Control Engineering
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Lecture - 46

What about the concept of a time invariant system or a time varying system. As in the case of linear system or linearity which consisted of additivity and homogeneity. We can talk about time invariance or time varying nature of a system from the point of view of a external measurement or what is normally called the black box approach. You apply an input and observe a response and on that basis you try to draw some conclusions.

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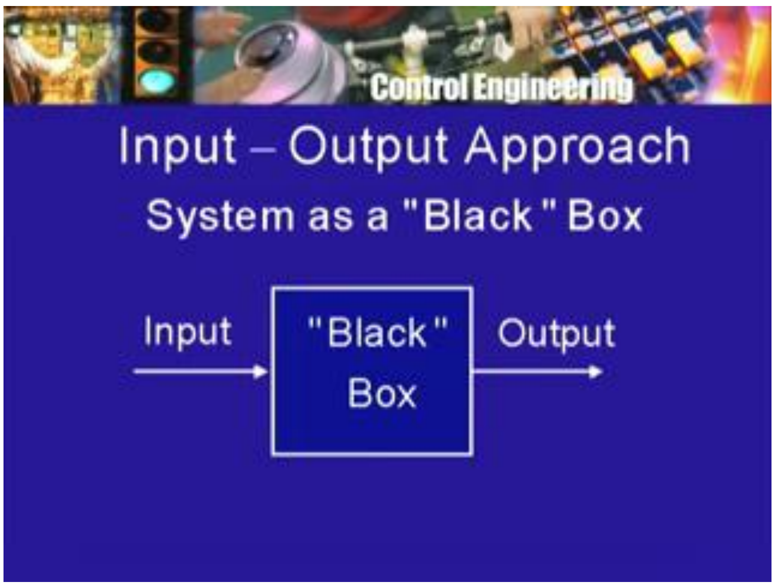

So, what do you do with regard to time varying or time invariance property? Well as the phrase suggest what we have to do is we have to what is called time shift or time delay usually in practice an input. To take a simple example, one may apply to a system a pulse of a certain duration of a particular shape let us say, starting at some particular instant of time and then observe the response for some period of time then, after some time you may again apply the same pulse that is pulse with the same shape, same width, etcetera and observe the response.

Now you are applying in a way the same pulse in the sense the pulse shape is the same but you are applying it starting at a different moment of time. So, that this is what is meant by a time delay or a time shift. The response of course for the shifted or the delayed pulse will begin later but what you can ask is you whether the shape of the response is the same as the previous response except for the time shift. Now of course this can be done with the help of some notation and it is a idea that is important and not the notation given any function of time say, f or f of t, one can talk about its delayed version and several notations are in use since, we have already used letter D to indicate derivative I cannot use D for delayed but one may use the let a delta for a delay.

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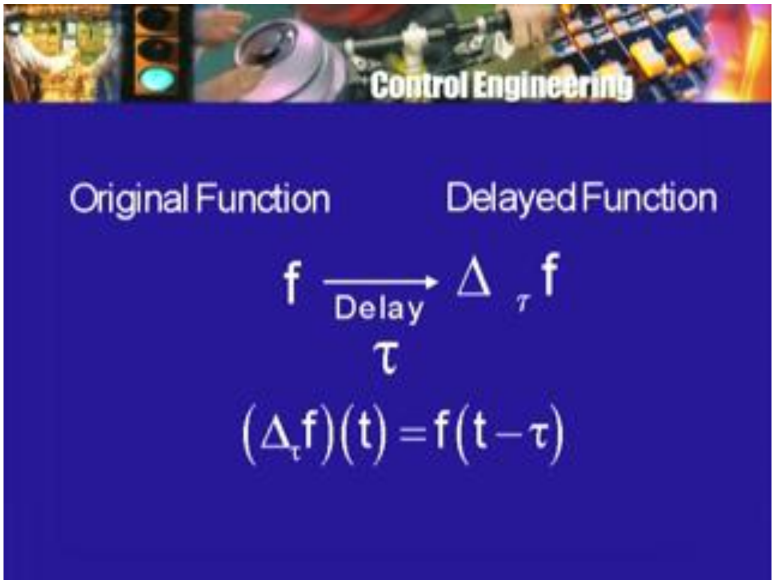

So delta is the delay operator and then you have to indicate by how much time interval is the function going to be delayed, so I could write it as delta with a subscript tau, tau being the time interval operating on f. Now this produces the delayed function, the function f which is delayed by an amount tau to take a specific example, let us say here is the time axis and here is the pulse that I talked about and let us say it goes from 0 to 1 second.

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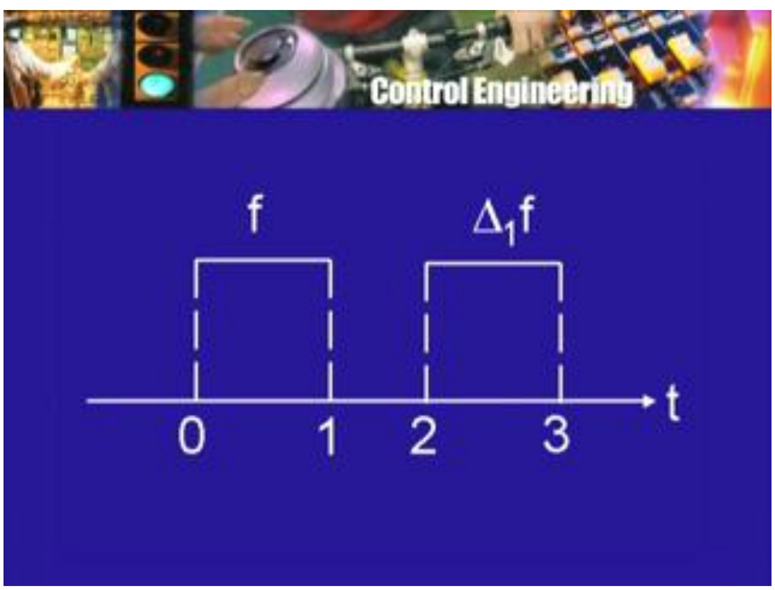

So this is the function f as it where and now, I have to delayed it by let us say 2 seconds then the pulse will start here at 2 seconds instead of starting at 0 and then, it will last for 1 second as before and go to 0 at 3 seconds. So this function will be denoted by delta tau f or in this case since tau is equal to 1, delta 1 f that is the function f delayed by 1 second. So, if this is the input there will be a certain response then you apply this input and you observe the response. Now just as this input is a delayed version of this input, you ask or you try to see whether the the response to the delayed input is a delayed version of the response to the original input, if this is so the system is said to be time invariant and if it is not so, the system is said to be time varying.

Now this does not require you to know what is inside the black box, what kind of devices is, what are the differential equations or algebraic equations or whatever that relate the response and input variables. It is a test which can be perform externally so to speak but on the other hand, as you are already familiar one can look at the set of equations or the model and by looking at the form or appearance of the model, one can say something about time invariant or time varying nature of the system, again to start with a very simple example, if the relationship between the input u and the output y is simply y is a scaled version of u y equal to a u then, it is not at all difficult to see that this system is time invariant and of course, the coefficient a is just a constant where as if I had y (t) equal to a t into u (t), where a (t) was a function which was not constant then, one can see that the relationship between the input and the output is going to vary with time and therefore this the system described by this relation is not going to be time invariant.

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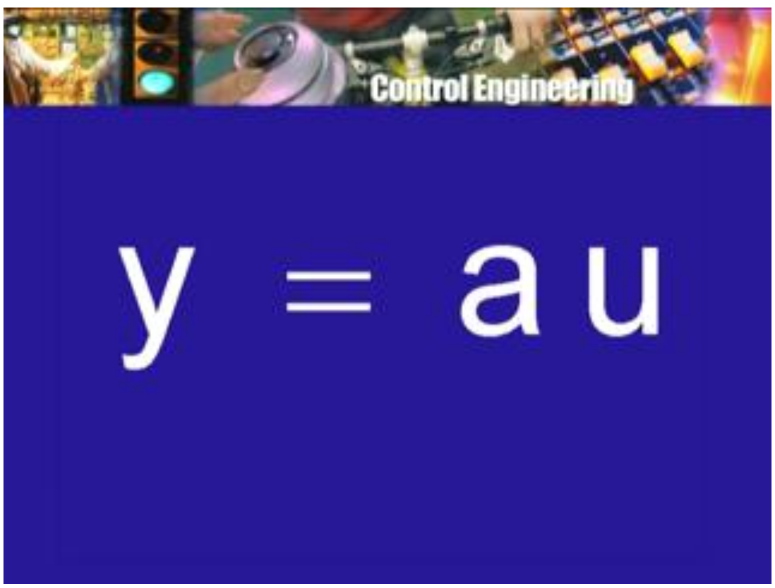

Similarly, if I a write down a differential equation let us say, D y equal to a y plus b u as we have done already. A little argument will show you that if the input is delayed the response will be delayed and as in the case of linearity provided the initial condition is 0 or the initial condition remains the same that is the response to an input starting with an initial condition 0 is 1 and response to the delayed input starting with the initial condition after that delay also 0 is another response then, the response to the delayed input will be the delayed version of the response to the original input.

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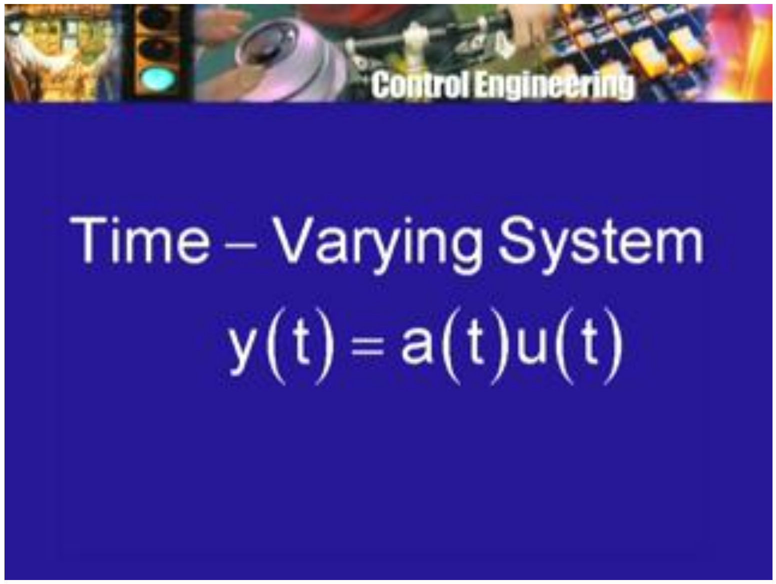

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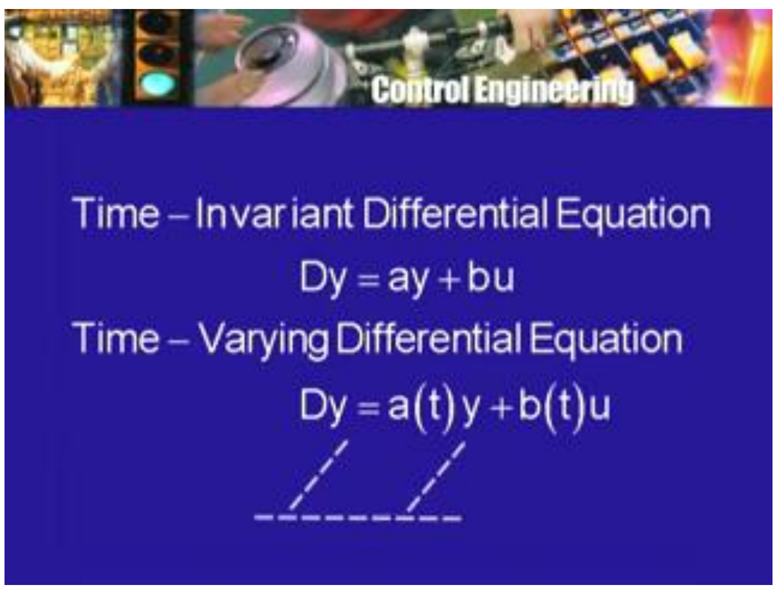

This can be seen by actually making use of the definition of the delay operator and substituting in the differential equation and see that this is so. This is because of another property of the derivative which is a little difficult to talk about although it is not very difficult to see and that is the following that is, if I have a graph here like this let say and if I shift the graph by a certain amount then, the derivatives also get shifted by the same amount that is shifting the graph horizontally along the t axis does not change the derivative values, it only changes the instance of time at which the various values are taken by the derivative. Of course the shape of the wave or the function remains the same.

So this is the property of differentiation that differentiation is invariant with respect to time shift or to use a more sophisticated word translation in time so, so much for time varying or time in variant and linear or non-linear systems and as I said earlier, the method of Laplace transformation and therefore the s domain methods using that such as the root locus method, the Routh criterion or test or for stability or location of roots over polynomial and the frequency domain methods such as the methods based on Bode plots Nyquist criterion, Nichols charts, etcetera are not applicable for systems which are non-linear that is which are not linear or for systems which are time varying or not time invariant.

Now of course as I told you earlier, non-linear systems and even time varying systems do occur in nature and in engineering practice. The pendulum whose swing is not sufficiently small or infact for no amount of swing is the pendulum exactly described by linear differential equations. So it is a non-linear system, the Fly-Ball governor would be another example of a system where there would be a non-linearity. Now for quite some time people tried to work with the non-linear models and to a limited extent they could do some work. One method of handling a non-linear model or a time varying model was to approximate it by a linear model or by a time invariant model. In the case of non-linear systems when they are approximated by a linear model the process is known as linearization.

Now of course this only results in an approximation and unfortunately, one may not be able to know or say much about how good the approximation is. This is one of the major problem which persists even today. Infact the non-linear model of the system itself is an approximation to the actual behavior of the system because you may have neglected, so many things you may be have assumed that some things remain constant and so on. So the physical system is one thing the non-linear model of it is only a model that is it is only an approximation to describe the behavior of the system and then, the non-linear model now you want to replace by a linearized version.

So there is a further degree of approximation and unfortunately, it is not always very easy to find out how good or how bad the approximation is. However, so it was not as if the things were that hopeless. Infact around in 1900, a Russian mathematician by name Lyapunov did some work on non-linear systems and essentially those were systems which also involved inputs that is therefore his work was applicable to a non-linear control systems. He introduced to approach or a method which is known as the Lyapunov method or Lyapunov approach and he showed that under certain conditions if you replace a non-linear system by an appropriate linearization and then study the stability properties of the linearized system.

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You can infer something about the original non-linear system and he introduced a function which has become known as a Lyapunov function after him and there was a lot of work which was done at that early time but it was done in Russia, the language of the work of course was French, somehow he did not catch the attention of people and therefore not much progress took place following the Lyapunov approach till the late 1950's. When people realizing that you could no longer make approximations or no longer could only work with linear systems, when back to problem of non-linear or time varying differential equations and re-discovered or discovered the work of Lyapunov and made a bit of progress using the Lyapunov approach or the Lyapunov method.

So this was the kind of thing that lead to a reconsideration of the differential equations that describe the system rather than transfer functions or frequency response ideas. In particular, if the differential equations were linear then the matters became easier to deal with and one could deal with linear differential equations with constant coefficients without using the Laplace transformation and such methods became known as time domain methods.

So these sort of started being studied from the 1950's time domain methods for control systems not just single input, single output control system but multi input, multi output control systems, to be modeled by differential equations rather than by transfer functions or through frequency response, even though the system may be linear and time invariant. There are some things that you can do without using the concept of the Laplace transform or transfer function and the ideas of frequency response and there are some advantages in doing that there are also in a way more direct or simple, once you have understood or mastered the required mathematics, I gave you an example of such a system already namely writing it in the vector notation D y equals A y plus b u, where y is not a single variable but it is a vector of several variables, say y 1, y 2, y n.

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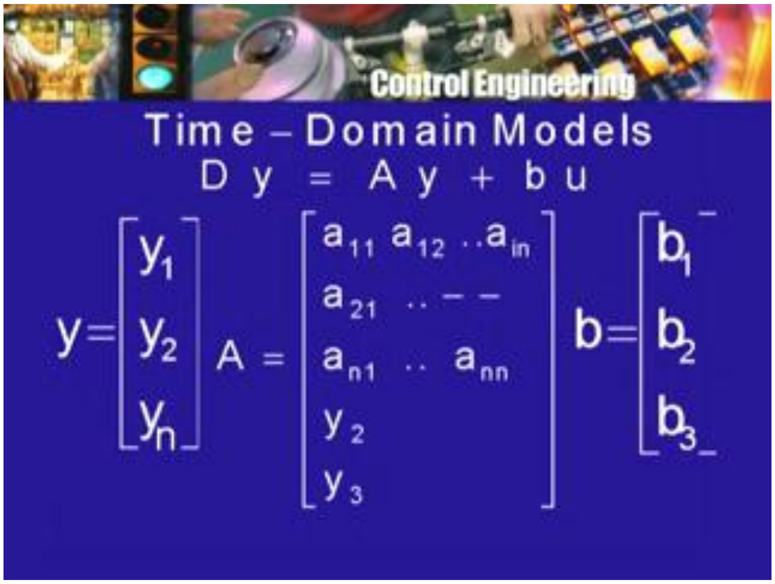

So there may be n different variable not just one response variable and u may be a single input in this case the notation, I use this such that u is a single input but again u can be a vector input that is the system can have more than 1, input A is a matrix of coefficients a 1 1, a 1 2 and so on and b is a constant vector again involving a number of components. Now such a system of equations which made their appearance and began to be studied and used as the model starting with the 1950's are known as state variable equations and such a model is known as a state variable model.

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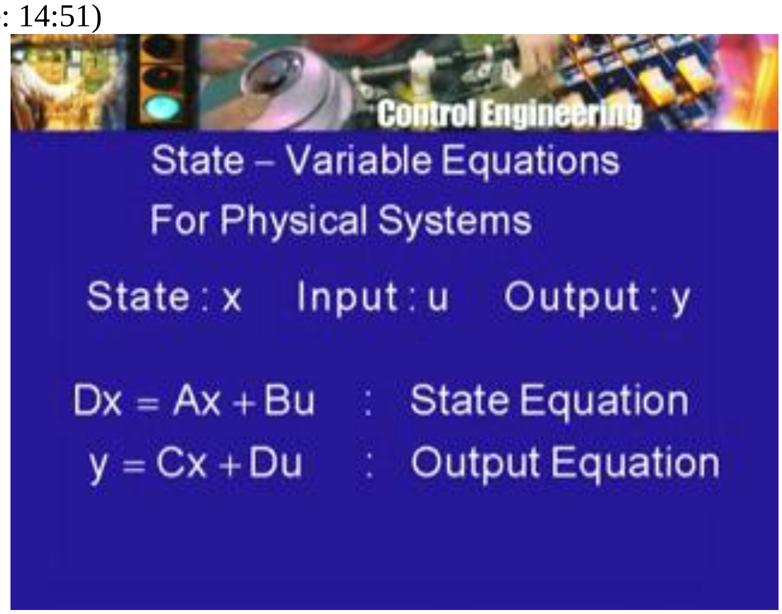

Infact, usually what happens is the y that I have talked about as I told you the letter y in control theory practice is usually limited for an output, u for an input then what is this state variable, the state variables may be will not be either input they may be an output variable or they may not be an output variable, I do not have time to go into these details usually your textbook will have a chapter on state variable formulation of system equations and in particular of control equations.

So you should take a look at that in that case the state variables will be a set of variables x again, this may be more than 1, so this is the vector the input variable u may be one or more the output variable y, may also be one or more variables and the state variable equation model which is very commonly used is then D x is equal to A x plus B u, D denotes the differentiation operation, A is a matrix, B is a matrix, in general the output vector as this called y is related to x and u through an equation like this. So this pair of equations is referred to as a state variable, set of equations these are known as the state equations and this set of equation is known as the output equations.

So this is the kind of model that one can start with instead of starting with transfer function or frequency response ideas. In fact, if you recall when we started our discussion of the dc shunt motor, driving the load, the simplest load that we talked about. We in fact wrote down differential equations of course in the beginning we said, let us consider the operation of the motor in the steady state, so the current is constant voltage is constant torque is constant speed is constant in that case, there are only algebraic equations that relate the various quantities. But when since, we are interested in the transient behavior what happens when the motor is switched on or load changes suddenly or the supply voltage changes suddenly, we have to write down the differential equations for the system and this we did and let me recall them, the armature circuit of the motor gave rise to the equation e a of t and now, I am writing as a function of time explicitly to allow for the fact that the armature voltage may vary.

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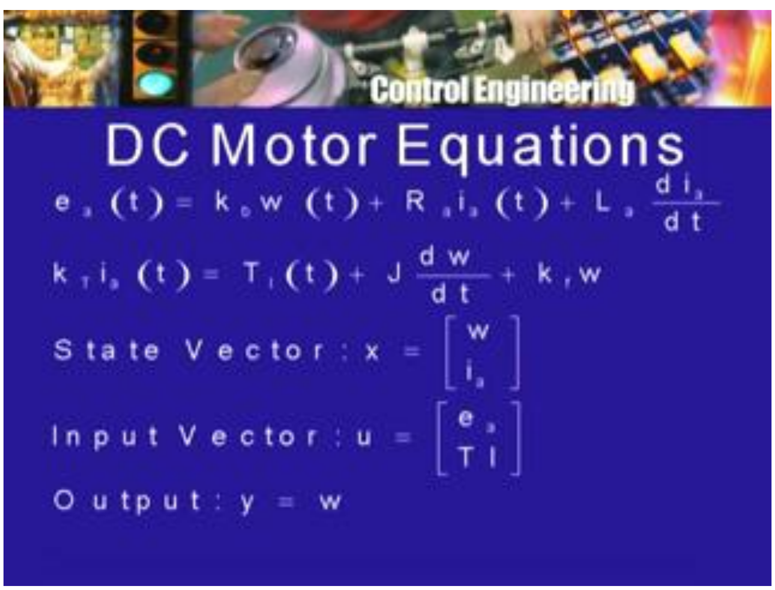

So this equals the back emf which is k b into omega t now because the speed also may vary with time plus the 2 terms which are R a, I a of t, the armature resistance drop plus the term due to the inductance of the armature L a d ia dt. Now already there is a derivative of the current appearing here. So we have a differential equation the second equation the torque equation or of the mechanical circuit equation is also differential equation. The generated torque the torque produced by the motor k T into I a of t part of it is used to overcome the load torque. So, I am writing it as T l of t and a part to overcome the moment on inertia.

So j d omega dt and a part overcome the viscus friction, so k f into omega. So the derivative of the speed also makes an appearance here and in fact, it is not difficult to rewrite these 2 equations in such a way that they appear in the so called state variable form, what will be the state variables as we can see armature current derivative appears in one equation, speed derivative occurs in the other equation. So we could choose as state variables x let us us say I write down the speed first omega and I a, so the 2 state variables for our control system will consist of the 2 variables omega and I a, x is sometimes referred to as the state vector.

So this, we said that the state vector has 2 components and is made up of the motor speed and the armature current. I will leave it to you to rewrite these 2 equations, so that they look like D x equal to A x plus B u but what is going to be u in this case. Now, if we look at the armature equation the armature voltage appears there and if we look at the mechanical equation, the load torque appears there in addition to terms involving I a and omega therefore we can choose as the input u, the vector consisting of the applied armature voltage and the load torque.

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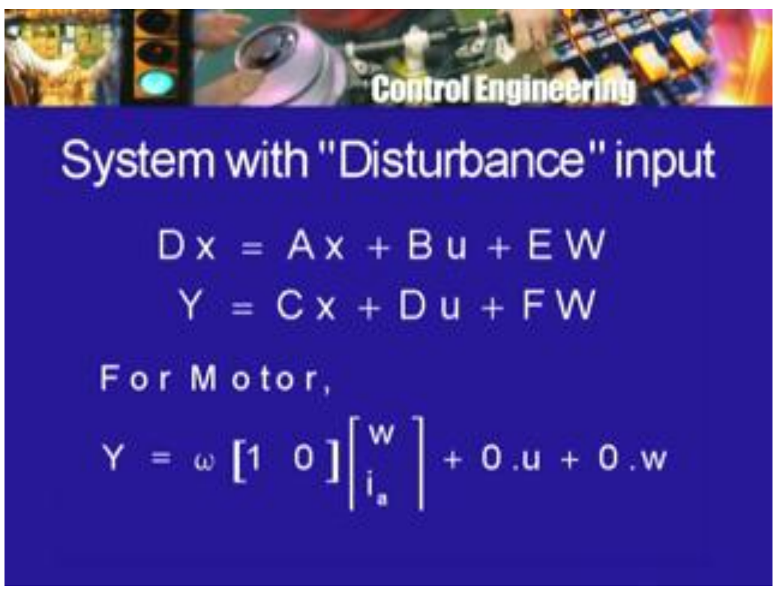

So our u will consist of 2 variables E a and T l with this then, you should rewrite these 2 equations so that they look like D x that is d omega dt and d ia dt equal to a matrix multiplying omega and ia plus another matrix multiplying e a an T l. So find out what those 2 matrices A and B will be for this problem. Sometimes these 2 E a and T l may be kept separate in the sense, e a is an input variable in a way because this is providing the power to the drive whereas the T l, as we saw earlier is more part of it is disturbance variable that is the uncertainty in T l is likely to cause changes in speed whereas the power supply voltage can perhaps be maintained reasonably constant. You want to drive to function in spite of load torque variation therefore, this instead of taking them together as one single vector, you may split it into 2 parts in that case the part that corresponds to the disturbance is usually denoted by the symbol W and so the equation may look like D x equal to A x once again plus B u and now I need one more letter and so E W.

So x is the state vector u is the input vector, W is the disturbance input vectors or the disturbance vector and what about the output equation Y equal to C x plus D u in this case and of course, the disturbance may makes it appearance in the output equation as well, what will be the output variable for our problem. Well, basically we are only interested in omega, we are not interested in I a. So the output variable will be simply omega and therefore Y will not be a vector really but a single variable omega can be written as C x, so that is easy one 0 omega I a, so 1 into omega gives omega 0 into I a give 0. So I have omega equal to omega plus nothing therefore D is equal to 0 and F is equal to 0, find out what these matrices A, B and E would be if I treat the load torque as a disturbance variable separately from the armature voltage as the as an input variable.

So we have we can write down the equations for the motor control problem in the state variable form or as a set of state variable equations, state equations and output equations
and then, start our study from these equations and not take apply the method of the Laplace transformation that is as we did earlier, take the Laplace transformation of both sides that is functions on both of each equation then, make use of properties of the Laplace transformation then, make some manipulations and finally, we get something like a transfer function and so on that is certainly possible in this case and because the number of state variables may be more than 1, the number of inputs may be more than 1, the number of outputs may be more than 1.

Instead of just a transfer function or 2 transfer functions as we had earlier, one transfer function relating the apply armature voltage the other transfer function relating the load torque to the speed of the motor. In general, we will have what are called transfer function matrices that is we will have a matrix whose entries are transfer functions. So there is more than 1 transfer function because there is more than 1 state variable or output variable and more than 1 input or disturbance variable. Now of course that is one way of handling the problem that is just write down the equations and then, apply the Laplace transformation and get back to the s domain and then use the ideas of the s domain.

Of course it was tempting to do that because the method of Laplace transformation had already been worked out for say, a single input single output with a single state variable as we saw earlier that kind of a system. But then, this is not really a time domain method
it is a transform domain method or a S domain method and of course, ideas which were derived for the single input, single output case could be extended to such cases but what about time domain methods only that is no Laplace transformation will be used and we will not assume that the input u or the disturbance W is a sinusoidal function, what can we say about a such a system.

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Now as I said it is possible to say something, it is possible to infact say quite a lot, it is possible to find out various things about the system explore properties like stability of the system and so on. Of course this requires new concepts and these new concepts as you can guess are going to be some what related to the kind of model that we have namely, we have matrices and therefore a lot of matrix theory becomes applicable here. Matrix theory had been introduced earlier I, we so referred to it to some extent and we talked about the signal flow graph and determinants and so on, matrices are used in that connection there are already being used.

Now they we have to going to be used in the context of differential equation. So when a lot of matrix theory results have been developed or available, one can use them to apply to linear differential equations in the state variable form. Now before we proceed further with motor control problem, now in the time domain formulation let us just briefly take a look at the following, what I have written down in the matrix form is only for a linear system, the matrices A, B etcetera may not be constant matrices that is they may have functions which change the time therefore it can represent linear time varying systems as well but linear systems.

However, one can think of differential equation representation of non-linear systems and even time varying systems and a fairly general way of doing that is as follows symbolically, I will write X dot or the same as D equals f of X u, may be w and t, what do we mean by this, X once again is a vector consisting of several state variables, u is a vector consisting of several input variables, similarly W of disturbance variables, what is this f? Well, this f stands for a vector of functions not just 1 function. So in other words this this represents compactly a set whole set of differential equations as follows X 1 dot let us say the first state variable, its derivative is a function f 1 of all the state variable perhaps X 1 to X n, all the inputs variables u 1 to u p perhaps all the disturbance variable say W 1, W q and also a function of time itself, this is what it meant by function f 1.

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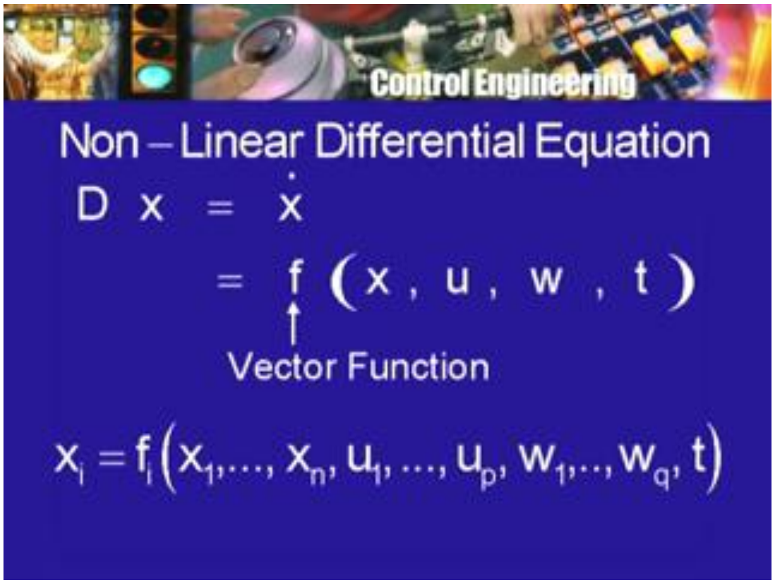

Similarly, X 2 dot will be equal to function f 2 of all these variables and so on. Now these functions f 1, f 2, f 3 etcetera put together constitute the vectors f. So this is a more general formulation, state variable formulation of a system which could be non-linear because this function f 1, there is no restriction put on it. So the system could be nonlinear t appears here explicitly, so the system could be time varying and therefore as you can expect a large number of systems can be represented by a set of first order differential equations like this. So this is the state variable formulation for a large number of systems, the systems which may be non-linear, the systems which may be time varying.

Of course, writing the description is one thing then being able get some information out of it being able to solve them is different thing and as we can expect solving of non-linear systems or time varying systems or getting some conclusions or findings some results about non-linear or time varying systems is not going to be easy because we do not have any longer matrices, a number of the results from matrices are no longer applicable. So some new techniques have to be developed and so on and these techniques where developed because already mathematicians had been looking at this kind of a problem, not a control problem but the problem of solution of differential equations for a long time and had developed some methods which then became or could be applied to our control problem.

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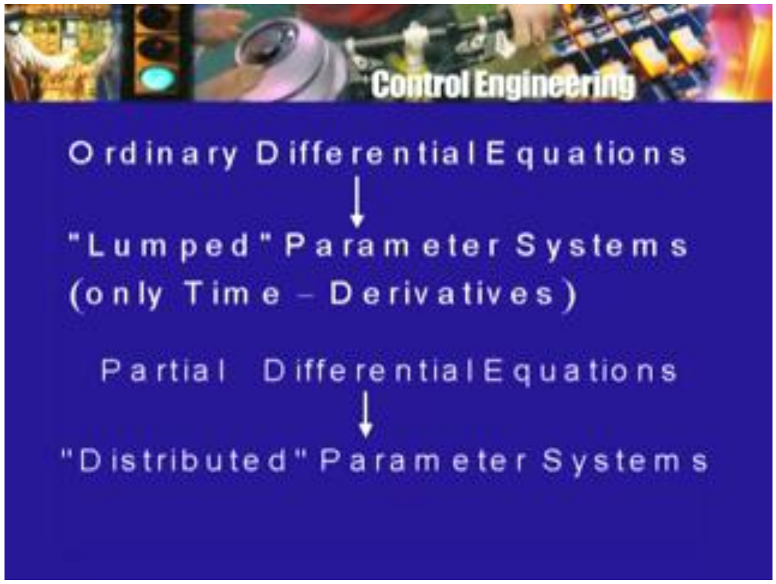

Of course all this is for what may called Lumped parameter systems because was have only ordinary differential equations, there is only one independent variable t, there are finite number of state variables X 1, X 2, X n or whatever number they are Lumped parameter systems. Our motor is an example motor control system is an example of a Lumped parameter system, in the sense we are looking only at 2 state variables, speed armature current and we are having only ordinary differential equations.

I mentioned to you earlier that one can encounter, what are called distributed parameter systems that is systems which are extended in space and cannot be considered to be Lumped. A resistor or an inductor for that matter is also distributed in space but a resistor is represented by a simple relation like v equal to r I, terminal voltage v, current I. Similarly, an inductor is distributed in space but we can represent it by a differential equation like voltage equal to l di dt.

Now of course, this representation ignores the distributed nature of the inductor but in many field problems, we cannot neglect this variation in space or over space region. In many problems arising in process control problems of heat transfer mass, transfer, problems involving flow of a fluids and so on, things vary in time they vary also in space or in other words, the space variables are also equally important therefore, we just do not have one independent variable t, you have additional independent variables and as a results the differential equations that you get are not ordinary differential equations but they are partial differential equations. The subject of partial differential equations as you can expect is more difficult than the subject of ordinary differential equations but one could talk about a similar kind of formulation for partial differential equations as well, although is not as general as one may want it to be.

Let me illustrate the time domain method for our motor control problem, I will not be able to go into full details but I just want you to get an idea of what exactly is involved and how the time domain approach will differ from the s domain or the frequency domain approach. So, we a have pair of differential equations describing the control system and they could be rewritten in the state variable form. Now there is one way of handling the equations in the state variable form which as I said involves lot of results from matrix theory, I will indicate them to you a little later, after we look at a somewhat simpler approach.

Now here are the 2 differential equations which involve armature current and the motor speed omega. The variable of interest to us of course is the motor speed omega. So a natural thing to think of is to eliminate the armature current. In fact, for the steady equations this is exactly what we did earlier to find out the relationship between steady state motor speed load torque and applied voltage. The armature current was not of particular importance although it was relevant because the armature current should not become very large and things like that.

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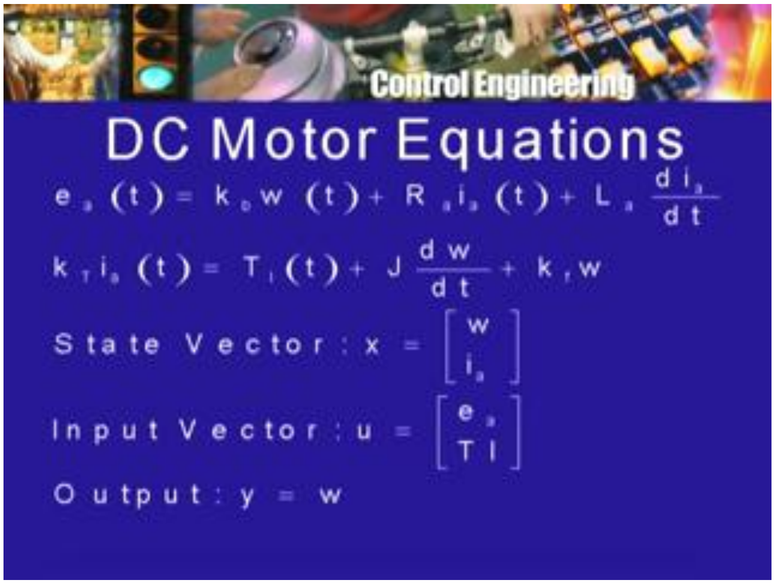

Now something similar can be done with the differential equations that is from these 2 differential equations, we can eliminate the variable I a, the armature current and then what is going to happen? As you can expect we will get a second order differential equation for the speed omega I am not going to find out what that equation is going to be you should try to carry out this elimination for yourself and get the second order differential equation. The elimination is not going to be that straight forward because each equation has both omega and I a in it.

So here is the equation which has derivative of current but it has both speed and armature current. Here is the second equation which has a derivative of speed but it also has the 2 terms armature current and motor speed therefore the elimination is not that simple but it is possible and after the elimination, one could obtain the following type of or kind of equation as I said it is going to be a second order differential equation. So what will it look like it will look like D square omega the second order derivative makes its appearance plus a coefficient let say a 1 times D omega plus a 0 times omega, this is the kind of thing that we will have when we have eliminated omega, what happens to the input terms, we have the input e a, the armature voltage and the load torque T l.

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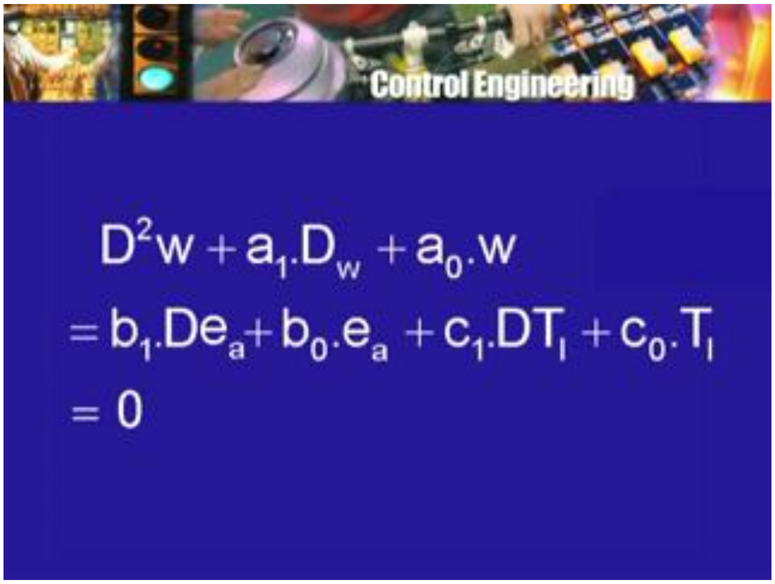

Now, what is going to happen in general is that when we eliminate the variable I a, derivatives of these 2 input variables may also appear and so we may have a set of terms like this b 1 into derivative of e a plus b 0 into E a plus c 1 into the derivative of the load torque plus c 0 into the load torque. So on the left hand side, I have the variable omega and its first and second derivative on the right hand side, I have the input variable ea and its first derivative and the disturbance variable T l and its first derivative. So this is the kind of differential equation that I get and of course, we had looked at such a differential equation earlier, when we got introduced to the method of Laplace transformation and that time we talked about several parts of the solution or response omega namely, the one that corresponds to 0 input or 0 initial conditions and so on.

Now we can talk about this equation in exactly the same way although the method of solution need not be the one based on the Laplace transformation and infact, we had seen the method of solving a differential equation without involving Laplace transforms. So let us assume for the moment that we are looking only at the part of the response that will remain when the applied voltage is 0 and the load torque is 0. For example, motor has been running for some time and then, we suddenly change ea to 0 and you suddenly the change the load torque to 0 that is there is no longer any load and there is no longer any power supply but still the motor was running therefore because of its momentum, it will keep on running for some time. So we imagine that we are studying the motor under that condition therefore the right hand side will become equal to 0 and then as we saw earlier the equation can be rewritten as D square plus a 1 D plus a 0 acting on omega equal to 0 that is we have an operator acting on omega equal to 0 what was this operator called it was called a polynomial differential operator, in this case the operator is of degree 2.

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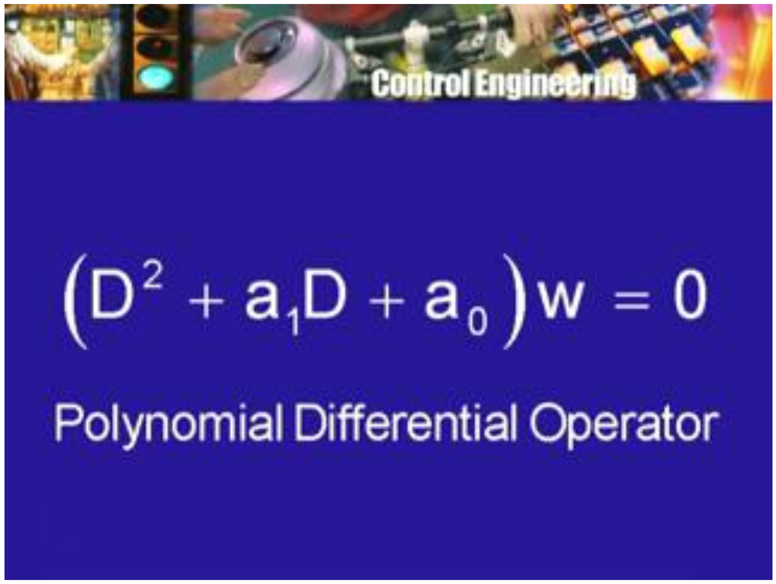

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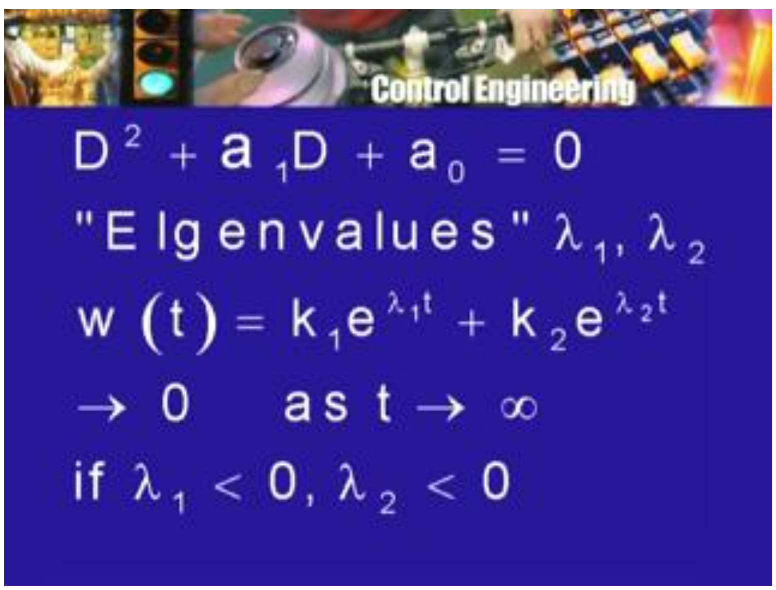

So, what was a way of finding the solution of this, the way to factorize this polynomial differential operator and then make use of certain basic facts like the exponential functions satisfies a simple first order differential equation and so on and then, as we saw the nature of the factors of this or the roots of the equation corresponding to this determined the behavior of the nature of the response. So the corresponding equation as it were would be D square plus a 1 D plus a 0, if we looked at this as an algebraic expression and try to find out the roots of this equation, if the roots of this equation were let us say lambda 1 and lambda 2 then, the solution omega t could be written as k 1, e raised to lambda 1 t plus k 2, e raised to lambda 2 t. This equation of course could be called the characteristic equation or since of course there is an operator D what I mean is an algebraic equation.

So think of D, I just an algebraic variable, so the roots of this equation are lambda 1 lambda 2 they are called the characteristic values or another term that I had introduced at that time was Eigen values and the solution therefore looks like this. In other words, when the motor voltage is switched off the load torque is made to 0, the variation of motor speed will obey a law like this. Of course, it may happen that the variables lambda 1 lambda 2 are not the Eigen values lambda 1, lambda 2 are not real numbers but they are complex numbers, we will looked at those kinds of things earlier but the nature of the solution is given by this and so, we saw that if lambda 1 and lambda 2 have a negative real parts or they are negative real then this will go to 0 a t tends to infinity.

So there will be no problem with the motor racing away that is its speed increasing in theory indefinitely in practice becoming very high that depends on the nature of the Eigen values which interms depends upon the equation D square plus a 1 D plus a 0 equal to 0. Now if we could change this equation in some way then, we would be able to change the behavior of the motor for example, this goes to 0 but not fast enough the time constant that is motor is not small enough. So the motor slows down but not rapidly enough, we may want to motor to come to a stand still as quickly a possible because in the other direction when the motor is turned on, the motor will pick up the speed perhaps very rapidly.

So we may want to change the time constants associated with the motor change lambda 1 and lambda 2 therefore change a 1 and a 0 but how can we do that. Our differential equation is given like, so what is it that we can change. Now this is where we can use some of the ideas from feedback theory that we have studied earlier namely proportional feedback, derivative feedback, integral feedback and so on. Well, we have the input variable ea now and of course, the input variable ea we had put equal to 0 but now as in the case of feedback we could have the input variable coming from not just the applied one was the reference voltage, let us say r t and the other was the signal obtained as feedback based on the speed and this difference could be then e a of t.

So E a of t could be r t minus this feedback signal and even if r t is set equal to 0 the feedback signal is still there and therefore although the external voltage is reduced to 0, internally to the motor the applied is not going to be necessarily 0. So, in other words we can express E a as say r t, r minus what did you have earlier we had the k tacho generator omega of course, there was a gain k that we had introduced because this difference may be small enough and therefore we really had e a equal to k, the gain of the amplifier multiplying r minus k tacho generator omega. So this is what one may think of doing.

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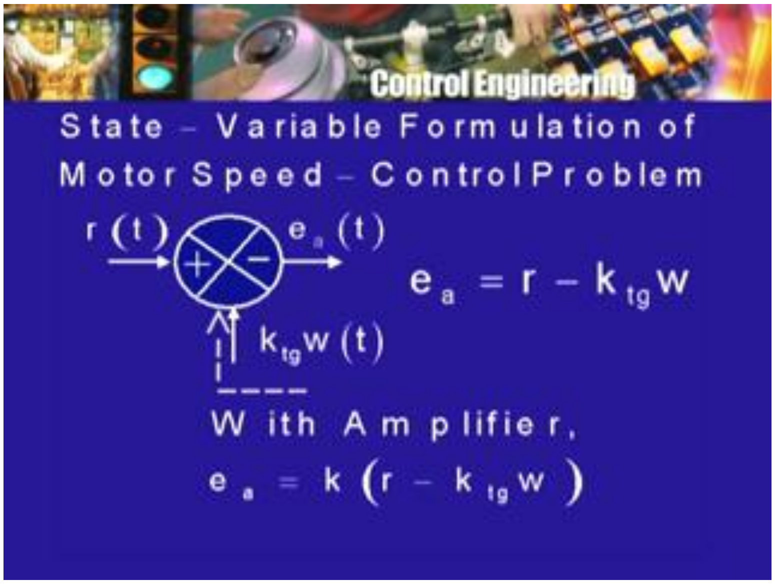

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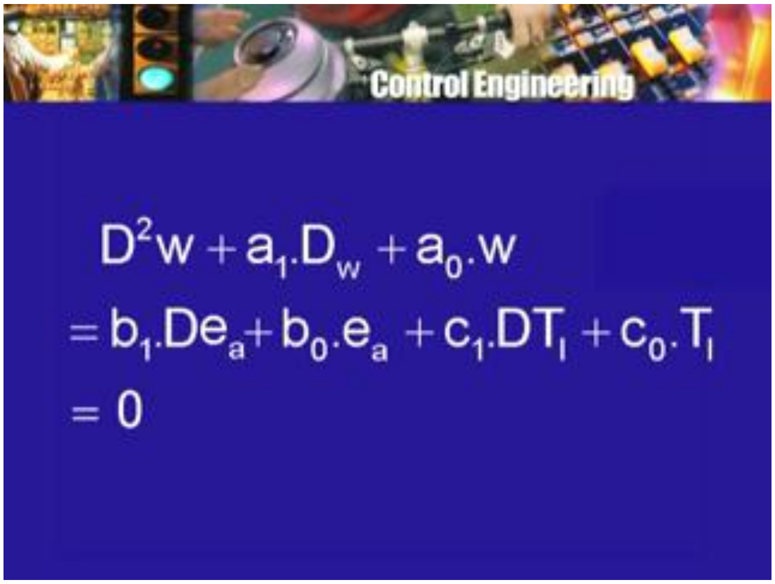

Now, if you take the differential equation that we had here this one and now, E a is going to be replace by or e a is going to be obtained as k time r minus k tacho generator omega. You will see that because there is this multiple of e a here, there will be a multiple of omega which will be introduced on the right hand side because there is a derivative of e a and multiple of it here, the derivative of omega also will appear on the right hand side of course, in addition r and D r will appear on the right side. So what is going to happen is and I am going to show it only symbolically, I am not going to write down the exact expressions we have on the left hand side D square omega plus a 1 D omega plus a 0 Omega.

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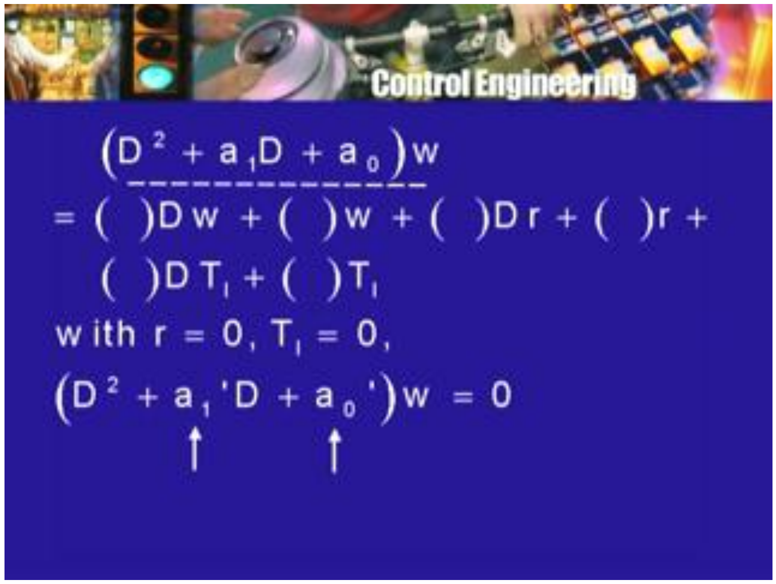

So I am just going to write all that stuff as simply this operator acting on omega equal to what will happen is now because of the feedback e a involving omega, there will be now a term which has D omega on the right hand side, there will be a term which has omega on the right hand plus there will be a term involving D r another one involving r and then, as before the torque turn D T l and T l. So with feedback and what kind of a feedback, it is proportional feedback because we are taking a multiple of the variable omega as the feedback signal and then taking the difference between the reference signal and this and amplifying it to generate the applied voltage armature voltage E a.

So this is what the equation is going to look like and now we can set, once again r equal to 0 T l equal to 0 that is the external voltage is made 0 the load torque is reduced to 0, what will happen now we have these additional terms on the right hand side and of course we transfer these terms to the left hand side therefore the left hand side, now with this change made will look like D square as before but not a 1 but a 1 prime D or a 1 dash D that is the coefficient a 1 will change plus not a 0 any longer but a 0 dash omega equal to 0. So what is it that we would have done in this case, we have changed the operator from D square plus a 1 D plus a 0 to D square plus a 1 dash D plus a 0 dash and how as this change taken place, this change has taken place through the proportional feedback.

Now, what this a 1 dash will be and a 0 dash in terms of a 1 and a 0 and the feedback equation, the tacho generator coefficient and the gain of the amplifier, it will depend on all of these and I would like you to work out the exact expressions for a 1 prime and a 0 prime in terms of the old coefficients a 1, a 0 and the parameters of the feedback namely, the tacho generator coefficient and the gain of the amplifier k but the point is that the differential equation is changed, the characteristic equation is changed and therefore the solution of the equation will also changed.

We expect therefore that the Eigen values will have changed, the solution may still go to 0 as t tends to infinity but it will do in a different way at a different rate and so we can now pose the problem. Well, is it possible by using this feedback choosing k tacho generator and the gain k, to change a 1, a 0 in such a way that the behavior of the system is improved, the response of the system is changed to what we want originally the time constants may have been large typically one of them is large the other one may not be so large, we want to reduce them, can it be done.

We can also ask whether the system can be become unstable by means of this feedback that is the roots of this may become such that they have a positive real part or they become purely imaginary. So the system response becomes oscillatory or even tends to go exponentially, can that happen? The answer in this case is no as we saw earlier proportional feedback using the method of Laplace transform and transfer function we saw and the root locus approach that there will not be any instability but there could be oscillations, there could be oscillatory behavior of the system but damped oscillations. You can check using this approach that that can happen in this approach of course, I have not taken Laplace transforms anywhere, I have only looked at at the characteristic equation of the system then, I know the fact that if I factorize the characteristic equations certain coefficients or Eigen values appear and they determine the nature of the response of the system and making use of that and answering questions which we have looked at and answered earlier using the method of the Laplace transformation.

So this is something I would like you to carry out for yourself and this is an example of a time domain method. Now this is about proportional feedback, what about derivative feedback, can we use derivative feedback, what about integral feedback, can we use integral feedback and as we can see, the things are not going to be very different from, as far as the computations of the work is concerned of course, the result will be different.
So for example suppose I want to introduce integral feedback then, what did we do well we introduced the integrator after the output of the difference device.

So here is r that is the feedback signal which is K tacho generator omega, here was the difference signal and then, we had K divided by S, when we used the transfer function notation, this was the integrator. Now therefore what is it that I am doing, I do not want to use S variable anymore because I am not thinking of Laplace transformation, I am thinking of an operation of integration. So what should we do we let us call this once again, this is still the motor armature voltage e a, let us call this variable, we have to give it some other name, let us call this variable e as it is traditionally done.

Now of course in terms of transfer function e a of S is 1 th K by S into e of S but what about as functions of time, what is the relationship between e and E a, E a is the integral of e and therefore the derivative of e a is e. So D, E a is e, so this error signal e or the difference signal e is the derivative of the output of the integrator, the output of the integrator is the integral of the input, the input of the integrator is the derivative of the output therefore, this is the relationship that holds D e a equal to e, e itself as before consists of in this case of course, I have this factor K, so I have to be careful here, I have to write here D of what will it be D e a equal to k e, e will be equal to r minus k tacho generator omega.

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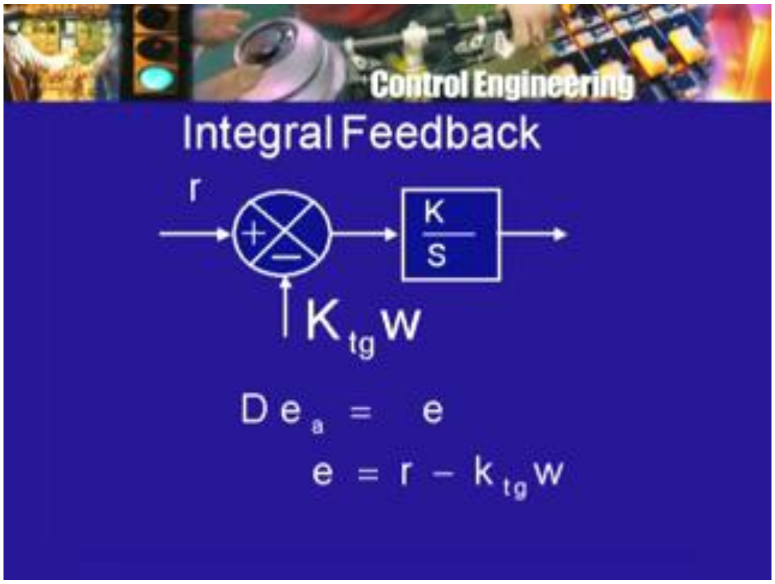

So go back to the second order differential equation describing the behavior of the motor or the speed of the motor in which e a and the load torque terms appeared and now, make use of this fact that we have e equal to r minus k tacho generator omega D E a is equal to k e. Now we have to eliminate the variable e a or we cannot no longer, we any longer set the variable ea equal to 0. So have to do some further manipulations and the nature of the system or the differential equation coefficients will be changed by the presence of this coefficient K and by this operation of integration which of course here appears like a differentiation because I am writing the derivative of the output equal to the input rather than the output equal to the integral of the input.

So just as proportional feedback changed the differential equation coefficients, integral feedback will also change the coefficients of the equation or it may even change the order of the equation and infact, this is exactly what happens, why do I say so, when we looked at the root locus method, what did happen in that case, this S introduced an additional pole in the transfer function. So instead of 2 poles, we had 3 poles as a result of the root locus changed and stability properties also changed.

So what is going to happen here is in this case when we do this that differential equation will no longer be of second order in omega but the differential equation will be of third order in omega. We would like to eliminate the variable E a, set r equal to 0 finally, set T l equal to 0. So obtain that differential equation, check for yourself that it is a differential equation of third order. So the order of the system goes up by 1. So we have more coefficients now but again we can factorize the characteristic equation look at the Eigen values whether see, whether they all have negative real parts or not and so on and so forth.

So this is something I would like you to do and then we will complete example as an illustration of one approach, one time domain approach. The other time domain approach will involve some matrix theory and some matrix manipulations, so I will not be able to go into great detail as per as that other approach is concerned.