Control Engineering
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Lecture - 44

We have taken a look at the Nyquist criterion or Nyquist test for stability of a closed loop feedback control system and we saw that the criterion nor test involved plotting, what is called the polar plot or the Nyquist plot for a system which is essentially the frequency response of the system or rather frequency response of the loop transfer function, KGH
what we me mean by frequency response, we mean the following apply a sinusoidal input and in the steady state the output will also be sinusoidal but of a different amplitude and different phase.

So the gain or the ratio of the amplitudes and the difference in the phase is what is measured or calculated knowing the transfer function of the system and then, it is plotted to obtain what is called the polar plot. Now this is an example of what are known as frequency response methods or frequency domain methods. It involves frequency response involves the idea of a sinusoidal signal or the input and response to such a signal. So methods like this are known as frequency domain methods and as I told you earlier, these methods were developed starting in the 1930's because of study of and problems associated with amplifiers and in the case of amplifiers, the sinusoidal signal was a very natural input candidate. These methods were applied to control system problems because it was in fact, the other way round the idea of feedback was used in amplifier design to stabilize the amplifiers as we saw earlier.

So that was the contribution or gift from control systems to amplifier design, the idea of feedback and then in the other direction the idea of frequency response was contributed by amplifier study to control systems study and naturally, therefore a large number of people worked on this problem and associated ideas and whole range of techniques known as frequency domain methods or techniques were developed and we have only looked at one part of it which is essentially the Nyquist criterion and then, of course because it requires the frequency response, we looked at Bode plots that is a quick way of obtaining an approximation for the amplitude and phase variation with frequency.

Now, the subject was developed further along the same lines but we do not have time to look at all of that moreover, what was perhaps very appropriate and natural in the 1930's and 40's is no longer necessarily appropriate today and so in my opinion, one does not need to spend that much more time on frequency domain methods. However, there are some interesting engineering ideas that is ideas which have what one may call an engineering aspect rather than a purely mathematical aspect or purely physical aspect and therefore will spend a little more time looking at some of those ideas.

I have not gone into any detail of design of compensators using the frequency response method. I just mentioned some compensating networks and left it you to find out their transfer function and also find out their frequency response, one other things that people try to look into and try to develop techniques or formulas was the relationship between frequency response and what may be called time domain behavior or time domain response in particular response to inputs like step input, one talks about the step response, response to a ramp input or response to a pulse very often, what is called a rectangular pulse a single pulse of this kind or a response to a square wave input and the like.

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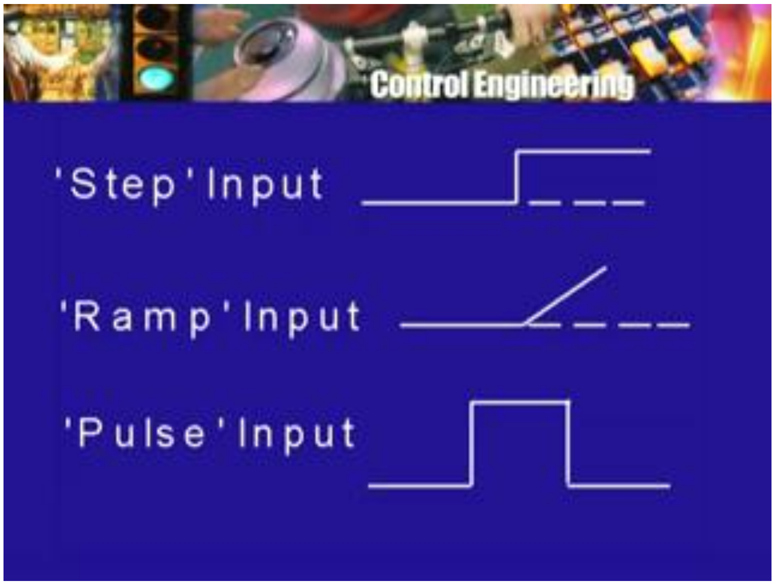

Now a step input of course has nothing to do with periodicity, it is just a signal which starts at sometime and then keeps on being at a particular level for all time to come, a pulse appears only for a short time and then it is 0. It is a really a transient input but these inputs are important they occur in control practice for example, step input may correspond to a sudden change in one of the variables associated with the system such as supply voltage or load torque as we have been seeing for a long, long time.

Similarly, a pulse could be a transient disturbance in the same thing there could be sudden change in the supply voltage which last only for the short time goes back to the original value or certain increase or decrease in torque. So step inputs pulse inputs are quite frequent and natural in control system applications, not so really sinusoidal inputs. Nobody really wants the motor speed to vary sinusoidally or motor position perhaps also to vary sinusoidally in the case of a follow of system or tracking system, the output of the system is to be follow the input and the input need not be sinusoidal. Let me remind you about the radar tracking an aircraft and then controlling a gun, so that the gun is directed at the aircraft as accurately as possible. The movement of the target is not at all going to be sinusoidal.

So such responses are quite natural and have to be studied in control system practice but having spent lot of time on frequency response, the following question naturally arose that is is there a relationship between these 2. For example, if I knew the frequency response of the system that is if I knew the amplitude and phase variation for all values of frequency from 0 going all the way up to as high a frequency as you wish, from that input from that information would I be able to calculate or determine the step response of the system and of course one can ask the converse question that is, if I know the step response of the system from that can I calculate the frequency response and a lot of work was done in this direction relating the frequency response to the time domain response or a response to a step input or to a pulse input and so on.

That is an interesting topic but once again we do not have enough time to look at it but in your system theory course or signals and systems course, you may have devoted sometime to this. The frequency response typically would be associated with the Fourier transform whereas the response to a step input or a pulse input would be typically associated with the Laplace transform and these 2 tools are not totally unrelated as you know both of them involve an integral and the integrals also look very similar. So it is quite natural to expect that there would be some relationship between the frequency response and the time response or the time domain response.

So that is one aspect of a lot work which was done with regard to the frequency response itself, you know you have 2 parts of it, one is the amplitude variation, the other is phase variation and a question that can be asked is, are these 2 independent of each other that is if I know that the gain of the system varies with frequency in a particular way, if I have that curve available then, do I have to separately calculate or measure the phase variation
from the gain variation, can I find out what the phase variation is going to be? So this was again a subject of investigation and quite a few results were found out of course much would depend on what kind of system you are looking at.

So if you are looking at linear time invariant system describe by an ordinary differential equations such as we have been studying then there is a relationship between the gain frequency curves or characteristic and the phase shift versus frequency behavior or characteristic. Of course, in practice from a practical point of you if you are determining the frequency response then it is not more difficult or does not really involved too much work to measure phase shift side by side with gain.

So from the point of view of applying the Nyquist criterion, you can measure the gain and phase variation simultaneously without involving too many difficulties. But there is the theoretical relationship which is useful and it could be important in some applications.
Another line of development was the following, the polar plot refers to KGH or if in the feedback path, we do not have a transfer function but we simply have a gain then, it is something like KG, this is called the loop transfer function, G is the forward path transfer function, H is the feedback path transfer function, k is the gain in the forward path and so KGH is the loop gain. But the transfer function of the closed loop from the input to the output, say from the reference input to the controlled output is given by KG divided by 1 plus KGH. This is the closed loop transfer function or it is sometimes also known as the overall transfer function from input to output now.

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Just as we looked at the frequency response for the KGH part of it that is opening the loop, applying an input at particular point and measuring the output at another point, the input being sinusoidal, one could in principal think of the overall control system, when the feedback is in place being applied a sinusoidal input and therefore you expect that the output will also be sinusoidal with a different amplitude and a different phase.

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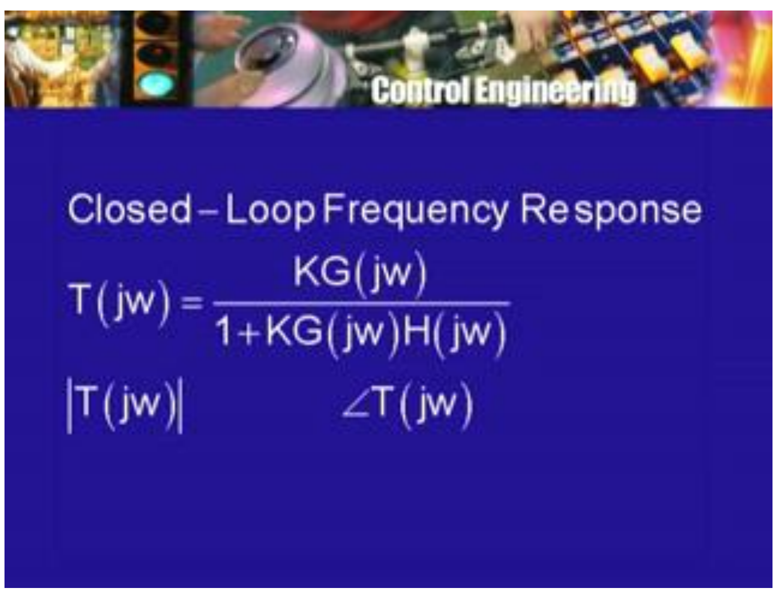

So one can ask, what is the closed loop frequency response that is the frequency response corresponding to the closed loop transfer function and of course then, a natural question that will arise is from the loop transfer function frequency response or from the Nyquist plot or the polar plot of the system, can one determine the closed loop frequency response. Of course the answer, one answer is immediate because TS is KG or 1 plus KGH therefore when we are talking about frequency response, one way of obtaining it immediately is to replace s by j omega, you get a complex number then in the transfer function, from the transfer function whose amplitude is the gain and its argument is the phase shift therefore, we have to look at T J omega this is equal to then KG j omega divided by 1 plus KG j omega H j omega.

Now, if I already know G of s and H of s then, it is just a question of evaluating this at different values of frequency omega or angular velocity omega. So from the calculation point of view from the point of you computation there is not anything very difficult,
this is a simple calculation of course G is itself a ratio of 2 polynomials. Similarly, H may be a ratio of 2 polynomials, so this will involve some complex arithmetic and then one can find out T j omega as a complex number and then of course look at its absolute value to get the overall gain and look at its argument to get the overall phase shift.

Now this is where, what I have I called an engineering approach or attitude comes into the picture. Of course, it is true that this can be calculated and today everyone of us can hold a calculator in our hand or in our palm for that matter or we have access to a computer we have such a program packages where in fact, the whole calculation will be done at a very simple command, you do not really have to write down any program statements and so on all you have to do is input appropriate data. So today there is not a big problem at all but that is not engineering, engineering looks at the problem in different ways and in intuitive ways that is in ways, in which one can see something going on and get a feeling for what is going on.

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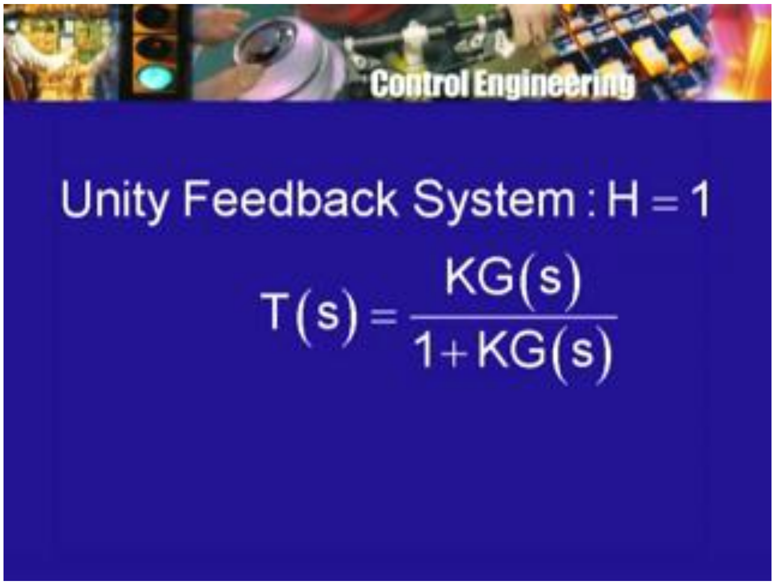

So some methods were developed. Now it is easier to work out a graphical method of obtaining the gain and phase shift for the closed loop transfer function from the polar plot for which the feedback transfer function should be a constant. It may even be one that is what called a unity feedback system that is a output is directly compared with the reference input as we saw earlier.

So we will assume that H is simply one and therefore we are looking at a loop transfer function which is given by KG divided by 1 plus KG and what you are now trying to develop or think of is the following. Here is the complex plane in which there is a polar plot as we saw earlier, the polar plot would depend on the number of poles and number of 0s of the system and their location and so forth.

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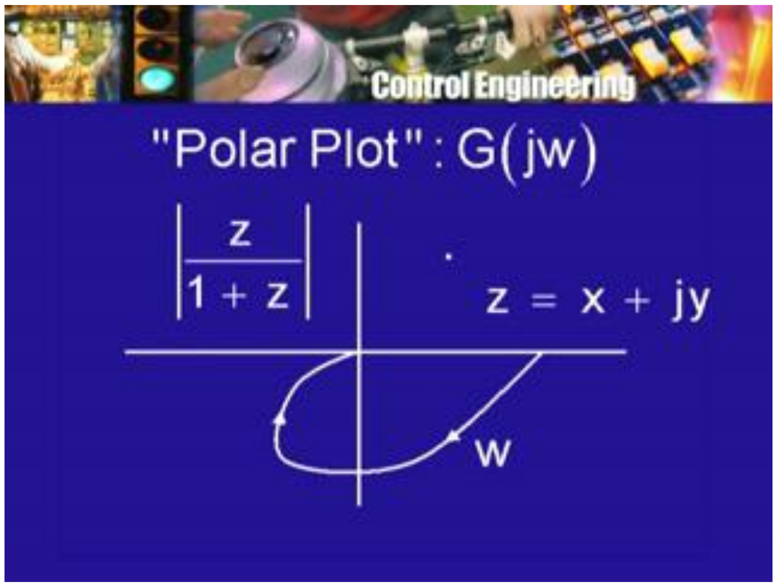

For the sake of simplicity, I am just drawing a simple curve say a polar that plot looks like this it could be just some arbitrary polar plot, really does not matter. Each point of this curve is what is a value of G j omega that is each point of this curve corresponds to a complex number? Remember, this is the complex plane which is the complex number G j omega. So if this is for some value of omega and I want to find out T j omega at this point or for this omega what I have to do I have to take this complex number that is G j omega here then, multiply it by the gain K which I have selected whatever is the value gain K and then add to that 1 in the denominator and then, calculate this and then take the absolute value and the argument of this whole number.

Now, instead of this arithmetical operation one looks at the following idea. Now this idea is to be found in many places not only in engineering but in also in physics in civil engineering those of you who may have had some idea of what is called surveying and leveling that is finding the variation of altitude in a given terrain, one talks about what are called contour maps that is you look at heights above let say, mean C level or above some benchmark at a number of points in a particular area and then, show them by means of a set of what are called contours.

You imagine that you are looking at the portion of land from a satellite as it were that is from the top, in other words it would be what one could call plan view of the terrain or a part of the terrain and then, all the various points or the various locations these could be some landmarks, it could be the top of a building or it could be the bottom of the building or it could be the top of a tree or the bottom of a tree and so on and so forth, the various landmarks or it could be just the surface of the earth or soil. These are points with the same altitude or height are joined by curve and these are called contours and then, you have a set of such curves for various values of the height and such a map is known as a contour map that is each curve or a contour is the result of joining all those points which I have given height or altitude.

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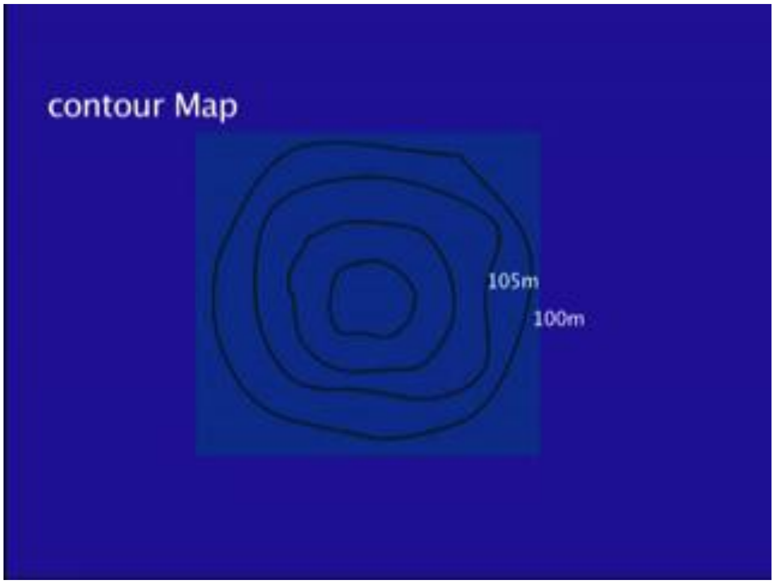

Another contour corresponds to a different altitude and of course, these heights or altitudes are shown alongside these curves for example this may be a height of 100 meters; this may a height of 105 meters and so on. Of course, this cannot take care of each and every point in the terrain and you you cannot really measure the altitude at each and every point of a terrain but it gives you a very of good idea of the nature of the terrain. For example, this is increase in height then probably there is some kind of a peak here and this may be the contoured map of the part of a hill for example, where this point may correspond to something like the peak and then, there is the land sloping downwards in all directions, from contour MAPS we can make how values for example and so on.

So that is one engineering way of conveying information, instead of just tabulating graphically you show it by means of contour maps. In physics, in electricity and magnetism which you have studied one has a notion of an equipotential. Again, you are considering a region perhaps which is a part of a plane and at each point, there is an electric potential then, you imagine joining all those points in the plane where the potential has a given value, if the potential is expressed in say holds then all the points at which the potential is a certain number of holds are joined together to get a curve and such a curve is called a equipotential curve. Of course, you can talk about an equipotential surface if you are working with 3 dimensional regions which is of course usually the case but it is not easy to show surfaces on a plain piece of paper.

So, equipotential lie in sorry, equipotential curves can be more easily depicted. So these are equipotentials curves along with something has the same value it may be the height of a point above a certain datum level or it may be the potential of a point above a certain datum or a reference point. This idea can be used in this case also, in this case what can be done is the following. We can think of all those points on the polar plot, this is the polar plot at every point there is a value of G of a G omega and a corresponding value of T j omega and therefore a corresponding value of the gain.

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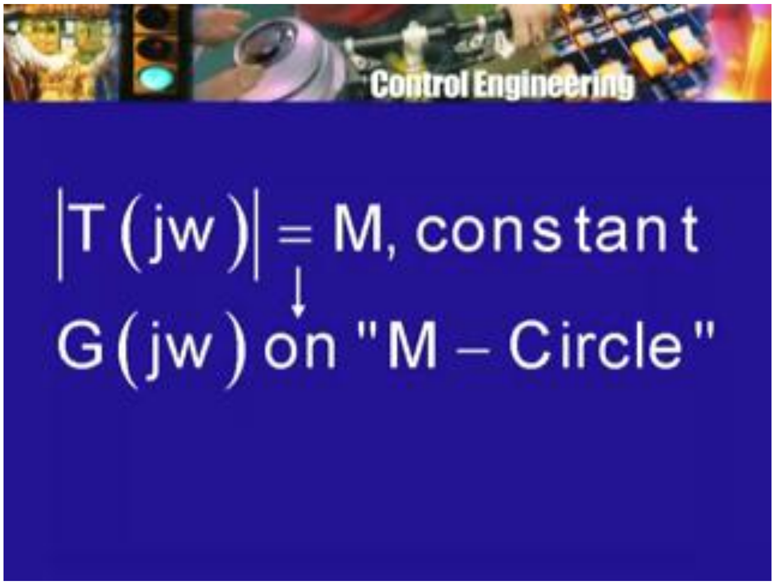

Now, what you do is you look at all those points in the complex plain which will correspond to a given value of absolute value of T j omega that is what you do or what you think of is the following. Of course this is the particular polar plot I may have another plot which looks different for a different system or if I change K, the polar plot will get scaled. So think of every point in the complex plain as a potential example of G j omega. It could be the value of G j omega for some frequency omega, for some system G. So every point in the complex plane is a candidate it can be on the polar plot of some system for some value of the frequency right.

Now for this value of G j Omega therefore, let us call it simply z you can immediately think of z as x plus j y if you want in the rectangular form. Now for this point that is for this complex number z, I can calculate the corresponding absolute value of T j omega which will be what z divided by 1 plus z absolute value of this. So corresponding to a given point regarded as the value G j Omega for some G and for some omega for that system, one can say that this number corresponding to it would be the gain of the closed loop system at the same value of frequency.

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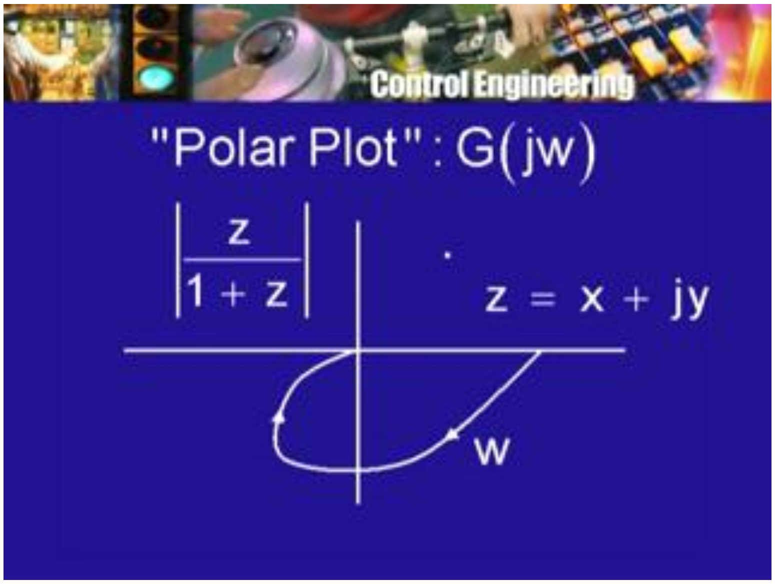

So now you say all right what are all those points or values or values of complex number z in the complex plain for which absolute value of z divided by 1 plus z is a constant. Of course because it is an absolute value it must be a constant greater that equal to 0. Now such a set of points can indeed be determined and this is the transformation really from z to z or 1 plus z or modulus of it, what you are looking at is the set of points z, where this function or this transformation has a constant value which is some positive number of course because it is gain, we can always think of it in terms of db's rather than in terms of the absolute gain.

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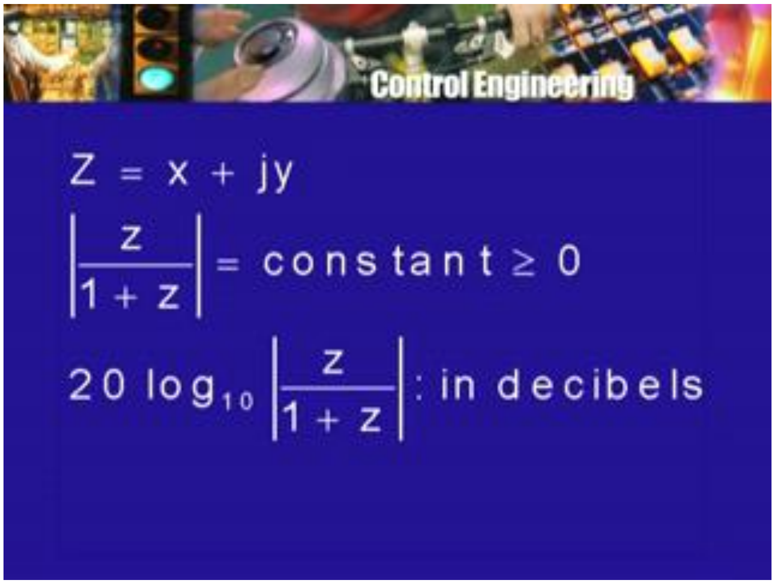

Now it turns out that the set of such points lie on a circle. Now this is something which is not very difficult to prove and you have done enough in complex variables to be able to prove it. So you should try to do it on your own what is it that I am saying that is think of the complex number z, look at this equations equal to constant equal to let us say alpha greater than 0, what is the set of points z for which this is 2, show that the set of all these points lies on a circle.

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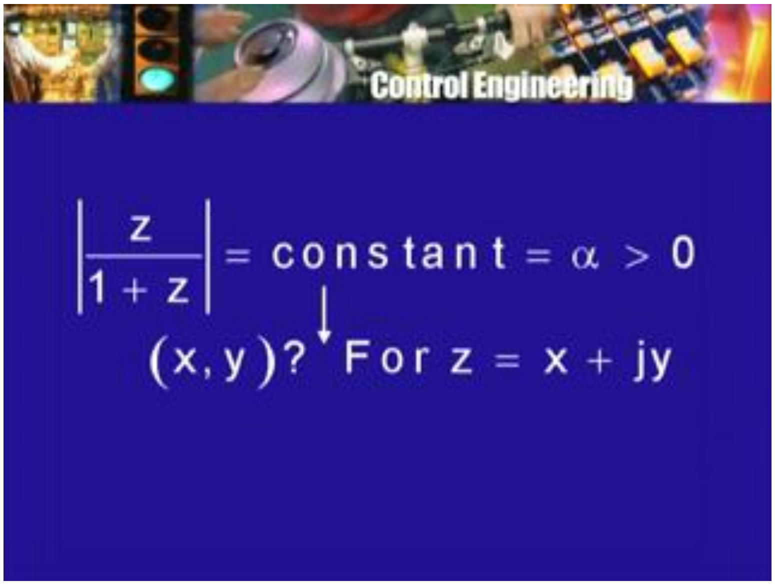

Now because this circle corresponds to the constant value which is the magnitude of absolute T j omega, which is of course the closed loop gain such a circle is known as an M circle. So for each constant value that is for each value of the magnitude of the closed loop gain, you have a circle one can find out the centre of this circle and the radius of this circle that will of course depend on this alpha and so, you get a family of circles say it is somewhat corresponds to the contour map of course the contour map shows you the actual height or the potential equipotential show you the actual potential, whereas here these M circles if I can draw the whole set of them would then enable me or would give me the value of the closed loop gain at a particular corresponding to a particular point in the complex plane which may be the value of G j omega for a specific G and a specific omega.

So in other words, once you have these M circles or a number of them drawn on a chart or on the complex plane on that you can draw the polar plot and then, almost for every point on the polar plot, you can read of the value of the gain. Let me illustrate this with the help of the contour map, now here is the contour map of a place or of an a of a terrain and let us say somebody starts moving and the path of the person is something like this. Let us say the person is going in the direction indicated by the arrow then, what can we say from the contour map let us say at a particular moment of time, the person was at this place on the hill where the altitude was 100 meters a little later the person went up a little bit climbed 5 meters to another place and still later, another 5 meters to a third place, till finally perhaps the person reached the top of the hill.

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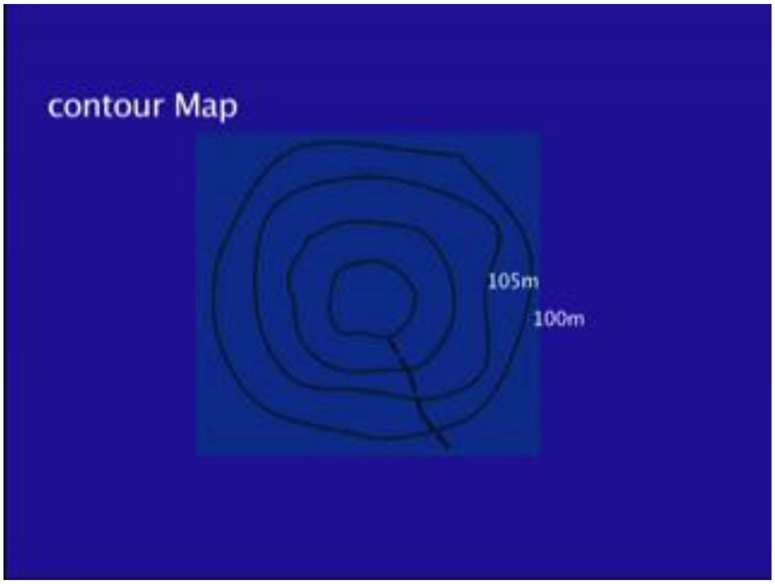

So here time would be the parameter as you move along the curve, it corresponds to the movement of the person or climbing of the hill, time is the parameter. For each value of T the intersection with the contours of the path of the person gives you the altitude of the height reached by the person at that moment of time. Imagine a team or a individual mountain climber climbing the Mount Everest. Now if you have this kind of information displayed you would essentially sort of follow the climb of that person that is in 1 hour, he climbed so many meters in the next hour, so many meters and so on.

Now it is something like this in the case of the M circle and N circle, we have the polar plot and we have a set of circles then, we can think of tracing the change in the frequency response as omega increases from 0 upwards and for the intersection of this polar plot with the radius circles because on the circle just as on the contour map along the contour map I have the altitude labeled labeling it. For the M circle, I will have the value of alpha that will appear along side it and so from this one can read out the change in the gain, the overall gain as a function of frequency and then, if necessary of course one can plot it but as I told you it is not necessarily always to do exact calculations to do plots, one can get a qualitative idea of what is happening?

So this is the set of M circles, now this was for constant amplitude of the overall transfer function which involved really the function z divided by 1 plus z modulus of it equal to constant. Of course, you can do the same thing or something similar for the phase shift and we can look at angle z plus z divided by 1 plus z equal to constant. We put here angle 5 which of course, we will have to let it range over say 0 degrees to 360 degrees or if you wish minus 180 degrees to 180 degrees because the argument can vary over a total range of 360 degrees. Now one can show and again you should try to show it that the set up valve point z of the complex plane for which the argument of z divided by 1 plus z is a given constant 5, where if 5 is interpreted as degrees in degrees lies between 0 and 360 is again a circle.

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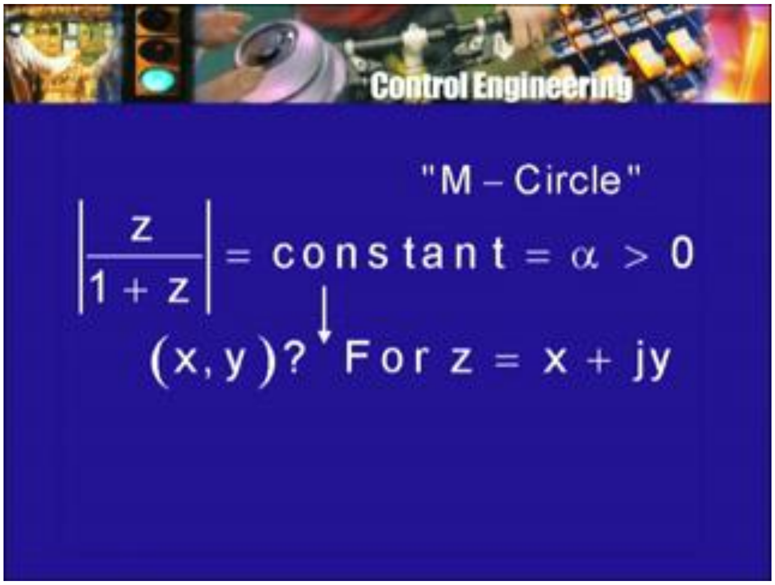

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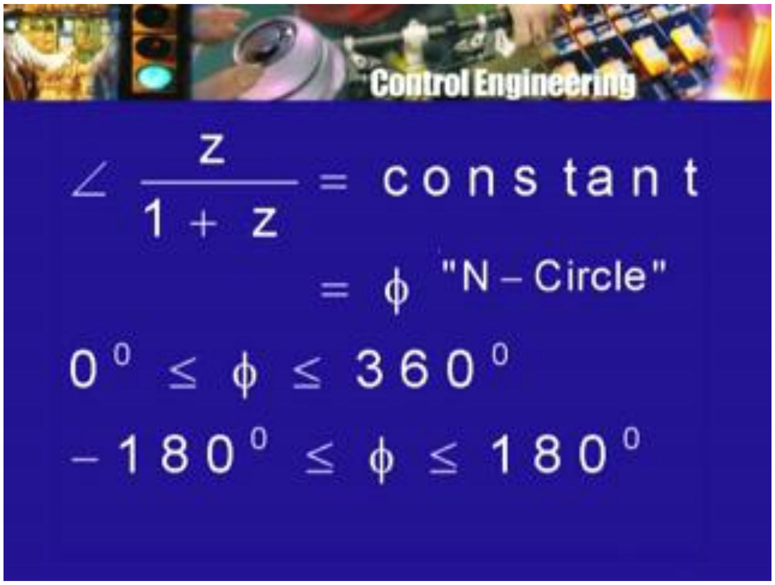

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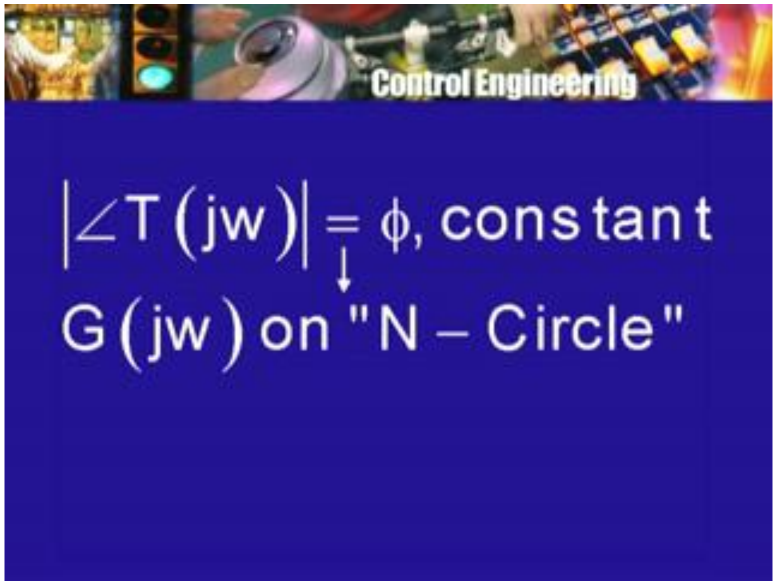

Now because the earlier circles were called M circles M for magnitude people called these N circles. So for each value of the phase shift of the over all transfer function, there is a circle or an N circle. So there is another contour map so to speak for the phase shift there is the contour map for the gain and there is a different contour map for the phase shift and then, in exactly the same way as you move along the polar plot that is as you trace the polar plot function frequency what is G j omega at each point intersection with the N circle will give you the phase shift at that point.

So this the set of M circles and N circles, you can calculate the location of the center and radii of these circles depending on the value of the gain alpha or the phase shift 5 and you can actually draw a small number of them, when these ideas were introduced of course just as you have now graph paper available or coordinate paper available and then, I told you also that we have a semi logarithmic paper that is available people prepared and sold charts containing these M and N circles.

So you had a readymade set of M circles and readymade set of N circles, on that, on those charts one with say M circles on it. You could plot the polar plot either from experimental data or from theoretical calculations and then, determine the closed loop gain as a function of frequency or get some qualitative idea about that variation. Another chart of N circles would give you the variation of phase with a phase shift with frequency.

So there are some further interesting things for example, between M and the N circles there is a nice relationship and that is each M circle is what is called orthogonal to each N circle that is whenever an M circle intersects an N circle, they intersect at an angle of 90 degrees that is if you were to draw tangents to these 2 circles at the point of intersection the angle between the tangents would be 90 degrees that is these 2 sets of curves or orthogonal to each other. The simplest case of which is not of course closed curve of something like this a system of lines which are orthogonal to each other is what of course you have not perhaps you have been told that this is an example of a set of orthogonal curves, these are not closed curves but these are orthogonal. They are nothing but the lines on our co-ordinate paper. The co-ordinate paper lines are orthogonal say x axis lines and y axis lines and there is a whole bunch of them.

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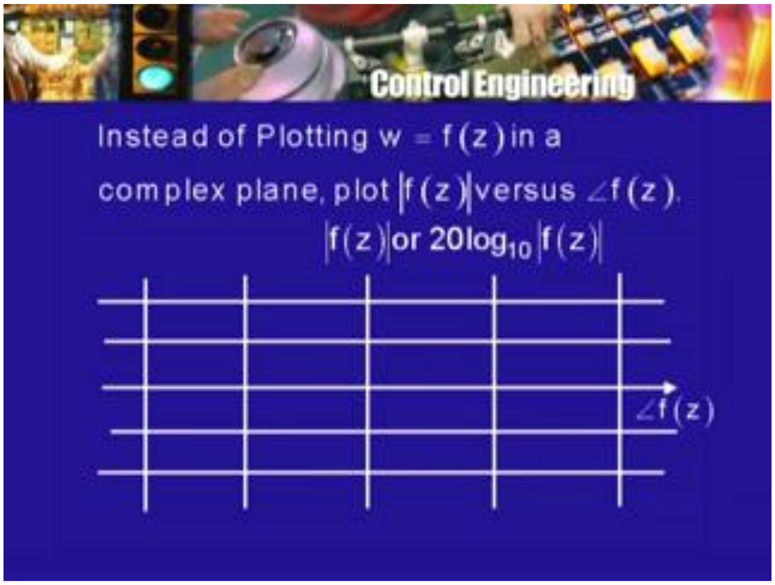

So here is an example of a set of lines the x axis line so to speak and the y axis lines which are orthogonal to each other. Similarly, the M and N circles are orthogonal to each other of course that is related to something like the following that is you have a function of the complex variable and you are looking at the modulus and the argument of that function. Now it is happening for this function z divided by 1 plus z, so one can ask what about a more general function f of z, if I look at mod fz equal to constant I will get a set of curves one for each value the constant if I look at argument f z equal to constant I will get another set of curves would these set of curves would be orthogonal to each other or not that is a questions that one can ask and it has been studied in some detail.

Of course, it is not necessary to have the M circles and the N circles on 2 different sheets of paper or on having making 2 different charts. We can have them on one and the same piece of paper although it might become a little confusing but as I said the M circle and the N circles are orthogonal to each other and just as you know how to use the co-ordinate paper in order to draw a graph or you would by this time know, how to use the semi logarithmic paper to sketch the Bode plots. So by practice one can recognize which are the M circles and which are the N circles and use that combined chart paper properly
and as I told you these things were actually done years ago, when these ideas were introduced,

Today of course, you will find it very difficult to get hold of even one copy of this chart containing M circles and N circles although it is quite easy to make your own chart and I have told you that you should try to do it, for say different values of M 10 different values of M actually draw the M circles similarly, for perhaps 10 different values of N draw the corresponding N circles. Now here we use the polar plot just as it was and just enter the polar plot information on the M and N circle chart and one is able to read of from that chart the closed loop gain and the closed loop phase shift.

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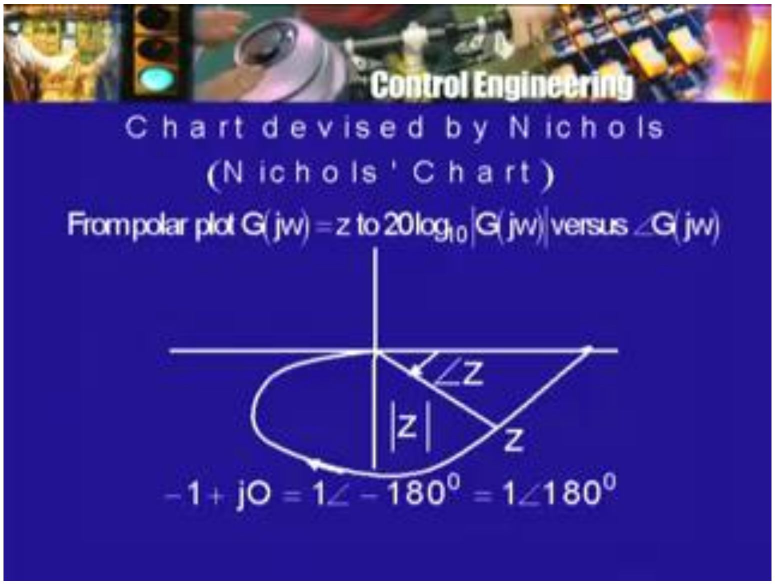

Now there was engineer by name Nichols who had a somewhat different idea and the advantage of his scheme is that it is more, you easy to use or more helpful in design. Determining the closed loop frequency response from the open loop frequency response is one thing and determining what kind of controller should be used in the forward path to achieve a desired performance is quite another. In the second case, when you are trying figure out what kind of controller one should use, what kind of compensating network one should use phase lag, phase lead, lag lead etcetera. You are going to try about various things and so therefore you are going to change KGH and you want to see the effect of that on the closed loop performance.

So Nichols had the following idea in the polar plot, what essentially you have is the complex plane and each point of the complex plane say think of it as a number z is simply nothing but G j omega for some g and at some angle of velocity omega. So here of course the distance 2 z has the significance of modulus of g and the angle made by this vector is the angle or the phase shift corresponding to g. So when you want to draw the polar plot from say experimental data, what you will do is you will determine the gain experimentally, you will determine the phase shift and then knowing this length of the vector and the angle made by the vector, you locate the point z corresponding to given frequency omega and then, you will do it for a number of different points and then join them by a smooth curve and that is the polar plot.

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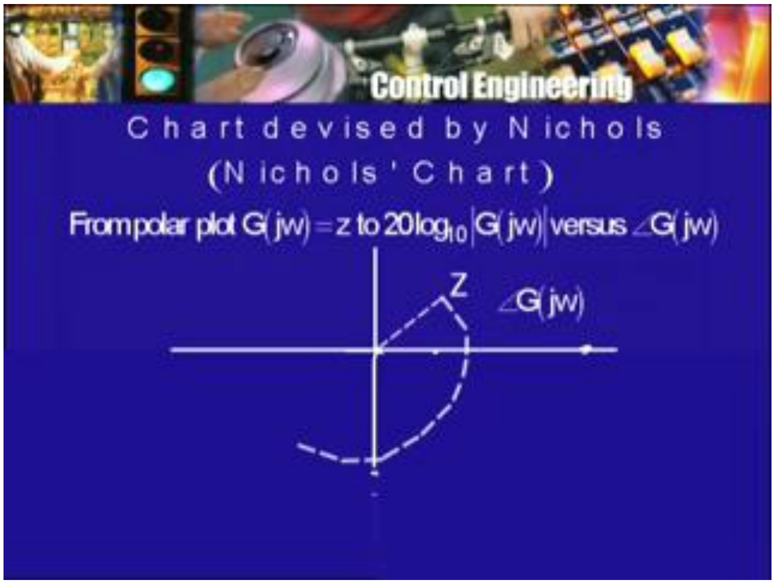

So this is how the polar plot is actually plotted this of course can be done on the MN circle chart paper. So, that you do not have to draw it separately once again. So here we are entering the modulus of gj omega and the argument of G j omega directly as the modulus or the length and the argument of a vector or a complex number in the plane. Now instead of entering this information this way, we can enter it in a somewhat different way. Now think of again 2 axis but now along one access what we have is angle or phase shift if you wish and along the other axis, we have what is gain either absolute gain or gain in decibels.

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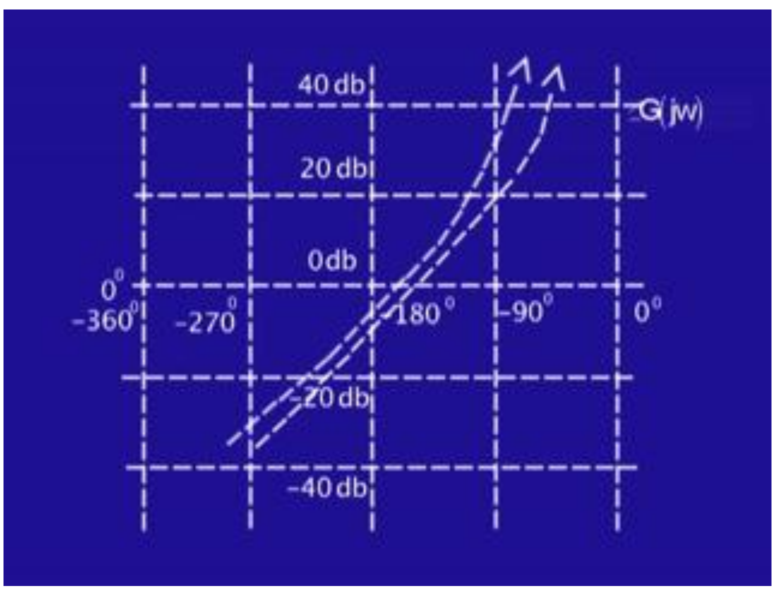

Now it turns out that and we will see the reason for that very soon that the phase shift variation usually will have minus 180 degrees as the central point and then, it will range towards one direction of increasing phase shift and another direction of decreasing phase shift and like the semi log paper, you can go up to any value that you like of course with minus 180 degrees at the center, I need to go only up to 0 degrees let us a here and up to minus 360 degrees which will also another way of writing 0 degrees okay.

So the phase shift range will be from 0 degrees to minus 360 degrees with minus 180 degrees at the mid point and of course, there will be other values of phase shift for example here will be the minus 90 degrees and here will be the minus 270 degrees phase shift lines. So the horizontal axis the term is going to represent or along the horizontal axis, you are going to represent the phase shift of G j omega. The angle of G j omega will be represented by location in the horizontal direction. The horizontal access is the phase shift access for G j omega. Along the vertical access you will represent the gain and since, the gain may vary from very small number to a very large value. As we saw, one takes the logarithmic gain or the db gain usually this point which is which may be the thought of as the origin of this set of co-ordinate axis corresponds to 0 db or absolute gain of 1 that is output amplitude equal to input amplitude and then, you have lines which occur at equal db increments of gain.

So for example, this may be the 20 db line, this may be the 40 db line and so on. This is minus 20 db minus 40 db and so on, okay. So now you have a graph paper in fact one can use the ordinary graph paper all you have to do is have an appropriate scale for angle, an appropriate scale for the db gain has to be choosen and you can mark points like thi. Now suppose I am looking at the gj omega transfer function multiplied by k, if you wish of a particular system either from the pole 0 diagram or from the experimental data. So what do I get, I get the modulus of G j omega which is the gain I will take 20 log the based then I will get the db gain and I have measured the phase shift or calculated the phase shift.

So I will get 2 numbers, the gain, db gain and the phase shift that will determine a point in this plane or a point on this chart for example, what is this point? This point corresponds to gain of 20 db, what is the gain of 20 db absolute gain is 10 because 20 log to the base 10 is the db gain. So mod of G j omega at this point will be 10 and the angle of the G j omega at this point will be minus 90 degrees. So this will determine a point in this plane and now, you do this for various values of omega and you will get a curve in this plane. It may or may not be a closed curve because this is not quite the Nyquist image of the Nyquist contour, this is simply the set of values G j omega plotted in different way, angle of G j omega along the horizontal axis and the gain, the db gain part of G j omega along the vertical axis, several points are determined and joint by means of smooth curve.

So this is a transformation from G j omega to angle of G j omega and modulus of G j omega. This is not that z over 1 plus z the transformation is still z over 1 plus z but now one is looking at plotting of in a different way and now, you can ask the question similar to the question that we asked earlier for the gain and phase shift, for the closed loop transform function where we had the polar plot. Now, we have a plot which is longer the polar plot and therefore since the Nichols introduced this idea, it is sometimes call the Nichols plot and corresponding to the contour idea or the constant thing idea for the M and N circles, we can now think of the set of all points in this plane, where the x coordinate so to speak or the horizontal coordinate represent the phase shift, the vertical coordinate represent db gain corresponding to any g at any frequency omega think of that as complex number z now, z over 1 plus z is the overall transform function think of the modulus of that now that is to be a constant.

So the same closed loop gain to be a constant the set of all those points on this chart or on this plot will be a set of curves 1 for each value of the constant. These curves are no longer circles they look like ellipses but they are not quite ellipses and they are fairly complicated. However, Nichols went ahead and plotted a number of these curves and after Nichols subsequently these curves or the charts containing these curves where obtainable commercially.

So instead of the M and N circles now you have a family of curves one corresponding to the modulus of the overall transform function the other corresponding to the phase shift of the overall transform function, I am not going to draw them here because they are fairly complicated and there is no need to remember really exactly what they look like but you can go through the exercise once again and find a equation for a curve like this for this is the actual variation in frequency. The set of all those points which correspond to a given gain of the overall transform function.

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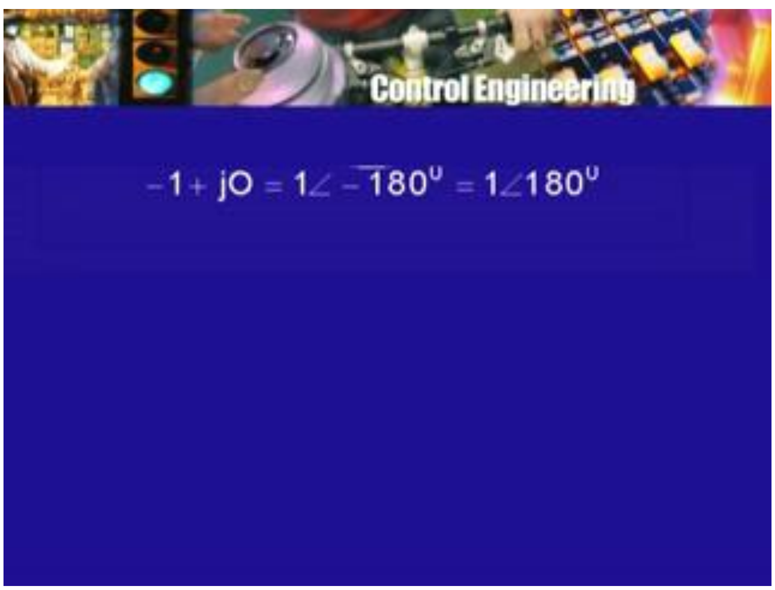

The point here is coordinates are what they are the horizontal coordinate is a phase shift and the vertical coordinate is the modulus. In the polar plot we are not doing anything like that in the polar plot and the reason for calling it the polar plot is that it is as if, we are plotting the polar coordinates that is G j omega is plotted as a set of polar coordinates.

Here, we are plotting G j omega as a set of rectangular coordinates the 2 coordinates being the phase shift and the modulus and the convention is to have the phase shift along the horizontal axis and the db gain along the vertical axis. Now what about the origin of this point of this chart it corresponds to 0 db, so gain of 1 and a phase shift minus 180 degrees.

Now what is the complex number 1 angle of minus 180 degrees that is nothing but the number minus 1 plus G 0 and this slope, we saw was the critical point in the application of the Nyquist criteria. Of course, from this chart when I shown G j of omega as a function of omega on this Nichols chart and I have not shown you the curves, it is not possible to conclude anything about stability immediately because this is not the image contour of the Nyquist contour for that you have to look at the Nyquist plot which is the polar plot. But still the location of this point minus 1 plus j 0, this point as if represents minus 1 plus j 0 whereas each one of these represents some other point in the complex plane. It is not difficult to think of the relationship between the 2, for example here this curve is going rather far away from this point where as there could be a curve that goes very close to this point, this point is the critical point for the Nyquist criterion.

So the same kind of information can also be inferred from this kind of a chart such a plot is known as Nichols plot or a Nichols chart. Now what is the advantage of this, the advantage is the following. I told you that we have the compensating networking which can introduce a gain and phase shift of its own. So if I have this G j omega plot if I now think of introducing a constants phase shift by putting something in the forward path or in the feedback path, a compensating network. Now what is going to happen, the G j omega curve will simply shift if I have phase shift the G j omega curve, the gain is the same at every point but there is only a phase shift.

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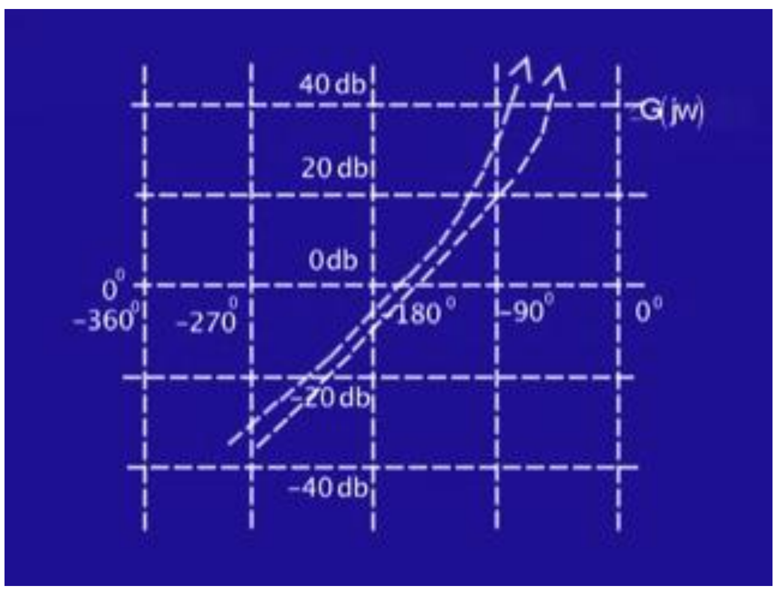

So if I know the compensating networking phase shift I can plot the new G j omega curve rather quickly if there is also varying gain for the network, I can take care of that by moving up or down. So at the given frequency omega the compensating network introduces an additional gain let us say and introduces a phase shift, so I go to a new point. I can do that for a number of points now I will get a new G j omega curve which then I can plot. So the compensating network effect on the closed loop performance specifically the overall gain and the overall phase shift can be figured out a little more easily than it could be in the case of the polar plot. So this is the main advantage of the Nichols plot or the Nichols chart and this was used by control system designers and of course feedback amplifier designers, after Nichols introduced the idea quite sometime ago in the late 1940’s and as I told you commercial charts where available and people did make use of it control system designers and of course amplifier designers did make use of these charts for quite some time.

Today, one can say that these charts have become absolute in the sense you cannot buy them in the market you can still buy a semi logarithmic paper but Nichols chart and M and N circle paper, you will not be able to buy. They are still useful control system design aids but I wanted to emphasize more the engineering way of looking at things through the help of set of equipotential, through the set of contour lines M N circles or constant gain curves on the Nichols chart and so on. There is this engineering idea of representing data, not in one way but in several ways, some are like tables, a table you would hardly call it a picture.

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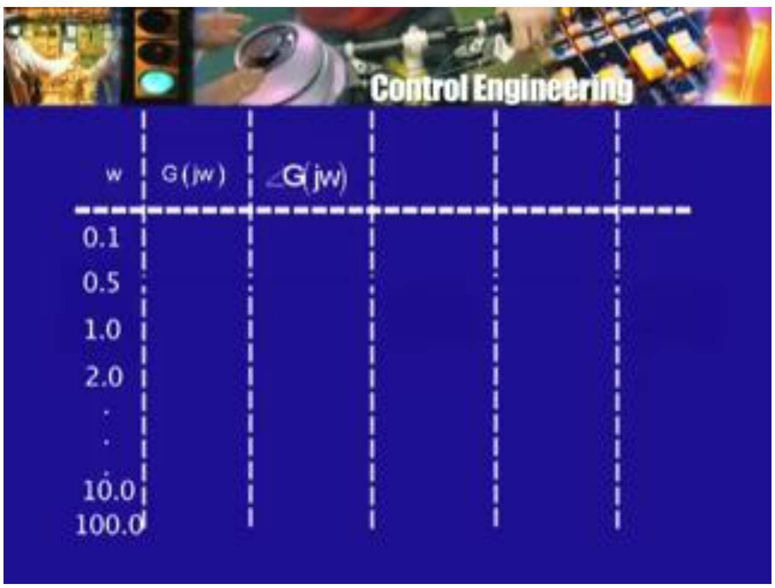

So there is data representation in a tabular form. I can have simply a table which gives omega verses gains or absolute value of G j omega and I can have table and I can have another table giving the argument of G j omega and of course, if you are doing an experiment then, this is what you will start with you will prepare a like this say omega equal to point 1 hertz, point 5 hertz, 1 hertz, 2 hertz and so on, may be going up to 10 hertz or 100 hertz corresponding gain. In fact, you make have an have 2 columns input amplitude may be kept constant, output amplitude then, gain then, db gain phase shift.

So this is the tabular form of the data but from this it is very difficult to figure out what is going on in fact, for studying instability we need to plot the polar plot. So this is same information or this information will now be re-presented in a different form by the help of or as a polar plot. Now, once you have polar plot you again try to think about stability what is going to be effect of the gain k on this stability and so on. You can re-present the information on the M and N circle chart that gives you something else and something different, you can re-present the information on the Nichols charts and that gives you again information about the closed loop behavior in different way.

So these are various alternating ways of representing a given set of data and this is what is helpful to an engineer and the engineering way of thinking, alternate ways of representing the same data, so to speak. There are some good points about a particular representation that is it helps you to do this operation or back operation or it gives you this aspect or that aspect more quickly than some other representation but one should learn, wherever possible a number of representation of data depending on the various aspects of the problem that you are studying and this is my main reason for really talking about M and N circles and Nichols chart, right now.

We do not have time to go into all the details as to how M circle and N circle Nichols chart can be used in design. I am stressing the engineering idea of alternate representations of data that is what is very important to learn and make use of whenever possible.