Control Engineering
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Lecture – 07

The electrical rlc circuit, rlc and source of emf that circuit is said to be analogous to the mass spring dashpot and force or displacement driver, mechanical system and the analogy comes out when we write down the equations for the 2 systems component by component and then combine the equations for each system separately into may be one single system equation that is when the analogy or the similarity between the two different physical systems comes through.

We can spend a few minutes on the electrical system with the rlc and the electromotive force as the 4 components. You will be all able to write down the equations for the 4components separately. We had started of with one of the equations namely the equation for the resistor but one can write down and I will do that the equations for the remaining 3 components. So here are the equations V R equal to R i, V $_L$ equal to L di dt I C equal to C dV dt and V for the source V E is equal to some function e actually I should be a little more careful here.

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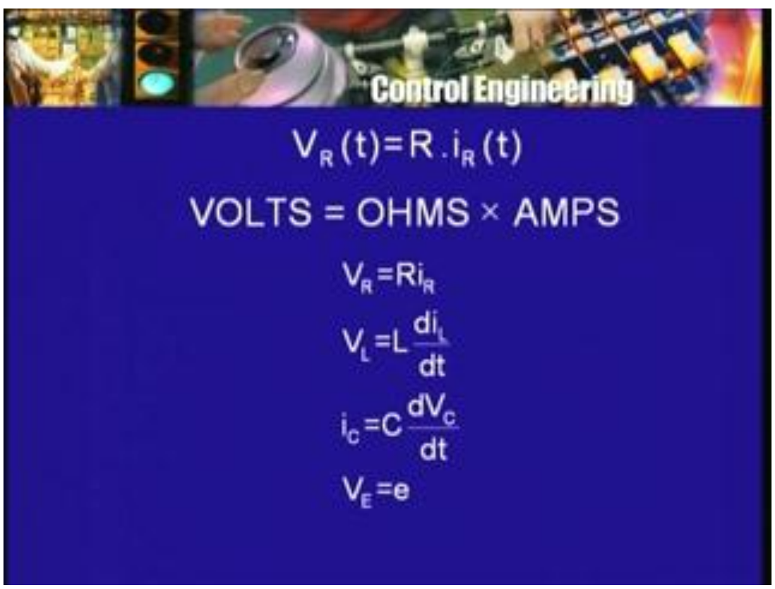

I should have written use the symbols which indicate what component or element of the circuit or network they are associated with thus I have written V R indicating there is a voltage associated with the resistor R of the resistor physical device resistor R and this voltage at any moment of time is given by the product of the resistance of the resistor a positive number expressed in units of ohms multiplied by the current in the resistor and therefore I should write strictly speaking I R, I subscript R meaning it is the current in the resistor and again to be absolutely correct I should write V R of t equal to R into I R of t. The voltage across the resistance as we say at any moment of time t is equal to this parameter of the resistor called the resistance multiplied by or multiplying the current in the resistor at time t, I Rt. So this is the equation for the component of the network a component which is called a resistor. Now in this point note one thing very clearly and I will go on emphasizing this a number of times because otherwise a lot of confusion can occur.

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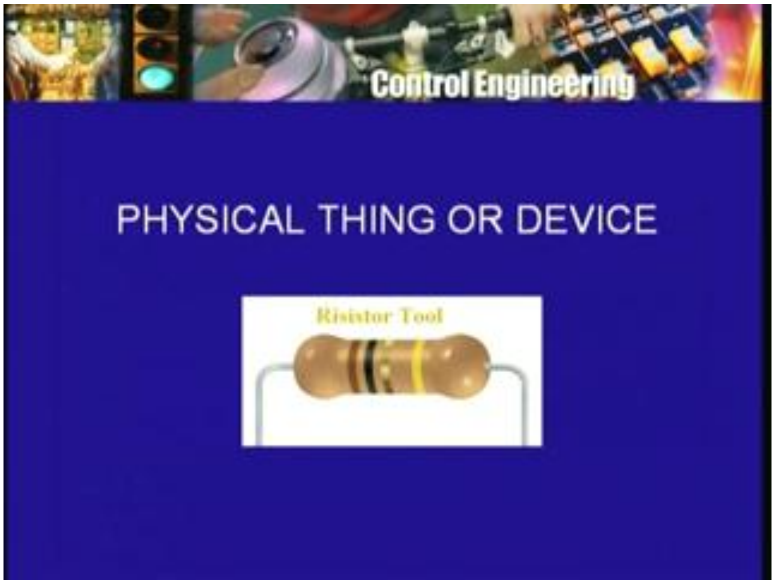

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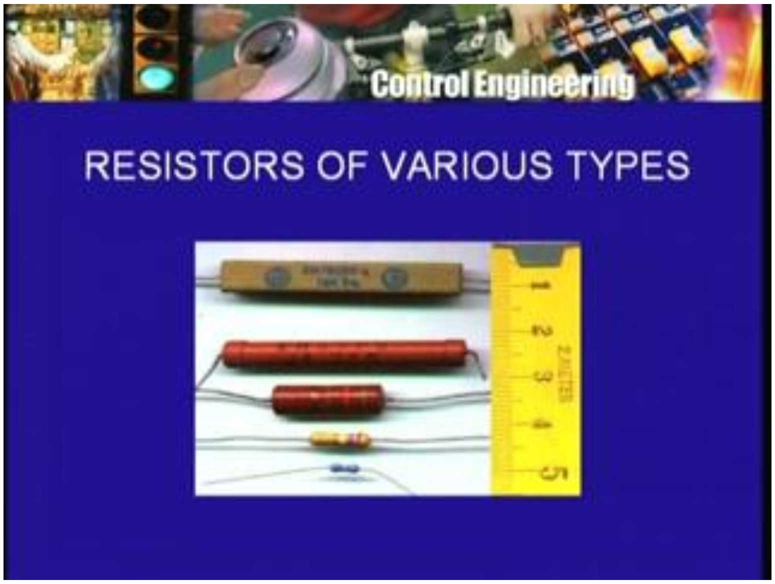

Now first of all there is the actual physical thing or device called a resistor. Here for example is a particular resistor of a particular kind this is what is looks like. Now you know that resistors can have various appearances here are some more resistors which look different from the one that I started with here is a one that is known as a wire wound rheostat it is a much bigger thing and one can see that wiring more or less very clearly there is here is a very small resistance which looks like a very small cylindrical thing it is not one does not know what is inside it and if you did not know what it might be you would not even guess that it is a resistor or it is a resistor certainly there are there is no wound wire inside that.

So there are those physical objects or things or components called resistors which one finds in the laboratory on the shelf or one can go to the market and buy them, there are made or manufacture also and as you know each one of these resistors comes with a value what we call the value of the resistance of the resistor typically it will be expressed in ohms but in some cases kilo ohms or even mega ohms and very rarely may be milli ohms.

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So here for example it is a resistor whose resistance value is 1.3 kilo ohms and all of you should be familiar with the way in which this value is marked on the resistor not numerically as 1.3 kilo ohms but in terms of some bands of particular colors. So here is a 1.3 kilo ohm resistor which is very small in size and as you can expect you cannot pass a very large current through it otherwise the heat that is produced in it will burn it up. So here is the practical thing which has a number called a resistance or the resistance associated with it expressed in some unit such a ohm or kilo ohm and so on having a particular appearance it has some weight, size and also along with it go some limitations like the current in it should not exceed may be 10 milli amperes or 100 milli amperes certainly the current in this tiny resistor cannot be maintained or kept at a level of say nk ohm 10 amperes for even a moment of time the heat produced will be so large that it will just heat it up and burn it.

So here is a physical device which has all these aspects it looks like this, it has weight, it has volume and there are limitations associated with this use. Of course it cost something but for us from the electrical engineering point of view what matters at the moment is only its resistance a particular number denoted by R or R with a subscript like R 1, R 2 if there are 2 resistances and so on and we are making the assumption here. This is the important fact that one should be aware or note that this physical thing can be described or part of its behavior can be described by means of an equation like V R of t equal to R I R of t, where in this case R would be the number 1.3 into ten raised to 3.

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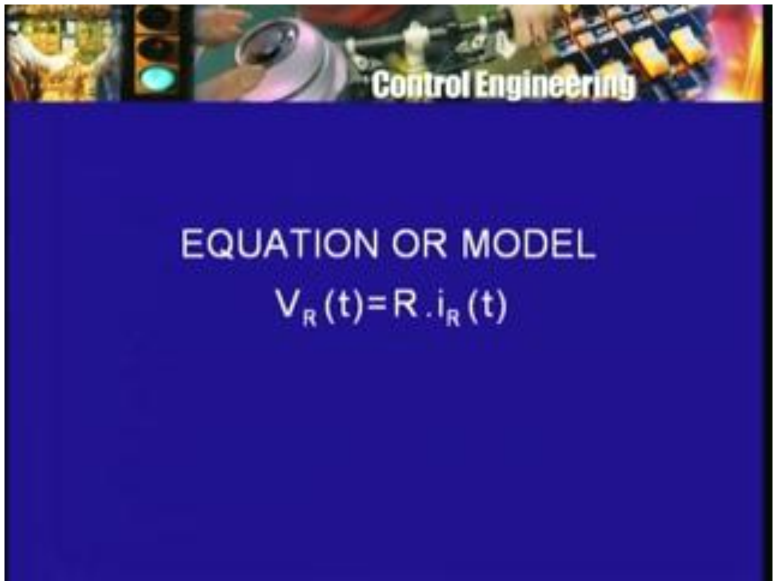

If I is a number which represents the current in the resistor in amperes and V R is a number which represents the voltage across it in volts and so we have volts equal to ohms into amperes I should not have said one point I said 1.3 into 10 raised to 3 because the resistance was 1.3 kilo ohm, if I have the current in milliamperes the resistance in kilo ohms then the products comes out to be voltage in volts.

Now if this resistance is say heated up either because of heavy current flow in it or you keep it near some other source of heat then will it continue to be described by this equation V equal to R i t with R equal to 3 into 10 raised to 3 or if I cool it if I put it in the contact with a very cold mass of ice let us say, will it continue to have a resistance of this same value. As you all know this is not the case the resistance value of resistor will depend upon the temperature of the resistor also.

So we are making the assumption that the resistance is operated in such a way that its resistance is given by 3 into 10 raised to 3 ohms. This is only an idealization in the sense, this is only in our concept or idea about it that is we like to think that the resistor behaves like this V equal to R i with R equal to 3 into 10 raised to 3, what we think will be correct of course or not too far wrong hopefully most of the time and so there is another aspect of it which we must remember that is this value 3 into 10 raised to 3 is to be taken with a pinch of salt in the sense it is not asserted that the resistance value is exactly 3 into 10 raised to 3 ohms even if the bands on it may indicate that it is say 3 into 10 raised 3 or rather 1.3 into 10 raised to 3 ohm, no matter how good a quality a resistance one may or a resistor one may buy and this is associated with what is called the precision. This value 1.3 into 10 raised to 3 by the very nature of it cannot be 100 percent precise in a course on electrical measurements of course you would have looked at methods of measurement of parameters like resistance, inductance, capacitance and also a physical quantities like voltage current frequency of a alternating supply and so on a magnetic flux and what not?

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So suppose we go to a laboratory and make actual measurement, so set it up. So that some current flows in it measure the voltage across it measure the current in it will we find that the voltage number in volts will be exactly equal to 1.3 into 10 raised to 3 multiplied by the current number no and in fact for more than one reason because just as there is this problem of what is exactly the resistance of the resistor is it that specified value there is a question of precision. In fact when you buy a resistor you should ask about the degree of precision or you should buy it knowing what will be the precision is the value going to be one point three into ten raised to 3 within plus minus 10 percent or 5 percent or one percent because this is all that can be guaranteed of a readymade resistor.

Of course if you wanted to make a resistance of resistance 1.3 into 10 raised 3 ohms and want it to be more precise of it you could chose wire and wind it yourself chose an appropriate length of the wire and so on and so forth and therefore you may have more precision than is commercially available in principle you can make a resistor of resistance value which is within plus minus .1 percent of what you think it could be. But commercial resistances or resistors do not come with that kind of precision obviously it is going to cost you more if you want a more precise resistor.

So that is only one part of the difficultly the other part of the difficulty is that in order to find out whether V is equal to 1.3 into 10 raised to 3 into I, I need to measure both current and voltage also and there is precision associated with those measurements perhaps you can measure the current within a milliampere the current may be one ampere but within one milli ampere this is because of the nature of the instrument the dial reading on the dial of if it is a digital display, the number digits that you have in the display. If you set it up in the ampere mode and it can go up three places of decimals then the milliampere is the lower limit, you cannot go lower than that.

Of course you could use a more precise instrument or you, you could use a setup where as there by you could measure the current within one micro ampere and again one can if one wants to go further, go further even and talk about a nano amperes similarly, with voltage, volts or millivolts or microvolts but ultimately there will be a practical limit where you will stop. So you will measure voltage say within one millivolt a voltage will be of the order of a few volts but within a few millivolts similarly current may be of the order of one ampere but within plus minus 1 milli ampere.

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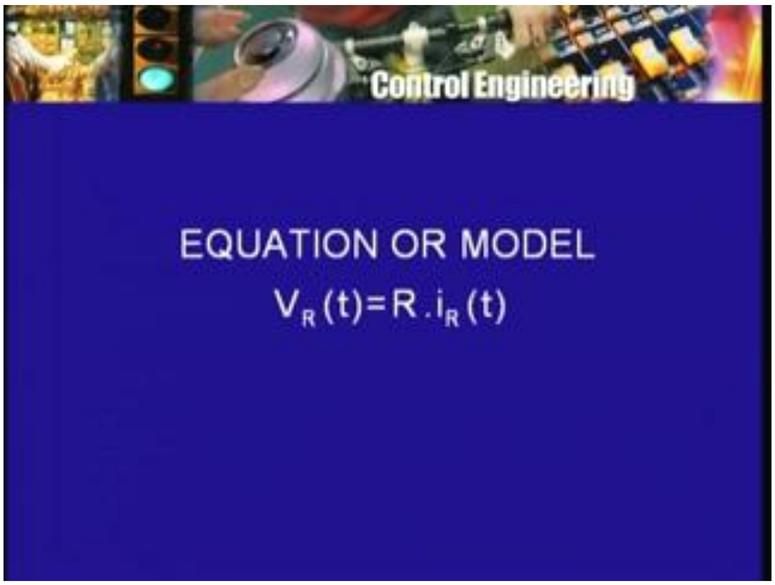

As a result the resistance value you wanted to measure it this way by finding out the voltage and the current and then dividing the voltage by the current will have its own precision. Of course the voltage current method is not the only method of measuring the value of a resistance in fact it is not necessarily the best method although it may be a simple method in principle there are in an instrumentation course or measuring instruments course you would have studied some other methods or some methods of measuring resistance other than simply measuring voltage and current. But to come back to our point then an equation like V R of t equal to R into I R of t with a particular value of R, R equal to 1.3 into 10 raised to 3 assuming V is in volts and I is in amps.

An equation that we write for this physical device here this particular resistor is only an approximation or an idealization and that that is about all that we can say, we do not know of any better relation but at the same time we will not insist that this hold exactly to 10 places of decimals or even 3 places of decimal. One can sometimes use the word and this is the little better word than modeling and that is the word representation. So instead of saying that this resistance is modeled by V equal to R i or V R of t equal to R I R of t.

So this equation is said to be a model of the resistor we could say that this equation represents the behavior of the resistor with the understanding that it is only under certain conditions and it is only within certain limits that the equation represents the behavior of the actual resistor. In fact right now, I use the word model in a third sense earlier we talked about a model of an aircraft a scaled down model which is physically similar works on similar or the same principle but has much smaller size weight etcetera, it is one kind of model. The other is what we just started with that is the rlc circuit and the mass spring dashpot force or displacement source system, one of them can be said to be a model of the other and in place of the rlc of course we can have an operational amplifier or analog computer system setup which is the model of the mechanical system.

The third use of the word model is when we say that this equation is a model of that physical thing the resistor. Sometimes one uses the word mathematical model implying for example that in this model or in this representation what I have written V equal to R i V and I are numbers, R is a number there is a product operation involved etcetera not much of mathematics here in V equal to R i it is really simple arithmetic a multiplication and that is about all. We must distinguish models in this case equations from the actual thing same thing holds for the model aircraft. The model aircraft is only a model of the actual aircraft if you want fly from here to Delhi, you cannot use the model aircraft for that purpose.

Similarly, if you want a spring to do something you cannot setup a rlc, emf circuit and hope that it will do the work of a spring the model is different from the system, the model is model of the system, it represents the system or the device. The model is mathematical in place of any equation I may draw graph. So V equal to R, I could be shown by means of a graph of voltage versus current in fact for some devices we may not be able to write down any equation so nicely, it is easier and in fact that is what is you start with by giving graph, a graph of the behavior of V and I such graphs or curves are known as device characteristics and probably you know we already come across them in connection with diodes, transistors and the light.

The simplest and experimentally, the simplest to obtain representation of a diode will be by its VI characteristic, a curve of V and I for different values of V and I. The equation for it it may not be very easy to write in fact there is no nice single formula for the whole curve as you know of course one can with some theory or from a purely practical point of view find out an equation that describes some part of this characteristic and even use it. So in that sense the VI characteristic is also a model but it is still more a mathematical or an arithmetical model in the sense it involves physical variables like voltage and current and gives you the pairs of values of voltage and current that you may find associated with a diode or with a resistor.

So when we use the word mathematical modelling then we can include such graphical methods also in fact they are useful as we will see in control work very often one can get some idea of what is going to happen by drawing graphs or pictures rough sketches even.
So you you do not have to really write down only equations and then find that you cannot do much with them because you do not know what how to solve it.

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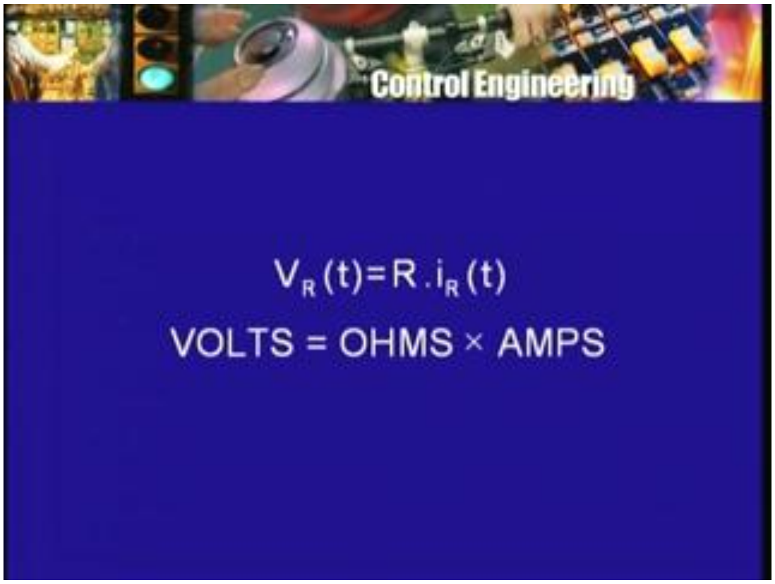

So here is one device resistor for which there is this model V equal to R i or to be very precise resistor voltage V Rt equal to R into resistor current I Rt, what about the second another component in the circuit it is the inductor. Now the inductor has a parameter associated with it namely the inductance the standard symbol for that is of course L just as the resistance is the parameter associated with the physical device resistor, inductance is the parameter associated with the physical device inductor and once again like the resistor and inductor or a coil can come in different sizes and shapes and material and so on.

Here are a few examples of inductors of various shapes and of course various values of inductance one can have an inductance with a small inductance value like few milli Henries or one can have a much bigger inductor or coil with an inductance of the value of few Henries. Just as you can have resistors with the few ohms resistance or resistors with mega ohms of resistance. Now just as in the case of the resistor and here it is even a much stronger approximation or one should say really a weaker approximation much cruder approximation namely the relationship between the voltage across the inductor and current in the inductor is given differently V L and I will write here.

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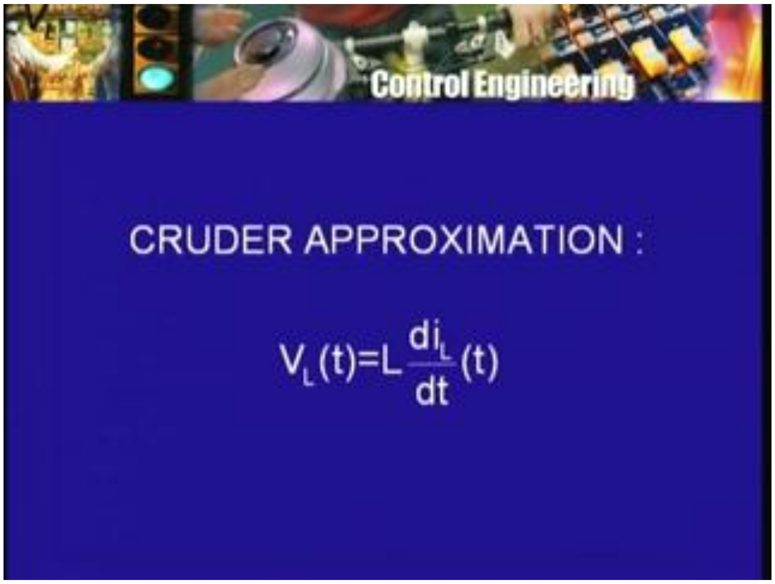

Now V L of t once again V voltage L because L is the standard symbol for the parameter inductance V L of t equal to L, the inductance value multiplied by now this time it is not the inductor current as you know but the rate of change of the inductor current D I L dt or to emphasize that the rate of change also may vary with time d I L dt at time t and later on we will use a somewhat different symbol which is not used in many books but you may find it in a few mathematics books.

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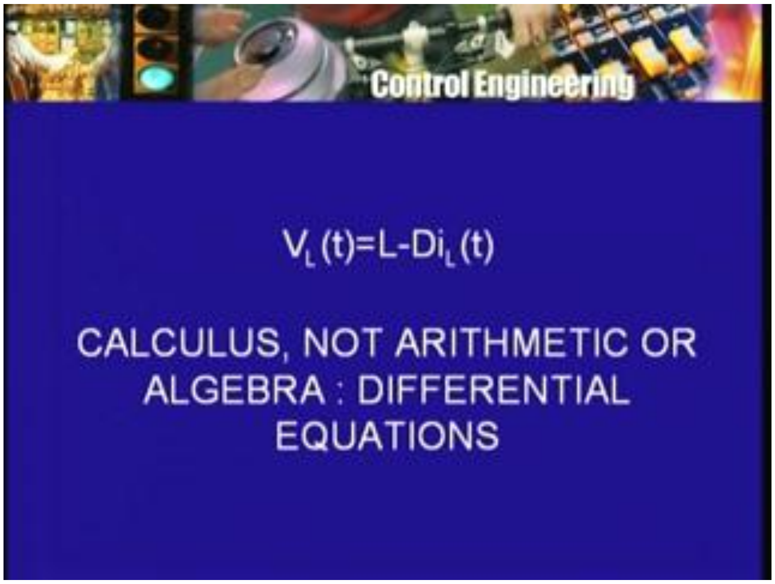

We will write it also as L into D, D standing for the derivative operation derivative of the current, so D I L, I L is the current in the inductor as a function of time D I L is the derivative the rate of change of it as a function of time and its value at some particular moment of time t. So the representation of the model for the inductor will be D VL of t equal to L into D I L of t, D I L at t. The voltage of the inductor is L times the rate of change of the inductor current and that same moment of time.

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Now this is not arithmetic anymore because it is not just multiplication but there is the idea of derivative or rate of change and so to some exchange this means that we have gone beyond arithmetic and algebra we have gone into calculus and in fact it is this kind of a thing that gives rise to a differential equation finally when we begin to model a system. It is because there are devices like inductor which can be represented approximately somewhat crudely but still it may be adequate from many purposes by means of a relations like this which involves derivative in electrical engineering of course one talks about this and the other relationship for the resistor as a VI relationship or VI characteristic.

So the VI relationship for an inductor is given by V equal to L Di dt, if I is in amperes, if the time is measured in seconds. So the rate of change of current is measured as amperes per second if the inductance L is in Henries then the product L Di dt will give you the voltage in volts. Now what about the third component, the capacitor like the resistor and the inductor, capacitors also come in various types sizes, shapes and capacitance value, here are the few of them, what is the model or a model a useful model for a capacitor. Well it is given as iC of t current I in the capacitor whose subscript C at time t equals C the capacitance multiplying the rate of change of the voltage associated with the capacitor or d VC dt at t or using the D operator notation it will be C times D V C (t)

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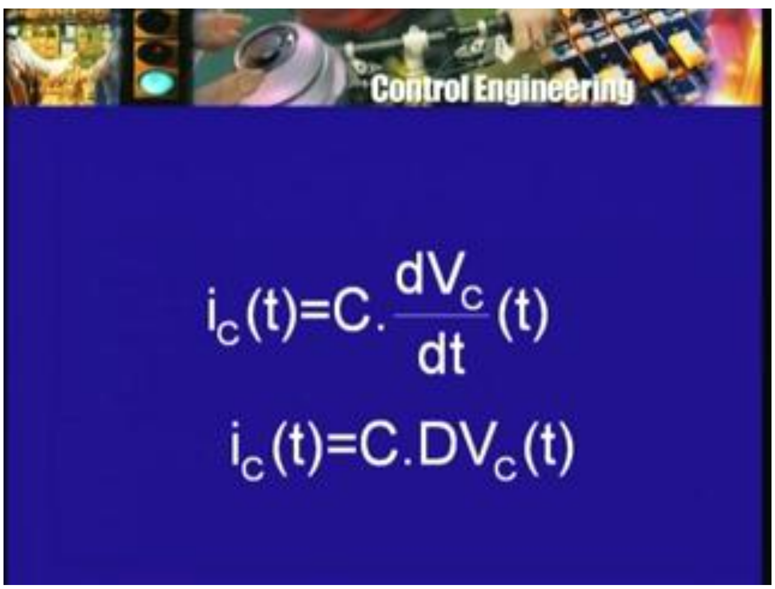

Here again, we go beyond arithmetic and algebra, we go into derivative or calculus the current in a capacitor or associated with the capacitor depends on the rate of change of voltage associated with the capacitor and is obtained from the rate of change of voltage if the voltage is in volts, rate of change is measured the time in seconds and capacitance in farads then the current will be obtained in amperes. So as you know the values of capacitance or capacitors that one is likely to meet in the laboratory is not of the order of farads but very likely of the order of microfarads may be 100 microfarads or 1000 microfarads or even much smaller than that a Pico farad. But the Faraday the farad named after Faraday is the unit and therefore volts per second into farads only gives you current in amps but if you took volts per seconds multiplied it by microfarads, you will get the current in micro amps.

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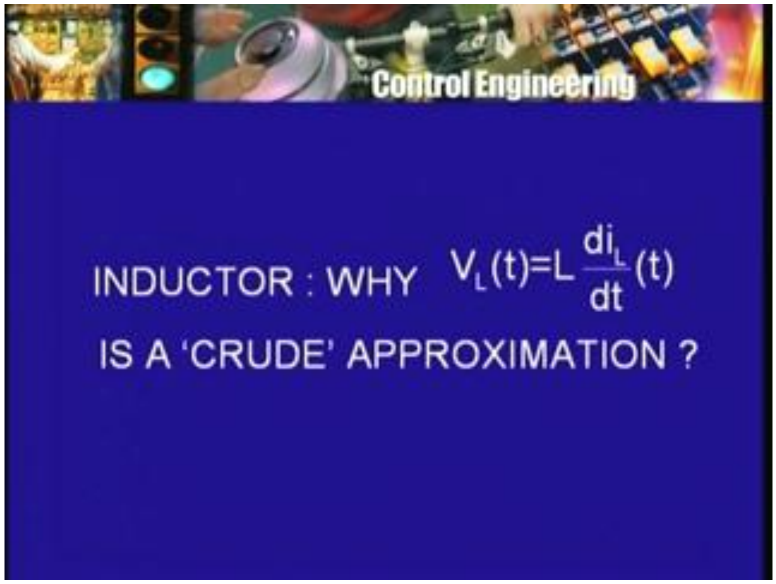

Now I said that the approximation for an inductor or the model for an inductor VL equal to L di, L dt is a crude approximation or I use the use the word strong and as I said that may be one should say weak because in many cases it is not really a very good approximation, why, why because the inductor or it is many inductors involve a long length of wire it is called a coil simply because there is a wire which is coiled or wound around what is called a core?

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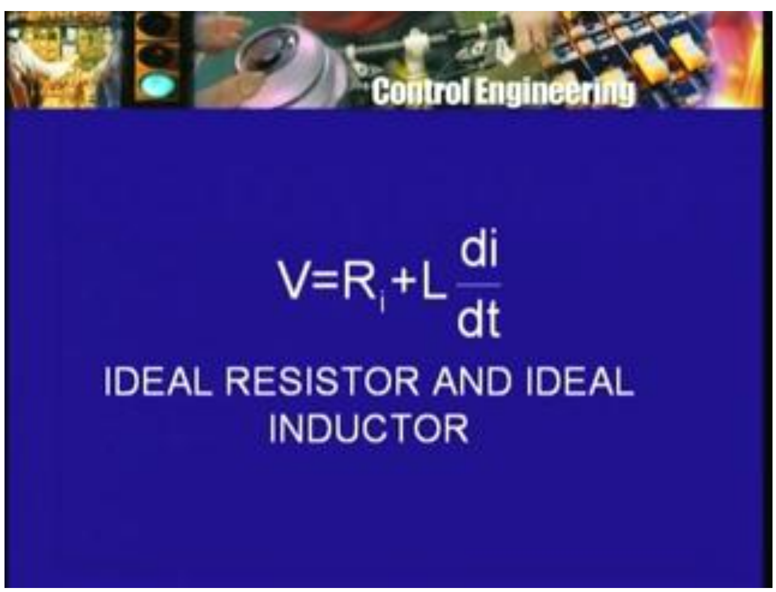

Some magnetic material and so there is a good length of wire there now this wire is a piece of resistor also it has resistance even if it is good conducting material like copper still it is not a very thick wire and there is long length of it. So it has a good bit of resistance associated with it by good bit of course I mean not meg ohms but it could be a few ohms. So this inductor is not just L that is it is not that current in it produces a magnetic flux in the core and therefore the rate of the change in the flux will produce a counter emf or an induced emf in the coil which is what this V is about but it has also a resistive part of it as it were and if so, if one wanted a little better a less crude approximation or model for it, one possibility is to think of it as series combination of a ideal resistor with an ideal inductor and so write down an equation like V for the voltage as not L di dt only but as consisting of 2 terms R i plus L di dt where the term R i is coming from or taking around account of the fact that there is a lot of length of wire there and the L di dt parts comes by taking account of the fact that the wire is wound round and magnetic core current flow gives rise magnetic flux and change in magnetic flux leads to the phenomenon of a counter emf or a back emf in the wire and so on.

But once again we have to remember that this is the model, nobody has connected actually a resistor so called ideal resistor and an ideal inductor in series to make a practical inductor the practical inductor is wire wound around a coil and how good this approximation is V equal the R i plus L di dt will depend on how you are using this inductor. For example, if the current and the voltage are not changing very rapidly and by very rapidly again one has to mean something very specific. Let us say the voltage and varies sinusoidally may be at few hertz or a few kilohertz perhaps then the representation like V equal to R i plus L di dt may be okay. But I want to operate it at frequencies of the order of megahertz and if I want to be very ambitious gigahertz then this model will be useless; it will just not be useful.

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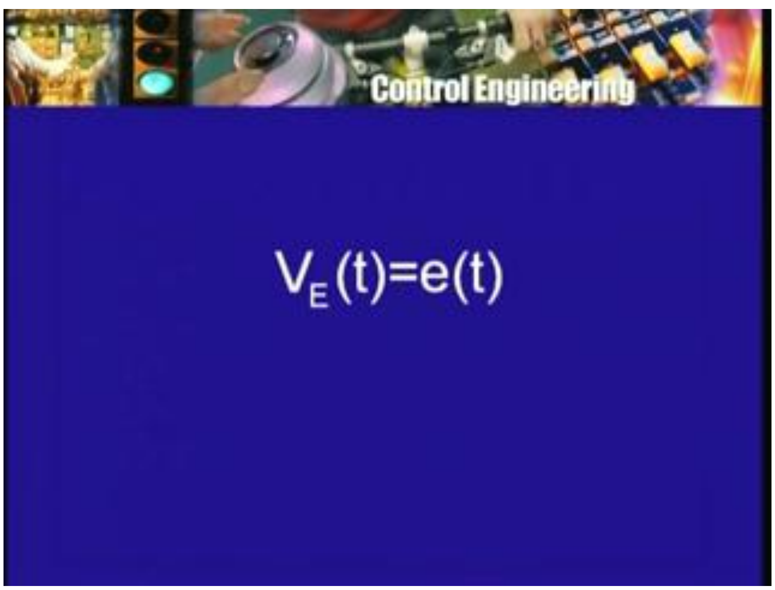

So this emphasizes the fact that the model is not this device of the system the model is only an approximation it is crude unfortunately, we do not know exactly what the true mode is. So in that sense it is an approximation to something that we do not know and we have to make use of it and see whether it gives you good results or not if it gives good results, it is okay, if it is does not then our modeling was not adequate. We will have to look for a better model that is the model which will give you results which fit what one actually finds by experimentation the last of the devices is the voltage source or source of emf or electromotive force for that the equation will be V again voltage E a symbol for the electromotive force E like RL and C small e or capital E. It is value at any moment of time t is equal to e of t and what do we mean by e of the t on the right hand side what is meant is that e of t is going to be some known function of time. For example if I have this cell freshly bought, what freshly prepared not very old then I can expect that this e of t will be nearly 1.5 volts all the time.

So this e of t will be really remain constant, will remain constant as I operate the cell for a fairly long amount of time. So if I can say confidently about this piece this cell that look the voltage across it is going to be 1.5, no matter of how long you go, no matter what the t is then V E of t equal to e t and in this case e t equal to 1.5 volts is a good model for the cell and once again we know that this model can cease to be good. If you use a cell for the long time then it does not behave like this anymore if you actually measure the open circuit voltage of the cell we will find that its not 1.5 but it has come down to 1.4 volts or even less and not only that when you actually use it in the circuit, when you have current in it so voltage may be even much less than 1.5 volts.

So that is something which is not written here when I write V E of t equal to e of t equal to 1.5 the claim is that this cell or battery will be described by this no matter, what current flows in it what value of current and whether any current flows in it or not. Now that is certainly not true of a practical cell from this small cell you should not draw perhaps a very large amount of current its voltage will no longer be then 1.5 volts not only that the cell can get even ruined. So may be if you draw a current or of the order of a few milli amperes or 10’s of milli amperes as when you put it in a transistorized radio receiver not one cell but may be 2 or 4 of them, the current that the cell supplies could be of the order of few 10’s of milli amperes under that condition if the cell is fresh it is not been used for too long a time the cell will be described by an equation like this. This is of course the simplest in the sense there is no arithmetic here the voltage is just some number in this case 1.5.

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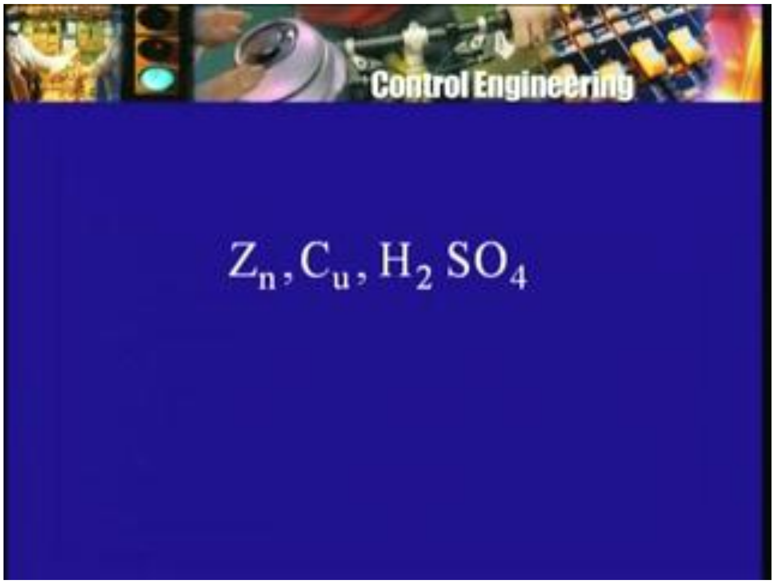

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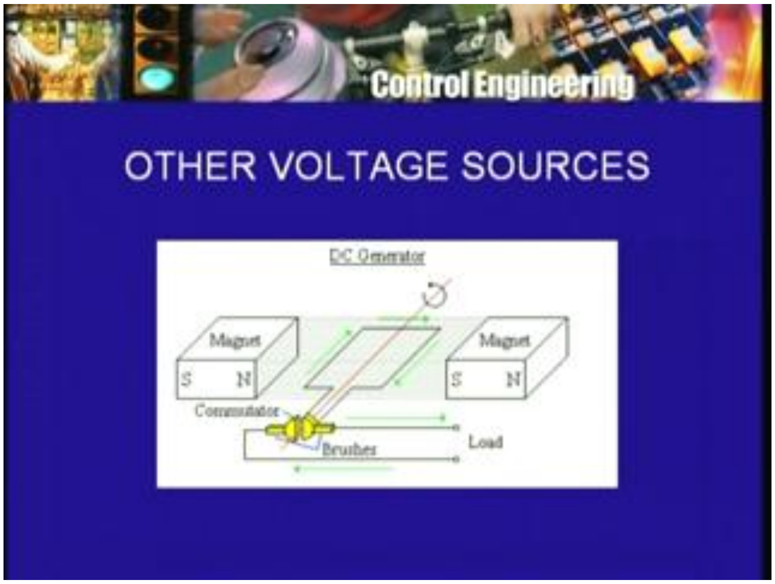

Now like the rlc voltage source says also come in various forms a cell of this kind or dry cell is one in your physics course earlier we have heard about the wet cell, the cell that goes back to volta in way or the cell with zinc and copper dipped in hydro sulphuric acid or hydrochloric acid and various other kinds of cells that you have studied earlier but it could look quite different a dc generator is also an example of this kind of a thing. Since we are talking about an electric dc motor which requires power supply for the field winding as well as the for the armature winding therefore a dc power supply.

Now this dc power supply itself can be modeled under certain conditions by simple equation like V of t, V of the source at time t is equal to not may be 1.5 volts dc but 230 volts dc. Now like the rlc this kind of an equation is referred to as an ideal characteristic of course it is ideal in a difference sense also that is if you could have something like this then perhaps some things other thing would be very simple. So in that sense this is something which is also desirable and it is true that you when you talk about the power supply, the power supply should be such that no matter what current you draw from it the voltage should be guaranteed to remain at some definite value in the case of the ac power supply let say 230 volts it should remain in a RMS value, the wave form should remains sinusoidal and the frequency should remain at 50 hertz or 50 cycles per second.

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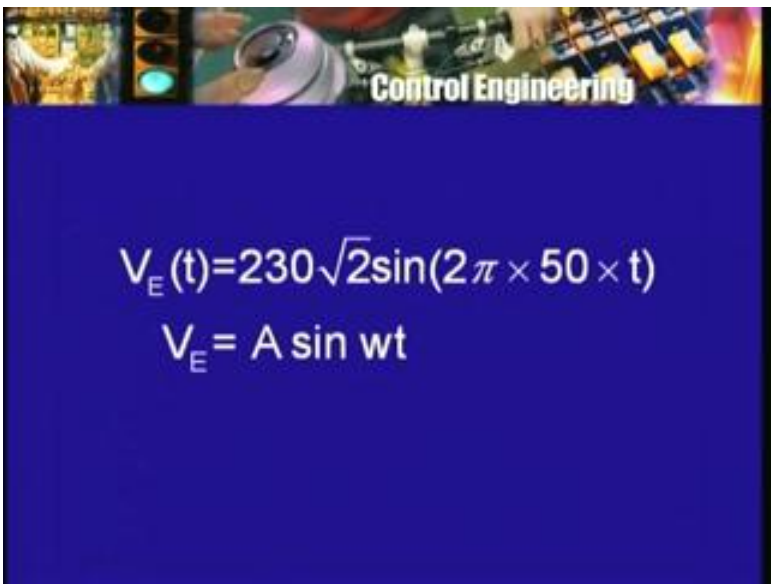

So in this sense when talks about an ideal voltage source but here the word ideal also means that the voltage of the source does not depend on the current. The source that we have looked at is a cell in which the voltage is constant as a function of time that is it does not change with time but an ideal ac voltage source, we will have a voltage which is given by a different equation or relationship namely V E of t equal to may be 230 root 2 into sin 2 phi into 50 into t, where as you know the root 2 come because 230 is the RMS value and 2 phi into 50 comes because the frequency is 50 hertz.

So in order for it to look like A sin omega t, where A is the amplitude and omega could be called the angular frequency of the sinusoidal wave form, A is RMS value into root 2 and omega is 2 phi into the frequency as cycles per second or frequency in hertz. Now of course on the other hand I can look at the mechanical system look at the various devices write down the mathematical models for each one of them and of course I expect that all of you will be able to do it because the 3 particular devices that are involved here are all familiar to us that piece of matter or a block which already one has called mass implying that in its description a number will play a role that number will be denoted by the symbol m for mass such that a particular equation or a relationship hold between two variables associated with the mass and what is this equation just as V E equal to R I, you know some sense goes back to Ohm although he did not really write equations like this.

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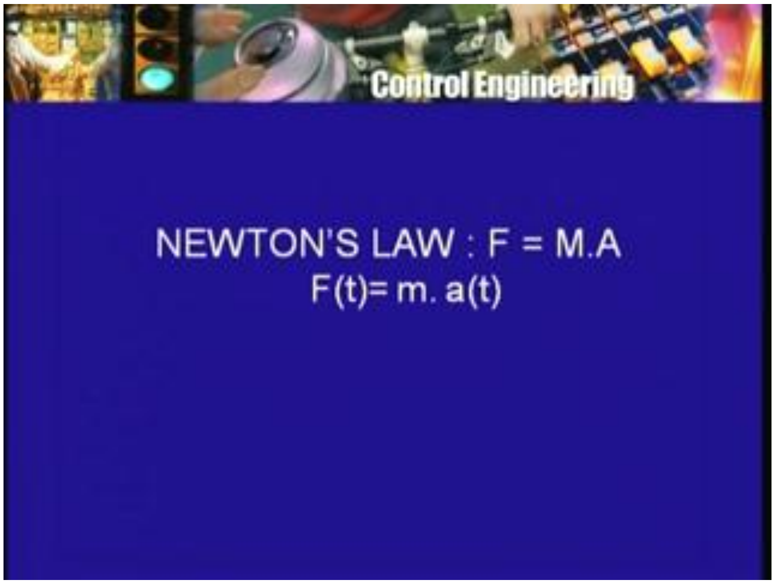

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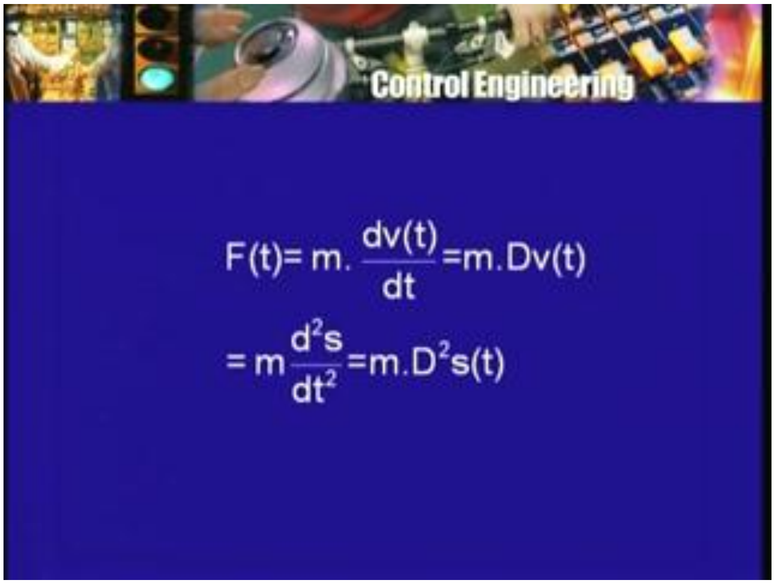

Similarly, the equation that one writes today for a mass goes back to Newton and as the familiar equation F equal to M into A, but since the acceleration may vary with time the force may also vary with time one will have to write F of t, the force associated with the mass or acting on the mass equal to the mass of the body multiplied by the acceleration of the body at that moment of the time t and because acceleration itself is a rate of change of velocity and velocity is a rate of change of displacement or position therefore ultimately we may write this as M dv dt at t and therefore with the D notation m into Dv at time t and because the velocity itself is a rate of change of displacement or position, we could write this as m d square s dt square at t or we could write this as m into d square s of t, where d square indicates that the differentiation operation is carried out twice successively.

First you differentiate s a displacement function or the position function, we get a new function that is the velocity function then you differentiate that to get the acceleration as a function of time that you multiply by the mass number m that will give you the force acting on the mass of the body at any moment of time t. So here is a model a mathematical model of the mass unlike the resistor perhaps and even much more so unlike the inductor we believe that this is a much better model, this is not as crude as the other models are. We believe that everybody has a mass and there is this kind of a equation associated with that body but then as you know perhaps if you have studied or heard about the theory of relativity and so on that one has to be careful when using this equation.

In other words there are situations when this equation no longer may continue to hold because I am not talking about the situation where the mass of a body so to speak changes as time proceeds because one could actually argue that is not the same body, that is the body itself is changing as time goes on and so no wonder its mass changes because it is not the same body, what could be a good example of what do you think of as a body whose mass is changing with time. In fact any device which consumes or uses fuel is a device of this sort. As you ride your motor bike or your car fuel is getting consumed. So, if you think only car as single body and if you include the weight of the fuel.

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As a part of that body then the mass of that whole body is changing as you drive the car or as you drive the scooter the mass of the scooter as a whole is changing. Now whether we want to say that the mass of the scooter is changing or we want to say the scooter itself is changing, although some parts of it remains the same and strictly speaking we are we are one would be right in saying that the scooter itself is changing because after all if one looks at the fuel tank one can see that fuel level is going down, if you look at the fuel gauge, the fuel gauge pointer is going down this is a home example but a much more important example is provided by a rocket because there the amount of fuel that the rocket requires to get itself up to a particular height is a can be a substantial part of the total weight of the rocket that is rocket with fuel and therefore as the rocket ascends there is less and less fuel therefore one has to think twice before applying the equation F equal to m a, how to change this equation that is a different story. But here is a situation when you are considering the ascent of a rocket and you want to be realistic about it we have to take account of the fact that the rocket is no longer a body with a constant mass and this is not because of any relativistic effect that the body is moving at a high velocity therefore its mass is not the same as rest mass and all that stuff it is simply that fuel is getting consume and fuel is a substantial part of the total mass of the whole package.

Now if we look at the fuel tank separately and the rocket separately as 2 bodies may be connected to either mechanically because after all the fuel tank is housed in the rocket. Then the rest of the rocket the mass will remain unchanged but the fuel tank mass of the fuel and the tank mass that will go on changing that will be a different way of looking at it but the equation F equal to ma will no longer be applicable. So even a time order equation like a F equal to ma there can be some reservations about it. The question of precision off course arises here you can measure mass within a gram the mass may be of few kilograms within a gram or within a milligram or a microgram but somewhere you stop.

We cannot be sure about the last milligram or the last microgram and the two quantities on the other on the two sides force and acceleration both of them also cannot be measured with any arbitrary amount of precision. Acceleration through displacement let us say displacement in meters could be measured to within one centimeter millimeter or even micron or a micrometer and therefore rate of change of position or rate of change of velocity acceleration will be measured within a certain accuracy. On the other hand the force a Newton will also have precision associated with it.

I should not have used the word accuracy one should use the word precision in this case. Accuracy arises when the instrument that you use is purely designed or it has some faults in it or is not very carefully designed or it could be replaced by another instrument which gives you a result which is closer to what you think is the correct one where as precision is something quite different, precision is almost inherent in all measurement simply because you know we know that earlier we could not measure things to within centimeter or within a millimeter because if one uses the scale then you can barely see a millimeter on a scale but then some person by name Vernier came with an idea with an idea that resulted in a vernier scale and using vernier scale now you can go down to 10th of a millimeter something which you cannot see directly but indirectly you say that your measurement length measurement is now precise within one 10th of a millimeter and if I combine it with a microscope and so on I may be able to increase the precision even more which makes us think that we have not reached the absolute end of it. We have increased the precision of our measurement presently this is where we stand but we will not claim that this is the length is exactly equal to this up to 10 or 20 places of decimal.

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So F equal to ma therefore is an approximation or a representation of the body which is good for some purposes and it may not be good for some other purposes but for our mechanical system let us say F equal to ma is a acceptable description or representation or model, what about the other two components the spring and the dashpot, well here is a spring and here is a variety of spring of various sizes of various natural or free lengths various material and so on, what about their model it is interesting that like the resistors which differ from each other in appearance the inductors the capacitors the bodies or masses springs also will look so different from one another.

All of them can be described for many purposes by an single common equation and what is that that is related to a law, which is named after Hooke just as your Newton's law which is embodied in F equal to ma, you have Hooke's law which will relate force to extension that is there is the idea of what is called the natural length or the free length of the spring and actual length at any moment of time because the spring, the spring is being stretched or is being compressed and therefore with some appropriate sign system F will given as k, small k into the extension.

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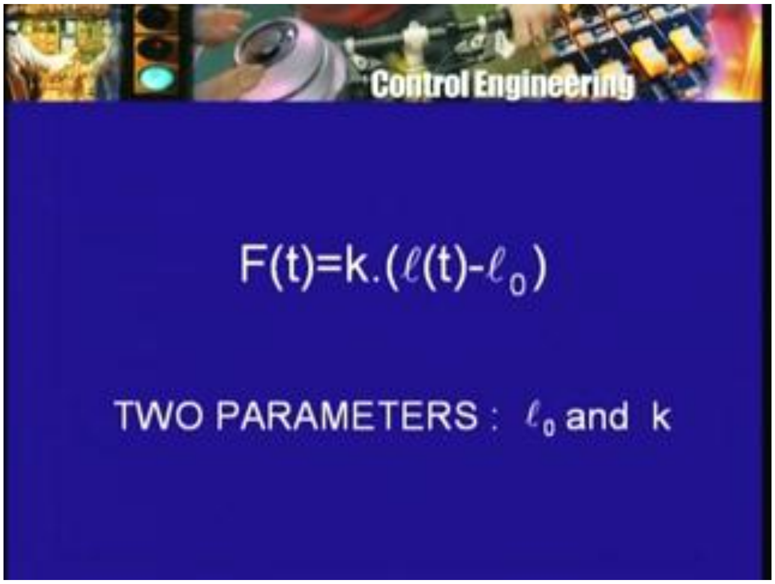

The extension could be written a length of the spring at any moment of time say l of t minus l, 0, 0 is to remind you that this is the length of the spring when there is no force acting on it it is free or the natural length of the spring. So if l (t) is a length at any moment of time l 0 is the free length then this difference is positive if the length of the spring is longer is greater that its natural length which means the spring is being stretched and therefore this extension which is positive multiplied by a constant which is the elastance of the spring it is connected with a property of elasticity of the spring.

This product will give you the force in this case the force of tension somebody is pulling at the two ends or at one end of the spring the force with which the spring is being is pulled is given by an equation like F equal to k into l (t) minus l 0. If l (t) the length is less than l, 0 which means this spring length is actually less than its free length therefore the spring is being compressed. So this difference has a negative sign and as a result the force also has a opposite sign and therefore it in the first case it was tension in this case it is the force of compression somebody is pushing, squeezing the spring with this force and of course I should once again put F of t is the force acting on the spring, the force of tension or the force of compression is given by k, the coefficient k into l (t) minus l 0 and the spring has 2 parameters associated with it one is the free length of the spring and the other is the coefficient k which is also sometimes called the spring constant because it is associated with the spring, the spring constant or the coefficient of elastance or elasticity of the spring or elastance of the spring.

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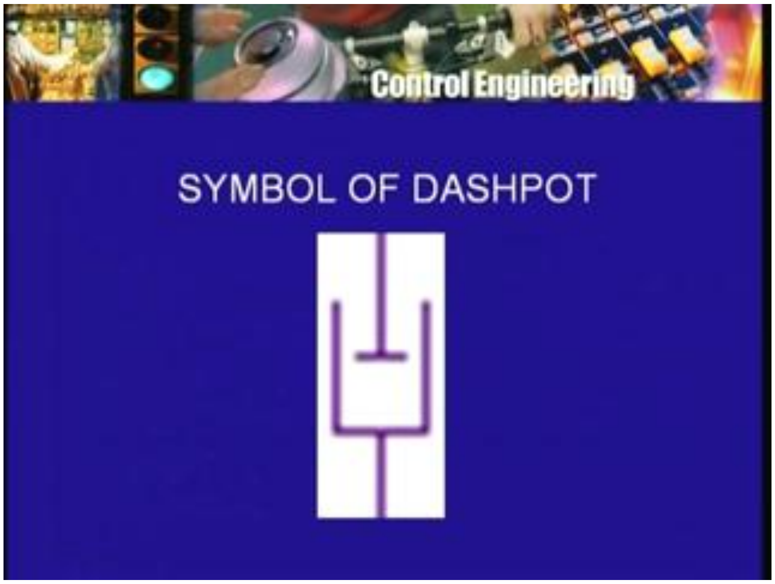

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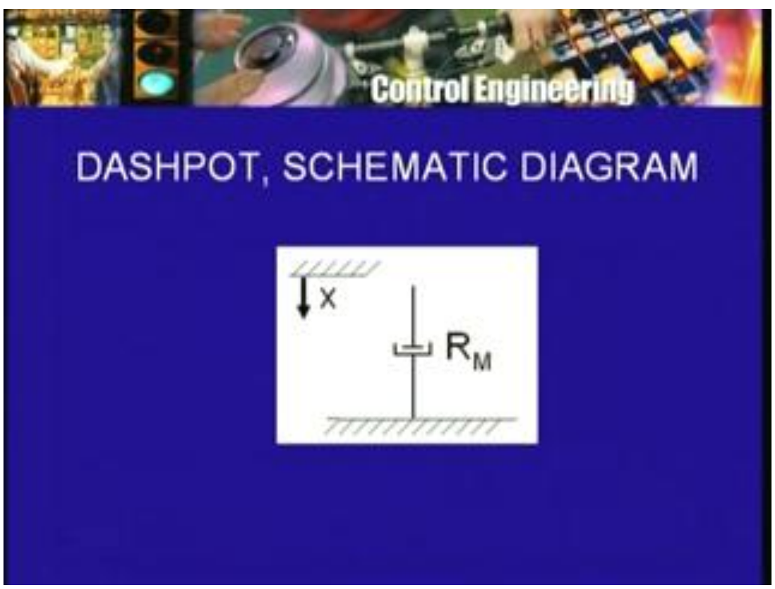

So that is the spring of the mechanical system the third thing is the dashpot with the symbol that we are seen already. Now one may not come across a dashpot very commonly as an explicit component simply because what the dashpot does is to be found without really asking for it the dashpot is based on the idea of resistance to motion but of a different kind from F equal to ma, the force of inertia so to speak. F equal to ma says that in order to accelerate a body you need force, you do not need force to keep it in constant velocity at a constant velocity, you need it if you want to change the velocity either in magnitude or directional or both.

In the case of a dashpot, dashpot will oppose any motion but here is a practical dashpot which we can think of really as a cylinder not this one necessarily but think of cylinder with a piston at the one end of it and with air under the piston and assumed that the piston moves in an air tight manner then obviously as the piston moves in the air is getting compressed. So it offers some resistance to the movement of the piston in place of, of course the air one have oil and one has to make some proper arrangement and so on. For our purposes of modeling in this particular example the equation relating two variables associated with the dashpot will be the force or what may called the plunger of the dashpot the other part of it is fixed.

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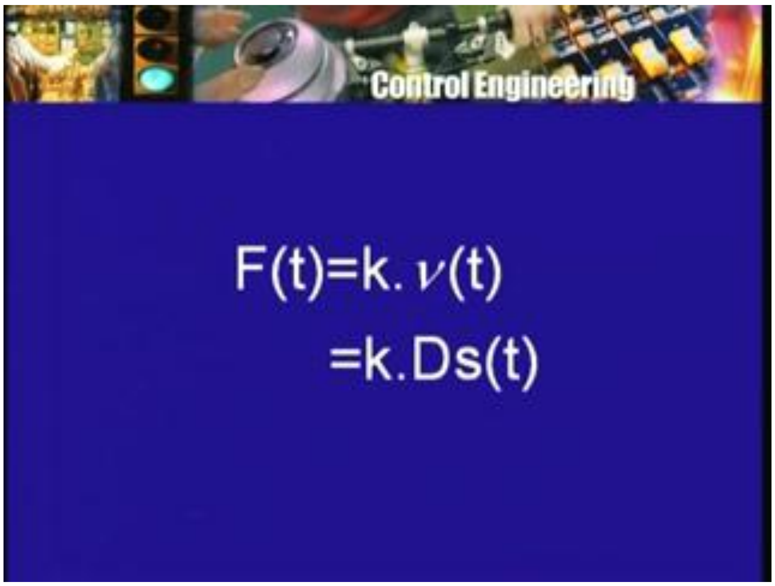

So if the force acting on the plunger this is given by a coefficient multiplying the velocity of motion of the plunger. So v of t into a symbol that will denote this resistance to motion of the dashpot I will insert the statement giving the actual name of this particular constant is I do not remember this standard name for it. So this is something which we will add in the video, so this is the equation for the dashpot F at any moment of time t is equal to this coefficient into vt and as before in place vt we may write D the derivative of the differentiation operation acting on the displacement function Ds of t.

So these are the 3 components of the mechanical system the mass the spring and the dashpot with their models which are all the equations it may be good for some purposes may not be some good so good and there is one more namely a force, source or a displacement source. Some external force may be acting on the mass of the body or one end of the of the mass itself of the body may be moved physically by some other device and undergoes some displacement.

So like the ideal voltage source we may be told that somehow the position of the mass say, x of t varies in this sinusoidal way with time that is the mass is made to move somehow in this fashion move back and forth along the line or alternately on the mass may be exerted force somehow may be by something attached to it or some remote force of attraction may be exerted on that mass whatever way that is the force source that is there is an external force acting on the mass which is guaranteed to behave in such and such manner may be a constant force or again an oscillatory force may be sinusoidal or whatever.

So that will be the fourth component of the mass spring dashpot system and in fact one can have a special situation and the corresponding situation in the rlc circuit also namely there is no force source or displacement source except that at what one may call the beginning that is when you start observing things or start looking at the system typically we call our t equal to 0 initial time start your watch or clock and starts looking what is going on and this initial movement of time let say the spring has been compressed or somebody is holding the spring with his hands compressing it and then at t equal to zero as you see the watch strike 0 he lets the spring go removes the force which is holding the spring in a compressed state then as we can expect the spring will vibrate it was compressed.

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So it will expand as its property of elasticity but unfortunately it will over do it or it may overdo it. So it gets pressed on its own but that gives rise to a force of restoring force of tension so it becomes compressed again and so it may go on vibrating like this for quite sometime therefore after sometime the comes to stand still with its natural length. So in this case one would have put the external force, source, force or displace the force source equal to 0.

Similarly, in the rlc circuit one may have the circuit initially and their will be one more element may namely a switch and I keep telling my students that this is one very important part of electrical system the switch, without the switch life would be difficult if you could not switch off the fan we did did not want it or you could not switch it on when you wanted it life would be difficult, the switch is very important device.

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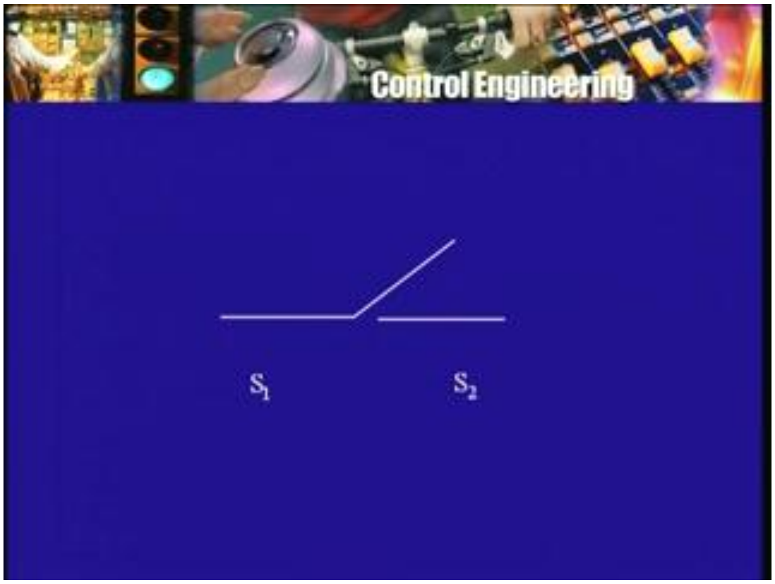

So in the rlc circuit suppose I had a switch initially and the switch was closed and as I see my clock strike 0 or here its strike 0, 12 o'clock or whatever t equal to 0, I call it, I open the switch or rather things will not happen if I do it or something will happen but I would like to think about what will happen but instead of this kind of a thing we have this alternate arrangement where now what will happen is there will actually 2 switches there will be one switch and there will be another switch and this one switch S 1 will be opened at t equal to 0 and almost simultaneously with it this other switch S 2 will be closed, S 1 will be open so that the battery or the cell is removed and S 2 is closed so that the circuit remains close, rlc circuit remains closed.

So that a current can flow in the circuit if you do this you do find that after you have opened S 1 and closed S 2 the activity does not stop immediately and if one connects an ammeter in the circuit to see whether there is a any activity going on or if one connects this to an oscilloscope one will find that indeed activity goes on in the circuit even after the source has been removed, current continues to flow for quite some time and under certain conditions the current may be in fact oscillatory that is it will go through positive and negative values and keep on oscillating for a sometime before the current becomes almost 0. So the force source, emf source could be set to 0 but the current the circuit will have some initial current, the capacitor will have some initial voltage and so on.

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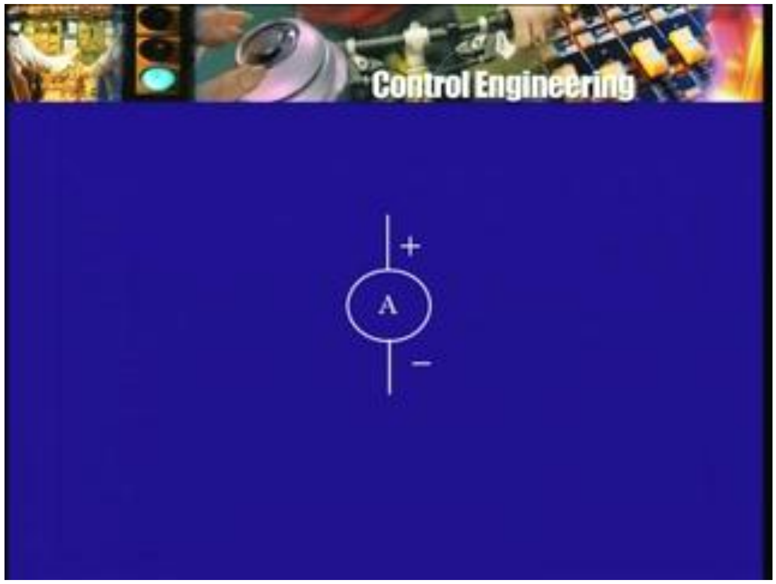

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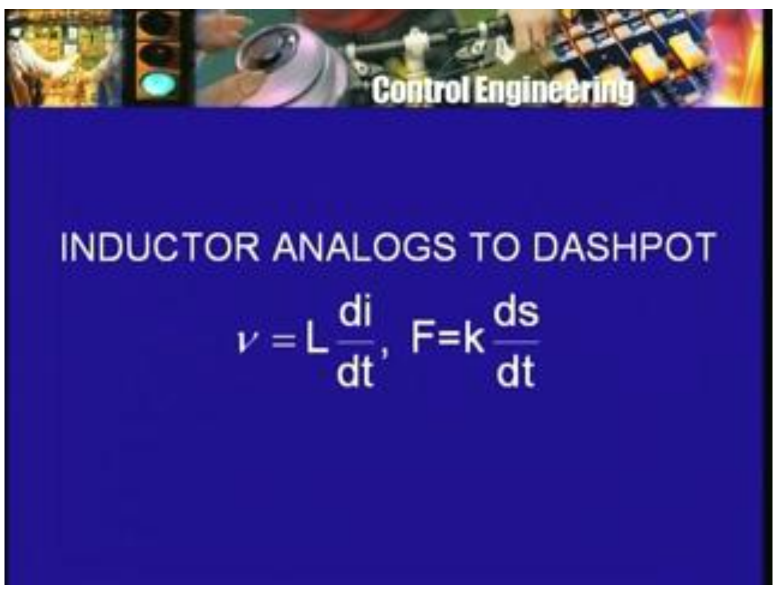

Whenever it as if that time t equal to 0 prior to that you had an excitation or a force or something acting on it after t equal to 0, you have removed it but this earlier activity of the force put the system in some initial condition or initial state where by the system can cool on for some time, before it comes to rest once again before it becomes dead whether it is the electrical circuit there I closed and open switches in a particular way but the system the circuit was earlier connected and carrying current or in the mechanical case I hold the spring in this fashion with my hands and then I let it go at some moment of time and watch it oscillate before finally it becomes still once again. So a system may have a force source or an emf source or a displacement source or the system may not have and still it may have a some activity associated with it.

Now going back to the analogy then I have setup or I have written down we have looked at the equations for the electrical components, the equations for the mechanical components. Do they look similar? May be yes, may be no for example the inductor equation v equal to L di dt looks very much like the dashpot equation perhaps because F is equal to the coefficient into derivative of displacement.

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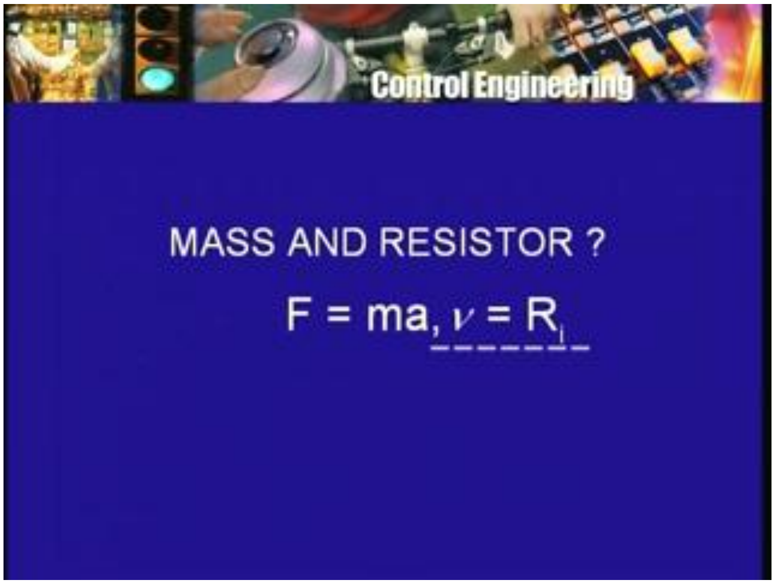

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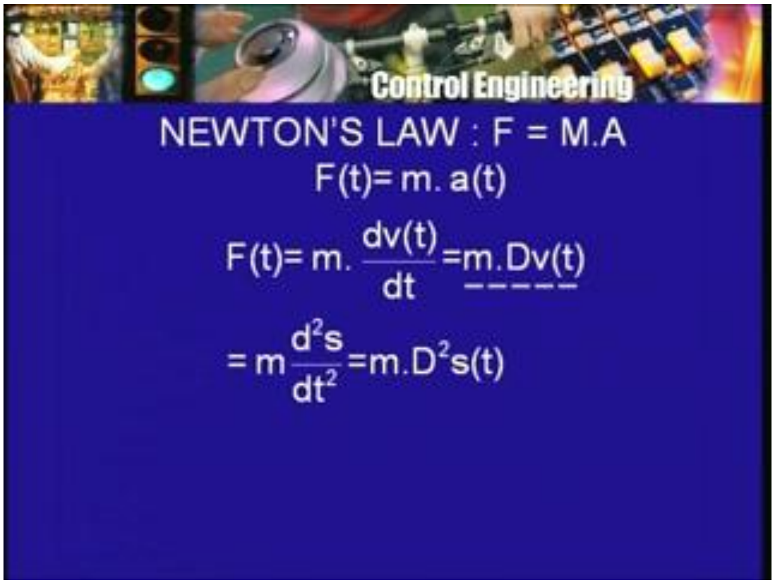

So if I take current to be analogous to displacement then the voltage across the inductor is analogous to the force on the dashpot, this is one analogy but may be one can look at it differently for example if I look at the mass equation then off course in F equal to ma there is no derivative. So it looks like v equal to R i but if I look at F equal to m Dv dt then it becomes the look more like an inductor because I have a derivative but if I look at the second model m D square s (t) then it is looking like something else altogether it is second derivative now instead of a first derivative, what about the electrical side? Do I have anything like that on the electrical side.

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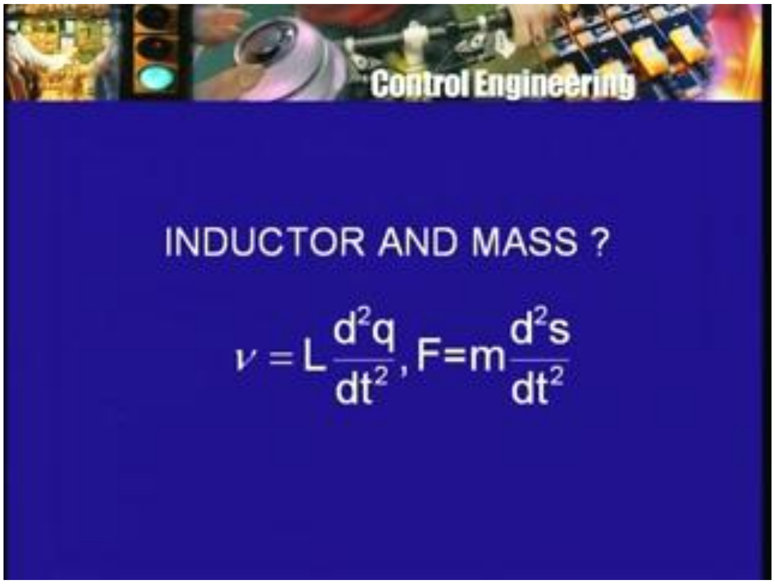

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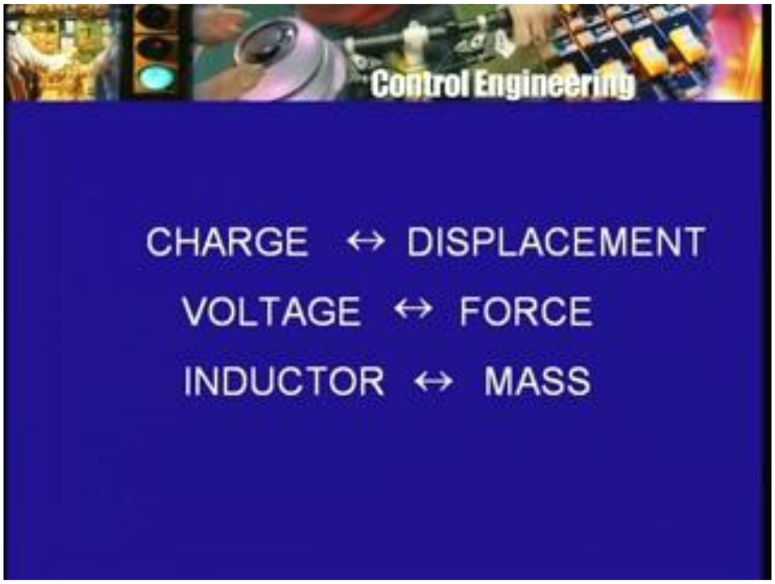

So the answer is yes because of I if I look at v equal to L di dt once again and if I say that ultimately the current is a result of motion of some charge or some charge at some place in the circuit is changing or charge is flowing across some cross section in the circuit then R becomes the rate of change of flow of charge across the circuit of the rate of flow of charge across the cross section, it charge for charger use the symbol q then current itself is the derivative of charge flow and therefore voltage of a inductor will be L times the second derivative of the charge flow rate.

Now this begins to look like the mass equation F equal to m d square s dt square. So if I take displacement to be analogous to charge then the mass becomes analogous to an inductor and the force on the mass becomes analogous to the voltage across the inductor. So this analogy can be studied in several different ways there are several analogies therefore that are possible. Now of course we have to thing of putting these 4 devices electrical devices together or the mechanical components together to get the electrical circuit or network or the mass spring dashpot with a force or a displacement source.

On the other hand and continue with these equations of course but write down some more equations where by one will see that the analogy will extend further and finally we may be able to write down one equation for the electrical network and one equation for the mechanical system. The two equations will look different, one will have voltages current charge may be rlc, emf, the other will have m elastance friction or coefficient of viscus friction and what not and of course displacement, acceleration, velocity force but except for these symbols the two equations will look very similar to one another and that is what will make the two systems analogous to one another and therefore one we will say that one is a model of the other and in this case there is another word which has becomes very fashionable now and that is the word simulation, simulation.

So one will say that the electrical network or circuit can be used to simulate the mechanical system or the other way around the mechanical system can be used to simulate the electric circuit or the electric network and this works simply because both of then under certain conditions are described by equations which look very much alike except for a change of symbolism instead of v, I will have F may be instead of q, I will have s the displacement time t fortunately is the same in both. But apart from that the two equations are similar or almost identical of course parameter values will be different instead of 1.3 into 10 raised to 3 I may have 10 their and so on and so forth but the but the 2 equations are similar.

In fact they are of the same kind known as a second order linear differential equation with constant coefficient and a forcing function. So in both cases we are led to this common model, a second order linear differential equation with constant coefficient and a forcing function or an input function.