Control Engineering
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Lecture - 47

Just as there was a close relation between the transfer function based or S domain approach and the frequency response method, in particular if you remember to obtain the frequency response one simply replaced S by j omega to get a complex number whose absolute value gave the gain and whose argument gave the phase shift. So, just as there was this relationship between the transfer function or the S domain and the frequency domain approaches. Similarly, there is a close relationship between the S domain approach and the time domain approach and the reason for that is simply that you really start with the time domain equations, you start with the differential equations of the system and then apply the Laplace transformation to the differential equation to obtain the equations in terms of S.
For example to calculate the transfer function and what happens is that because of the derivative property of the Laplace transformation wherever, you have a derivative occurring or derivative of a function, when you take the Laplace transform, you get a term which is S multiplying the Laplace transform of the function.

So in a way S takes the place of D and therefore you can expect a close relationship between what one does using S domain or transfer function based approach and the time domain approach. In particular look at the differential equations of our motor the 2 equations, I will rewrite them in a symmetrical form involving the 2 variables, motor speed and armature current on the left hand side and the input variables, the applied voltage and the load torque on the right hand side.

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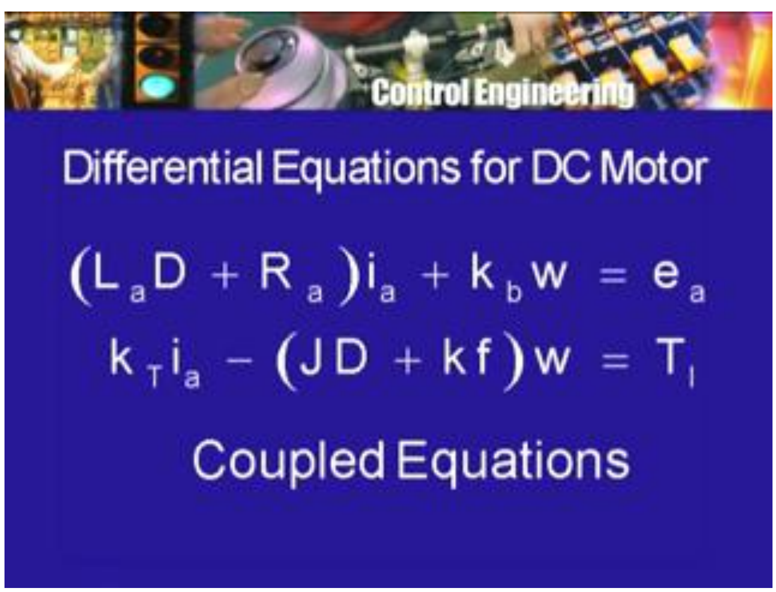

So the armature circuit equation was L a D plus R a operating on the armature current, these are 2 drops in the armature circuit, one due to inductance another due to resistance of the armature plus the back EMF voltage K b omega equals the applied voltage E a, this is one equation and the second equation is the torque equation which can be written as, let us say K T, I a which is the generated torque minus the torque required to overcome inertia and friction. So that can be written as J D plus k f acting on omega, this difference is used to drive the load. So this is equal to the load torque, note the operator D appearing in 2 places, one operator acting on I a, another operator acting on omega and the 2 equations are what are known as coupled equations. In the sense, each equation involves both the variables omega and I a.

Now, what we did earlier was to take the Laplace transformation apply the Laplace transformation to these equations where by the D was replaced by S, on the left hand side.
Of course because of the derivative property of the Laplace transformation involving initial values, we did obtain initial conditions or initial values of omega and I a, those terms to be transferred to the right hand side and said that we will not look at them or we will assume that the initial values are 0, equivalently. Now in the time domain approach of course, I am not going to take the Laplace transform of these equations and then do any further manipulation.

So, how do I reduce or from these 2 equations, how do I eliminate, let us say the armature current I a is to be eliminated. So that we obtain an equation for omega now, what one can do here is very similar to what one does in algebra and these differential operators, polynomial differential operators can be used or operated upon in exactly the same way that you operate upon on things in algebra. So in particular, I want to eliminate armature current which occurs in these 2 places. So, if these were some numerical coefficients or these were symbols then what would one do, one would say that okay here is K T, I a, here is this multiplying I a.

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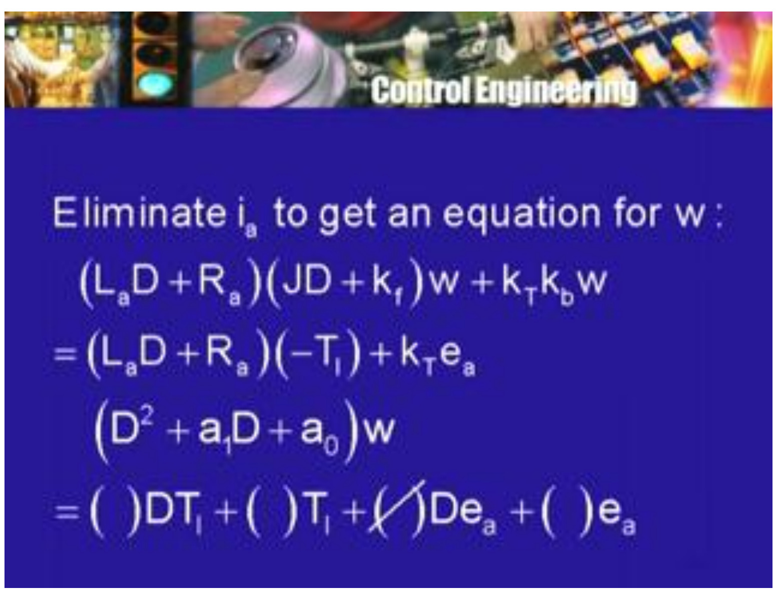

So multiply this by some suitable quantity and multiply this by a suitable quantity and then, choose those quantities in such a way that I a term cancels out and that leaves you only with the omega term and of course, on the right hand side you will be acting upon e a and T l respectively. Now, when you do that and I had asked you to carry it out for yourself, you will realize that you get a second order operator acting on omega and the operator, I will write as follows L a D plus R a composed with or following J D plus K f acting on omega plus K T, K b omega, this equals La D plus R a acting on minus T l or with a minus sign, I could have put it outside plus k T in to e a.

So on the left hand side, we have a second order differential operator which could be written as D square plus a 1 D plus a 0 acting on omega. This equals on the right hand side, I have the T l and e a terms and it, so happens that the T l term has an operator acting on it. So I wrote it earlier as some operator some coefficient multiplying DT l plus some coefficient multiplying T l plus some coefficient multiplying De a plus some coefficient multiplying e a. In this particular case, this coefficient is 0 that is the derivative of the armature voltage does not appear in the equation whereas the derivative of the torque appears in the equation. So to go from the 2 first order differential equations, in 2 unknowns to a single differential equation in one of the unknown requires manipulation of the equations and these manipulations are almost like algebraic manipulations.

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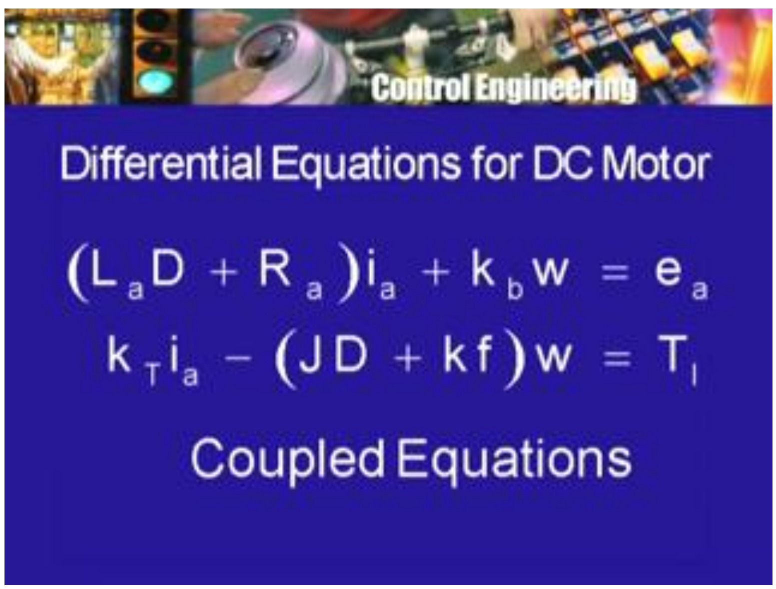

In fact, using the Laplace transform method in place of La D plus R a, one would have S L a plus R a which is a polynomial in S. Similarly, in place of J D plus K f you would have of J S plus K f another polynomial in S and you will have an equation in transforms of armature current and motor speed and similarly, applied voltage and load torque in which the coefficients are polynomials in S and polynomials of course are algebraic entities, we have been operating on them from school days and so, you do exactly the same kind of thing multiply one equation by a suitable polynomial, another equation by another suitable polynomial and add up to cancel of I a, of course this is done for when you have only 2 equations, when you have more than 2 equations, you may prefer to work either with determinants or with matrices and their inverse.

So the something similar has been done here to get the second order differential equation. Now we saw the effect of proportional feedback here by seeing that in case of proportional feedback, the armature voltage e a is derived from an amplified version of the difference between the reference voltage r and the tacho generator output voltage, K tachogenerator omega. So we are setting e a equal to K in to r minus K tacho generator omega.

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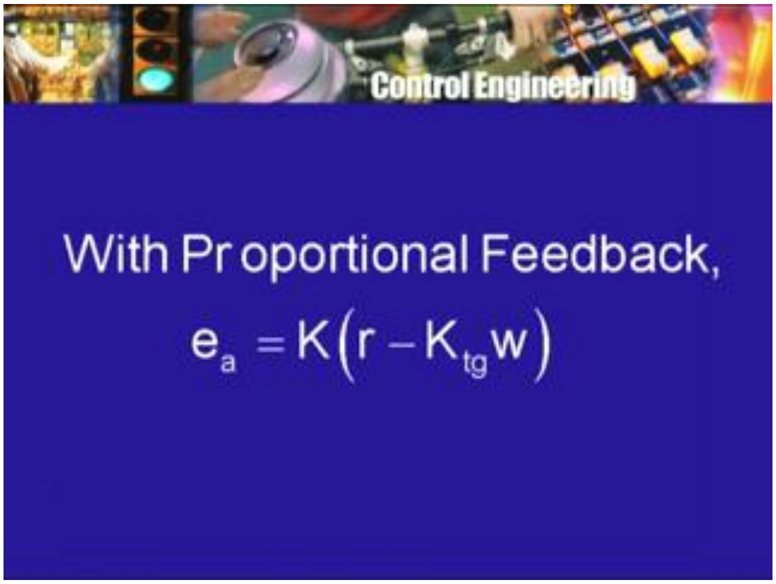

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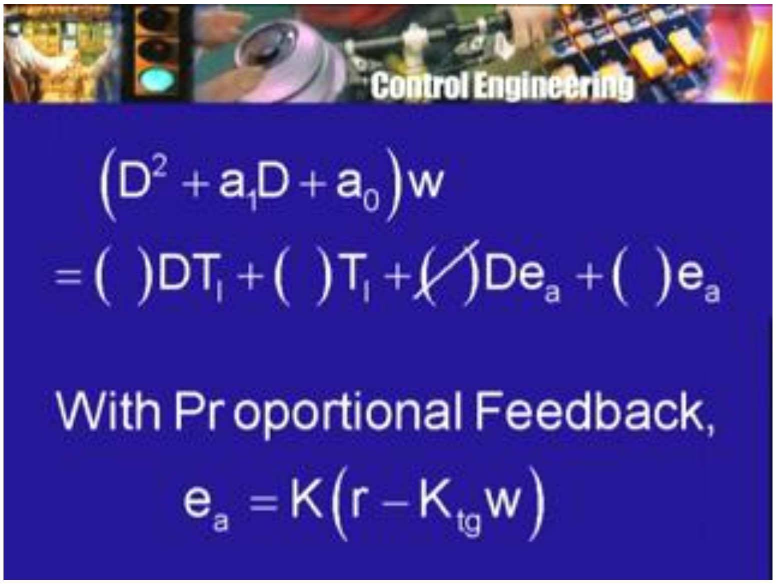

Now when you substitute that in to this equation, you will see that there is an omega term which is because of this term and the presence of e a here that this omega term will appear on the right hand side and therefore, I can transfer it to the left hand side. So, what is the effect of all of this, what this is doing is it is changing the coefficient a 0, in this operator D square plus a 1 D plus a 0. This operator D square plus a 1 D plus a 0 or in the S domain case S squared plus a 1 S plus a 0 was called the characteristic polynomial, in one case it is a polynomial in D, in another case it is a polynomial in S but both are polynomials never the less.

So this is the characteristic polynomial and its roots determine the nature of the transient response. Now, what is happening is that because of this proportional feedback, we are changing this coefficient a 0 of the characteristic polynomial. Now by changing a 0 of course the roots will change and therefore one can find out what is effect of this change. Now this is the kind of the thing for which of course, the root locus method is an appropriate one and therefore in this case the polynomial is only a quadratic. So, one can write down explicit formula for the roots of this polynomial and see the effect of this gain coefficient K on the nature of the roots but in general one can use a method like the root locus although the root locus method was obtained on the basis of transfer function that is in the S domain case, it is equally well applicable to the D domain if you may call it or the time domain case.

So something similar could be done here also now, the second case that we considered was integral feedback which, of course was expected to make a significant difference to the performance or the to the transient response of the system, as we had seen earlier. Now, what about the feedback integral feedback or proportional plus integral feedback in the time domain approach. In the time domain approach once again, the armature voltage e a is written as integral of what, integral of the output of the difference device of course, integral multiplied by gain K once again of the output of the difference device, the output of the difference device was simply r minus K tachogenerator omega as before.

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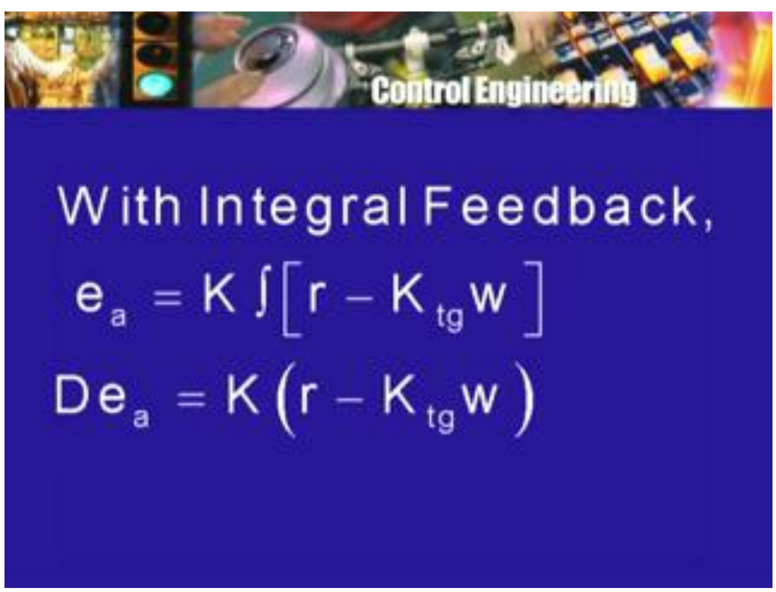

So the difference between the reference signal and the tachogenerator signal is integrated and amplified to produce the armature voltage. Now here we have an integral operator appearing and we can get rid of it by differentiating both sides and this is what, I told you earlier that the output of an integrator is such that the derivative of the output is the input. The output is the integral of the input, the input is the derivative of the output and of course that is simply obtained by from this equation by differentiating.

So differentiating what do I get? I get D e a equal to K times r minus K tachogenerator omega. Now this is the relationship I have because of the feedback and of course, I have the second order differential equation that relates omega to load torque and e a. Now we have e a appearing in both the equations and therefore we will have to use the second feedback equation appropriately. Now one way of doing it is the following, in this equation we have e a, in this equation we have D e a, so what can we do. Well, we can take this equation and differentiate it.

So, if I differentiate it that is operate on both sides by D or it looks like multiply both sides by D, what I will get is on the left hand side D in to D square plus a 1 D plus a 0 whole thing acting on omega equals. Now of course there will be a D square T l term, there will be a D T l term and this D e a therefore coefficient multiplying D e a, substitute the expression for D e a, so K in to r minus K tachogenerator omega. So this is going to be the new expression, now again omega term appears on the right hand side. So we can transfer it to the left hand side and now, if you look at the left hand side, what is the polynomial operator it is D cube plus a 1 D square plus a 0 D and this term on the right hand side contributes a multiple of omega.

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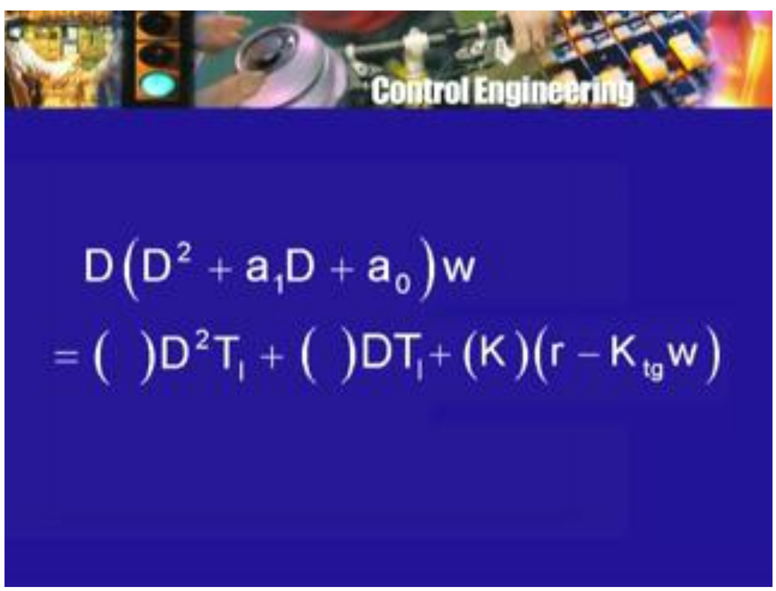

So we get a cubic polynomial on the left hand side or we get a cubic operator or we get a cubic characteristic polynomial therefore, there will be 3 roots that we will have to think about and the transient response will depend on the nature of the 3 roots, for a cubic also there is a formula but it is too tedious. So once again its better to use an approach like the root locus method to investigate where the roots of a polynomial lie as a particular parameter varies. The root locus method can be looked upon as a method for finding out the variation of the roots of a polynomial whose coefficients depend on a parameter in a linear fashion.

So we can see here that there is going to be some effect and what the effect will be of course will require a study of the cubic polynomial and how changing its constant term in the cubic polynomial is going to change the roots. So this is an example of a time domain approach where we have used a method of elimination to get rid of all the variables except one of interest. Now it is not necessary to do this or rather there is another way of doing it which involves the apparatus of matrices and as I told you, you have probably learnt a little bit of matrix theory and some ideas from matrix theory. They can be used for a control system purposes as follows. The 2 first order differential equations as I mentioned to you earlier, can be written as a single vector equation D x equals Ax plus B u and in case, we have, we are going to treat the load torque as a disturbance variable, so we might write this B u into 2 parts that is the input u may be split in to 2 parts, the armature voltage input and the disturbance input for the moment let us put them together.

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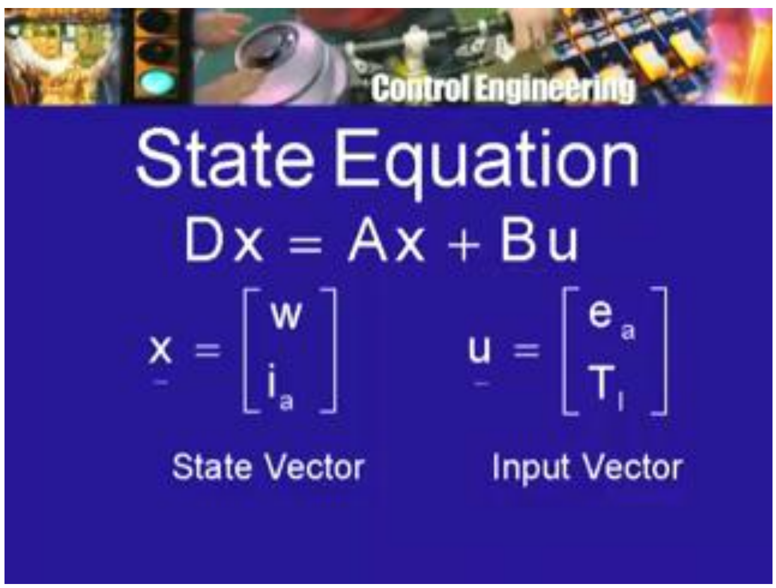

So x is a vector, u is a vector this x vector will consist of what in our problem, it will consist of omega and armature current. These are 2 variables which constitute the state vector, this is the state vector, what is the other vector u, the other vector u will consists of the armature voltage ea applied armature voltage e a and the load torque T l, this is the input vector. So these are the state equations or this is the state equation, now this is a differential equation it involves a matrix A and another matrix which happens to be a column vector that is it consists of only 1 column, a column vector B this is one of the standard forms in to which many control system or system equations can be written and this is called the state variable form.

Of course in this case, I have to add it is they are linear and they are time invariant and of course, they are ordinary differential equations, all right. So we have these equations, now one could think of methods of solving this equation by that we mean of relating the state vector as a function of time to the input vector as a function of time. Now this is something which needs to be developed and most probably, your textbooks may have a chapter or a section that deals with this, writing down or obtaining the solution of a set of differential equations in the state variable form for the linear time invariant case. Then, what you to get the variable of interest in which in this case is only the speed, one has what is called the output equation and the general form of it denoting the output which may also be a vector not one or they may be more than 1.

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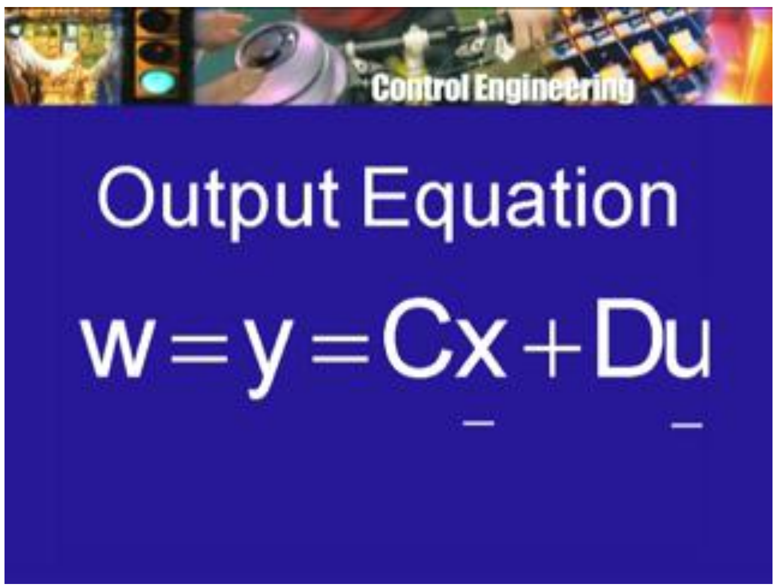

The output vector Y will be equal to C x, where C is another matrix and D u, where D is a 4th matrix and this is the output equation. Now when you do this in trying to solve this equation differential equation, you come across what we have come across earlier, you come across the characteristic polynomial. This time the characteristic polynomial is associated with the matrix A.

So you need or you introduce the idea of characteristic polynomial of the matrix A. Earlier, I talked about a single higher order differential equation and talked about the associated characteristic polynomial, we also talked about the feedback equation 1 plus G H equal to 0 and talked about a characteristic polynomial or a characteristic equation arising out of that. You see that no matter what approach you use, you run in to these polynomials and therefore the algebra of polynomials, how to solve or find roots of polynomials is a very important topic relevant to control system analysis and designs.

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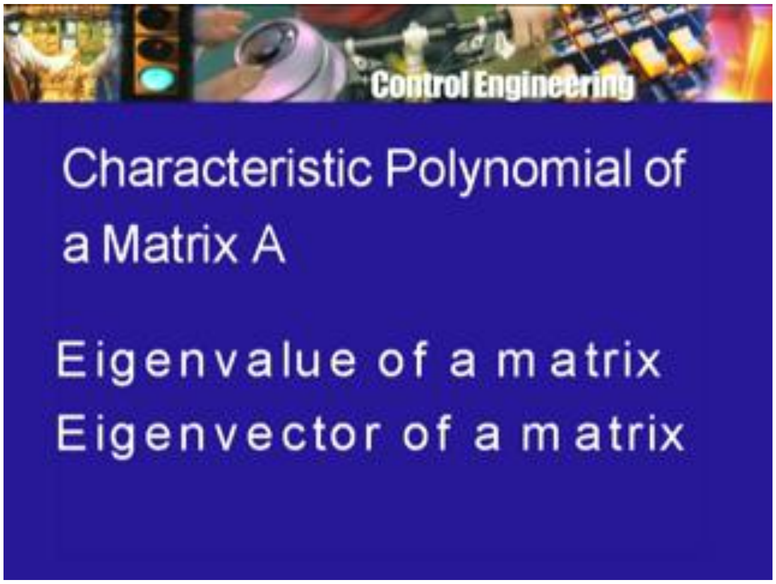

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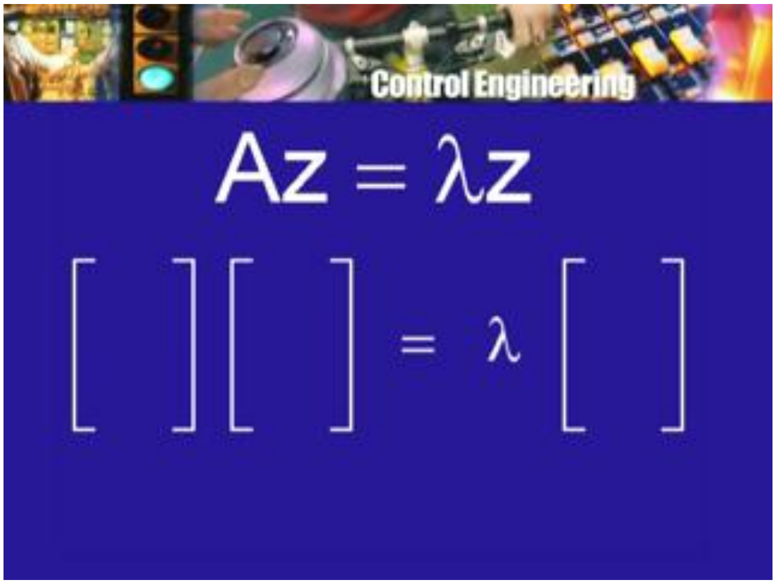

Now the way this comes in this case, it can be introduced in several ways one is through the concept of what is known as an Eigen value of a matrix, this is pronounced as Eigen, this is a German word which means in English something which is characteristic, very special or particular about something. So it is called the Eigen value or characteristic value of the matrix A and there is also associated with the characteristic value, the corresponding Eigen vector or characteristic vector. I will give you the definition only, I will not go through how one can introduce this concept and how one can use it, given the matrix A one says that a vector let us say z of appropriate size a constant vector z of appropriate size is an Eigen vector of the matrix A corresponding to an Eigen value lambda, if the following equation holds, A acting on z, this is a matrix which multiplies a column vector therefore it produces a column vector. On the right hand side, you have a scalar or a number multiplying a column vector therefore it also produces a column vector.

Now, if the pair lambda number, scalar to contrast with vector z, if the pair lambda and z is such that A z is equal to lambda z that is these 2 turn out to be equal then, this pair is said to be an Eigen value, Eigen vector pair for the matrix A. Lambda is said to be an Eigen value of A, z is said to be an Eigen vector of A and remember, that the Eigen value and Eigen vector go together that is you cannot really think of an Eigen value usually without also thinking of what is the Eigen vector that goes with it or Eigen vector without thinking of what is the Eigen value that goes with it.

So it is better always to think of them in pairs. So such pairs are known as Eigen value, Eigen vector pair and this is an important problem, in the study of matrices when there we applied that study to the study of differential equations in particular system equations or control equation, given a square matrix A, how to find out its Eigen value Eigen vector pairs, how many there are and find to find out all of them. This is known as the Eigen value problem, the concept of Eigen value to not of a matrix but of some specific vector perhaps and again when it was done at that time, there was no clear idea of a vector but a set of numbers constituting a vector like z and another number lambda related to a particular system of equations because essentially, we have come to this only through the differential equation D x equal to A x plus B u.

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This it seems was done by a mathematician whose name, you may have heard in mechanics it is a French name, so you have to pronounce it a little carefully you should not pronounce it as if it is an English word, so D’ Alembert, D’ Alembert’s principle in mechanics is the same person D’ Alembert, who was looking at some equations in mechanics and trying to find out way of solving them. It seems that he introduced this concept although he did not explicitly talk about a vector and an Eigen value but today, we can see that what he was doing the way he was trying to solve a set of simultaneous equations in mechanics looks or finally, leads you to this Eigen value problem.

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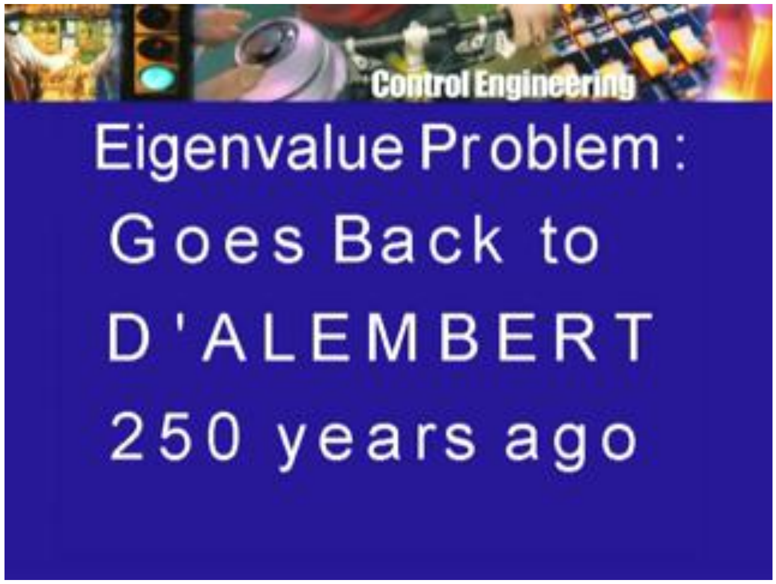

So in way the Eigen value problem is almost 250 years old but its application in the solution of differential equations is not that old. However, there has been a lot of work done on the Eigen value problem. In fact there are even whole books devoted to the Eigen value problem because as you can expect different people have developed different methods of finding out the Eigen values and Eigen vectors. The size of the matrix is one thing you know to find out the Eigen values Eigen vector pairs of let us say a 2 by 2 matrix or the 3 by 3 matrix that you and I can do it without too much difficultly but if the size is say, 5 by 5 or 10 by 10 then, doing it by ourselves writing down the necessary equations doing all the manipulations is just so tedious and one is likely to commit mistakes.

So that it is better to use a computer program and of course as you can expect there are readymade computer programs or packages which will solve the Eigen value problem, all you have to do is input the data about the matrix A and use an appropriate command and you will get a display of the Eigen values and a list of the corresponding Eigen vectors. But remember, that there are many methods which have been developed. So depending on the particular method that the computer program is using the accuracy and the time required may vary, certain matrices have some special forms and therefore some special methods have been developed for matrices having those special forms. For example, there were matrices in which many entries are 0, actually 0 not just negligible or small but exactly 0 such matrices are called Spars matrices.

So when you have Sparsh matrices then you take advantage of this fact that there are lots of 0s and try to develop methods for solving the Eigen value problem. So this is a very interesting area which somehow, now has become applicable to control system analysis and design after the turn towards time domain methods starting in the 1950's. Now the Eigen value problem leads to a characteristic equation or a polynomial equation once again that is associated with the matrix A, you derive a polynomial which is called the characteristic polynomial of the matrix A and the roots of this characteristic polynomial turn out to be the Eigen value of the matrix A.

So once again, we come across the characteristic polynomial and roots of the characteristic polynomial one can then, show that the nature of the solution or a part of it at least will depend upon the Eigen values of the matrix A and all the Eigen values which are just numbers, which may be real or complex, if they are real parts are all negative then, the corresponding terms in the response will go to 0 as T tends to infinity and therefore it will make or it will contribute to stability of the system. So the stability of the system gets related to the location or the values Eigen values associated with the matrix and whether all the Eigen values have negative real parts.

Now this is a problem which again goes back to Routh or the work done by Routh and the Routh algorithm. So you go back to some of the earlier discoveries made by Routh and Hurwitz's and others. Now this whole approach can of course be used for design and let me just tell show you, how one may try to do it very crudely but then, one sees that that is not a very good way of doing it and therefore one devices alternative methods.

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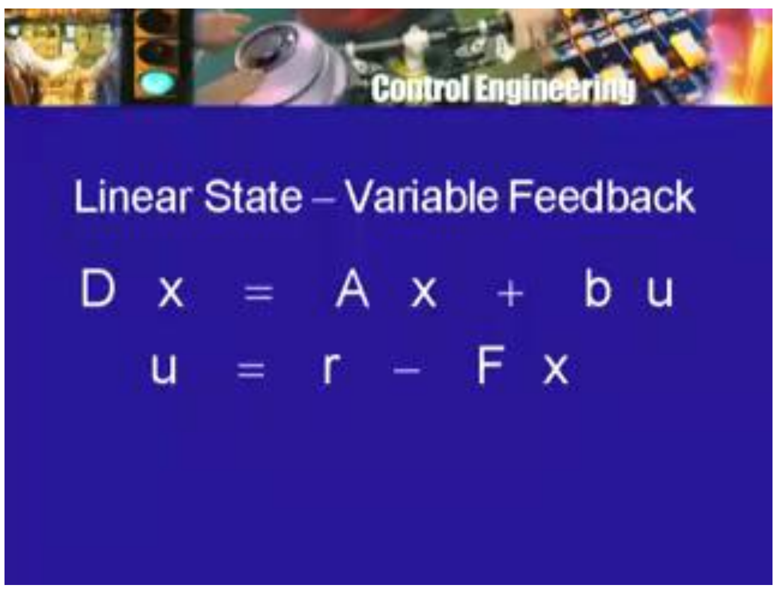

So let me write down the differential equation once again Dx is equal to A x plus b u and let us say for the moment, we leave aside Y, we look at the full state vector x, let us say we are interested in each and every component of the state vector x okay. Now as I said just now the if you look at the response that is the solution of this then, it can be split in to several parts one which depends on the initial values of x and the other which depends on the input.

Now, if you look at the solution which the part of the solution which depends only on the initial values of x then, that will consists of exponentials whose exponents are the characteristic values or the Eigen values of A. So, suppose that the control system that this set of equations is the set of equations for the control system and my matrix A is such that the characteristic values are not good enough, you remember time constants that we talk about the characteristic or Eigen values are ultimately, related to the time constant.

So let us say that the system time constants were too slow one or more of the time constants were rather large. So you would like to change or reduce the time constant. So that the system response due to initial conditions only goes to 0, as fast as possible or faster than it was doing earlier. For this purpose, we want to use feedback, so how shall we use feedback, well exactly the way we thought about it earlier, the input to the system u instead of getting it directly, you obtain it as a combination of what is called a reference input and the feedback signal.

So in this case, this can be done and represented in matrix terms as follows. The input vector u will not be applied directly but will be obtained from a reference input vector r, remember earlier diagrams had a reference input on the left hand side, the reference vector r minus I will still keep the minus sign although, the minus sign can be replaced by plus minus something that depends on x. The variables which we are trying to control or whose behavior we are trying to improve. So one can think of it as let us say F x, some linear combination of the state variable.

So this is known as state variable feedback. Earlier, we talked about proportional feedback of course there was only one signal namely, the speed and therefore we took a feedback signal based on the speed of course, then later on we introduced the integral feedback. So we had a term that depended on the speed and a term that depended on the integral of the speed. So proportional plus integral feedback, this is now called state variable feedback for an obvious reason. The feedback signal which may be a vector in this case depends on the state of the system of course, this means that I have to measure the state variables just as we had to measure the speed of the motor indirectly of course, we did it by putting a tachogenerator on the motor shaft, the output voltage of the tachogenerator served as a measure of the speed of the motor, it was simply proportional to the speed of the motor.

So similarly here, one will have to device some physical system or set of components which will produce signals which are given by F x. So this is state variable feedback signal that is subtracted from the reference signal and that is the output which is then, applied to the system. So if you do that it is simply is combining the 2 equations, so I have D x equal to A x plus b acting on it of course, I will now replace this b by capital B because the input u may be more than 1, it may be a vector input the underlining usually indicates a vector rather then a single or a scalar.

So B in to u but what is u? U is r minus F X all right. So now of course there is x here, so I can pull out this term and I can write this as A minus B F multiplying x plus B multiplying r. So what has happened now the differential equation which was D x equal to A x plus b u and therefore the matrix A determined the nature of the response. The differential equation has changed the differential equation is now, D x equal to some other matrix acting on x plus B acting on the reference input. So now the characteristic values or Eigen values of this matrix will govern the nature of the response.

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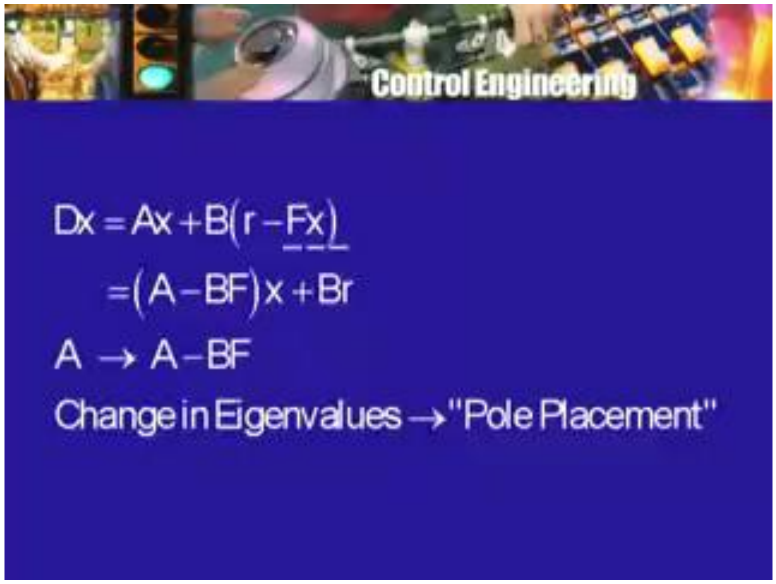

So, what we have changed is essentially, we have changed the system matrix A as it were from A minus B F. This F is the effect of feedback, so by using feedback we are able to change the matrix, if you are able to change the matrix, we will be able to or we will be changing the Eigen values. The Eigen values are associated with the time constants of the system so then, by changing A that is by introducing F, we will be able to change the time constant. Suppose, if F is equal to 0 that is there is no feedback, we just have A that is we have the system without feedback.

Now, as I said this is a crude way of doing it rather that is if you said that all right now, let me try a different some F and see what Eigen values I get and see whether they are good enough, if they are not let me choose another F and then find out the Eigen values and see whether they are good and so on. This is going to be a very crude approach, so naturally people have developed methods where by you do not solve the problem this way, although in a sense you are really doing this. The matrix is changed using feedback and we are going to choose the feedback matrix F in such a way that the Eigen values of the new matrix are somewhat better that is in fact, if possible you would like to choose them to be any numbers that you like and this, then would constitute the designs and it is known as pole placement. The Eigen values roots of the characteristic polynomial, we all ready saw where associated with the word pole in the transfer function approach.

So this called pole placement, pole placement using feedback and what we discussed just now of course, I assumed that one could measure all the state variables associated with the system remember that I had mentioned also a set of variables known as output variables.

In fact a system practical system, may have a large number of state variables but may have only a small number of outputs in which one is interested. It is not always very easy to measure the state variables some of them cannot even be measured directly of course some mechanical variables like position or displacement, velocity and even acceleration can be measured without too much difficulty but there are many variables which cannot simply be measured directly or easily. So there is this problem of measurement of state variables.

Secondly, whenever you do any measurement or whenever you use any measuring system or instrument, there is associated with it some error. So we will have to consider the effect of errors in measurement, thirdly we have the outputs in to which, we are really in in which we are really interested and therefore it is quite likely that the outputs can be measured physically, much more easily because somebody is probably watching a display of the output variables. So it is being, they are being measured anyway.

So is there something that can be done in this case that is we do not have access to all the state variables of the system, we are not able to measure or we do not want to measure all the state variables of the system but we will assume that we have available outputs. So we go back to the equations we have D x equal to A x plus B u and now, I have to take cognizance of the output equation Y equal to say, C x plus D u. Now what I will do is when I use feedback, I will use feedback which is based only on the output Y rather than on the state x and of course continuing the earlier idea for feedback, the error detector or the difference device and so on. I can now think of building up my u from the reference input r from which I subtract not F of x, F acting on x or multiplying x but F multiplying Y, so u is equal to r minus F Y, so this is the equation that I will have.

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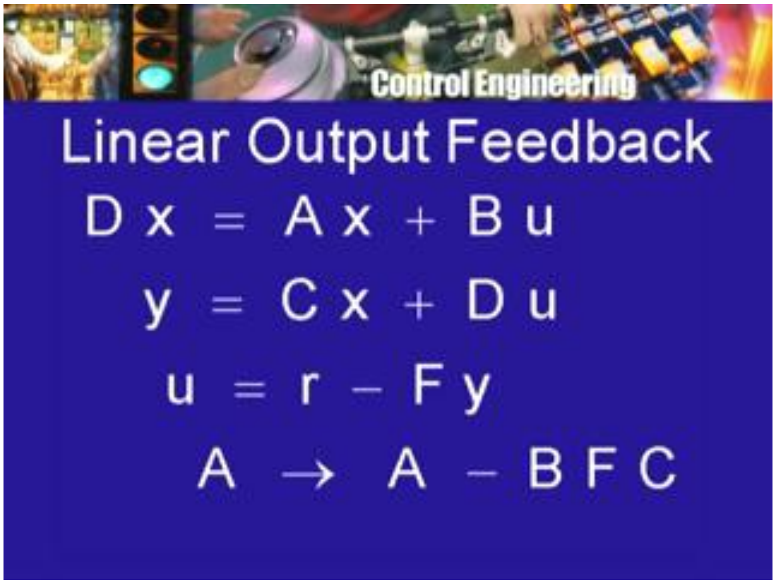

Now, when you have this equation and when you substitute this in the state variable equation then, the situation is not as easy as it was before. As you can see the situation becomes a little more complicated and therefore, this has to be studied as a separate problem and this is known as output feedback problem, if you actually carry out the substitution, you will see that the matrix A will be changed to the matrix A minus B F C. So it is not A minus B F anymore but it is A minus B F in to C. Now I have to choose the matrix F which determines the feedback in such a way that the Eigen values of this new matrix A minus B F C, I have appropriate values. So this is going to be a different problem now and this has been studied quite extensively over the last 30, 40 years.

Now this time domain approach and the time domain approach of course in simpler cases turns out to be the study of linear time invariant differential equations of the state variable form usually. This is sometimes referred to as modern control theory or modern approach in control theory but remember, that it is not really that modern, it goes back to the early 1950's, when control engineers and designers had started looking at differential equations rather than thinking of frequency response or transfer functions all the time. Gradually, they started using techniques which were developed by mathematicians in the study for differential equations for linear time invariant equations.

Of course, all the techniques from matrix theory were imported so to speak and so this modern approach emerged and it is a very important and powerful approach today, especially because computer programs can be developed for doing a lot of these problems and therefore, you have at your disposal computer aids and therefore one really talks about computer aided design of control systems, many packages have been developed for example, a package can be found which will take care of the problem of state variable feedback, another package to care of the output feedback and there are of course, questions that do arise as to what can be done with state variable feedback, what can be done with output feedback and so on.

One does not any longer talk about proportional feedback and derivative feedback and integral feedback in this context, PID controller of the earlier days and we have looked at of course P and I, we did not spend anytime on the derivative control but I said that one could introduce the derivative as a one more component to the total feedback signal. So this is quite different from the PID controller design, it proceeds, it is based on differential equations and it proceeds in a different way altogether although one can try to relate the PID control to the modern control method, the 2 are basically they are not the same they are quite separate.

So this is where the modern approach to control system design stands of course that what I have illustrated is only for linear time invariant systems, differential equation is linear time invariant. The output equation is linear time invariant therefore matrix, Eigen values characteristic equation, Eigen value problem all these are tools which can be used. The moment the differential equations become non-linear, the output equation becomes nonlinear then the situation is much more difficult and there are no generally applicable tools that are available even today.

Now before, I look at another relatively modern example of a control problem which I mentioned almost in the beginning of this course, let me spend a little time on something which is more classical control in fact, let us go back to the motor driving a load situation but this time we will look at a problem of not speed control but position control and this of course, arises or arose during the second world war, in the problem of tracking a target, a gun, tracking a target.

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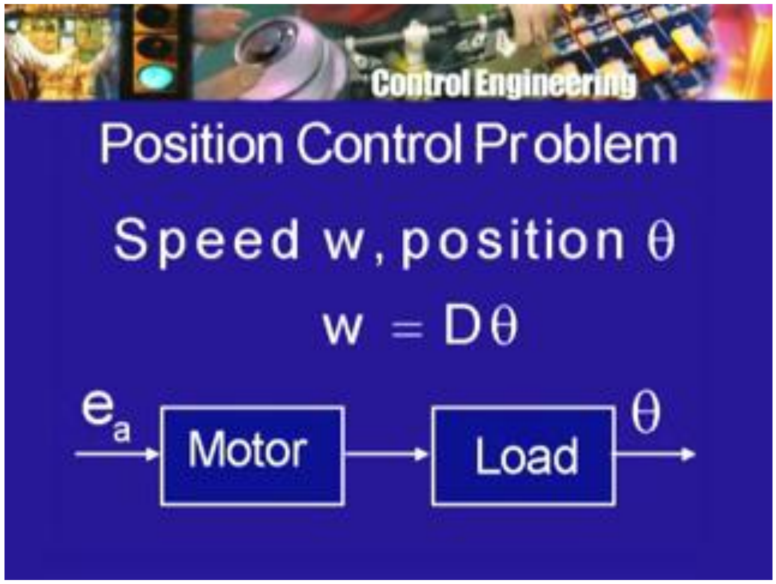

So you have to move the gun in such a way that the gun position is as close as possible to the target position or at appropriately, related to the target position. So that you can take a decision and lob a shell at the target. So this is a position control problem but in order to move the gun, you need a motor all the same and you could equally well, use the DC motor for this purpose. The output variable or the variable of interest will not now be omega, the speed of the motor but it will rather be theta let us say, the angular position of the shaft of the motor with respect to some reference position and of course, one expects in this application. For example, the movement of the gun at the gun is not going to move around through 360 degrees but the gun will move through some small angle.

So this angle theta will vary over a small range and it may, so we do not expect continuous rotation of the motor which drives the gun although because of the possibility of a gear train being introduced, the motor shaft may make several revolutions whereas the gun target or the shaft of the gun only makes a rotation through a small angle. So there could be rotation of the motor shaft through more than 360 degrees but the motor is not expected to be continuously running at a constant speed, if it did, the gun would be const continuously spinning and it would be of no use, I mean no target would move in that way, anyway.

So theta is a variable and not omega of course, omega is just the derivative of theta, derivative of the angular position. Now therefore let me draw the block diagram of the system, so we will have the motor and the motor will drive the load and it is the angular position of the load that one is looking at, so here is theta and there will be a signal, input signal given to the motor of course that will be the armature voltage as before. Now in the open loop case, what is that one will expect, what one will expect is that the motor will be turned on for a short period of time, the shaft of the motor will therefore start moving along with that the load shaft will move and then, if an operator is operating or controlling the system as soon as, he sees that the load is about to reach the desired position, he will switch of the motor.

So the motor will be switched on and switched off, if a new angular position is desired then again you will switch on the motor and, switch off the motor. Of course, the rotation required may be in one direction or in the other direction therefore the applied armature voltage may have to be positive that is in one direction or may have to be negative, making the motor turn in the opposite direction. So if you operate this system as an open loop device or as an open loop system then, the operator essentially has to turn on and turn off the motor or apply the motor in the voltage, in the positive direction or in the negative direction of course, he may also change or control the value of the voltage that is being applied for a small change in position perhaps a small armature voltage applied for say, one second is enough whereas for a large change and if you want that change to take place quickly, you may have a large voltage applied for that same duration 1 second, it will make the load, the motor will make the load move through the desired angle in a shorter time. So this is open loop situation.

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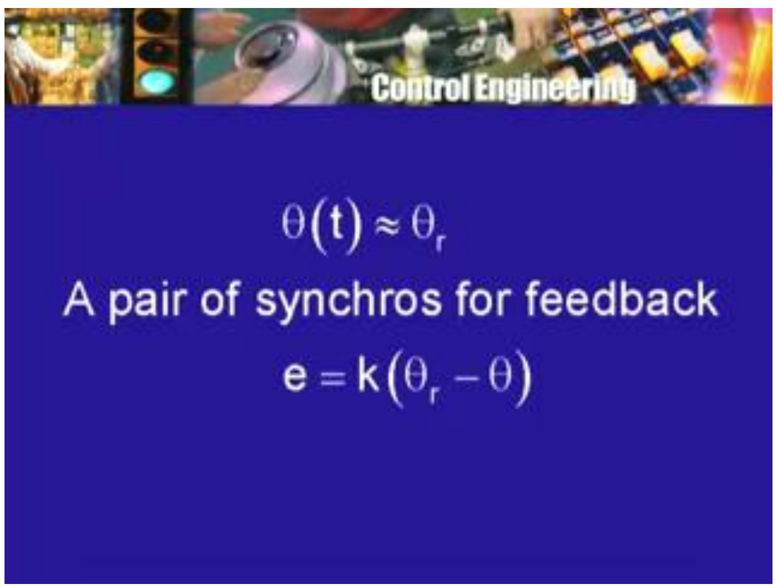

Now, what about closed loop or feedback control. The operator is going to look at a reference position theta r of the output that is the load shaft, the actual load shaft position is theta t let us say and so, the operator is actually comparing the 2, he is comparing the reference position, the required position with the actual position and therefore, one can immediately think of a feedback along the lines that we had done for the speed control problem. The desired speed and the actual speed, the actual speed measured by the tachogenerator, the desired speed represented by reference voltage. In this case the reference position and the desired position.

Now because speed is difficult to represent physically, directly, as I said you cannot have something rotating at the reference speed, serving as the reference therefore we replaced it by the tachogenerator but in this case, the reference position being an angular position it could well be represented by something physical and in fact, this is what was done in the early days and I mentioned this earlier that one uses a pair of synchros the synchros, there are 2 synchros, one synchros provides the reference position and the other synchros is mounted on the load shaft.

So effectively, you have 2 angular positions and by proper interconnection between the synchros, you can obtain a voltage that depends or is a function of the difference between the angular positions of the reference angular position and the actual angular position and therefore, the synchros output, if the difference is not very large and of course, the synchro output may be an AC voltage, rather than a DC voltage but if you look at its amplitude then it is represented by something like K in to theta r minus theta and therefore we can think in the block diagrams of the motor and the load producing the signal theta, the input reference position theta r being present here and a difference between theta r and theta being amplified by the factor K and fed to the motor load combination.

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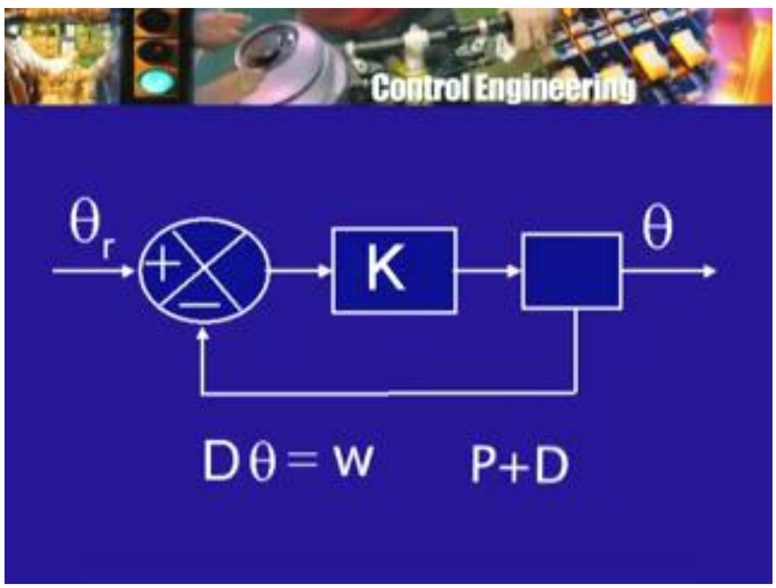

So the familiar feedback system block diagram will appear here. Now what kind of feedback is this well this is proportional feedback because I have K multiplying theta r minus theta. So effectively theta is what is appearing in the feedback, in this case, it is quite natural to think of derivative feedback because derivative of theta is speed and speed can be physically measured by a tachogenerator voltage. So in addition to this theta term or K theta term or minus theta term which is occurring here, I could also introduce the derivative of theta or omega as one more term in the feedback loop.

So here, we can proportional plus derivative feedback in quite a natural way because the derivative of the quantity of interest, angular position is a physical quantity which is measurable quite easily, speed by means of a tachogenerator. So what we will have is in addition to the synchros on the shaft of the load, we will mount a tachogenerator which with measure the angular velocity or D theta and then, introduce a term proportional to D theta in the feedback path and then, see what is going to be the effect on the performance of the system.

So we will have proportional plus derivative feedback, of course we can as before introduce an integral element in the forward path although it is not quite natural to do it because to maintain the load at a desired angular position, there is no steady state input required, the voltage required for the motor armature to maintain its shaft stationary is 0 whereas to maintain the motor speed at a constant value, you require a non-zero voltage at the motor armature.

So there is a difference between the 2 because of which in this case integral feedback is not quite natural to think of whereas in the other case we thought of integral feedback for what reason, the main reason for introducing integral feedback was to reduce the error due to load torque variation or that is the effect of the disturbance variable that is where we introduced the integral feedback. In this case, one can see that it is not necessary to introduce the integral feedback to take care of the load torque disturbance because ultimately, the motor is to come to a steady position theta r, very close theta, you do not require any movement of the motor, you do not require any voltage to be given to the motor therefore, no integral feedback is necessary.

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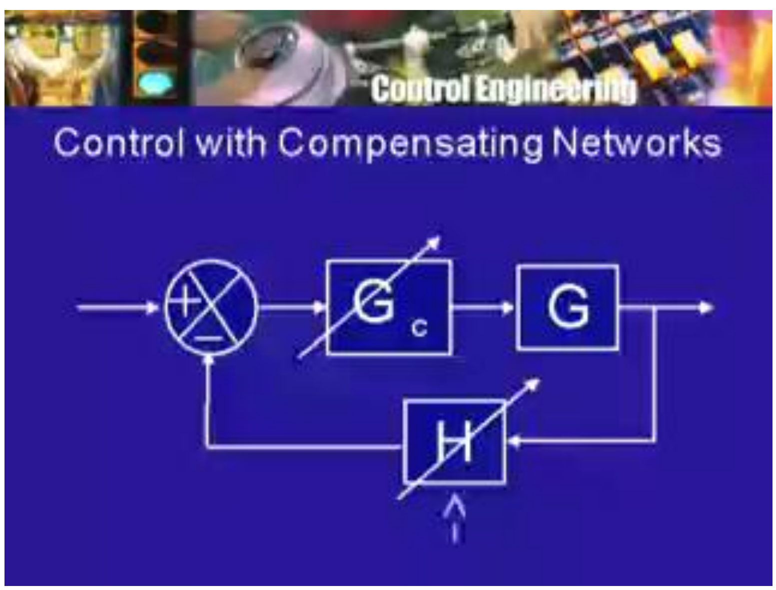

However, one could think of using it and see what is going to be the effect of integral feedback. So this is with regard to the position control system where a derivative feedback may make sense. Now just quickly to go over classically what one thinks of you have the system, the motor, the prime mover and the load combined together of the plant represented by the transfer function G, you have the feedback path elements represented by the transfer function H and then, you have what I have called a compensator which is introduced in the forward path which produces the output quantity that drives the load classically, the control problem is to choose the compensator G c in the so-called forward path and to choose the feedback path transfer function and as we saw in the early days because almost all the signals except the one that involved the load itself were electrical signals, one could think of simply electrical networks or compensating networks to do the job.

So classical design techniques were heavily geared towards design of compensating the networks whereas the more modern technique based on state variable representation that I introduced just a while ago has nothing of this side, there is no structure of this kind that is visible, there is no question of any transfer function and therefore any compensating networks. So this is why the 2 approaches are different but of course one should look at as many approaches as possible for solving a given problem. So it will not be correct to say that the old approaches are no longer useful, for small order systems for a initial design, the classical approach may still be quite useful but the new techniques are available. So one should also make use of the new techniques whenever possible.