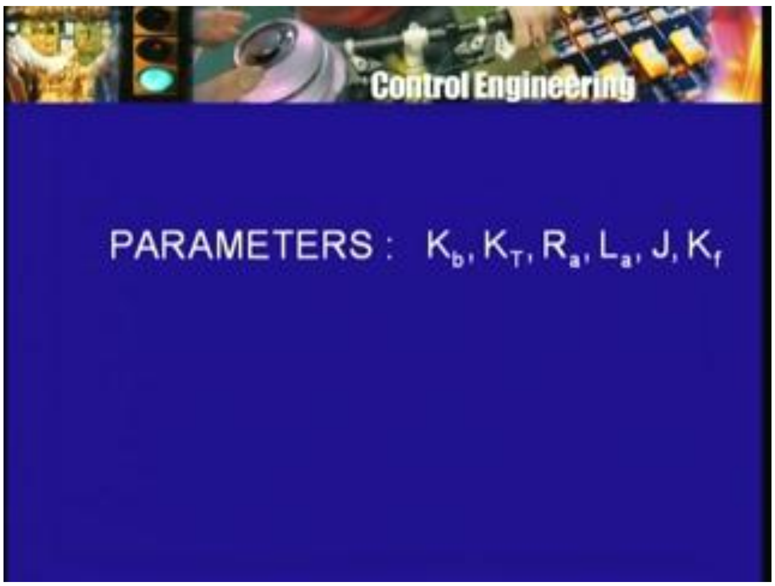

So these are parameters that we need to know before we can use these equations to design a control system. This is of course a very simple modeling of the separately excited DC motor. We have made many assumptions or a approximations but it is a good enough model to start with and it is not that simple because we have equations which are no longer simply algebraic or arithmetical equations, back EMF is proportional to armature current is simply arithmetic but the electrical circuit equation already involves a derivative of the armature current and so it is more complicated than a simple arithmetical equation. As you can therefore expect there is a little bit of calculus or derivatives are coming in here. Similarly, the motor torque produced is proportional to the armature current that is very simple but the mechanical equation is again something that involves a derivative, a derivative of the angular speed.

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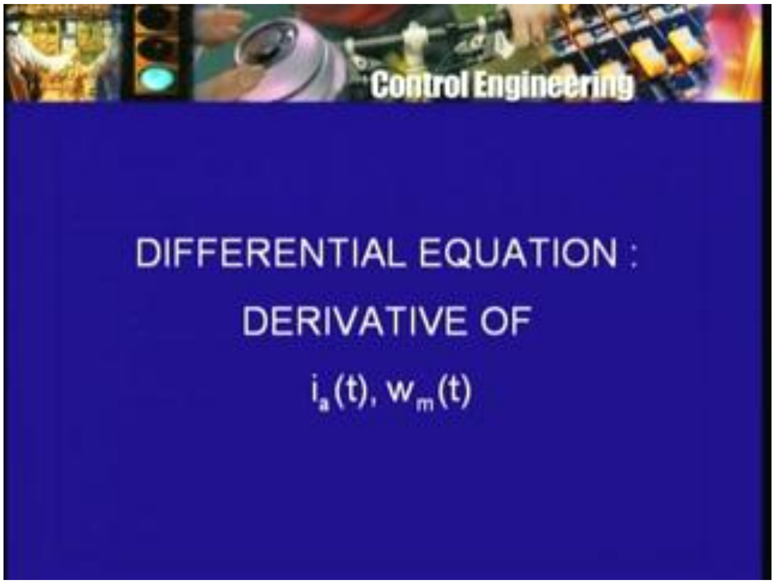

Of course the angular speed itself can be thought of as a derivative of the angular position but in our modeling we are not worried about the angular position of the motor shaft simple because I am assuming that the application is such where the motor shaft is to go on rotating, the whole idea is that the grinding wheel should be rotating and not simply to be put in some particular position. We are looking at a speed control system omega is our variable which is to be controlled the motor speed. Later on we will see an application were the same DC motor could be used for position control there the purpose of the motor will be not to make something rotate at some constant speed or change the speed but to put it in some particular angular position.

This should remind you of something we discussed earlier namely a gun which is being controlled with the help of a radar, an enemy aircraft is being tracked by means of a radar and the gun is to pointed as closely as possible at the target therefore the gun has to be moved the gun will not be rotating continuously but the gun will change its position and which can be achieved by a motor, in fact 2 motors will be required because the motion is in a horizontal plane, a motion is azimuth as well as well as a motion in a vertical plane that is a motion of elevation or altitude.

So not 1 but 2 motors will be required and that will be a position control system but right. Now we are looking at a speed control system. Let us right down the two equations which involve the derivatives using the notation that I had introduced earlier namely D as the operator or the symbol that denotes the operation of the differentiation or taking the derivative so if we do that then the electrical equation will look like E a of t equal time K b into omega m t plus R a, I a, t plus L a into derivative of I a at time t.

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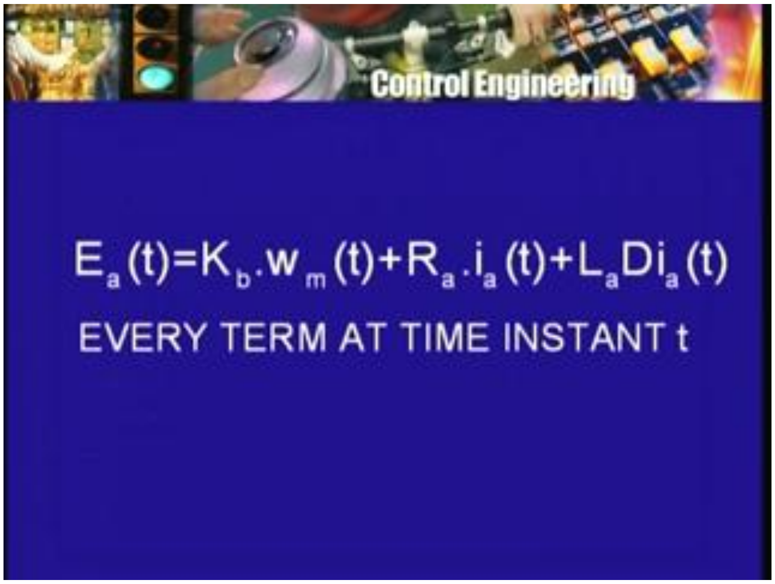

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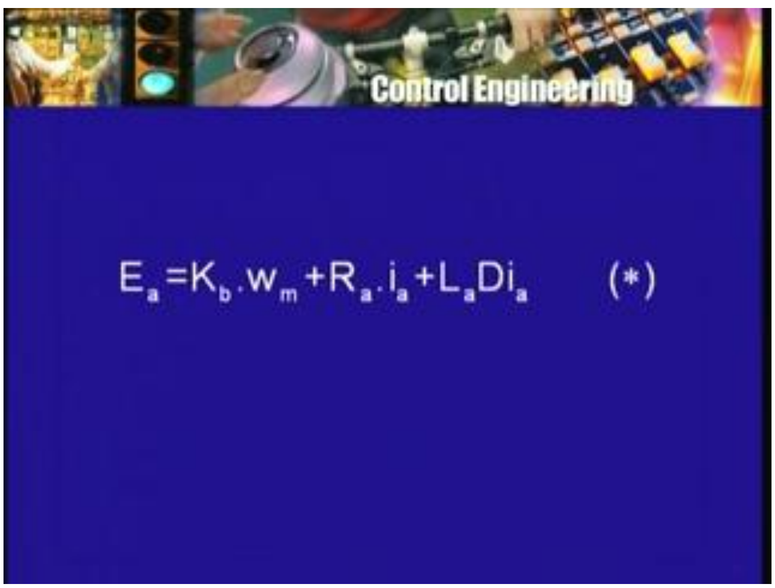

So everything is at every moment of time or instant of time t the applied voltage at time t at a given instant of time t is some of these 3 terms, instead of writing this t on both the sides. We could write this as an equation as follows E a equal to K b into omega m plus R a into ia plus L $_a$ into D I $_a$ such equations are not very commonly written in many control books you will find that the equations have that t standing out all the time. So why am I writing this equation, well this equation as we will see later on can be interpreted a little differently the earlier equation were that t appears says that at every instance of time something is happening whereas this equation in which the t has disappeared is now an equation that involves not instantaneous values because there is no t there but involves function of course these are functions of time that is we are interested in what happens with time and so we can think of ta as the applied voltage function of time in a particular application the applied voltage may be simply kept constant in that case E a will be just a constant function, omega m is the angular speed function, I a is the armature current function, D I_a is the function which is the result of differentiating the armature current function and so these equations tells you that one function the applied function is a sum of 3other functions. Omega m multiplied by a coefficient or constant K b, K b is not a function of time K b is unchanged with time I a into R a again is not changing with time that is one of the assumption that I am making of course in practice the armature will get heated up and depending on how much current it is carrying the armature temperature will vary therefore the armature resistance will not remain absolutely constant. The third term is the derivative of the current multiplied by the inductance and I am assuming that the inductance does not change with time but remains constant.

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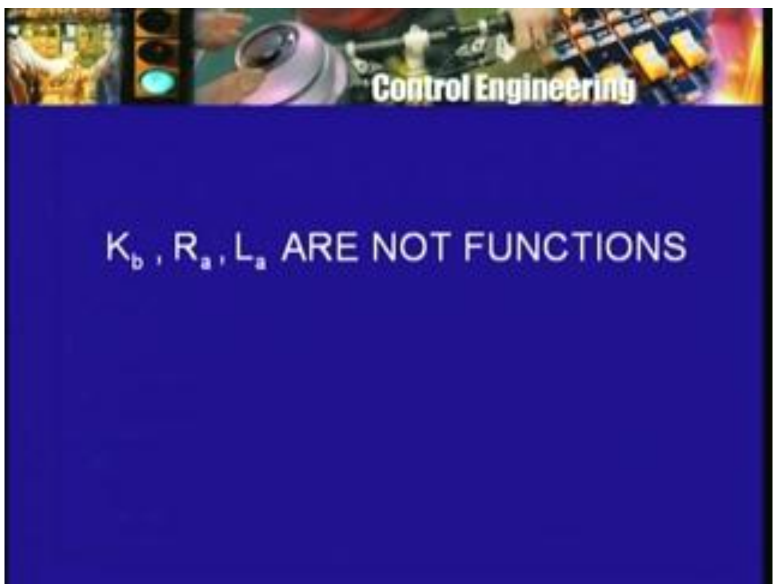

So instead of an equation relating the instantaneous values we have an equation relating function we will use tools like the Laplace transformation or the Laplace transform for studying, analyzing and for designing and at time this kind of a way of looking at it will useful. Instead of instantaneous values think of function behavior and time. So this is one equation and the other is the mechanical equation which says that the motor torque T m and I will not write therefore for T m at time t but the function T m equal to the load torque function T l plus the inertia related terms. So J into D omega m plus the third term which is K f into omega and of course I can instead of T m, I can directly have the armature current there and so write it as K t into I a because as I said T m is not going to be easy to measure nor is it necessary to measure it.

So we will have these two equations both of them involved derivatives, one of them involves a derivative of the current in other words when the current is changing you have to take care of that fact by with using that term and the other equation you know is the derivative of the angular speed therefore when the speed of the motor is changing for example when the motor starts from rest and goes up to a certain speed or the load is suddenly changed therefore the speed of the motor changes then we have to consider this inertia term which is related to the rate of change of speed and as you can see therefore these two equations together look as if they are differential equation, they are differential equations involving the variable I a armature current and the angle of velocity omega m of the motor and there are first order differential equations because only first order derivatives of the armature current variable and the speed variable are occurring in this equation.

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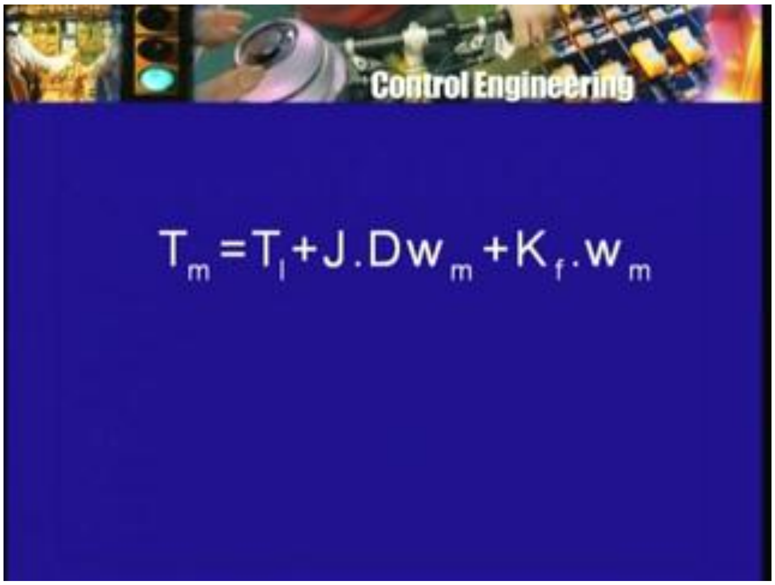

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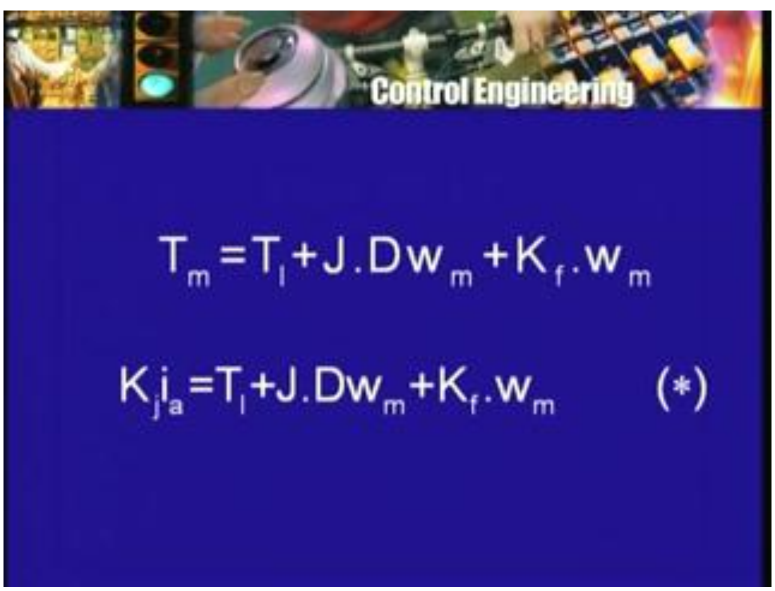

So they are both first order differential equation in both the equations the only quantities that appear are the two quantities ia omega m whose derivatives appear and the other two quantities E a and T l, T l is the load part of it now this will play the role or this is what can be thought of as a disturbance in the sense T l may change suddenly in some unknown way, if you know what T l is going to be then that is one thing but suppose something happens and the load torque changes suddenly.

So T l will be our disturbance variable although it is not really a disturbance it is the reason why the system is in place but about it we may not know as the drive actually proceeds something may happen when load torque may change suddenly. The other variable E a is the power supply voltage variable and this we can think of as an input variable because in our scheme and I have called it armature controlled drive we will assume now that we will control the motor by changing E a if necessary. The load is driven by a motor and not cranked by some handle the motor is controlled by changing the applied voltage E a.

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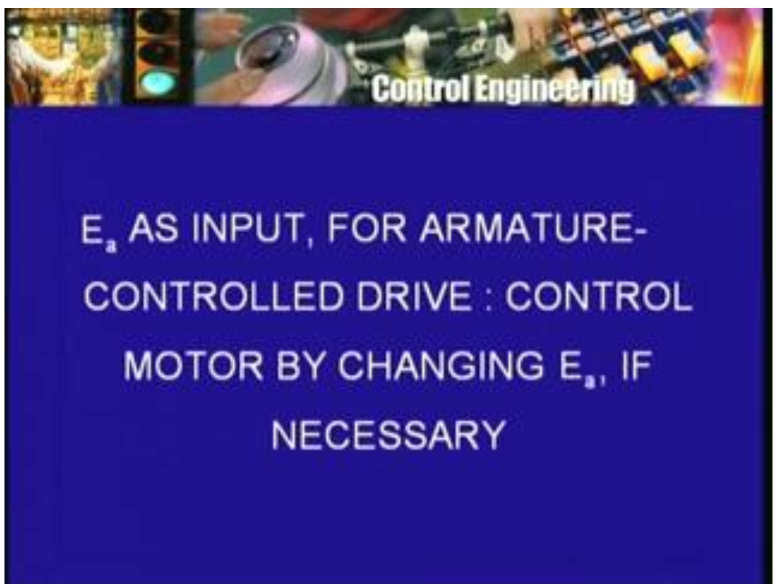

So the problem is now of how and when to change the applied voltage. We will see that this T l not being know before hand or the possibility that T l will change along with the fact that the motor parameters, the drive parameters that we have all listed may not be known exactly the back EMF and the torque constants, the armature resistance, armature inductance, the moment of inertia the coefficient of friction none of them of course are going to be known exactly with 100 percent accuracy, there will be some tolerance for each one of them.

So that is one thing the other is the load torque, uncertainty about the load torque if it cannot be ensured that you know T l what is going to be. These two things will necessitate the use of feedback that is if you want to the drive to perform satisfactorily it is these two the uncertainty in the parameter values and the disturbance aspect of the load which perhaps can be reduced the effect of these can be reduced by using feedback and we will see what kind of feedback can be used and what is the effect of using feedback.

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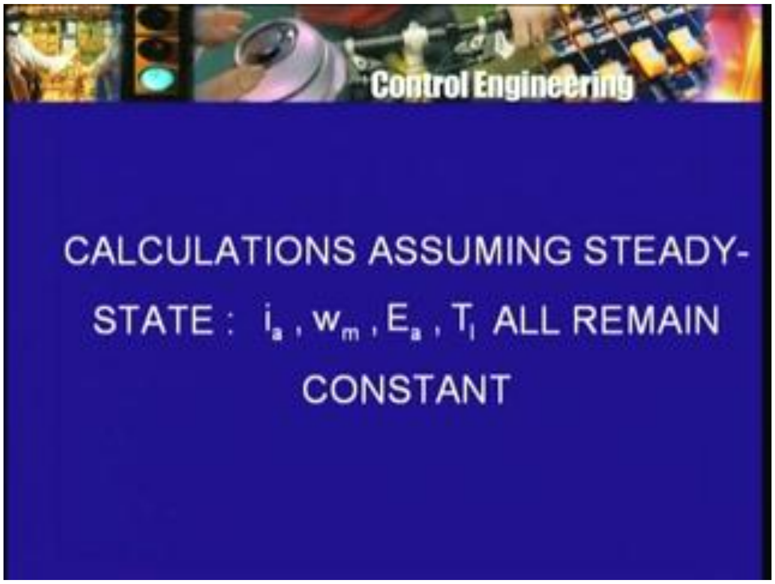

But before we think of using feedback, we can do some preliminary calculations to show that T l is known and omega the desired angular speed is known that is you want the load to run at a particular angular speed and the load at that angular speed requires a certain amount of torque and we will assume that somehow the motor has been put in what may be called a steady state the motor and the load that is the armature current, the velocity omega, the applied voltage E a and the load torque T l all of them are going to remain absolutely constant. It is like the DC case so to speak all the variables are not really varying but have remained constant. Thanks to the circumstances or supply voltage is remaining constant load torque is not changing and therefore the motor runs smoothly at some desired constant speed.

So under this assumption of constant speed, constant applied voltage, constant torque, constant armature current under these condition we can make some calculations then see what is the effect of changes or inaccuracy in the knowledge of the parameters changes in the load torque, changes in the applied voltage and so on and then go on to the possibility of using feedback, various kinds of feedback and so on.