Control Engineering

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Lecture - 45

Before closing our discussion of a frequency domain methods. I wanted you to take a look at a page from a book written by Nichols with 2 others in the 1940's where his chart appeared for the first time. I told you that the constant contours that is contours that correspond to various constant values in the DB gain versus phase shift plane are not simple circles, they are curves which look somewhat like ellipses some of them others are open curves and so on.

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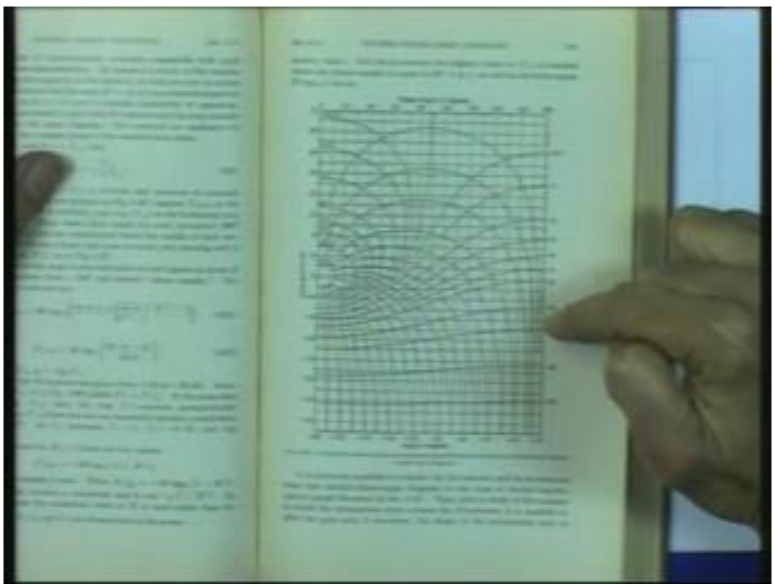

So this is the so-called Nichols chart that I wanted you to see. Now let us move on, I hope by this time the expression frequency domain method is clear to you. The Nyquist criterion of stability the method of drawing the asymptotic Bode plots, m circle, n circle Nichols chart, compensator design using networks, all of these are based on frequency domain methods or concepts that is one is thinking of a sinusoidal function as a typical input function and then, finding out what would be the response to that kind of an input therefore one talks about gain either absolute gain that is ratio of output amplitude to input amplitude or DB gain, 20 log to the base 10 of the absolute gain and phase shift or phase difference between the output and the input.

Earlier to that we had looked at the root locus method which made use of the concept of transfer function and poles and 0s of the transfer function, there is no specific mention of any particular input like the sinusoidal input or even the unit step of the step function input although from the control point of view, one of the things that one likes to study is the effect of the disturbances and a simple kind of disturbance which occurs very often is a sudden change from one value to another value and this is therefore thought of as a step input or a step disturbance but apart from that we did not really look at any other or any specific inputs.

We looked at the transfer function, the poles and the 0s are the transfer function and then, the idea of a characteristic polynomial or a characteristic equation associated with the closed loop system. Its factors are therefore the roots of the characteristic polynomial and their location in the complex plane were all the ideas that we have lead to Evan's root locus method, of course has stability investigation as one part of it. Now what kind of method is that it is not a frequency domain method, it is not quite what is called a time domain method which we look at very soon. It may be called a transform domain and specifically because the transform is the Laplace transform and the preferred symbol for the complex variable there is s and it may be called an s domain method.

So then, we have had s domain methods, the root locus method was a particular example of that then, we had look at the frequency domain methods and now, we will take a look at what can be realistically called time domain methods. In a way, in control system study the time domain methods are should have been the most natural methods to use, why is that so because the earlier control systems going back to the time of the Astronomer Airy and the problem of tracking the stars or James Watt's governor or the steering of shifts that is before servomechanisms and feedback amplifier came on the picture. The system to be controlled was described by one or more differential equations involving derivative and so the system description itself was as one could put it in the time domain.

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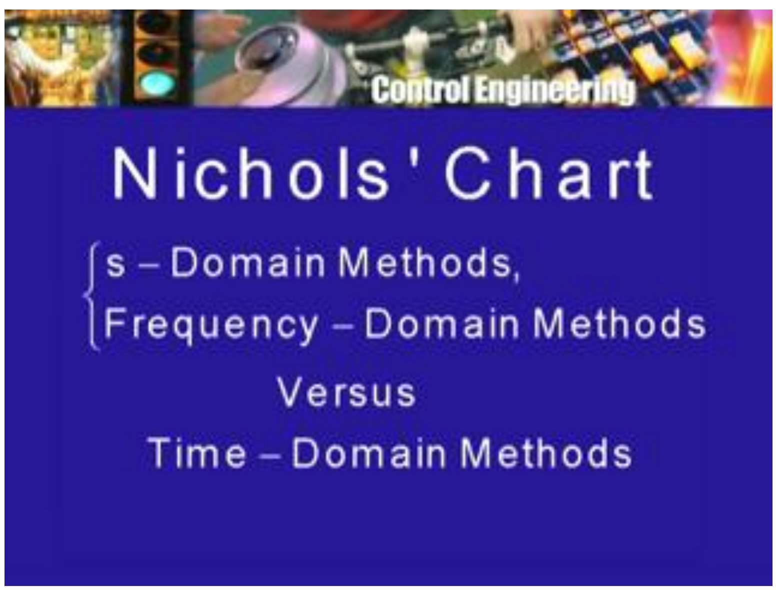

You wrote down the differential equation which involved some parameters of the system and that is what we did for our simple motor speed control problems, subsequently in order to deal with the differential equation making use of some other work or some other areas of mathematic such as the theory of the Laplace transformation, Fourier series, analysis and Fourier transforms and so on, people they relate 2 methods like the s domain method of which the root locus method is an example or the frequency domain method which preceded it of which the Nyquist criterion and the Nichols chart based design are examples. In the recent times and it is not really that recent, nearly 40 years ago investigators went back to the differential equations for several reasons, one of them is that the differential equations that led to concepts like transfer function, frequency response where linear differential equations and time invariant differential equations or as I used an abbreviation earlier, ordinary linear differential equation with constant coefficient. The Laplace and the Fourier transform methods could be used concepts like frequency response could be used for such differential equations only.

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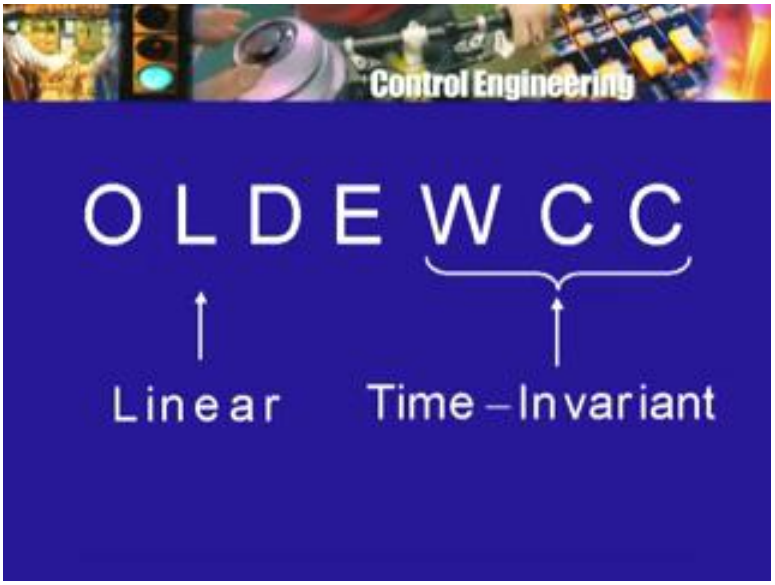

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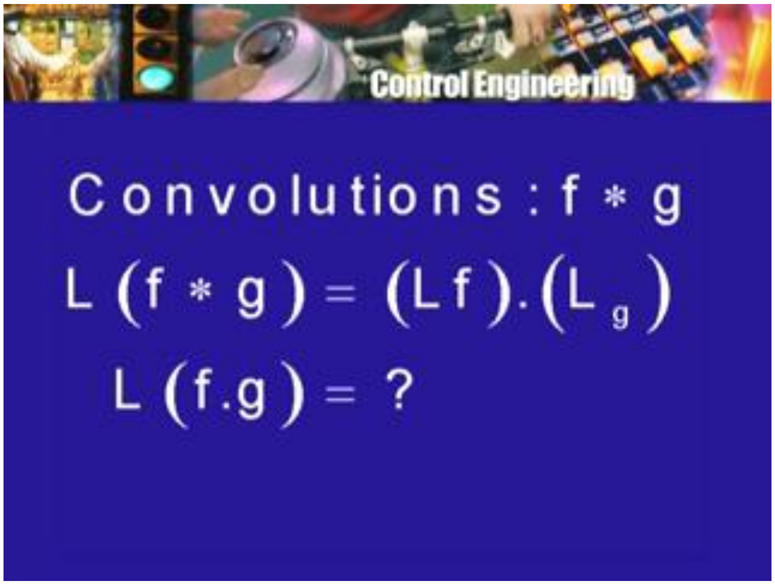

We saw some properties of the Laplace transformation and the first of the properties that we saw was the Laplace transformation is linear, the Laplace transform of the sum of 2 functions, it is the sum of their Laplace transforms. The Laplace transform of scalar multiple of a function is same the scalar multiple of the Laplace transform and the third property of the Laplace transform of course, concerned the derivative of a function, there is no simple theorem regarding the Laplace transform of the product of 2 functions that is if I have 2 functions f and g and I look their product f times g then, there is no nice and simple expression for the Laplace transform of the product of f and g, it is possible of course to write an expression for the Laplace transform of f times g, in terms of the Laplace transforms of f and g separately, but that is a relation which is not very convenient to exploit. Of course, for the Laplace transform of the convolution of 2 functions, this idea I had briefly mentioned the convolution operation is denoted by a star usually.

So the Laplace transform of the convolution of 2 functions of course, has a nice alternate expression namely the product of the Laplace transform of the 2 functions, separately. But in the description of the system the convolution operation does not occur usually, a product may occur but not a convolution. So the usual Laplace transform approach was restricted to linear ordinary differential equations with constant coefficients. The Laplace transformation method cannot be used, in general if the system equations are not linear.

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The system equations one or more are non-linear that is one case where the Laplace transform method will not be applicable, another is with regard to this part of time invariants or with constant coefficients, the Laplace transform method cannot be used if the system is not time invariant or specifically therefore the system is time variant. This may happen because there are coefficients, what look like coefficients which multiply derivative terms which are time varying, so sometimes such equations are referred to as differential equations with time varying coefficients.

So if the differential equation is non-linear, if the differential equation is time varying or if it is both then the Laplace transformation approach will not work, neither will the frequency response approach work. For example, if the input to a nonlinear system an non-linear differential equation, say is a sinusoidal function the output or the response of the system in general will not be sinusoidal and therefore, you cannot talk about amplitude of the input and amplitude of the output because although the input is sinusoidal, the output is not. So the idea of gain is not applicable phase shift is not applicable which means you cannot do anything more in that direction and of course, it is known or it was known that many systems can be modeled only if you are willing to consider non-linearity or the time varying nature of the system that is there are many systems, some of them are have been known for a long time by system I do not necessarily mean a system which is a control system involved in a particular control application. I, now mean here any physical system so non-linearities cannot be ignored. It was known, say even 100 years ago and at that time also people had tried to develop methods for handling systems with non-linearities.

You might remember a chapter on differential equations in your mathematic courses where you may have looked at and solved some specific non-linear differential equations which give rise to or gave rise to some different kinds of functions. For example, Bessel's functions they arise when you investigate a non-linear differential equation of a particular type. But a simple example of a non-linear system will be a pendulum and this of course goes back to your early college days, when one deduced the formula for the period of oscillation of a pendulum 2 phi square root of l divided by g that derivation made the assumption that the angle of swing of pendulum was small, if the angle was called theta then theta was small and how small, well small meaning sign theta which occurs in the differential equation could be replaced by theta.

So the pendulum equation is actually non-linear, may be you should write down that equation starting from first principles consider, general position of the pendulum at an angle theta to the vertical look at the forces acting on the pendulum, the force of tension and the force of gravity and the resultant force being the force that accelerates the pendulum and you can then write down the differential equation for the angular position theta and you will see that it is non-linear. So there were physical systems as simple as a pendulum for which it was known that the description is non-linear, the pendulum can be described by an equation which is non-linear although approximations can be made.

Now after the success of the s domain and the frequency domain methods in the 40's and 50's, there arose some problems where you could not work with such approximations to non-linear systems or it was no longer acceptable to make those approximations. The requirements for precision had increased tremendously and therefore, interest went back to the study of differential equations. Now in this context first of all let us see exactly what we mean by a non-linear system.

Now there are 2 ways of defining a non-linear system, there is one way which is a little abstract in the sense I do not write down any particular differential equation and then, say that it is linear or it is non-linear but rather, I consider the differential equation and as we saw for a physical system or a differential equation that describes a physical system, you have something called an input something which you can vary over which you have some control and some other function or variable called the output or the response which you do not directly manipulate or control, you do it through the system.

So one can think of a system as a relation between inputs and outputs, very crudely we say input produces an output or this is a response to that particular input. So, I do not even have to say that there is a differential equation there there is some differentiation action taking place or what have you, this is sometimes referred to as a black box approach to a system, there is an input variable, there is an output variable and there is something which has been put together such that I can change the input and I will get a change in the response. So this is a very broad concept of a system, what is inside, how do I describe it are there resistors motors pipes whatever of course, for a given practical situation I have to know what is inside and that is why actually this is an abstract approach.

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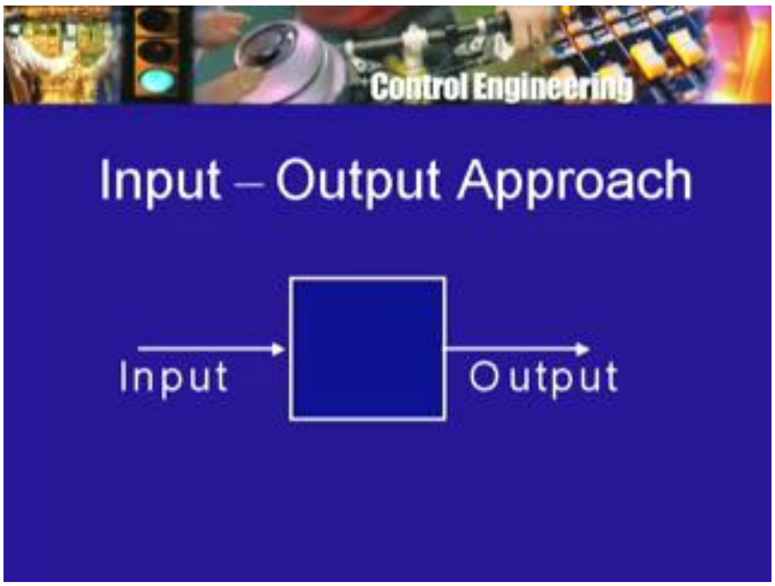

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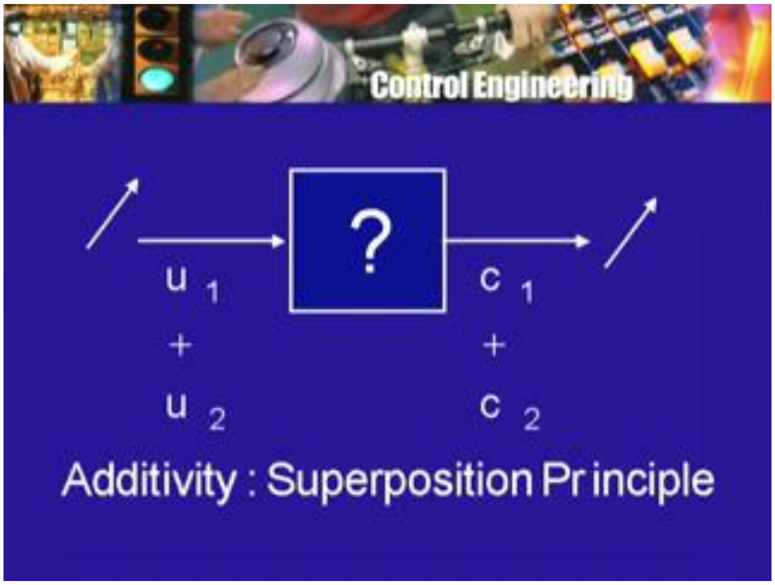

We will concentrate only on the input and the output and the relationship between the 2 specifically each particular input produces some definite output. With this concept of a system one can define linearity quite easily and in fact, when we talked about the Laplace transformation and its linearity or the 2 properties something exactly like this was being done. In other words, you have let us say an input u 1 which produces a response let us call it using control literature c 1. So u 1 is the input corresponding output is c 1, suppose we apply some other input u 2, it results in a different output c 2.

Now, if the system is such that applying u 1 plus u 2 produces a response c 1 plus c 2 then, we say that the system is additive or we say that the system obeys or satisfy the superposition principle. So, this is one part of linearity additivity or the superposition principle note that we do not apply 2 inputs together, what we apply is we apply input u 1, look at the response then change the input to u 2, look at the response and then, go back and apply a new input u 1 plus u 2. The sum of the 2 inputs and see if the response looks like the sum of the original responses, the system still has 1 input, 1 output.

Now, if for some choice of u 1 and u 2, this fails to happen then we will say that the system is not additive or the system does not obey the superposition principle. So this is one part of linearity or correspondingly one part of non-linearity, a system is additive or a system may not be additive, a system obeys the superposition principle or it may not obey the superposition principle. The second is of course the scaling property or homogeneity. Here, what do you think of again one input let us say one output and of course and some of you who will go further from this course will realize that when I say one input, I can think of several inputs as one vector input.

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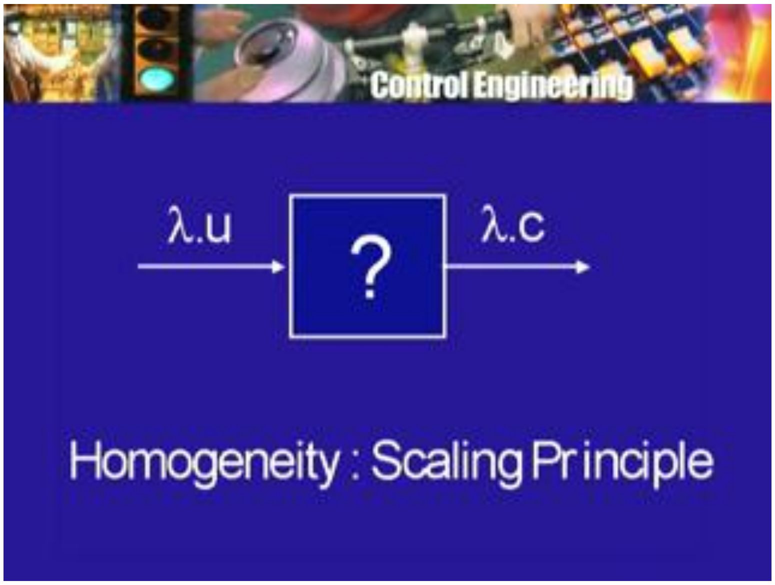

Similarly, when I say 1 output it need not be only one variable it could be a whole set of variables being considered together, so vector input and a vector output. So that is also possible, so I apply input u I get a response c then I scale by sum factor lambda, do I get an output which is also scaled by the same factor, if this is going to happen then the system is homogeneous or has the scaling property and if a system has both additivity and homogeneity then, we say that the system is linear. So, if we say that a system is non-linear what we mean is that one of these 2 properties or both of these properties do not hold.

Now this does not require you to know what is inside although very often to find out whether a system has this property or not, you will have to know what is inside but on the face of it and in a very simply way, you can sort of make an assessment whether a particular physical system is say homogenous by applying some input to it observing the output then, doubling the input see whether the output is doubled, if the output is not quite double of the original output then, the system does not have the homogeneity property.

Similarly, you could check additivity sort of experimentally something has input in a box and I have only an input and an output which I can measure and observe and these are the things that I do with the input and output. In principle, I can find out whether a system is additive or homogenous or linear or not. Of course, I may scale it by factor of 2 the output may be doubled but if I scale it by factor of 10, I may find that the output is not ten times, why does this happen or when this happens in what cases familiar to electrical engineers can this happen. You have a magnetic field which is produced in the presence of say, iron its particular current produces a particular flux density.

You double the current , the flux density may get doubled but if you scale the current by a factor of 10 or 100, the flux density may not get scaled by the same factor and so, we call this phenomenon, the phenomenon of saturation. So, if a device shows saturation then certainly it is not homogenous to go back to the example of the pendulum, scaling the swing of the pendulum of course that is not quite a system but a large swing and a small swing, things are not necessarily, the same.

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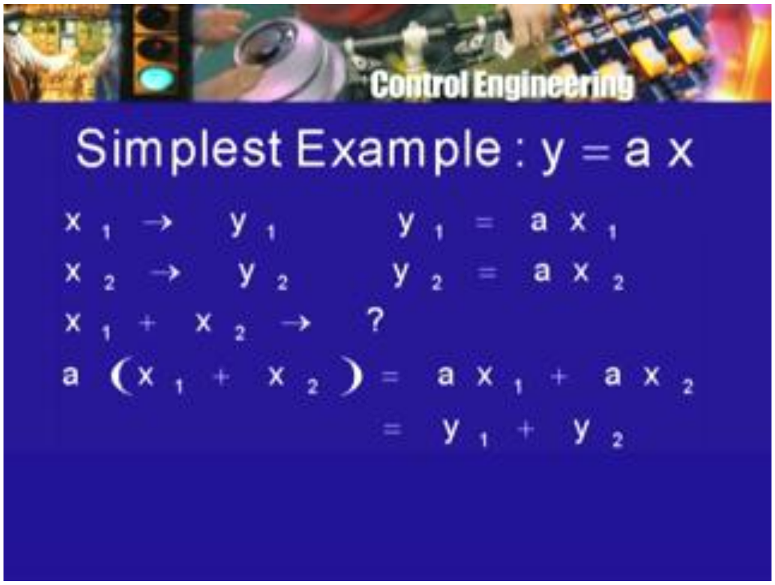

Now of course, if you are not going to follow this kind of an experimental approach to find out whether a system is linear or not and you look at the description of the system that is what is inside and you write down a set of equations then, how do we find out from them whether the system is linear or not. This concept of linearity is more familiar to all of us because we encountered that fairly, early in fact in school itself. When one talked about simple equations which were called linear equations or linear algebraic equations and the simplest of course will be simply say, the equation y equal to ax, say x is the input, y is the output a is simply some coefficient a scale factor and if this is the description of a system and do not think that there is no system for which a description as simple as this possible. A potentiometer or gain coefficient implemented is some physical way is exactly of this kind output is just a constant multiple of the input. Now is this linear, is this additive, does it have the homogeneity property.

Well, from the description, we can determine without much difficulty that it is so, so let the input x 1 produce the output y 1 which means y 1 is equal to a x 1, the input x 2 produces the output y 2, so y 2 is equal to a x 2. So one input produces 1 output another input produces another output, think of the sum of the 2 inputs as being applied. So the new input is not x 1 plus x 2 what is going to be the new output, the new output is a times x 1 plus x 2 but that is the same as a x 1 plus a x 2 but a x 1 is y 1, a x 2, y 2, so the new output is the sum of the 2 outputs. So this system or one can even say now because this you may not think of system, we may think of the equation itself, this equation is additive has the property of additivity.

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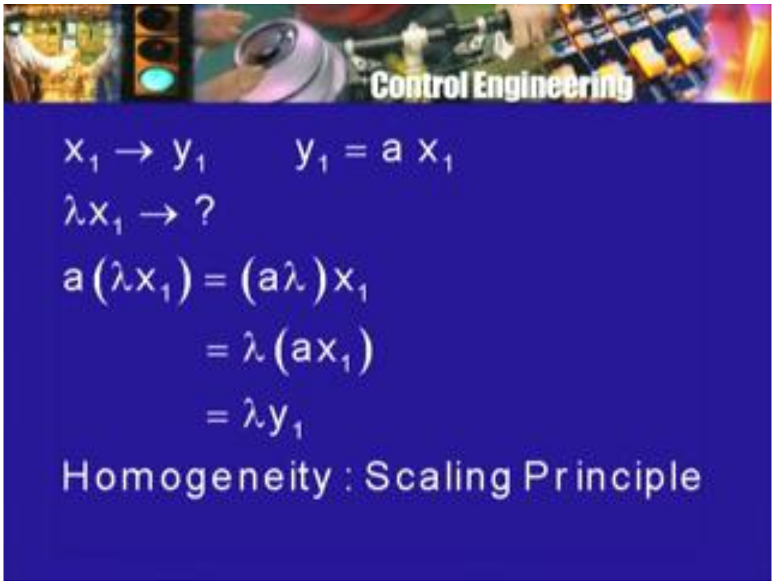

Similarly, with regard 2 homogeneity, if x 1 produces y 1 and therefore y 1 is equal to a x 1, what about lambda x 1, what will it produce. Well the output will be a times lambda x 1 but this can be rewritten as lambda times a x 1 or equal to lambda y 1. So, if the input is scaled by lambda the output is going to be scaled by lambda. So the system or the equation is homogenous and therefore the equation is linear, this is how in fact, we learned to use the word linear or we learned to recognize equations which were set to be linear. Notice that we can talk about a system being linear and an equation being linear and the 2 are not quite the same.

We can take another example and see that an equation can be found out to be not linear or non-linear, take the simple equation say y equal to a x square, x is the input, y is the output such a thing could be called a squaring device or power law device, the power law 2 and again, there can be some physical system for which this is a good model, y equal a x square, squaring devices, is this equation linear, is it additive, is it homogenous. Now one can quite easily verify that it is not so. For example, homogeneity, so x 1 produces y 1 or y is therefore a x 1 square what is the output for lambda x 1, lambda x 1 produces what output, lambda x 1 produces a times lambda x 1 whole squared but this is a times lambda square, x 1 square or it is lambda square times y 1.

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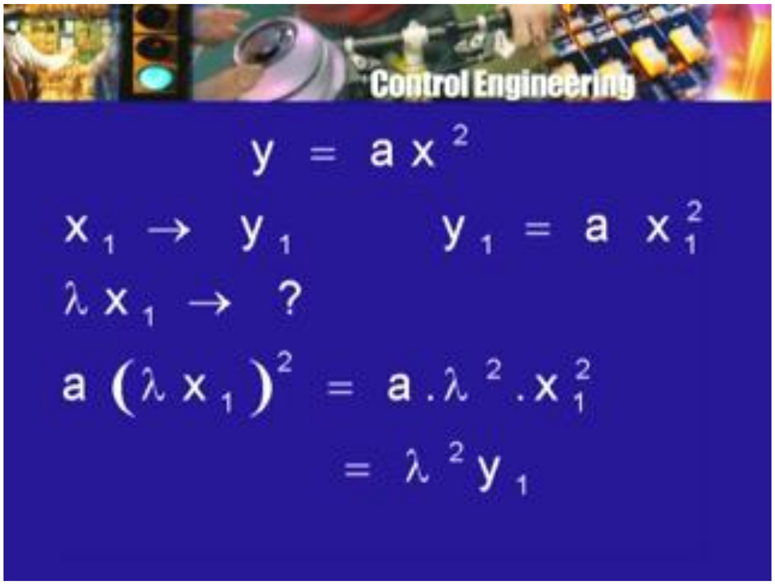

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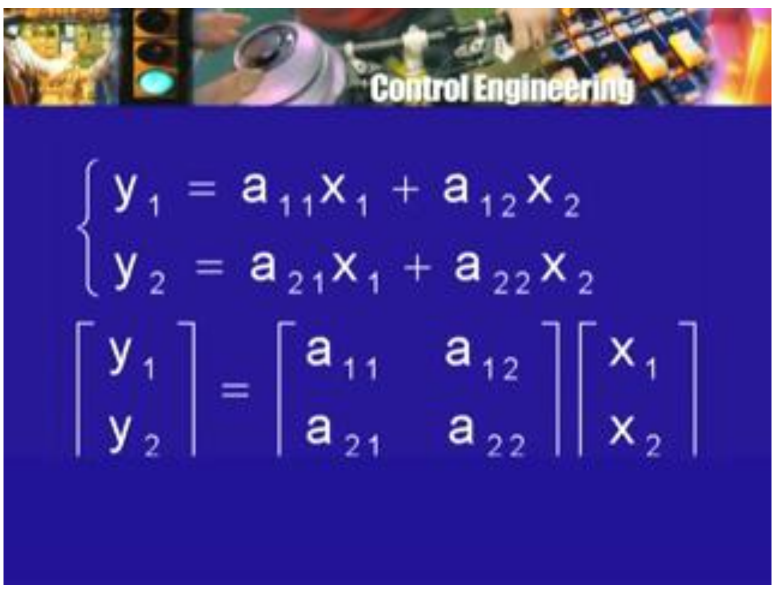

So, scale the input by lambda, the output is scaled not by lambda but by lambda square. So the system is not homogenous or the equation is not homogenous. Similarly, you can verify that the equation is not additive and therefore when we see an equation like this y equal to x square having gone through this experience. We see that no, this equation is not linear whereas the equation y equal to a x is a linear equation. So this is how we learnt to recognize or this was our first exposure to linearity and non-linearity. Of course a system may have 2 inputs, it may have 2 outputs the relationship may be more complicated but again one can by working out see that the system equation or description is linear or not. Let me take an example of a system with 2 inputs and 2 outputs and this kind of a description of course is already familiar to you, 2 inputs x 1, x 2, 2 outputs y 1, y 2 and there are given by 2 equations of this kind and as you know, one learns to write this in a more compact form or it is not really compact but it leads to very new ideas in the form of a matrix of coefficients. Here are the 2 outputs y 1, y 2 now put together as a single vector. So I think y 1 and y 2 as 1 but with 2 parts y 1, y 2, x 1 and x 2 has one with 2 parts x 1 and x 2. So here is the input, here is the output and here is the matrix of coefficients that is involved in the relationship.

Now it is a simple exercise to find out that then, the 2 inputs x 1 and x 2 are given some values and you have corresponding outputs then, you change the values given to the inputs, find out some new outputs and then, superpose them then, what happens, see easy to verify that additivity holds that homogeneity holds therefore the set of 2 equations or the input output relationship given by these 2 equations is also linear. So one linear equation, several linear equation, one recognizes it by the appearance of the equation, one need not worry about what physical system is it for which this equation is a good approximation or holds and so on. So, linearity of the equation rather than linearity of the system this is something also in fact, this what one learns earlier, linearity of a system is more advance and as I said the little more abstract concept. Now just like linear algebraic equations, you have learnt to recognize linear differential equations. For example, the differential equation and I am going to use the dot on the top of a variable to denote derivative.

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We have also seen earlier other ways of indicating derivative of course D by DT is the first method or way of denoting a derivative but I have also introduced the symbol capital D as denoting the operation of differentiation. So I can write down an example of a differential equation which is linear and will verify that it is linear although by its appearance, you know that it is linear.

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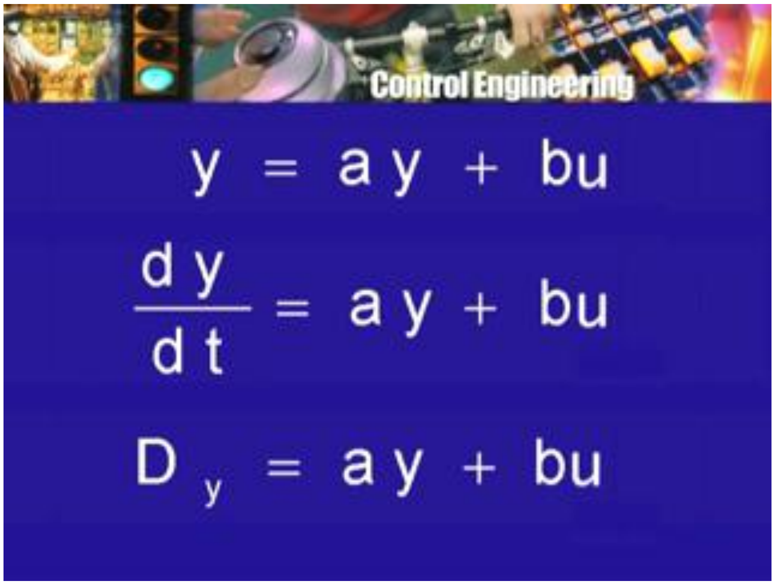

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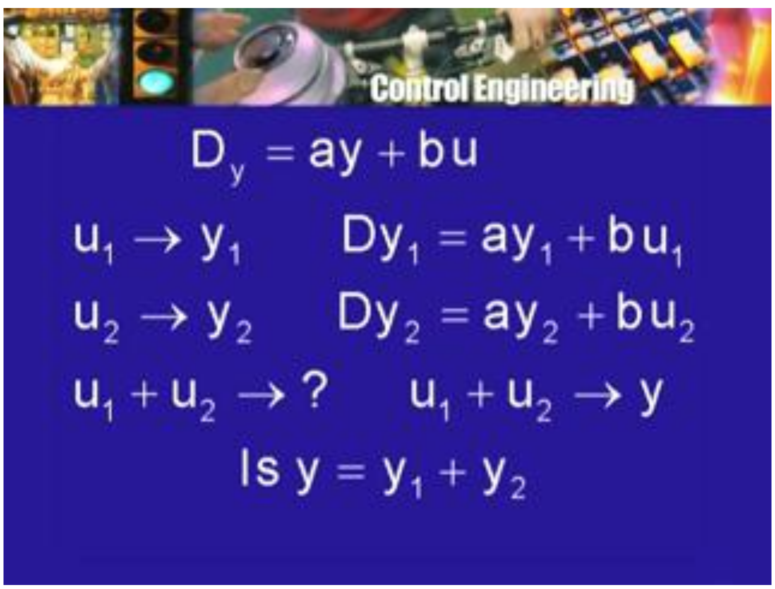

So, let us say we have an output variable y, so somehow in the system description its derivative appears. So y dot equal to let us say a y plus b u, u is the input y is the output in the description, a derivative of y appears, y dot equal to ay plus b u or if you raise dy by dt equal to a y plus b u or D acting on y operating on y equal to a y plus b u. So let us say this is the description of a system, is the system linear or the way we are going to find is not by giving inputs and measuring the outputs but by looking at the system description or equations. So is this equation linear, how do find out, well we apply exactly the same kind of test. Consider an input u 1 corresponding output y 1, so since y 1 is the output, what must be true D y 1 equal to a y 1 plus b u 1, for an input u 2 suppose the output is y 2 therefore Dy 2 must be equal to a y 2 plus b u 2. So u and u 2 are 2 inputs for which the corresponding outputs are y 1, y 2 and these 2 inputs and output satisfy these 2 equations. So now we think of an input which is the sum of the 2 inputs, so u 1 plus u 2, what is the corresponding output? Let the corresponding output be denoted by y, so I have to find out what is this y and I am going to ask whether this y is equal to y 1 plus y 2 or not if I can show that it is y 1 plus y 2 then, I have shown that the system or there differential equation is additive. But what what equation does y satisfy y is the output for input u 1 plus u 2 therefore D y must be equal to a y plus b u 1 plus u 2 right.

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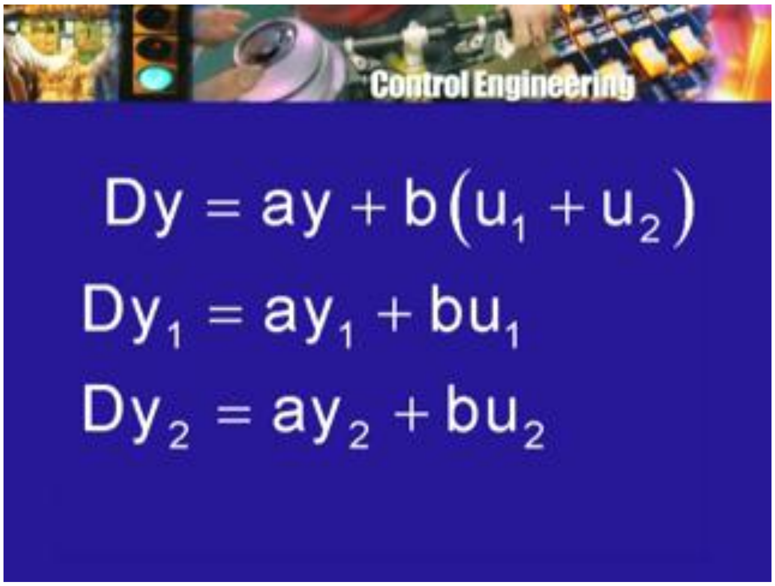

So this is what I have D y equal a y plus be into u 1 plus u 2 and I have the 2 equations for y 1, u 1 and y 2, u 2 from that now, I have to try to find out or try to show whether y equal to y 1 plus y 2 or not and you will see that there are some conditions or assumptions which I had mentioned earlier, do come into the picture now. I will write down the equations for y 1 and y 2 below, so I have a y 1 plus b u 1 and D y 2 equal to a y 2 plus b u 2. So these are the 3 equations for y 1 and y 2 and y inputs u 1, u 2 and u 1 plus u 2.

Now the appearance of the equations suggest that we can do the following, we can subtract from the first equation, the second the third equation. So if I do that I will get D y minus D y 1 minus D y 2, D y minus D y 1 minus D y 2 equals a y minus 1 minus a y 1, a y 2 and what happens to u 1, u 2. Well there is b u 1 here that cancels with b u 2 cancels with b u 2, so there is no u 1, u 2 appearing in the equation. So I have this equation and since, I am asking whether y is equal to y 1 plus y 2, let me call the difference between y and y 2 or y 1 plus y 2 z and so I am asking the question this is the z is 0 or not.

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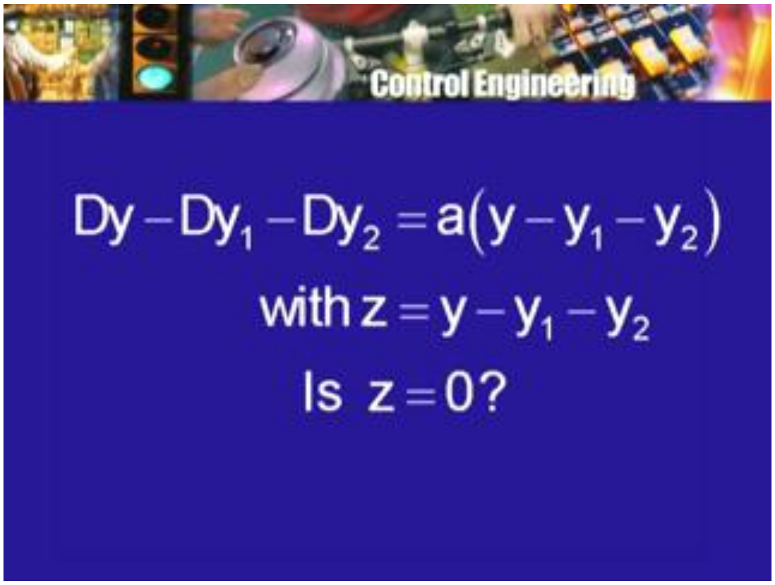

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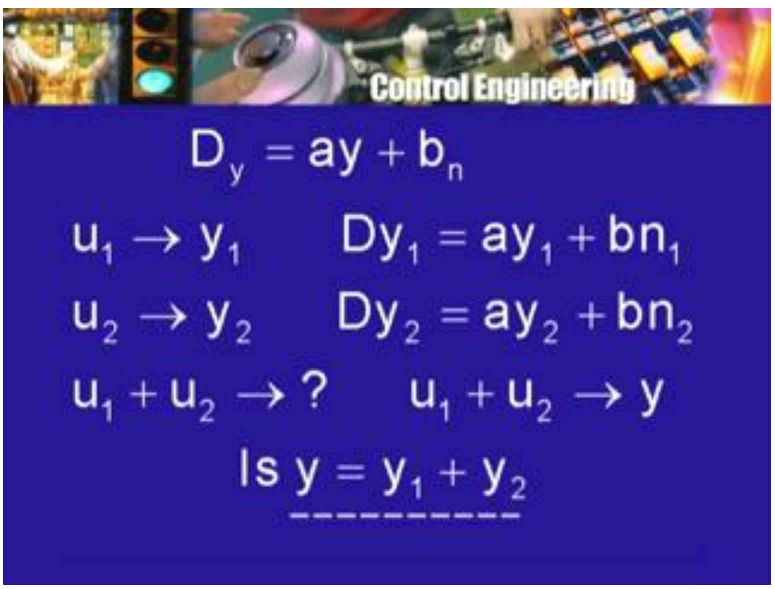

Well, we almost have an equation for z, we have the equation D y minus D y 1 minus D y 2 on the left hand side equal to a y minus a y 1 minus a y 2 on the right hand side. Now, we make use of a property of the derivative operation which in turn is a linearity or an additivity property. The derivative of the sum of 2 functions is the sum of their derivative or the derivative of the sum of 3 functions is the sum of their derivative or the derivative of the negative of a function is the negative of its derivative. The first 2 are examples of additivity, the third one negative being minus 1 times the function is an example of homogeneity.

So, here we are making use of the fact that the derivative operation is linear and therefore D y minus D y 1 minus D y 2 can be written as D of y minus y 1 minus y 2 and on the right hand side I have a y minus a y 1 minus a y 2. So I can write this as a multiplying y minus y 1 minus y 2 or since, I have called z, I had D z is equal to a z. So whatever this difference between y and y 1 plus 2 may be the z satisfies the equation D z equal to a z. Now from that does it follow that z is equal to 0 for all time t, the answer is no because we know their solution of this simple differential equation. You remember, in our discussion in the beginning, we saw or I reminded you that a solution of this differential equation a general solution of this differential equation involves the exponential function.

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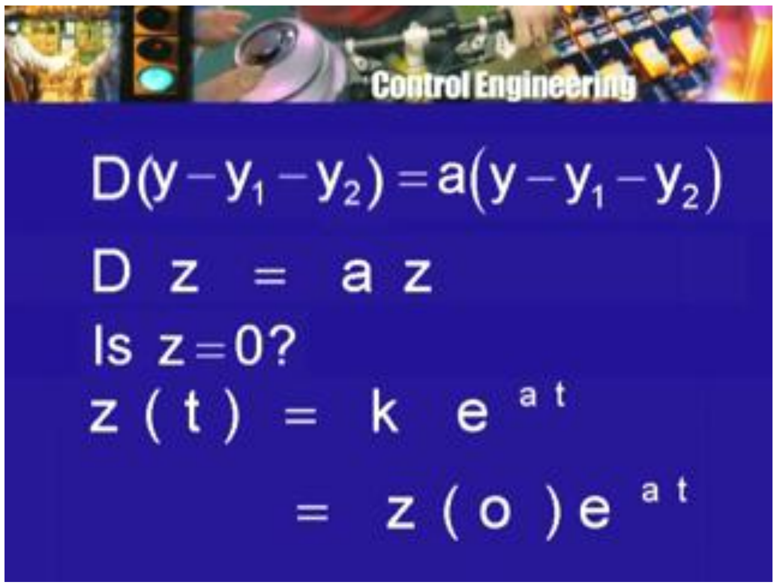

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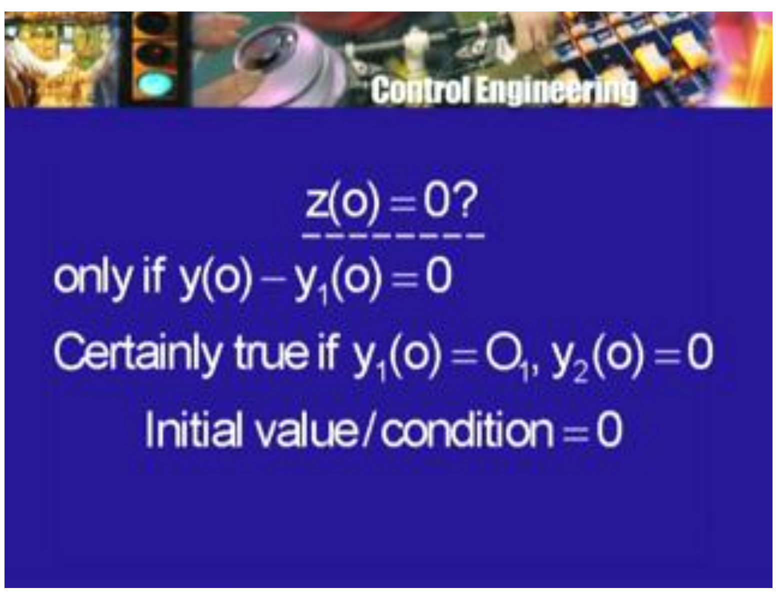

So the solution is actually z t equal to k times e raised to a t or by putting t equal to 0, we can see that z t is equal to z 0 into e raised to a t. So z t is equal to z 0 into e raised to at and therefore z t is not 0 unless z 0 is 0. Now this is something that brings us to an important consideration which we had seen earlier, reference to initial conditions that is when we say that this differential equation is linear, we will have to say in addition that under certain initial conditions. For example, what does it mean to say that z 0 is equal 0, when z 0 is actually what y 0 minus y 1 (0) minus y 2 (0).

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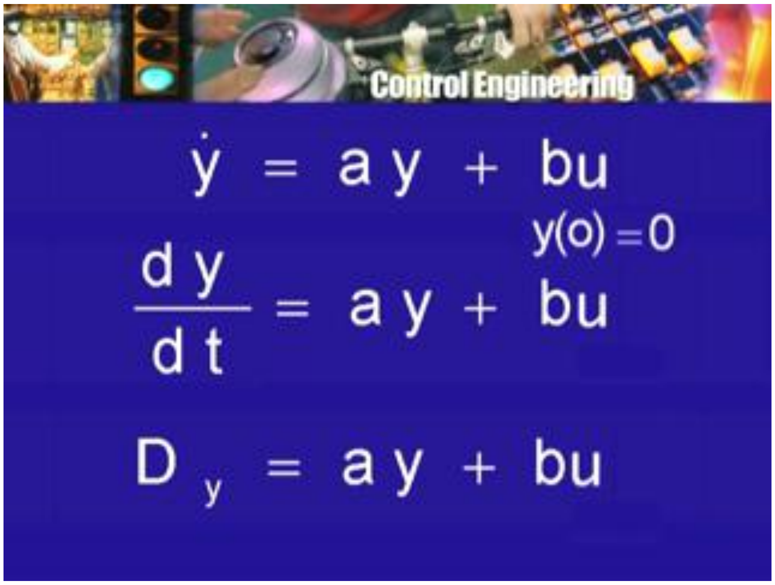

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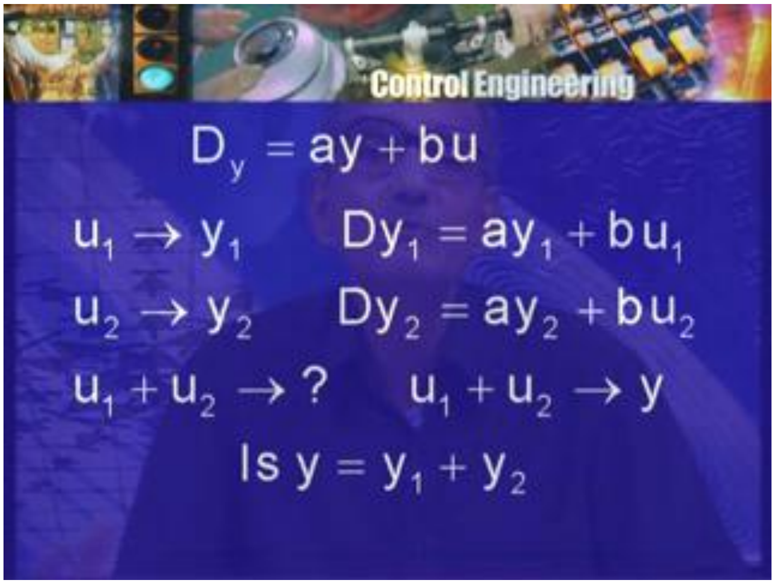

So this must be equal to 0 and one simple way in which this can be made 0 is by making all of them 0. So in other words now, we have to say that okay I am going to look at the differential equation y dot equal to a y plus b u but I will consider, it only under the condition that the initial value of the response is 0, y 0 equal to 0. So under 0 initial conditions, I will study or I will look at the differential equation. For one input I get one response, for another input another response. In both cases, the initial value of the response must be 0, third input u 1 plus u 2, third response again the response must be put to 0, in some way or the other. If that is done then, the system is additive or the differential equation is additive and of course, when you use the Laplace transform approach one sees that more explicitly because in the derivative property of the Laplace transform. The initial value makes its appearance, so the differential equation is additive with this restriction, is the differential equation homogenous. Now you can see that that also is going to involve this assumption of the initial value being 0.

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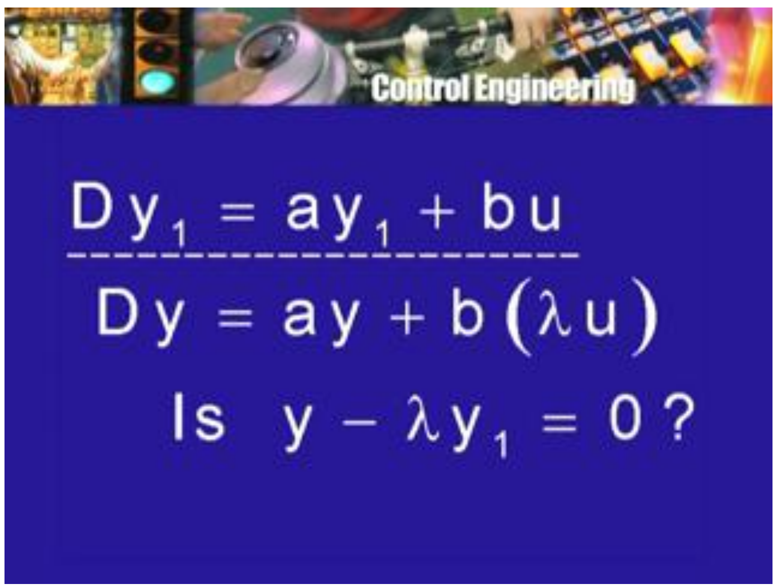

So D y is equal to a y plus b u with input changed to lambda y, is the response or this D y 1 equal to a y 1 plus b u, the response to lambda u is y. So D y is equal to ay plus b into lambda u is the difference y minus lambda y 1 equal to 0 if I call that difference z then, I will get the differential equation D z is equal a z once again and this has the solution 0, only if z 0 is 0. So it means for homogeneity to hold also the initial value of the response must be equal to 0. Of course we saw earlier that when we did the Laplace transformation technique that you could talk about components of parts of the response that is the response although physically, it is not a sum of 3 separate things, mathematically or in our model, we can think of it as coming in, coming from 3 different parts or 2 different parts. If you remember, I talked about 0 input response and 0 state response, response when the input is 0 but because the initial conditions are not 0, you get a non-zero response.

On the other hand, the initial conditions are made 0 and input applied you get a component of the response, what I am talking about right now therefore is the 0 state response under initial conditions being made 0, the relationship between response and input is linear. Of course one learns very quickly to recognize a linear differential equation like this as an equation which looks like this. In fact, this is how students answer the question now for them the concept of linear is the appearance of the equation and not input output relationship.

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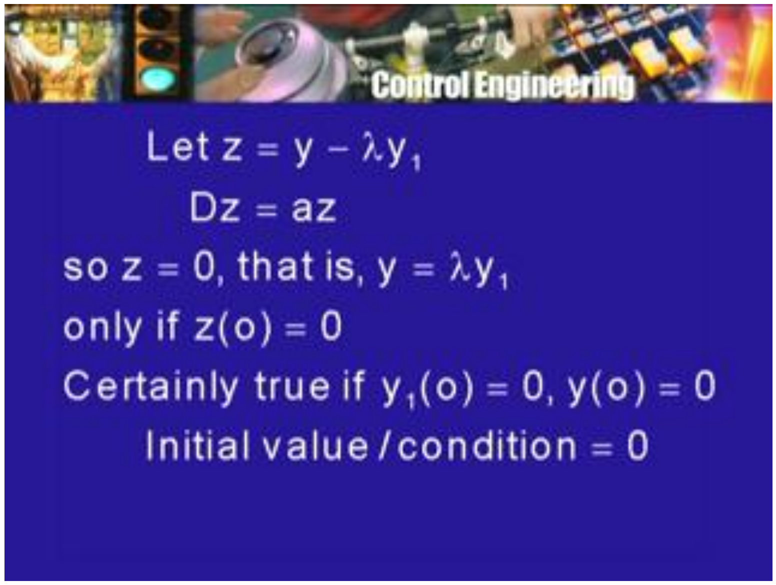

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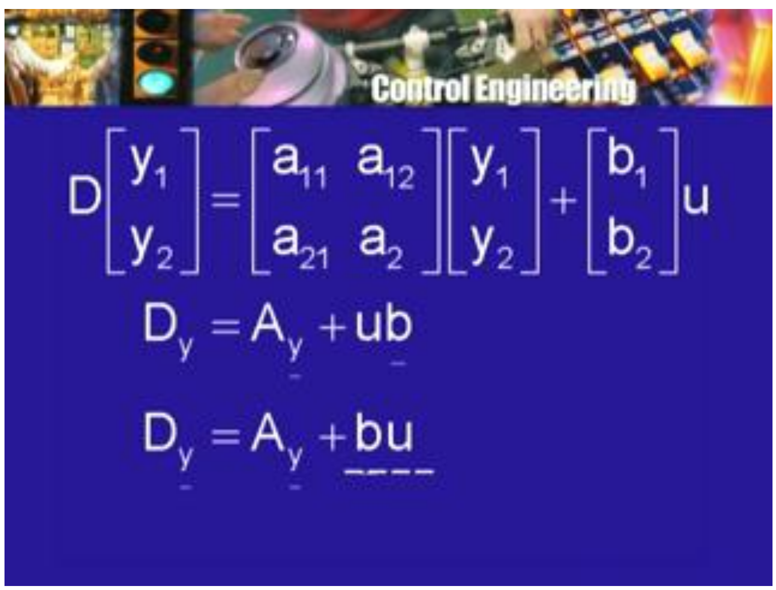

Of course, this can be extended to several differential equations in involving only 1 input or more than 1 input and this is again something which your control theory book has a lot of discussion of for example, I may have 2 outputs, 1 input and a relationship which looks almost like what we wrote for example, D y 1 is equal to a 1 1, y 1 plus a 1 to y 2 plus b 1 u, D y 2 is equal to a 2 1, y 1 plus a 2 to y 2 plus b 2 u or by combining or thinking of the 2 y 1 and y 2 together and parts or components of what is called a vector, I can write this as D acting on y 1, y 2 equal to a matrix again a 1 1, a 1 2, a 2 1, a 2 2 acting on y 1, y 2 plus b 1, b 2 multiplying u or u multiplying b 1, b 2 or to use a further compact notation this can be written as D y and one may sometimes underline y to to indicate that this not 1 but this is several like y 1, y 2 equal to some symbol that denotes this matrix a fairly standard symbol is capital A, multiplying the vector y plus again a symbol for the vector consisting b 1 and b 2 which is multiplied or scaled by u and therefore b u.

Although this u is a scalar multiple, so usually it is written in front of b but in control literature this is a fairly standard way of writing. So, this is in fact a linear model of a system with one input with may be 2 outputs in this case or this y may consist of 3 components or whatever. So again one recognizes by the appearance of the equation that the equation, is linear. It is not so easy to show that something which does not look like this is in fact non-linear, for example let me write down the differential equation y dot equal to a y square instead of y dot equal to a y or y dot equal to a y square plus let us say b u.

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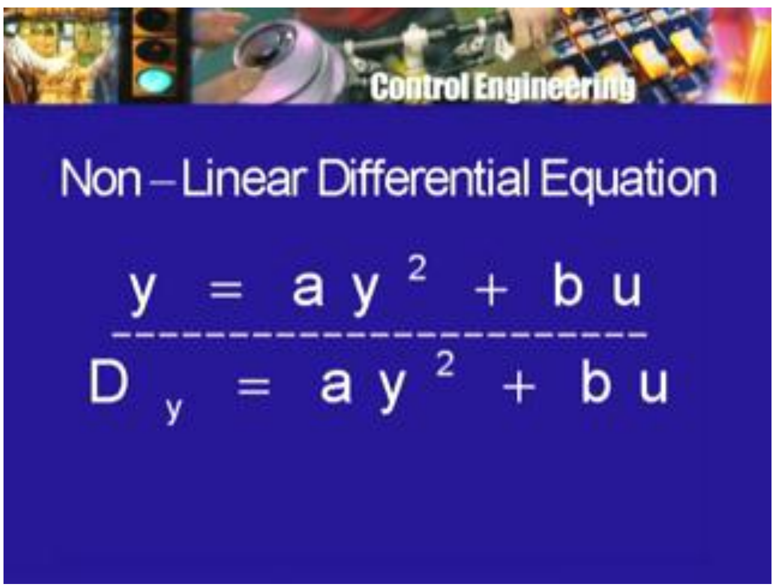

So there is an input u there is a response y and there is this differential equation or using the denotation D y is equal to a y square plus b u. Now you will see this does look like a y plus b u therefore it is non-linear, now that would not be a good answer or a really a correct answer because I would likely you to show that this is non-linear because homogeneity fails or because superposition fails. So you have to give me an example of an input and corresponding response and a scaled version of the input the response fails to be the corresponding scaled version of the output. You must either give an example or by looking at the equations and by manipulating them you must be able to show to me that this is the case. For example, if I choose an input y 1, if I choose a response y 1 corresponding to an input u 1.

So I have this equation and then, I take about scaling. So the corresponding response let it be y equal to ay square plus b into u 1 is now lambda u 1 then, I have to show that y will not be equal to lambda y 1. For that I will have to manipulate these 2 equations somehow and show that the difference between y and lambda y 1 is not 0. Now in this case it is not too difficult although it as we will see does involve some trouble, so going by what we did earlier I will take this equation and from that I will subtract lambda times the first equation and calling the difference y minus lambda y 1 equal to z then, I will get D of z equal to a, y square minus a, y 1 square because the u 1 term disappears and from this now I can express, let us say y in terms of y 1 and z.

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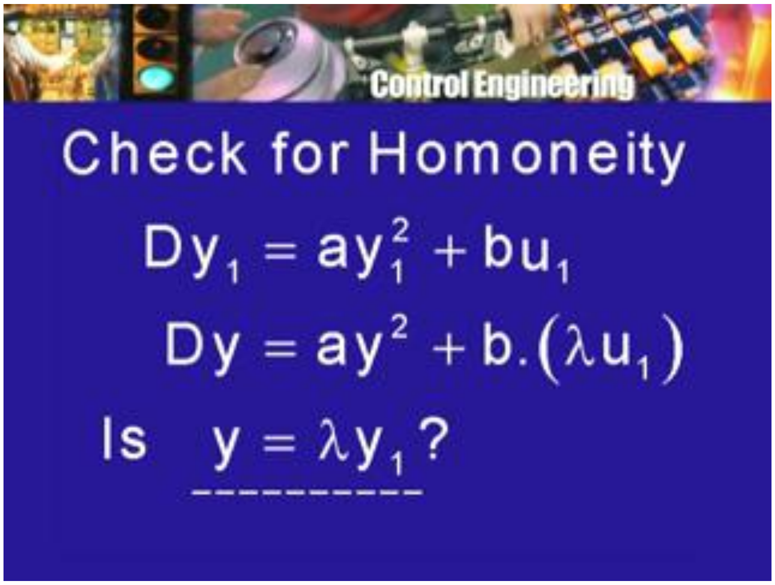

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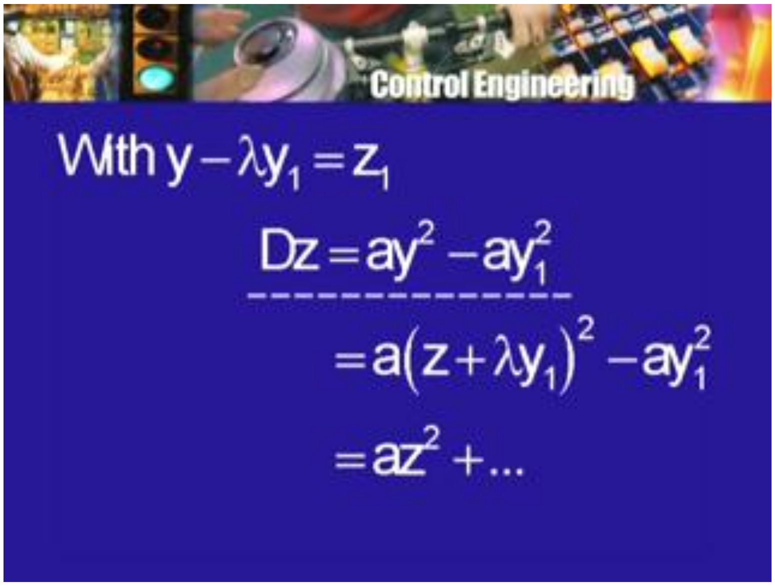

So if I write this as a y is z plus lambda y 1 minus a y 1 square and so D z equal to this of course is squared, so I will Dz equal to a z square plus something, something. I have show that this equation does not have a 0 solution even if the initial conditions are 0. Now that in general it is not easy to do. In other words, in general if you write down some equation which looks non-linear by having square terms or by having say sign of z and terms of this kind, the solutions of that equation are not very easy to obtain that does not mean that the solution is not 0 or 0.

So it requires a lot more effort to show that the particular equation is linear or is not linear, what therefore you have learnt is to recognize linear equations, what is non-linear is not simply, what does not look like what is linear, unfortunately. To show that something is non-linear, you have to do some more work and show that homogeneity does not apply or superposition principle does apply. I will not press this point any further but I did want to emphasize that a system being linear and non-linear or failing to be linear is one thing, a differential equation failing to be linear is another thing and the main point is that you can recognize something as being linear but by its form by its appearance, there is there is this nothing like an appearance of a non-linear system. For example, D y equal to y square is non-linear D y equal to y cube is also non-linear but that is not all D y equal to sign y is also non-linear and so on.

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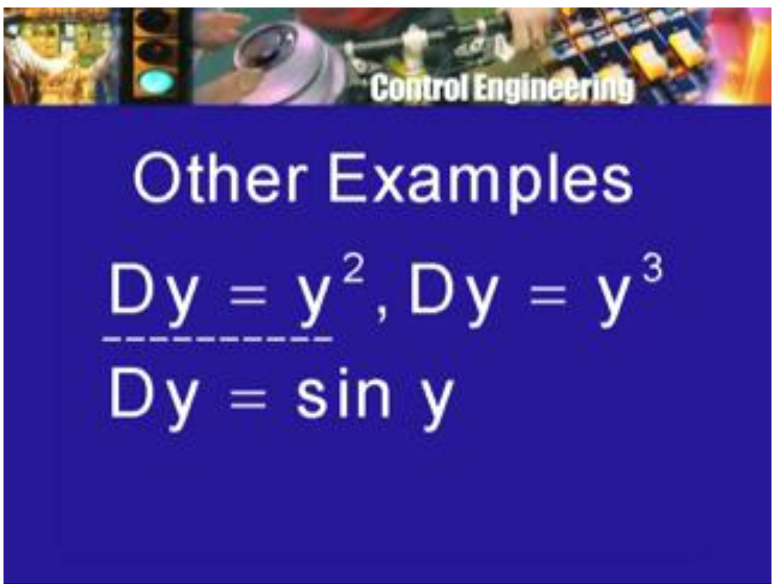

So in other words there is nothing like a general non-linear model whereas there is something like a general first order linear differential equation model. The model that I have written earlier D y equal to a y plus b u is very a general first order linear differential equation model and this is the reason why non-linear systems are very difficult to study because there is no particular form that the all of them have, whereas all first order linear differential equations by their very choice have a particular form that is why we recognize them as linear and therefore, they can all we studied together. Non-linear systems unfortunately cannot be all studied together. So there is no general statement that I can make about all non-linear systems whereas I can say something about all linear systems described by a first order differential equation. This is why the study of nonlinear systems has proceeded more slowly has been very difficult and therefore the s domain and the frequency domain methods were applied to linear systems and were developed quite well, before people went back to the study of non-linear systems.